Alternating series

- series that has positive & negative terms

Ex. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \cdots$

- Def: $\sum_{n=1}^{\infty} (-1)^{n+1} U_n$, $U_n \geq 0$

$= U_1 - U_2 + U_3 - U_4 + \cdots$

- Alternating series test

$\sum_{n=1}^{\infty} (-1)^{n+1} U_n$ is converge if:-

1- $U_n \geq 0$

2- $U_{n+1} \leq U_n \quad \forall n \geq N$ non increasing

3- $\lim_{n\to\infty} U_n = 0$

if not $\rightarrow$ No info

Note:- $\sum \frac{(-1)^{n+1}}{n}$ is an Alternat harmonic series That converges

- Alternating series Estimation Theorem

If $\sum_{n=1} (-1)^{n+1} U_n$ converges then $S_n = U_1 - U_2 + U_3 + \cdots (-1)^{n+1} U_n$

approximates $L =$ Sum with error $E = L - S_n$

$|E| < \boxed{U_{n+1}} \rightarrow \text{Sign}(E) = \text{Sign}\ (-1)^{n+2} U_{n+1}$

$\rightarrow$ The first unused term

Example: Estimate the sum of the first 8 terms of the series $\sum_{n=10}^{\infty} \frac{1}{2n} (-1)^n$

$= 1 - \frac{1}{2} + \frac{1}{4} \cdots\cdots$

The first 8 terms, so the first unused term is the 9th term

Alaa Etaiwi

- if $\sum |a_n|$ converges and $\sum a_n$ converges we say $\sum a_n$ converges absolutly

- If $\sum |a_n|$ diverges and $\sum a_n$ converges we say $\sum a_n$ converges conditionally.

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