Problem

Student C tries to define a function $h: \mathbf{Q} \to \mathbf{Q}$ by the rule $h\left(\dfrac{m}{n}\right) = \dfrac{m^2}{n}$, for all integers m and n with $n \neq 0$. Student D claims that h is not well defined. Justify student D's claim.

Step-by-step solution

Step 1 of 1

$h\left(\dfrac{m}{n}\right) = \dfrac{m^2}{n}$ where $h : Q \to Q$, $n \neq 0$, m are integers

Suppose that $\dfrac{m_1}{n_1} = \dfrac{m_2}{n_2}$

Then, $\dfrac{m_1^2}{n_1} \neq \dfrac{m_2^2}{n_2}$ for instance $\dfrac{3}{2} = \dfrac{6}{4} \not\Rightarrow \dfrac{3^2}{2} = \dfrac{6^2}{4}$,

i.e., $h\left(\dfrac{m_1}{n_1}\right) \neq h\left(\dfrac{m_2}{n_2}\right)$

$\therefore \dfrac{m_1}{n_1} = \dfrac{m_2}{n_2} \not\Rightarrow h\left(\dfrac{m_1}{n_1}\right) = h\left(\dfrac{m_2}{n_2}\right)$

$\therefore h$ is not well defined