Chap 11: Rolling, Torque And Angular Momentum

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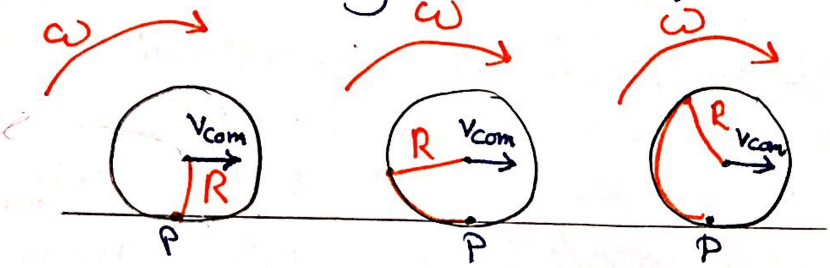
11-2: Rolling as Translation and Rotation Combined

Roll smoothly:- roll without slipping or bouncing

com



$S = R\theta$

$V_{com} = R\omega$

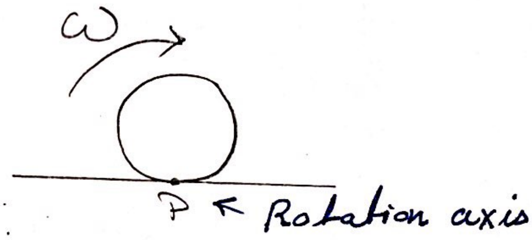
- when the object roll smoothly $V_{com} = R\omega$
- when V_{com} and ω are constants $\Rightarrow$ friction force $= 0$

11-3 The Kinetic Energy of Rolling

$K_{rolling} = \frac{1}{2} I_P \omega^2$

where $I_P = I_{com} + Mh^2$

$= I_{com} + MR^2$



$K_{rolling} = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M \underbrace{(R\omega)^2}_{(V_{com})^2}$

$$\boxed{K_{rolling} = \underbrace{\frac{1}{2} I_{com}\omega^2}_{K \text{ rotating around com}} + \underbrace{\frac{1}{2} M (V_{com})^2}_{K \text{ Translation of com}}}$$

Important

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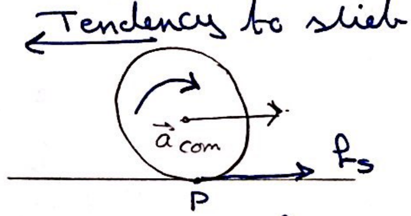
11-4: The forces of Rolling friction & Rolling

- If an object is Rolling at a constant speed The it doesn't slide at the point P and frictional force = 0

- But if an object is Rotating in a variable speed (There is a net force acting on it) The net force causes acceleration $\vec{a}_{com}$ and An angular acceleration α which means the object tends to slide at P $\Rightarrow$ frictional force $\neq 0$ to resist the sliding

Smooth Rolling $\rightarrow$ the wheel does not slide $\rightarrow$ the force is a static frictional force f_s $\rightarrow$

$$\boxed{\vec{a}_{com} = \alpha R}$$



Rolling down a Ramp (an inclined plane)

- here: f_s is necessary to prevent sliding

$\tau_{net} = I\alpha$ $\quad$ τ for mg & N = 0 cause $\theta = 0$

$\tau_o = I\alpha$

$f_s R = I\alpha$

$$f_s = \frac{I\left(\frac{a_{com}}{R}\right)}{R} = \frac{I\, a_{com}}{R^2} \quad \text{.....} ①$$

The object is sliding: $M \vec{a}_{com} = Mg\sin\theta - f_s$ ...②

$\boxed{1 \text{ in } 2}$ $\Rightarrow M \vec{a}_{com} = Mg\sin\theta - \frac{I\, a_{com}}{R^2}$

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...stely : $\boxed{a_{cm} = \frac{g \sin\theta}{1 + \frac{I}{MR^2}}}$ • where I depends on The geometry

11-7 Angular Momentum

• It's a vector quantity

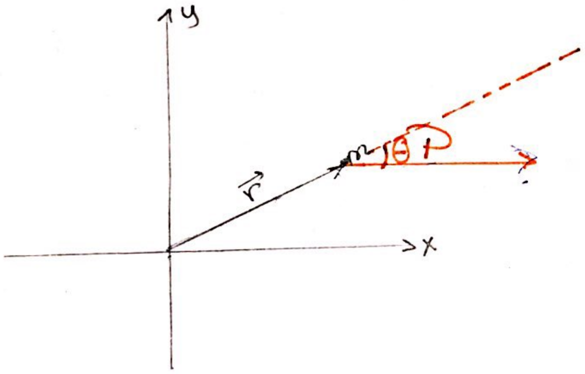
$$l = \vec{r} \times \vec{P}$$

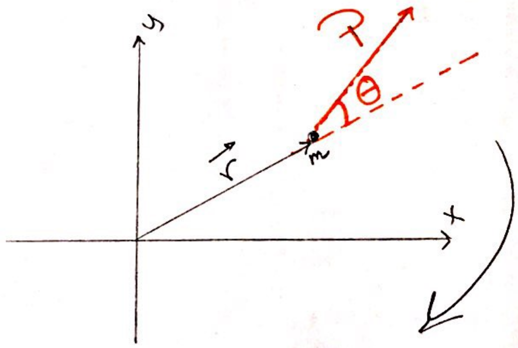
$\vec{r}$ → Position of The particle

$\vec{P}$ → كمية التحرك الخطية

$$\boxed{l = r m v \sin\theta} \qquad [l] = Kg \cdot m^2/s$$

θ → The smallest angle between r and P





11-8: Newton's second law in Angular form

$$\vec{\tau}_{net} = \frac{d\vec{l}}{dt} \quad \text{for a single particle}$$

$$\vec{F} \times \vec{r} = \frac{d\vec{l}}{dt}$$

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11-9 : The Angular Momentum of a system of Particles

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} \quad \text{(system of Particles)}$$

where $\vec{L} = \vec{l}_1 + \vec{l}_2 + \vec{l}_3 - - - - \vec{l}_n$

11-10 : The Angular Momentum of a Rigid body Rotating about a fixed axes

$$\vec{L} = I\vec{\omega}$$

11-11 Conservation of Angular Momentum

$$\boxed{L_i = L_f}$$

when $\tau_{net} = 0 \rightarrow \frac{d\vec{L}}{dt} = 0 \rightarrow \vec{L} = Cste$

L_i: net angular momentum at some initial time t_i

L_f: net angular momentum at some later tim t_f

Remark :- The system should be ~~to~~ isolated

$$I_i \omega_i = I_f \omega_f$$

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