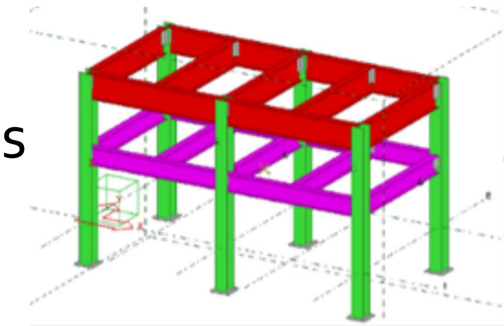
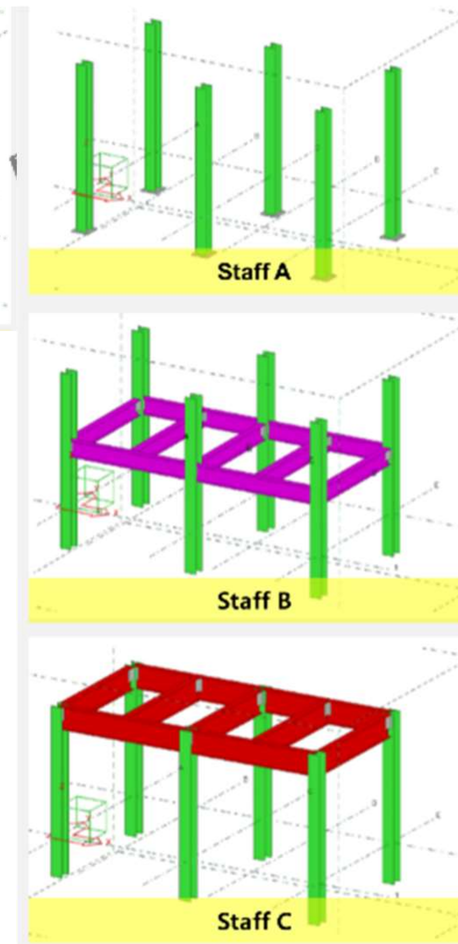
Analysis of Structures

Chapter 6

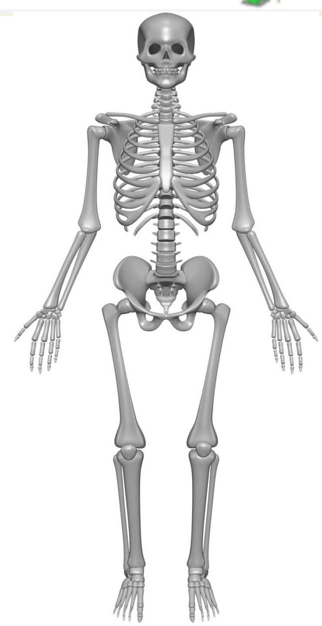
What is a structure?

- In general a structure is the arrangement of and relations between the parts or elements of something complex.





- In engineering a structure is intended to support forces that are applied to it.

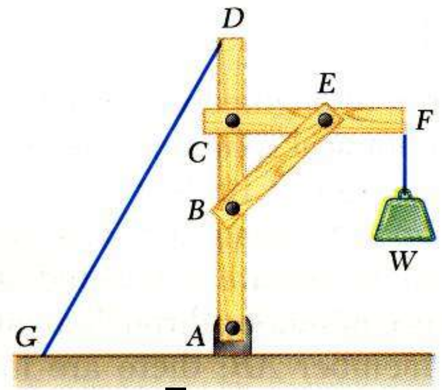


- To Design a structure, we must determine the force in each member, the internal forces or the forces that hold together the various parts of the structure.

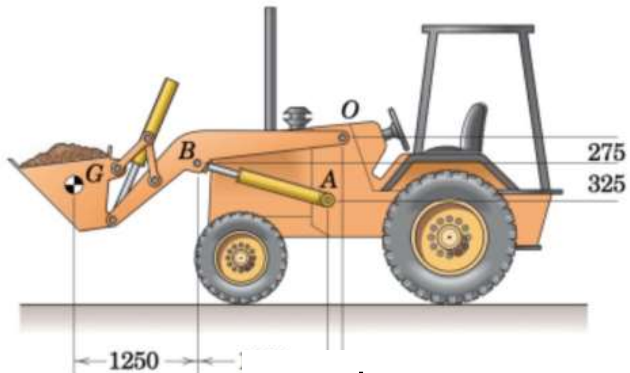
Engineering structures

- Three categories of engineering structures are considered:
  - Frames: contain at least one multi-force member, i.e., member acted upon by 3 or more forces.
  - Trusses: formed from two-force members, i.e., straight members with end point connections
  - Machines: structures containing moving parts designed to transmit and modify forces.

Truss

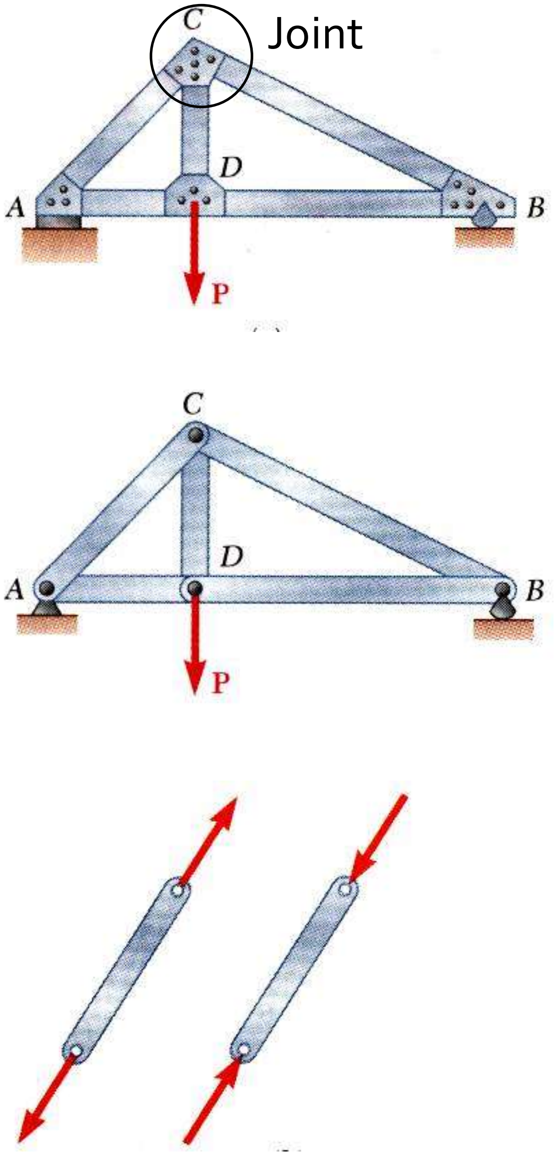


Frame



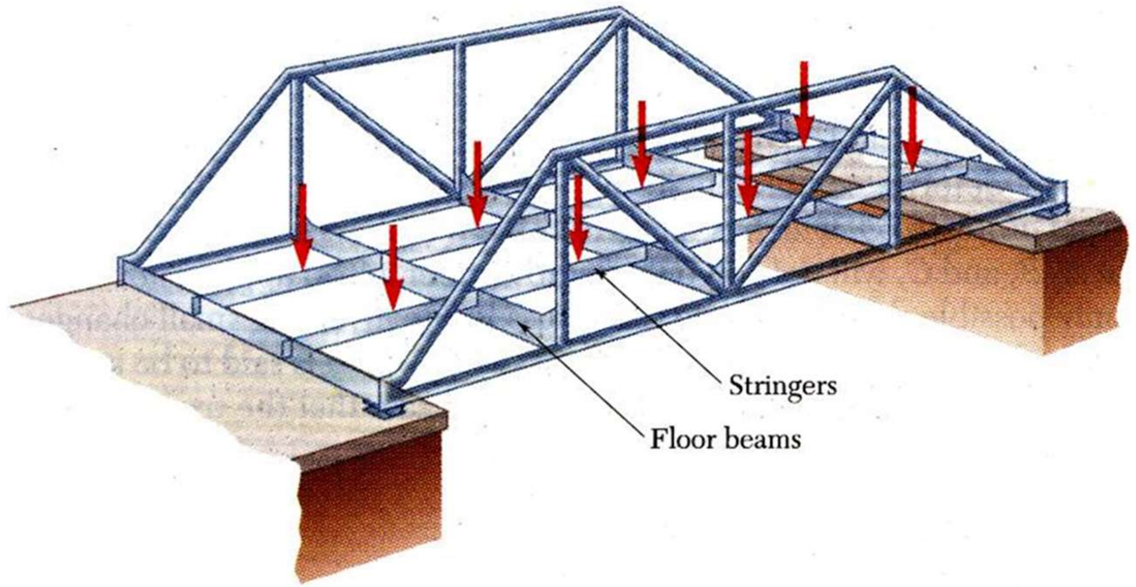
Machine

Definition of a Truss



- A truss consists of straight members connected at joints. No member is continuous through a joint.
- Bolted or welded connections are assumed to be pinned together.
- Forces acting at the member ends(joints) reduce to a single force and no couple. Only two-force members are considered.
- The weight of individual members must be negligible.
- When forces tend to pull the member apart, it is in tension. When the forces tend to compress the member, it is in compression.

Definition of a Truss

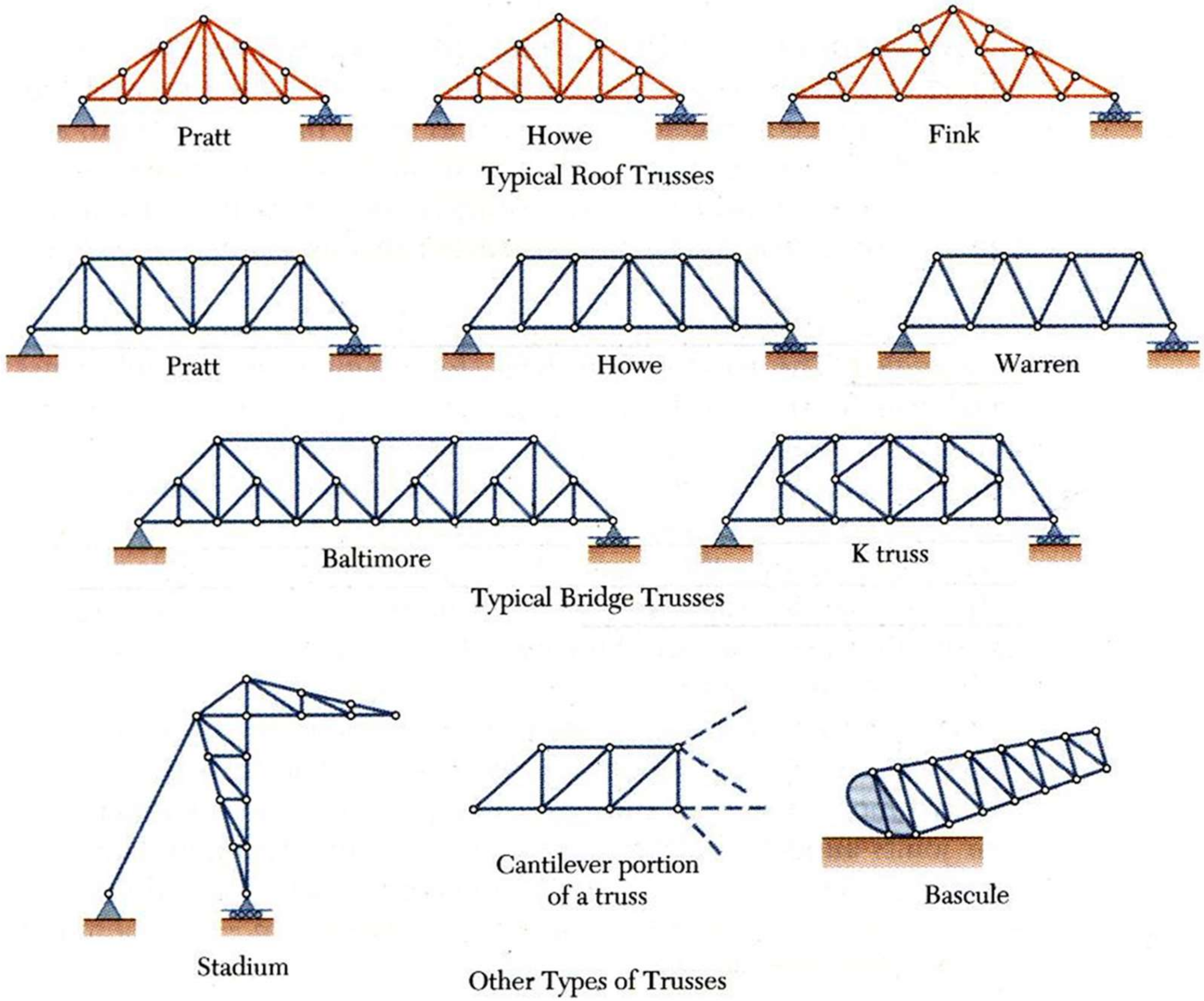


- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.

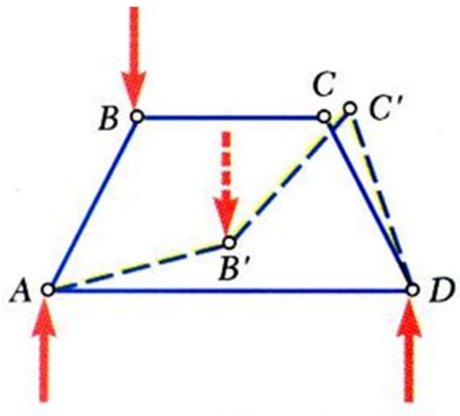
Definition of a Truss

- Trusses use very little material, yet they are very strong.
- Accordingly they are usually used in long span structures such as bridges

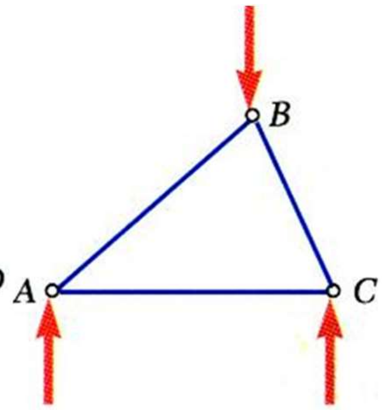
Definition of a Truss



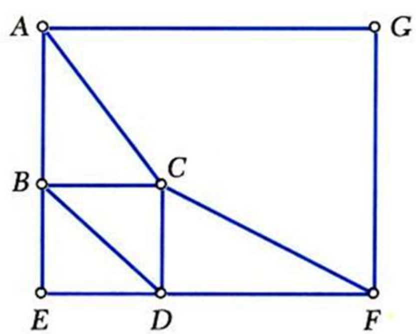
6.1A Simple Trusses

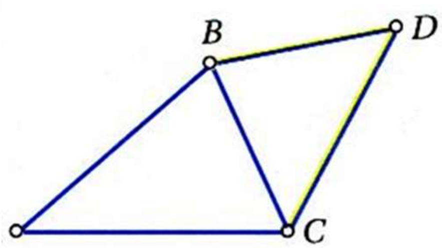


Non rigid frame/ collapsible



Rigid truss/ non collapsible





- The simplest truss can be built from 3 members connect in 3 joints.
- The truss can be extended by adding additional units of two end-connected bars, such as CD and BD or AF and DF ETC..
- In a simple truss, $m = 2n - 3$ where m is the total number of members and n is the number of joints.
- Not all trusses are simple.

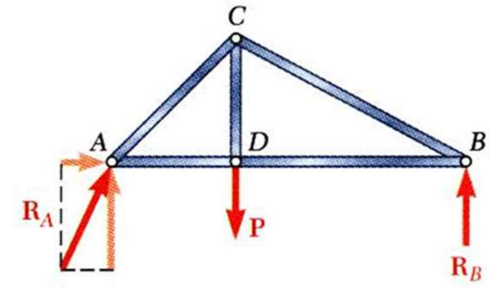
Static determinacy and indeterminacy

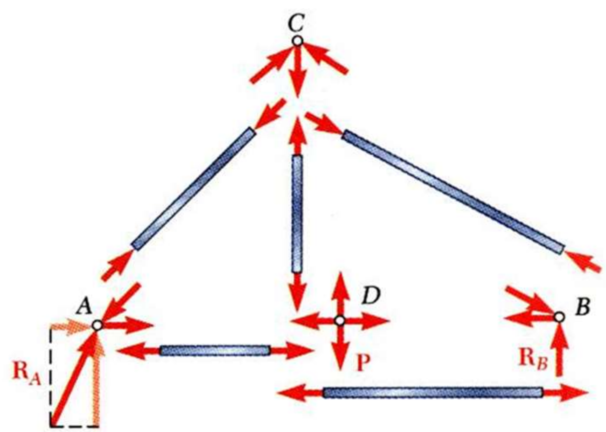
- Consider a plane truss having m = number of members; r = number of support reactions, and j = number of joints.
- Each member has one unknown force and each support reaction has one unknown force, so that m + r is the total number of unknowns.
- For a plane truss, each joint has two equilibrium equations so that (2j) is the total number of equations. Thus, the rule of thumb is as follows:

If $m + r < 2j$	The truss is a mechanism and/or has partial fixity.
If $m + r = 2j$	The truss is statically determinate if it has full fixity. The truss is statically indeterminate if it has partial fixity.
If $m + r > 2j$	The truss is statically indeterminate, and it can have full fixity or partial fixity.

Analysis of Trusses by the Method of Joints

- In general, the objectives of truss analysis are the determination of the support reactions and members forces.
- The important consequence of having only two-force members in trusses is that equilibrium analysis reduces to the analysis of a system of particles where the number of particles equals the number of joints in the truss.
- If the entire truss is in equilibrium, then every joint within the truss is also in equilibrium, so we can write two equilibrium equations as shown for every joint and solve for the unknows.





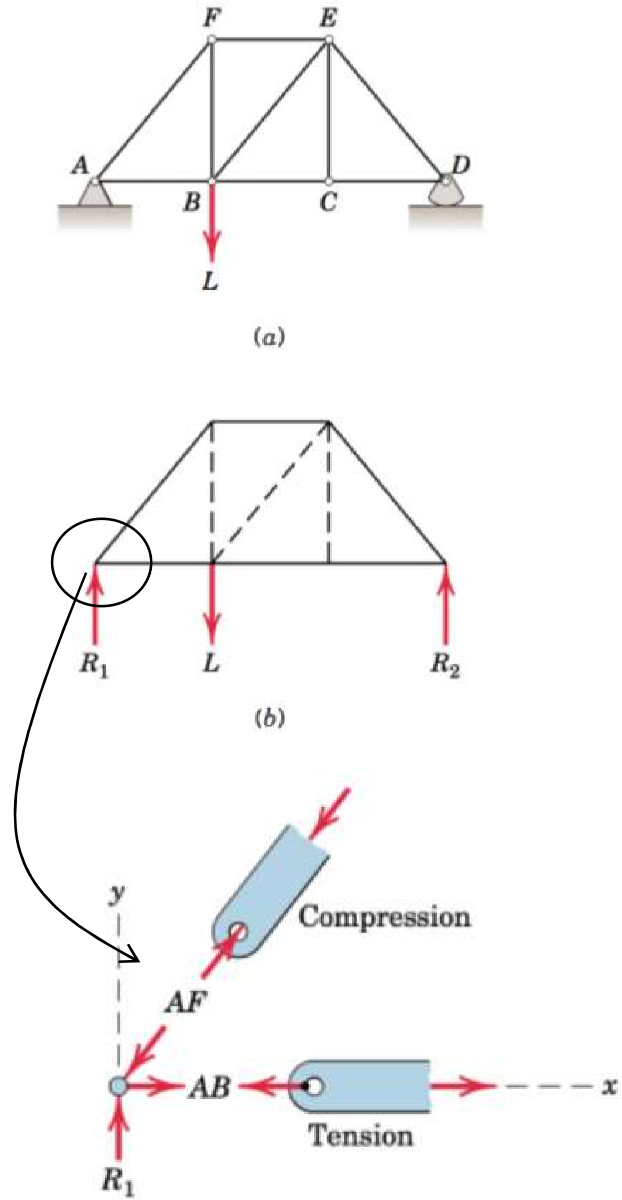
$$\sum F_{xi} = 0$$

$$\sum F_{yi} = 0$$

Analysis of Trusses by the Method of Joints

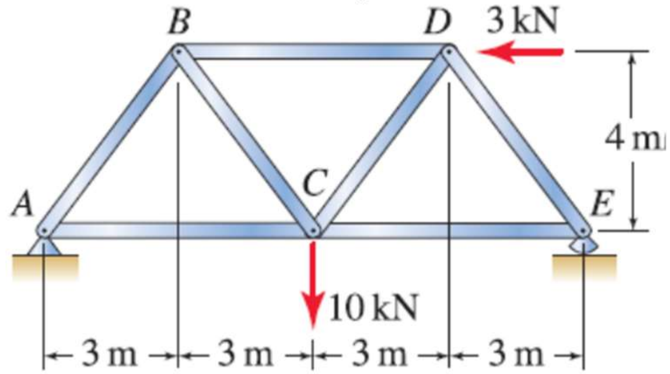
- Procedure:

1. Find the truss reactions (R1,R2).
2. Select a joint to start with where at least one known load exists and where not more than two unknown forces are present.
3. Isolate the joint, assume the unknown forces in tension and apply the two equilibrium equation to find the unknowns.
4. Repeat steps 2 and 3 for another joint, until all members forces have been determined



Analysis of Trusses by the Method of Joints

Example: Use the method of joints to determine the force supported by each member of the truss shown in the fiqure.

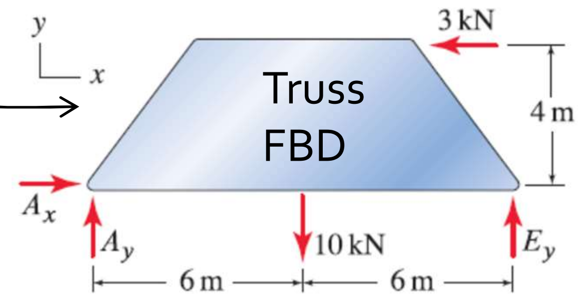


1. Reactions

$$\sum M_A = 0 \rightarrow E_y = \frac{10 \times 6 - 3 \times 4}{12} = 4\ KN$$

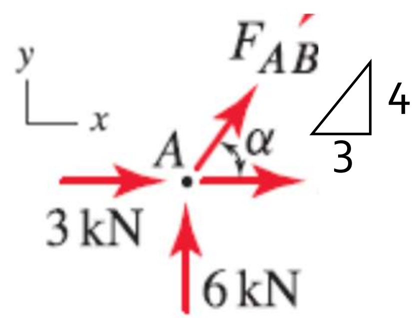
$$\sum F_y = 0 \rightarrow A_y = 10 - 4 = 6\ KN$$

$$\sum F_x = 0\ \rightarrow A_x = 3\ KN$$



2. Select starting joint: notice at this stage you can only consider joint A or E. Why?

Joint A:

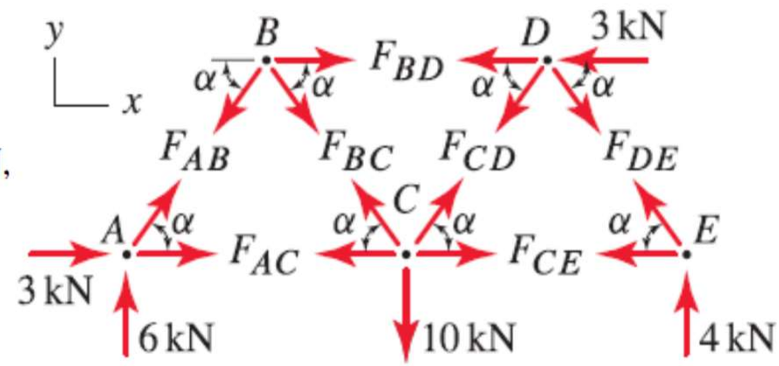


$$\sum F_y = 0: \quad 6 + F_{AB} \sin\alpha = 0\ \Longrightarrow\ F_{AB} = -7.5\ KN(\mathrm{C})$$

$$\sum F_x = 0: \quad 3 + (-7.5)\cos\alpha + F_{AC} = 0\ \Longrightarrow F_{AC} = 1.5\ KN\ (T)$$

Analysis of Trusses by the Method of Joints

Example Cont.



Joint *B*:

$$\sum F_y = 0: \quad -F_{AB}\sin\alpha - F_{BC}\sin\alpha = 0 \quad \Rightarrow \quad F_{BC} = 7.5\,\text{kN},$$

$$\sum F_x = 0: \quad -F_{AB}\cos\alpha + F_{BC}\cos\alpha + F_{BD} = 0$$

$$\Rightarrow \quad F_{BD} = -9.0\,\text{kN}.$$

Joint *E*:

$$\sum F_y = 0: \quad 4\,\text{kN} + F_{DE}\sin\alpha = 0 \quad \Rightarrow \quad F_{DE} = -5\,\text{kN},$$

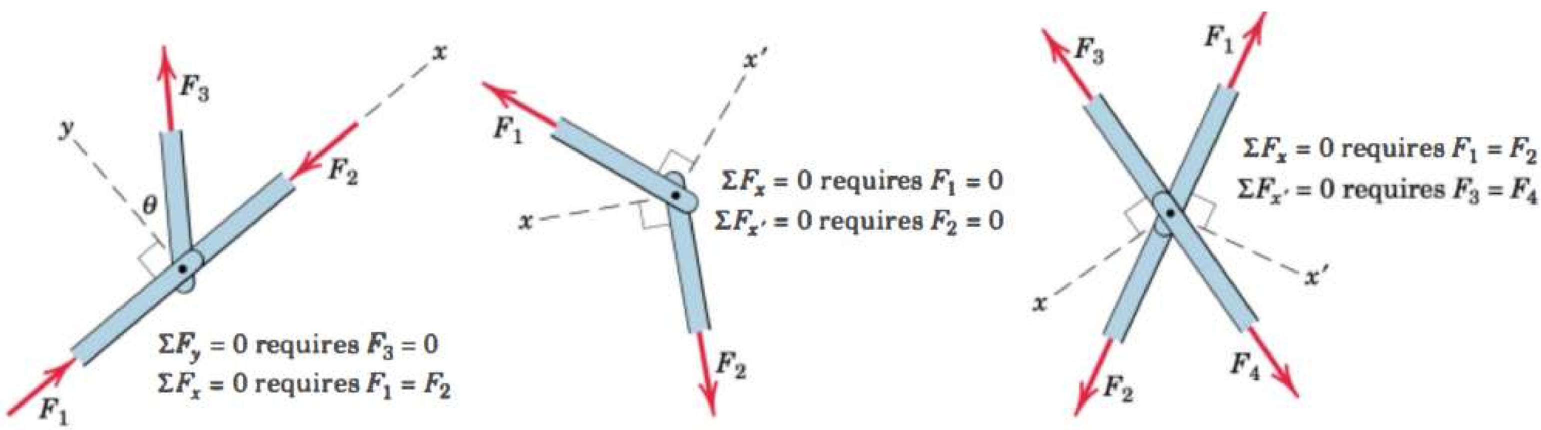
$$\sum F_x = 0: \quad -F_{CE} - F_{DE}\cos\alpha = 0 \quad \Rightarrow \quad F_{CE} = 3\,\text{kN}.$$

Joint *D*:

$$\sum F_y = 0: \quad -F_{CD}\sin\alpha - F_{DE}\sin\alpha = 0 \quad \Rightarrow \quad F_{CD} = 5\,\text{kN}.$$

Joints Under Special Loading Conditions

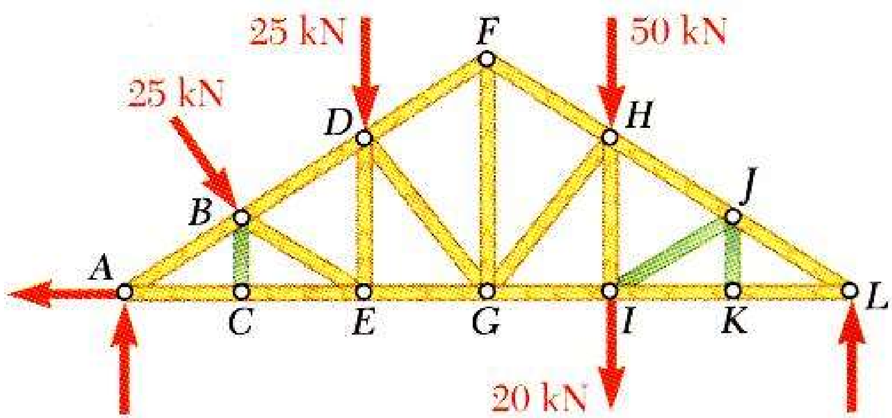
- In joint at fig. a, from the summation of forces in the y direction the force F3 in the third member must be zero and from the x-direction F1 = F2. If an external force with a component in they-direction were applied to the joint, then F3 would no longer be zero.
- In the absence of an externally applied load at joint in fig.b, the forces in both members must be zero.
- When two pairs of collinear members are joined as shown in Fig. c, the forces in each pair must be equal and opposite.



(a) (b) (c)

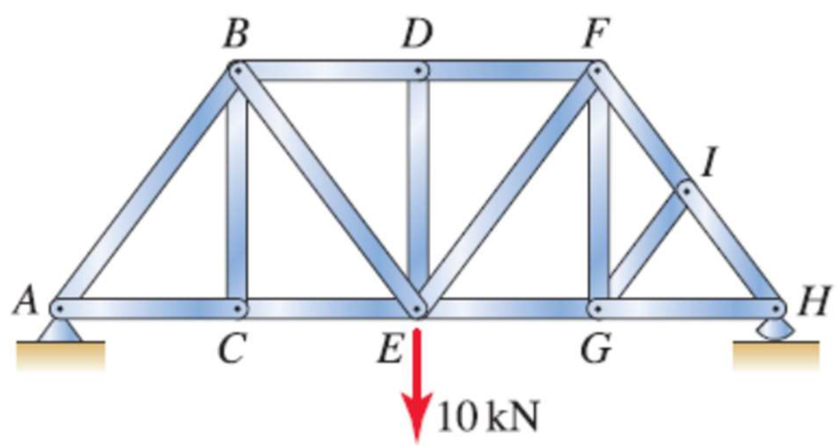
Joints Under Special Loading Conditions

- A truss member (or any member) that supports no force is called a zero-force Member.
- Zero-force members can be identified by inspection. examine the joints in a truss and try to identify the joint of special conditions as explained in the previous slide.
- Recognition of joints under special loading conditions or zero force members simplifies a truss analysis.



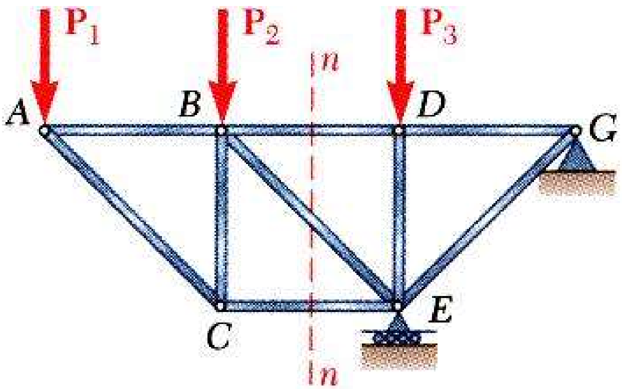
Example: Zero-force members

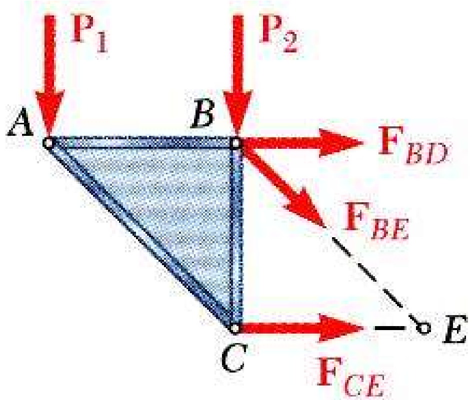
- Examination of joint C shows that member BC is zero-force.
- Examination of joint D shows that member DE is zero-force.
- Examination of joint I shows that member GI is zero-force.
- Because member GI is zero-force, examination of joint G then shows that member FG is zero-force.



Remark. For a member such as DE, the presence of the 10 kN force applied to joint E is a common source of confusion, as intuition may suggest (wrongly) that member DE must participate in supporting the 10 kN force. Note that our conclusion that member DE is a zero-force member is based entirely on the conditions at joint D.

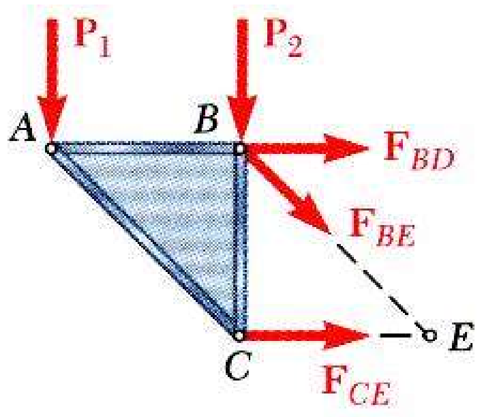
Analysis of Trusses by the Method of Sections

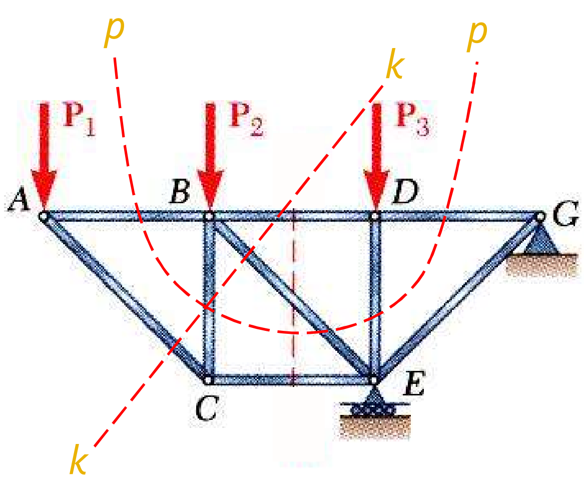




- When the force in only one member or the forces in a very few members are desired, the method of sections works well.
- To determine the force in member BD, form a section by "cutting" the truss at n-n and create a free body diagram for the left side. Assuming that all the exposed internal forces are to be in tension.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces including F_{BD}.
- An FBD could have been created for the right side, but why is this a less desirable choice?

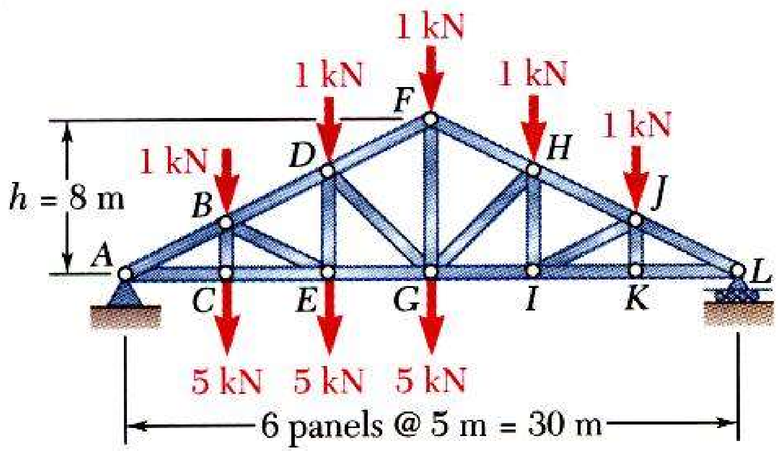
Analysis of Trusses by the Method of Sections





- Using the left-side FBD, write <u>one</u> equilibrium equation that can be solved to find F_{BD}.
- Assume that the initial section cut was made using line k-k. Why would this be a poor choice?
- Notice that any cut may be chosen, so long as the cut creates a separated section.
- So, for example, this cut with line p-p is acceptable.

Method of Sections - Problem 6.3

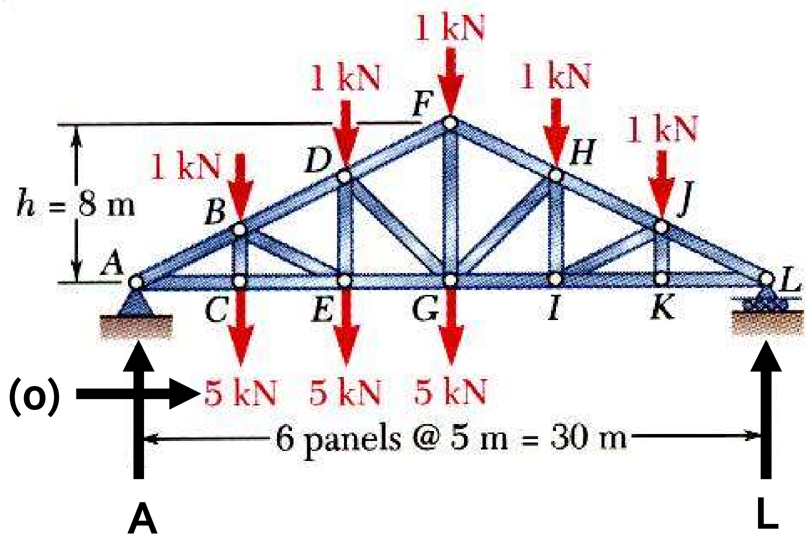


Determine the force in members *FH*, *GH*, and *GI*.

SOLUTION:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at *A* and *L*.
- Pass a section through members FH, GH, and GI and take the right-hand section as a free body.
- Apply the conditions for static equilibrium to determine the desired member forces.

Method of Sections - Problem 6.3



- Reactions at *A* and *L*

$$\sum M_A = 0 = -(5\text{ m})(6\text{ kN}) - (10\text{ m})(6\text{ kN}) - (15\text{ m})(6\text{ kN})$$

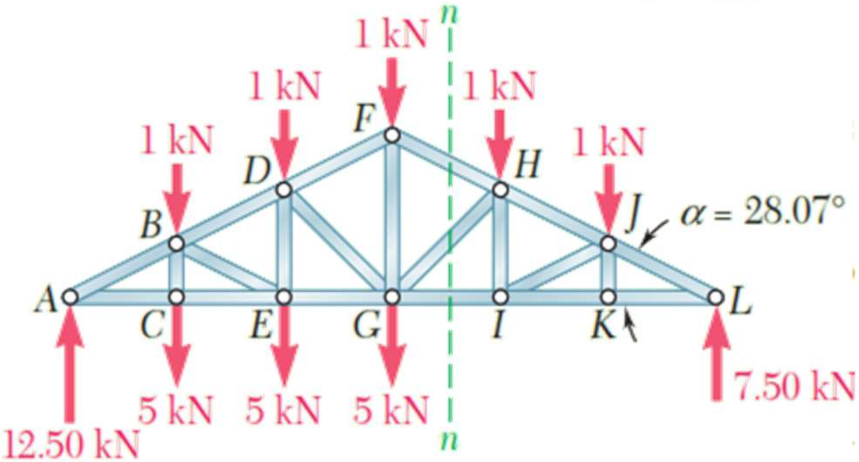
$$-(20\text{ m})(1\text{ kN}) - (25\text{ m})(1\text{ kN}) + (25\text{ m})L$$

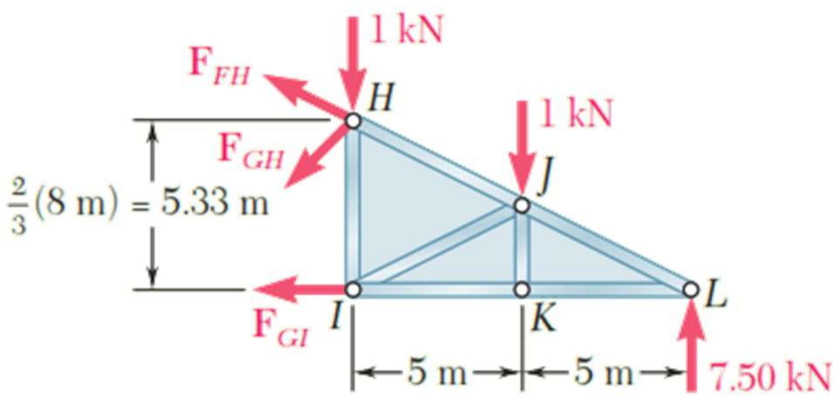
$$L = 7.5\text{ kN} \uparrow$$

$$\sum F_y = 0 = -20\text{ kN} + L + A$$

$$A = 12.5\text{ kN} \uparrow$$

Method of Sections - Problem 6.3





$$\tan\alpha = \frac{FG}{GL} = \frac{8\text{ m}}{15\text{ m}} = 0.5333 \quad \alpha = 28.07°$$

- Pass a section through members FH, GH, and GI and take the right-hand section as a free body.
- Apply the conditions for static equilibrium to determine the desired member forces.
- Force in Member GI.

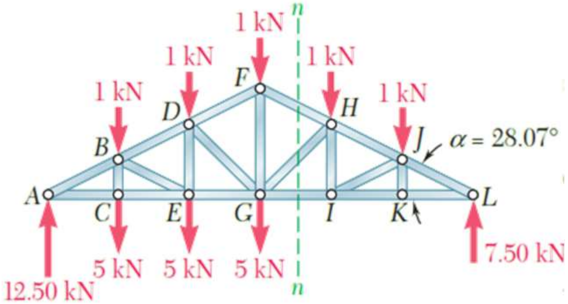
$$\sum M_H = 0$$

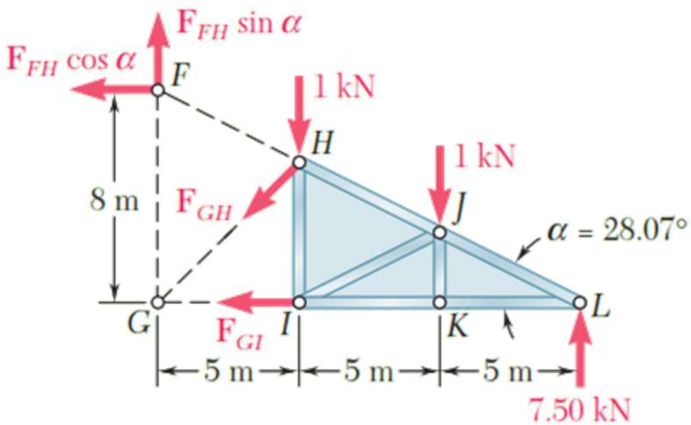
$$(7.50\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) - F_{GI}(5.33\text{ m}) = 0$$

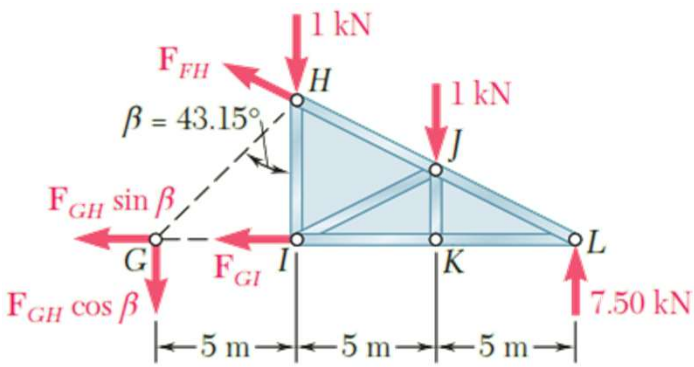
$$\Longrightarrow F_{GI} = +13.13\text{ kN}$$

$$F_{GI} = 13.13\text{ kN } T$$

Method of Sections - Problem 6.3







- Force in Member GI.

$$\sum M_G = 0$$

$$(7.5\text{ kN})(15\text{ m}) - (1\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) + (F_{FH}\cos\alpha)(8\text{ m}) = 0$$

$$\Longrightarrow F_{FH} = -13.82\text{ kN}$$

$$\boxed{F_{FH} = 13.82\text{ kN } C}$$

- Force in Member GI.

$$\tan\beta = \frac{GI}{HI} = \frac{5\text{ m}}{\frac{2}{3}(8\text{ m})} = 0.9375 \qquad \beta = 43.15°$$

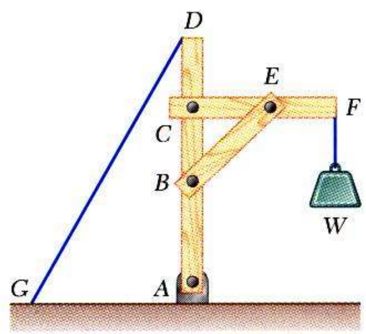
$$\sum M_L = 0$$

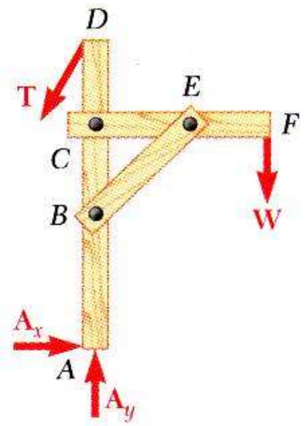
$$(1\text{ kN})(10\text{ m}) + (1\text{ kN})(5\text{ m}) + (F_{GH}\cos\beta)(10\text{ m}) = 0$$

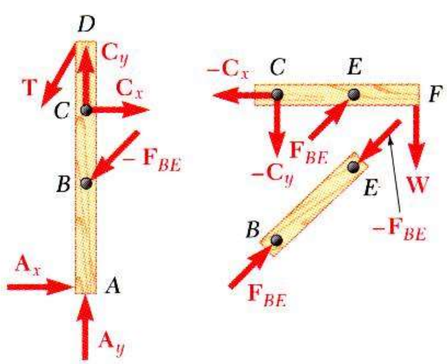
$$F_{GH} = -1.371\text{ kN}$$

$$\boxed{F_{GH} = 1.371\text{ kN } C}$$

6.3 Frames

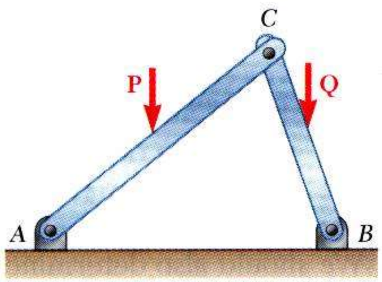


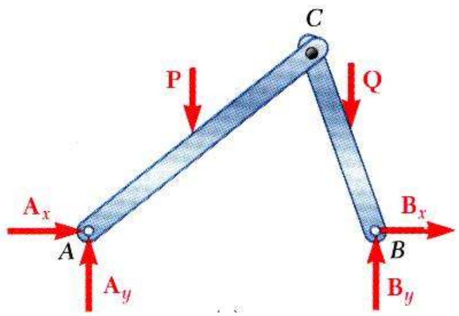


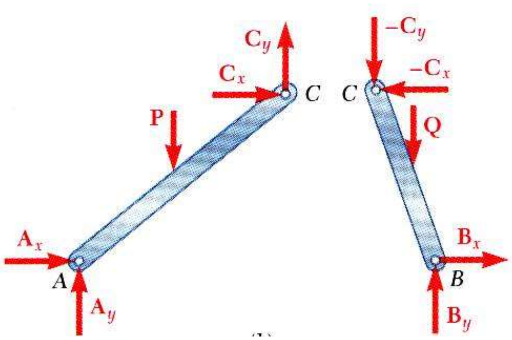


- Frames and machines are structures with at least one multi-force (>2 forces) member.
- A free body diagram of the complete frame is used to determine the external forces acting on the frame.
- Internal forces are determined by dismembering the frame and creating free-body diagrams for each component.
- Forces on two force members have known lines of action but unknown magnitude and sense, while forces on multi-force members have unknown magnitude and line of action.
- Forces between connected components are equal, have the same line of action, and opposite sense.

6.3B Frames That Collapse Without Supports

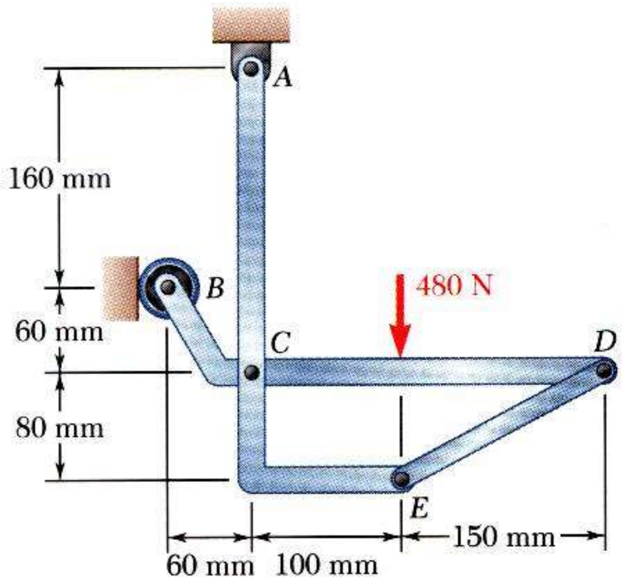






- Some frames may collapse if removed from their supports. Such frames can not be treated as rigid bodies.
- A free-body diagram of the complete frame indicates four unknown force components which cannot be determined from the three equilibrium conditions (statically indeterminate).
- The frame must be considered as two distinct, but related, rigid bodies.
- With equal and opposite reactions at the contact point between members, the two free-body diagrams show 6 unknown force components.
- Equilibrium requirements for the two rigid bodies yield 6 independent equations. Thus, taking the frame apart made the problem solvable.

Frames - Sample Problem 6.4

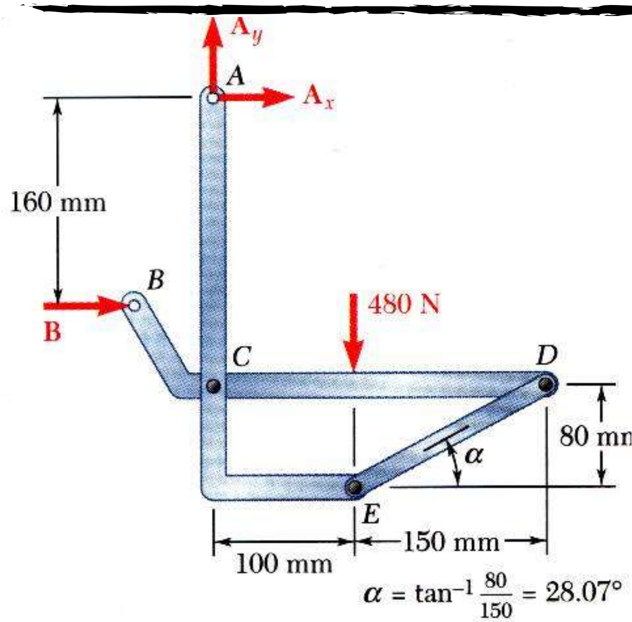


Members ACE and BCD are connected by a pin at C and by the link DE. For the loading shown, determine the force in link DE and the components of the force exerted at C on member BCD.

SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.
- Define a free-body diagram for member BCD. The force exerted by the link DE has a known line of action but unknown magnitude. It is determined by summing moments about C.
- With the force on the link DE known, the sum of forces in the x and y directions may be used to find the force components at C.
- With member ACE as a free-body, check the solution by summing moments about A.

Frames - Sample Problem 6.4



Solution:

1. support reactions:

$$\sum F_y = 0 = A_y - 480\text{ N}$$

$$A_y = 480\text{ N} \uparrow$$

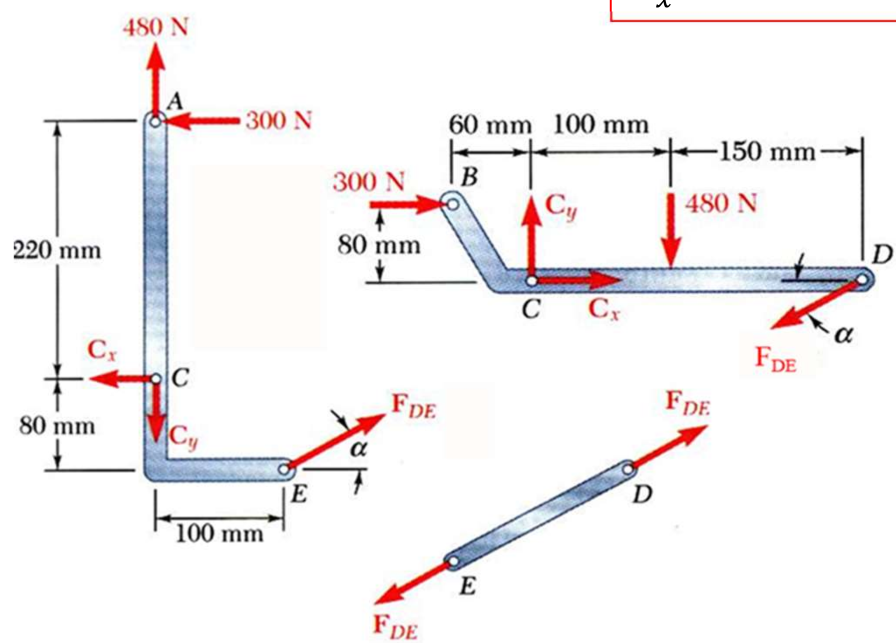
$$\sum M_A = 0 = -(480\text{ N})(100\text{ mm}) + B(160\text{ mm})$$

$$B = 300\text{ N} \rightarrow$$

$$\sum F_x = 0 = B + A_x$$

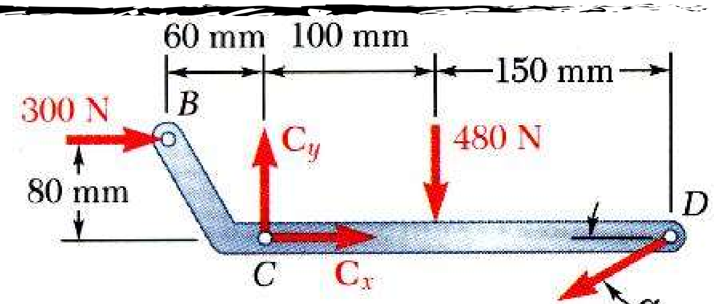
$$A_x = -300\text{ N} \leftarrow$$

2. Detach the members and Create the FBD for each member



Frames - Sample Problem 6.4

3. Using the best FBD for member BCD,



$$\sum M_C = 0 = (F_{DE} \sin\alpha)(250 \text{ mm}) + (300 \text{ N})(60 \text{ mm}) + (480 \text{ N})(100 \text{ mm})$$

$$F_{DE} = -561 \text{ N}$$

$$F_{DE} = 561 \text{ N } C$$

- Sum of forces in the *x* and *y* directions may be used to find the force components at *C*.

$$\sum F_x = 0 = C_x - F_{DE} \cos\alpha + 300 \text{ N}$$

$$0 = C_x - (-561 \text{ N}) \cos\alpha + 300 \text{ N}$$

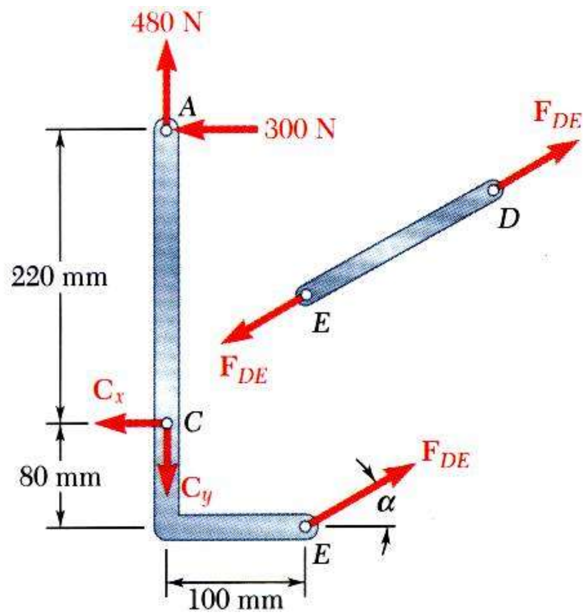
$$C_x = -795 \text{ N}$$

$$\sum F_y = 0 = C_y - F_{DE} \sin\alpha - 480 \text{ N}$$

$$0 = C_y - (-561 \text{ N}) \sin\alpha - 480 \text{ N}$$

$$C_y = 216 \text{ N}$$

Frames - Sample Problem 6.4

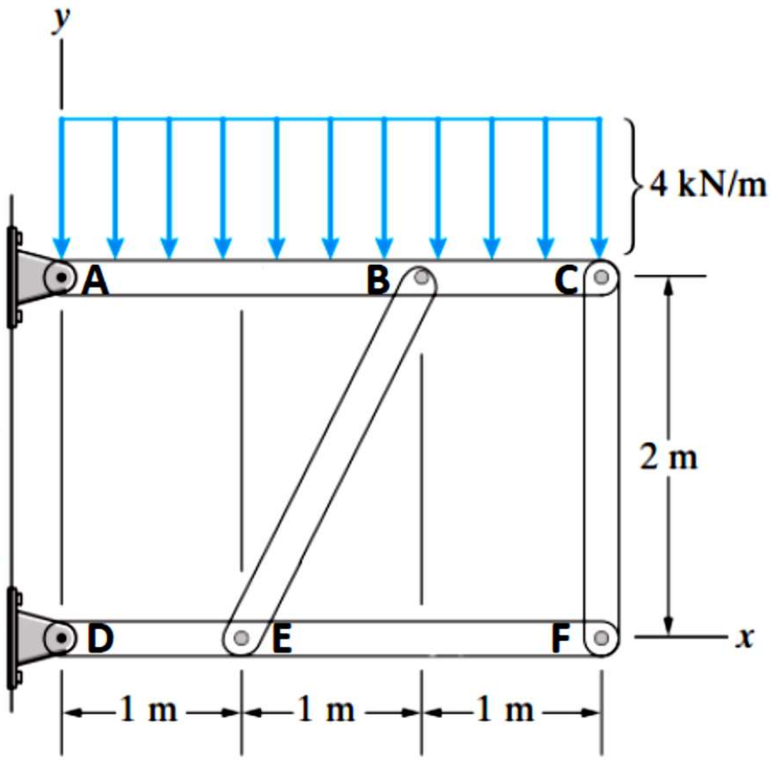


4. With member ACE as a free body with no additional unknown forces, check the solution by summing moments about A.

$$\sum M_A = (F_{DE}\cos\alpha)(300\text{ mm}) + (F_{DE}\sin\alpha)(100\text{ mm}) - C_x(220\text{ mm})$$

$$= (-561\cos\alpha)(300\text{ mm}) + (-561\sin\alpha)(100\text{ mm}) - (-795)(220\text{ mm}) = 0$$

Frames - Sample Problem



For the frame with the loading shown, determine all forces acting on member ABC and member DEF.

Frames - Sample Problem

1. Create a free-body diagram for the complete frame and solve for the support reactions if possible.

$$\sum M_A = 0$$

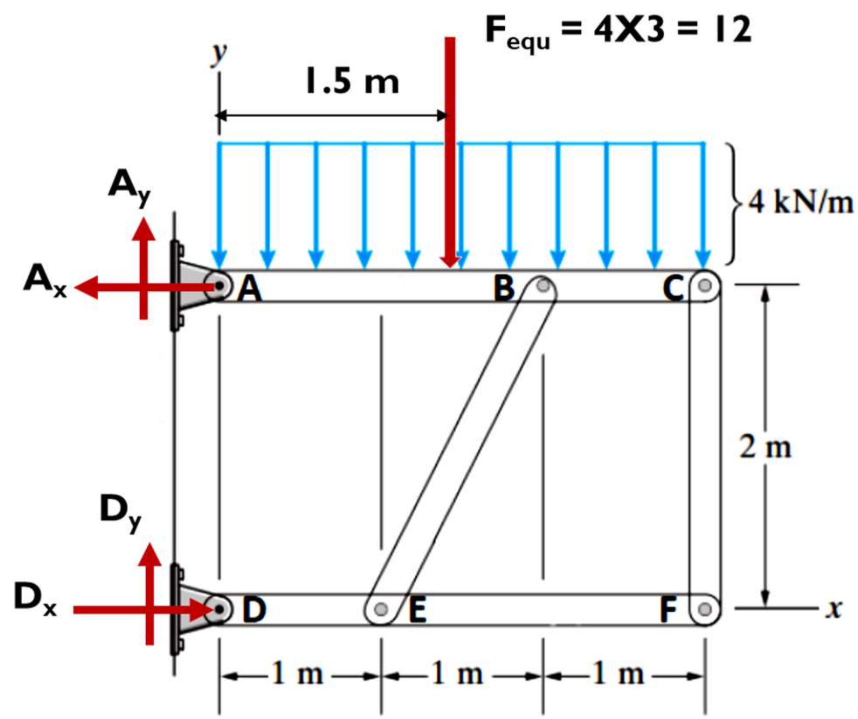
$$12 \times 1.5 = D_x \times 2, \rightarrow D_x = 9\ KN$$

$$\sum F_x = 0$$

$$D_x = A_x \rightarrow A_x = 9\ KN$$

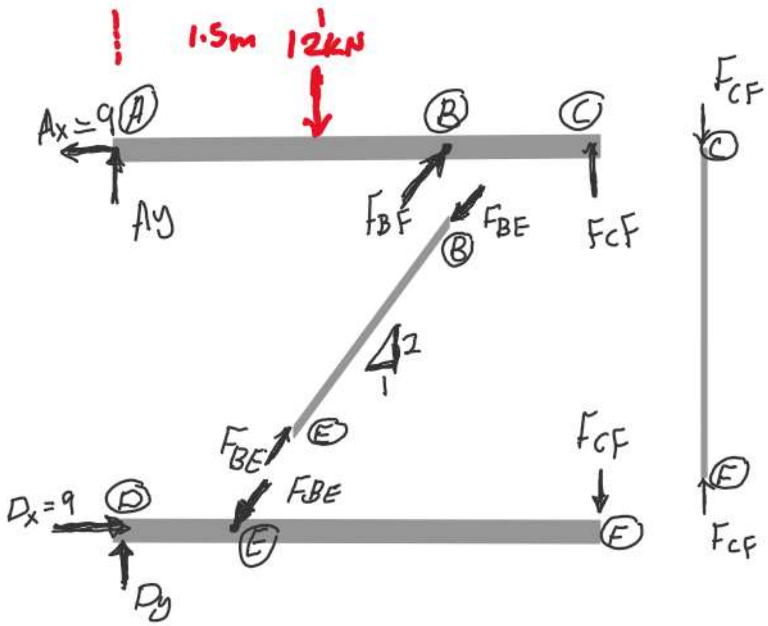
$$\sum F_y = 0$$

$$A_y + D_y - 12 = 0 \quad \text{... equ. (1)}$$



- Note That members BE and CF are two-Force Members

Frames - Sample Problem



Member DEF

$\Sigma M_D = 0$

$-F_{CF}(3) - F_{BF}\left(\frac{2}{\sqrt{5}}\right)(1) = 0$

$\Rightarrow F_{CF} = -F_{BF}\left(\frac{2}{3\sqrt{5}}\right) \;\ldots\ldots (2)$

$\Sigma F(x) = 0 \Rightarrow \frac{F_{BF}}{\sqrt{5}} = 9$

$\Rightarrow F_{BF} = 9\sqrt{5}$

from 2 then

$F_{CF} = -9\sqrt{5}\left(\frac{2}{3\sqrt{5}}\right) = -6\ kN.$

$\Sigma F_y \quad D_y - 9\sqrt{5} \times \frac{2}{\sqrt{5}} - (-6) = 0$

$\Rightarrow D_y = 12\ kN$

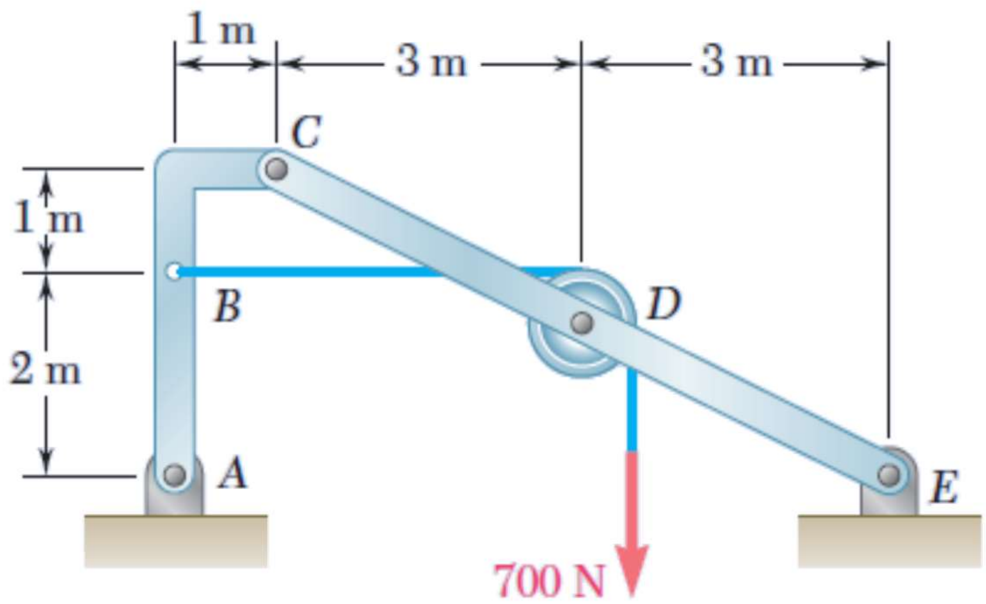
Back to Eq.(1)

$A_y + D_y - 12 = 0$

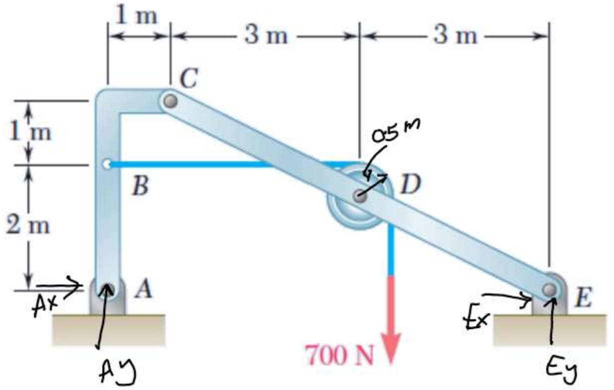
$\Rightarrow A_y = 0$

Frames - Sample Problem

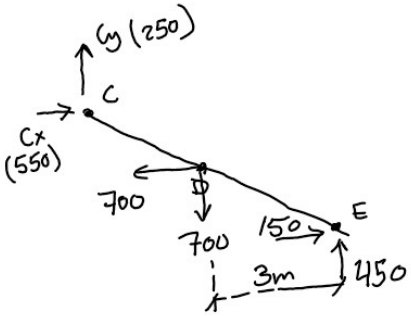
For the frame shown, Knowing that the pulley has a radius of 0.5 m, determine the components of the reactions at A and E.



Frames - Sample Problem



take member CDE to check!



$\Sigma M_E = 0$

$700(3) + 700(1.5) - 550(3) - 250(6) = 0$

$\Sigma F_x = 0$

$550 - 700 + 150 = 0$

$\Sigma F_y = 250 + 450 - 700 = 0$

take the whole structure.

$\Sigma M_A = 0$

$700 \times (4.5) = E_y(7) \Rightarrow E_y = 450\ N \uparrow$

$\Sigma F_y = 0$

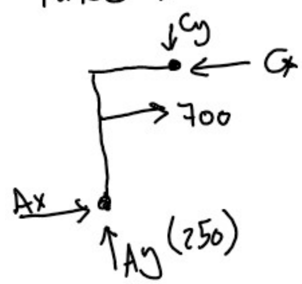
$A_y + E_y - 700 = 0$

$\Rightarrow A_y = 250\ N \uparrow$

$\Sigma F_x = 0$

$A_x + E_x = 0 \quad (1)$

Take member ABC



$\Sigma M_C = 0$

$700(1) - 250(1) + A_x(3) = 0$

$\Rightarrow A_x = -150\ N$

From (1) $E_x = -(-150) = 150\ N$

* $\Sigma F_y = 0$

$-C_y + A_y = 0 \Rightarrow C_y = 250\ N.$

* $\Sigma F_x = 0$

$-C_x + 700 + A_x = 0$

$\Rightarrow C_x = 550\ N$