Section 10.1 : Sequences.

Find the values of: a_1, a_2, a_3, and a_4 :-

$\boxed{6}$ $a_n = \dfrac{2^n - 1}{2^n}$

$$a_1 = \frac{2^1 - 1}{2^1} = \frac{1}{2}$$

$$a_2 = \frac{2^2 - 1}{2^2} = \frac{4-1}{4} = \frac{3}{4}$$

$$a_3 = \frac{2^3 - 1}{2^3} = \frac{8-1}{8} = \frac{7}{8}$$

$$a_4 = \frac{2^4 - 1}{2^4} = \frac{16-1}{16} = \frac{15}{16}.$$

#

Write the First ten terms of the sequence:

$\boxed{5}$ $a_1 = -2$, $a_{n+1} = \dfrac{n a_n}{(n+1)}$

$$1 \Rightarrow a_2 = \frac{(1)\, a_1}{(1+1)} = \frac{1(-2)}{2} = -1$$

$$2 \Rightarrow a_3 = \frac{2 a_2}{2+1} = \frac{2(-1)}{3} = \frac{-2}{3}.$$

$$3 \Rightarrow a_4 = \frac{3 a_3}{3+1} = \frac{\cancel{3}\left(\frac{-2}{\cancel{3}}\right)}{4} = \frac{-1}{2}.$$

$$4 \Rightarrow a_5 = \frac{4 a_4}{4+1} = \frac{{}^{2}\cancel{4}\left(\frac{-1}{\cancel{2}}\right)}{5} = \frac{-2}{5}.$$

$$5 \Rightarrow a_6 = \frac{5 a_5}{5+1} = \frac{\cancel{5}\left(\frac{-2}{\cancel{5}}\right)}{6} = \frac{-1}{3}.$$

⋮

$a_7 = \dfrac{-2}{7}$, $a_8 = \dfrac{-1}{4}$, $a_9 = \dfrac{-2}{9}$, $a_{10} = \dfrac{-1}{5}$.

#

...d a formula for the nth term of the sequences:

2, 6, 10, 14, 18, ...

$a_1 = 2 = 4(1) - 2$

$a_2 = 6 = 4(2) - 2$

$a_3 = 10 = 4(3) - 2$

$a_4 = 14 = 4(4) - 2$

$\vdots$

$\Rightarrow a_n = 4n - 2 \,,\quad n = 1, 2, \ldots$ #

$\boxed{6}$ 0, 1, 1, 2, 2, 3, 3, 4, ...

$a_1 = 0 = \left[\frac{1}{2}\right]$.

$a_2 = 1 = \left[\frac{2}{2}\right]$.

$a_3 = 1 = \left[\frac{3}{2}\right]$

$a_4 = 2 = \left[\frac{4}{2}\right]$

$a_5 = 2 = \left[\frac{5}{2}\right]$.

$a_6 = 3 = \left[\frac{6}{2}\right]$

$a_7 = 3 = \left[\frac{7}{2}\right]$

$\vdots$

$\Rightarrow a_n = \left[\frac{n}{2}\right] \,,\quad n = 1, 2, 3, \ldots$.

#

Convergence and Divergence :

$\boxed{31}$ $a_n = \dfrac{1-5n^4}{n^4+8n^3}$.

$$\lim_{n\to\infty}\left(\frac{1-5n^4}{n^4+8n^3}\right) = \lim_{n\to\infty}\left[\frac{\left(\frac{1}{n^4}\right)+5}{1+\left(\frac{8}{n}\right)}\right] = \frac{0-5}{1+0} = -5.$$

$\Rightarrow$ converges. #

$\boxed{35}$ $a_n = 1+(-1)^n$

$a_1 = 1+(-1)^1 = 1+-1 = 0$

$a_2 = 1+(-1)^2 = 1+1 = 2$

$a_3 = 1+(-1)^3 = 1-1 = 0$

$a_4 = 1+(-1)^4 = 1+1 = 2$

$\vdots$

$\Rightarrow \lim_{n\to\infty}(1+(-1)^n)$ doesnot exist. $\Rightarrow$ diverges. #

$\boxed{38}$ $a_n = \left(2-\dfrac{1}{2^n}\right)\left(3+\dfrac{1}{2^n}\right)$

$$\lim_{n\to\infty}\left[\left(2-\frac{1}{2^n}\right)\left(3+\frac{1}{2^n}\right)\right] = (2-0)(3+0) = (2)(3) = 6$$

$\Rightarrow$ converges. #

$\boxed{41}$ $a_n = \sqrt{\dfrac{2n}{n+1}}$

$$\lim_{n\to\infty}\sqrt{\frac{2n}{n+1}} = \sqrt{\lim_{n\to\infty}\left(\frac{2n}{n+1}\right)} = \sqrt{\lim_{n\to\infty}\left(\frac{2}{1+\frac{1}{n}}\right)} = \sqrt{\frac{2}{1+0}} = \sqrt{2}$$

$\Rightarrow$ converges. #

44 $a_n = n\pi \cos(n\pi)$

$\cos(n\pi) = 1$ for $n = 2, 4, 6, \ldots$ (even)

$\cos(n\pi) = -1$ for $n = 1, 3, 5, \ldots$ (odd).

$\Rightarrow \cos(n\pi) = (-1)^n, \quad n = 1, 2, 3, \ldots$

$\Rightarrow \lim_{n\to\infty} (n\pi)\cos(n\pi) = \lim_{n\to\infty} (n\pi)(-1)^n$ does not exist.

$\Rightarrow$ diverges. #

48 $a_n = \dfrac{3^n}{n^3}$.

(using l'Hôpital's rule).

$$\lim_{n\to\infty} \frac{3^n}{n^3} = \lim_{n\to\infty} \frac{3^n(\ln 3)}{3n^2} = \lim_{n\to\infty} \frac{3^n(\ln 3)^2}{6n}$$

$$= \lim_{n\to\infty} \frac{(3^n)(\ln 3)^3}{6} = \infty \Rightarrow \text{diverges.}$$ #

50 $a_n = \dfrac{\ln n}{\ln 2n}$

(using l'Hôpital's rule).

$$\lim_{n\to\infty} \frac{\ln n}{\ln 2n} = \lim_{n\to\infty} \frac{\left(\frac{1}{n}\right)}{\left(\frac{2}{2n}\right)} = 1 \Rightarrow \text{converges.}$$ #

54 $a_n = \left(1 - \frac{1}{n}\right)^n$

$$\lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^n = \lim_{n\to\infty} \left(1 + \frac{(-1)}{n}\right)^n = e^{-1} = \frac{1}{e} \Rightarrow \text{converges.}$$ #

* $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$

proof:

$$a_n = \left(1+\frac{x}{n}\right)^n$$

$$\ln a = n\ln\left(1+\frac{x}{n}\right)$$

Applying l'Hôpital's Rule : we have :

$$\lim_{n\to\infty} n\ln\left(1+\frac{x}{n}\right) = \lim_{n\to\infty}\frac{\ln\left(1+\frac{x}{n}\right)}{(1/n)}$$

$$= \lim_{n\to\infty}\left[\frac{\frac{(-x/n^2)}{(1+(x/n))}}{(-1/n^2)}\right] = \lim_{n\to\infty}\frac{x}{1+\frac{x}{n}} = \frac{x}{1+0} = x$$

$$\Rightarrow \lim_{n\to\infty} a_n = e^x$$ #

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[58] $a_n = (n+4)^{1/(n+4)}$.

$$\lim_{n\to\infty} (n+4)^{1/(n+4)} = \lim_{x\to\infty} x^{1/x} = 1 \Rightarrow \text{converges.}$$

[by theorem5 ③]. $\boxed{\text{Let } x = n+4}$

#

[60] $a_n = \ln n - \ln(n+1)$.

$$\lim_{n\to\infty} [\ln n - \ln(n+1)] = \lim_{n\to\infty} \ln\left[\frac{n}{n+1}\right] = \ln 1 = 0$$

$\Rightarrow$ converges.

#

[63] $a_n = \dfrac{n!}{n^n}$ (Hint: Compare with $\frac{1}{n}$) $\Rightarrow 0 \leq \dfrac{n!}{n^n} \leq \dfrac{1}{n}$.

$$\lim_{n\to\infty} \frac{n!}{n^n} = \lim_{n\to\infty} \frac{1.2.3.4\ldots(n-1)(n)}{n.n.n.n\ldots n.n} \leq \lim_{n\to\infty} \left(\frac{1}{n}\right) = 0$$

$$\Rightarrow \lim_{n\to\infty} \frac{n!}{n^n} = 0 \quad \text{converges.}$$

#

[70] $a_n = \left(\dfrac{n}{n+1}\right)^n$

$\lim_{n\to\infty} \left(\dfrac{n}{n+1}\right)^n$, Let $y = \left(\dfrac{n}{n+1}\right)^n$

$\ln y = n \ln\left(\dfrac{n}{n+1}\right)$

[using l'Hôpital's rule]

$$\Rightarrow \lim_{n\to\infty} \left[n \ln\left(\frac{n}{n+1}\right)\right] = \lim_{n\to\infty} \frac{\ln\left(\frac{n}{n+1}\right)}{(1/n)}$$

$\boxed{\ln\left(\frac{n}{n+1}\right) = \ln n - \ln(n+1)}$

$$= \lim_{n\to\infty} \frac{\frac{1}{n} - \frac{1}{n+1}}{\frac{-1}{n^2}} = \lim_{n\to\infty} \left(\frac{-n^2}{n(n+1)}\right) = \lim_{n\to\infty} \frac{-n^2}{n^2+n} = -1$$

$$\Rightarrow \lim_{n\to\infty} a_n = e^{-1} = \frac{1}{e} \Rightarrow \text{converges.}$$

#

$\boxed{72}$ $a_n = \left(1 - \frac{1}{n^2}\right)^n$

$\lim\limits_{n\to\infty} \left(1 - \frac{1}{n^2}\right)^n$ Let $y = \left(1 - \frac{1}{n^2}\right)^n$

$\ln y = n \ln\left(1 - \frac{1}{n^2}\right)$

$\Rightarrow \lim\limits_{n\to\infty} n \ln\left(1 - \frac{1}{n^2}\right) = \lim\limits_{n\to\infty} \frac{\ln\left(1 - \frac{1}{n^2}\right)}{(1/n)}$ ← (using l'Hôpital's rule)

$= \lim\limits_{n\to\infty} \left[\frac{\left(\frac{2}{n^3}\right) / \left(1 - \frac{1}{n^2}\right)}{\left(\frac{-1}{n^2}\right)}\right] = \lim\limits_{n\to\infty} \left[\frac{-2\cancel{n^2}}{n^{\cancel{3}}\left(1 - \frac{1}{n^2}\right)}\right] = \lim\limits_{n\to\infty} \left[\frac{-2}{\cancel{n}\left(\frac{n^2-1}{n^{\cancel{2}}}\right)}\right]$

$= \lim\limits_{n\to\infty} \left[\frac{-2n}{n^2 - 1}\right] = 0$

$\Rightarrow \lim\limits_{n\to\infty} a_n = e^0 = 1 \Rightarrow$ converges. #

$\boxed{76}$ $a_n = \sinh(\ln n)$

$\lim\limits_{n\to\infty} \sinh \ln n = \lim\limits_{n\to\infty} \left[\frac{e^{\ln n} - e^{-\ln n}}{2}\right] = \lim\limits_{n\to\infty} \left[\frac{n - \frac{1}{n}}{2}\right]$

$= \lim\limits_{n\to\infty} \left[\frac{n^2 - 1}{2n}\right] = \infty \Rightarrow$ diverges. #

$\boxed{82}$ $a_n = \frac{1}{\sqrt{n}} \tan^{-1} n$

$\lim\limits_{n\to\infty} \frac{1}{\sqrt{n}} \tan^{-1} n = 0 \cdot \frac{\pi}{2} = 0 \Rightarrow$ converges. #

$\boxed{86}$ $a_n = \frac{(\ln n)^5}{\sqrt{n}}$

$\lim\limits_{n\to\infty} \frac{(\ln n)^5}{\sqrt{n}} =$ using l'Hôpital's rule five times $= 0 \Rightarrow$ converges. #

$\boxed{92}$ $a_1 = -1$, $a_{n+1} = \dfrac{a_n + 6}{a_n + 2}$

In Exercise 92, assume that the sequence converges and find its limit.

Since a_n converges $\Rightarrow \lim_{n\to\infty} a_n = L$

$$\Rightarrow \lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \frac{a_n + 6}{a_n + 2} \Rightarrow L = \frac{L+6}{L+2}$$

$$\Rightarrow L(L+2) = L+6$$

$$L^2 + 2L = L+6$$

$$L^2 + L - 6 = 0$$

$$(L+3)(L-2) = 0 \Rightarrow L = -3 \text{ or } L = 2$$

since $a_n > 0$ for $n \geq 2$ $\Rightarrow$

$\Rightarrow \boxed{L = 2}$

$a_1 = -1$

$a_2 = \dfrac{-1+6}{-1+2} = \dfrac{5}{1} = 5 > 0$

$a_3 = \dfrac{5+6}{5+2} = \dfrac{11}{7} > 0$

$\vdots$

#

$\boxed{111}$ $a_n = \dfrac{3n+1}{n+1}$

In Exercise **111**, determine if the sequence is monotonic and if it is bounded.

$a_1 = \dfrac{3+1}{1+1} = \dfrac{4}{2} = 2$

$a_2 = \dfrac{6+1}{2+1} = \dfrac{7}{3}$

$a_3 = \dfrac{10}{4} = \dfrac{5}{2}$

$a_4 = \dfrac{13}{5}$

$\vdots$

$\Rightarrow a_n \leq a_{n+1}$

$$\Rightarrow \frac{3n+1}{n+1} \leq \frac{3(n+1)+1}{(n+1)+1}$$

$$\Rightarrow \frac{3n+1}{n+1} \leq \frac{3n+4}{n+2}$$

$$\Rightarrow \frac{(3n+1)(n+2)}{(n+1)(n+2)} \leq \frac{(3n+4)(n+1)}{(n+2)(n+1)}$$

$$\Rightarrow \cancel{3n^2} + \cancel{6n} + \cancel{n} + 2 \leq \cancel{3n^2} + \cancel{3n} + \cancel{4n} + 4$$

we have, $2 < 4 \Rightarrow$ the steps are reversible so the sequence is nondecreasing;

Now, $\frac{3n+1}{n+1} < 3 \Rightarrow 3n+1 < 3n+3$

$1 < 3 \Rightarrow$ the steps are reversible, so the sequence is bounded above by 3.

Note that:
$\lim_{n\to\infty} \left(\frac{3n+1}{n+1}\right) = 3$

#

Theorem 5: the following sequences converge to the limits listed below:

① $\lim_{n\to\infty} \frac{\ln n}{n} = 0$

② $\lim_{n\to\infty} \sqrt[n]{n} = 1$

③ $\lim_{x\to\infty} x^{1/n} = 1$, $(x>0)$

④ $\lim_{n\to\infty} x^n = 0$, $(|x|<1)$

⑤ $\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x$, (any x)

⑥ $\lim_{n\to\infty} \frac{x^n}{n!} = 0$, (any x)

In formulas (3) through (6), x remains fixed as $n\to\infty$

* Section 10.2 : Infinite Series.

Find a formula for the nth partial sum, then use it to find the series sum if the series converges.

$\boxed{6}$ $\frac{5}{1.2} + \frac{5}{2.3} + \frac{5}{3.4} + \cdots + \frac{5}{n(n+1)} + \cdots$

$$\frac{5}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} = \frac{A(n+1) + Bn}{n(n+1)}$$

$$\Rightarrow 5 = (A+B)n + A \Rightarrow A = 5 , B = -5 .$$

$$\Rightarrow \frac{5}{n(n+1)} = \frac{5}{n} - \frac{5}{n+1}$$

$$\Rightarrow S_n = \left(5 - \cancel{\frac{5}{2}}\right) + \left(\cancel{\frac{5}{2}} - \cancel{\frac{5}{3}}\right) + \left(\cancel{\frac{5}{3}} - \cancel{\frac{5}{4}}\right) + \cdots + \left(\frac{5}{n-1} - \cancel{\frac{5}{n}}\right)$$
$$+ \left(\cancel{\frac{5}{n}} - \frac{5}{n+1}\right) = 5 - \frac{5}{n+1}$$

$$\Rightarrow \lim_{n\to\infty} S_n = 5 .$$

#

Write out the first few terms of series to show how it starts. Then find the sum of it.

$\boxed{14}$ $\sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n}\right) = 2 + \frac{4}{5} + \frac{8}{25} + \frac{16}{125} + \cdots$

$$= 2\underbrace{\left(1 + \frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \cdots\right)}_{\text{this is a geometric series with } a=1,\ r=\frac{2}{5}}$$

$$\Rightarrow \sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n}\right) = 2 \sum_{n=0}^{\infty} \left(1 + \frac{2}{5} + \cdots\right)$$

$$= 2\left(\frac{1}{1 - \left(\frac{2}{5}\right)}\right) = \frac{10}{3} .$$

#

Determine if the geometric series converges or diverges. If a series converges. Find its sum.

$\boxed{18}$ $\left(\frac{-2}{3}\right)^2 + \left(\frac{-2}{3}\right)^3 + \left(\frac{-2}{3}\right)^4 + \cdots$

geometric series with $r = \frac{-2}{3}$.

$\left|\frac{-2}{3}\right| < 1 \Rightarrow$ converges to $\frac{(-2/3)^2}{1-(-2/3)} = \frac{4}{15}$.

Express the number as the ratio of two integers.

$\boxed{24}$ $1.\overline{414} = 1.414414414\ldots$

$$1.\overline{414} = 1 + 0.414 + 0.000414 + \cdots$$

$$= 1 + \frac{414}{1000} + \frac{414}{1000000} + \cdots$$

$$= 1 + \frac{414}{1000} + \frac{414}{1000}\cdot\frac{1}{1000} + \cdots$$

$$= 1 + \frac{414}{1000} + \frac{414}{1000}\cdot\frac{1}{10^3} + \cdots$$

$$= 1 + \sum_{n=0}^{\infty}\left(\frac{414}{1000}\right)\left(\frac{1}{10^3}\right)^n = 1 + \frac{(414/1000)}{1-(1/1000)} = 1 + \frac{414}{999}$$

$$= \frac{1413}{999} \quad \#$$

Use the nth-term test for divergence to show that the series is divergent.

$\boxed{32}$ $\sum_{n=0}^{\infty}\left(\frac{e^n}{e^n+n}\right)$

(using l'Hôpital's Rule)

$$\lim_{n\to\infty}\left(\frac{e^n}{e^n+n}\right) = \lim_{n\to\infty}\left(\frac{e^n}{e^n+1}\right) = \lim_{n\to\infty}\left(\frac{e^n}{e^n}\right) = \lim_{n\to\infty} 1 = 1 \neq 0$$

$\Rightarrow$ diverges. #

Find a formula for the nth ~~part~~ partial sum of the series and use it to determine if the series converges or diverges.

$\boxed{38}$ $\sum_{n=1}^{\infty}\left(\tan(n)-\tan(n-1)\right)$ ← Telescoping Series.

$$S_k = (\cancel{\tan 1} - \tan(0)) + (\cancel{\tan(2)} - \cancel{\tan(1)}) + (\cancel{\tan(3)} - \cancel{\tan(2)}) + \dots + (\cancel{\tan(k)} - \cancel{\tan(k-1)}) + (\tan(k+1) - \cancel{\tan(k)})$$

$$= \tan(k+1) - \tan(0) = \tan(k+1)$$

$$\Rightarrow \lim_{k\to\infty} S_k = \lim_{k\to\infty} \tan(k+1) = DNE \Rightarrow \text{diverges.}$$

#

Find the sum of the series:

$\boxed{44}$ $\sum_{n=1}^{\infty}\left(\frac{2n+1}{n^2(n+1)^2}\right)$

$$\frac{2n+1}{n^2(n+1)^2} = \frac{A}{n} + \frac{B}{n^2} + \frac{C}{(n+1)} + \frac{D}{(n+1)^2}$$

$$= \frac{An(n+1)^2 + B(n+1)^2 + Cn^2(n+1) + Dn^2}{n^2(n+1)^2}$$

$$\Rightarrow 2n+1 = (A+C)n^3 + (2A+B+C+D)n^2 + (A+2B)n + B$$

$$\Rightarrow \boxed{B=1}$$

$$A+2B = 2 \Rightarrow A+2=2 \Rightarrow \boxed{A=0}$$

$$A+C=0 \Rightarrow \boxed{C=0}$$

$$2A+B+C+D=0 \Rightarrow 1+D=0 \Rightarrow \boxed{D=-1}$$

$$\Rightarrow \frac{2n+1}{n^2(n+1)^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$\Rightarrow S_k = \left(1-\cancel{\tfrac{1}{4}}\right)+\left(\cancel{\tfrac{1}{4}}-\cancel{\tfrac{1}{9}}\right)+\left(\cancel{\tfrac{1}{9}}-\cancel{\tfrac{1}{16}}\right)+\cdots+\left(\cancel{\tfrac{1}{(k-1)^2}}-\cancel{\tfrac{1}{k^2}}\right)$$

$$+\left(\cancel{\tfrac{1}{k^2}}+\tfrac{1}{(k+1)^2}\right)$$

$$S_k = 1-\frac{1}{(k+1)^2}.$$

$$\Rightarrow \lim_{k\to\infty} S_k = \lim_{k\to\infty}\left[1-\frac{1}{(k+1)^2}\right] = 1 \qquad \#$$

Convergence / Divergence.

$\boxed{54}$ $\displaystyle\sum_{n=0}^{\infty}\left(\frac{\cos n\pi}{5^n}\right)$ $\qquad \cos(n\pi) = (-1)^n$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{5^n} = \sum_{n=0}^{\infty}\left(\frac{-1}{5}\right)^n \Rightarrow \text{geometric series.}$$

$a = 1, \; r = \frac{-1}{5}.$

$$|r| = \left|\frac{-1}{5}\right| = \frac{1}{5} < 1 \Rightarrow \text{converges. to:}$$

$$\frac{1}{1-\left(\frac{-1}{5}\right)} = \frac{5}{6} \qquad \#$$

$\boxed{62}$ $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n^n}{n!}\right)$

$$\lim_{n\to\infty}\left(\frac{n^n}{n!}\right) = \lim_{n\to\infty}\frac{n.n.\cdots n}{1.2.\cdots n} > \lim_{n\to\infty} n = \infty \Rightarrow \text{diverges.} \qquad \#$$

$\boxed{63}$ $$\sum\left(\frac{2^n+3^n}{4^n}\right) = \sum_{n=1}^{\infty}\frac{2^n}{4^n}+\sum_{n=1}^{\infty}\frac{3^n}{4^n}$$

$$= \underbrace{\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n}+\underbrace{\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n}$$

geometric series $\Rightarrow$ both converges.

since $|r| = \left|\frac{1}{2}\right| < 1$ and $|r| = \left|\frac{3}{4}\right| < 1$

$$\left.\begin{aligned}\Rightarrow \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n &= \frac{(1/2)}{1-(1/2)} = 1\\ \text{and } \sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n &= \frac{(3/4)}{1-(3/4)} = 3\end{aligned}\right\} \Rightarrow \sum_{n=1}^{\infty}\left(\frac{2^n+3^n}{4^n}\right) = 1+3 = 4.$$

$\#$

find the value of x for which the given geometric series converges.
Also find the sum of the series.

$\boxed{78}$ $\sum_{n=0}^{\infty} (\ln x)^n$ $\quad a = 1 \, , \quad r = \ln x$

↳ converges to $\boxed{\frac{1}{1-\ln x}}$ for $|\ln x| < 1$

$-1 < \ln x < 1$

$\boxed{e^{-1} < x < e}$

#

$\boxed{90}$ find the value of b for which:

$1 + e^{b} + e^{2b} + e^{3b} + \cdots = 9$.

↳ geometric series : $a = 1$
$r = e^{b}$

$\Rightarrow 1 + e^{b} + e^{2b} + \cdots = 9$

$\frac{1}{1-e^{b}} = 9 \Rightarrow 1 - e^{b} = \frac{1}{9}$

$e^{b} = \frac{8}{9} \Rightarrow \boxed{b = \ln\left(\frac{8}{9}\right)}$.

#

