ENCS3340 - Artificial Intelligence

Adversarial Search & Games

Game Playing and AI

- Why would game playing be a good problem for AI research?
  - Game-playing is non-trivial
    - Need to display “human-like” intelligence
    - Some games (such as chess) are very complex
    - Requires decision-making within a time-limit
  - Games are played in a controlled environment
    - Can do experiments, repeat games, etc
    - Good for evaluating research systems
  - Can compare humans and computers directly
    - Can evaluate percentage of wins/losses to quantify performance
  - All the information is available
    - Human and computer have equal information

How Does a Computer Play a Game?

- A way to play a game is to:
  - Consider all the legal moves you can make
  - Compute the new position resulting from each move
  - Evaluate each resulting position and determine which is best
  - Make that move
  - Wait for your opponent to move and repeat

- Key problems are:
  - Representing the “board”
  - Generating all next legal boards
  - Evaluating a position

Game Playing: Adversarial Search

- Adversarial: involving two people or two sides who oppose each other

- Different kinds of games:

	Deterministic	Chance
Perfect Information	Chess, Checkers Go, Othello	Backgammon, Monopoly
Imperfect Information	Battleship	Bridge, Poker, Scrabble,

- Games with perfect information. No randomness is involved.

- Games with imperfect information. Random factors are part of the game.

Games as Adversarial Search

- many games can be formulated as search problems
- Zero sum: my win is your loss, my loss is your win!
- the zero-sum utility function leads to an adversarial situation
  - in order for one agent to win, the other necessarily has to lose
- factors complicating the search task
  - potentially huge search spaces
  - elements of chance
  - multi-person games, teams
  - time limits
  - imprecise rules

Difficulties with Games

- games can be very hard search problems
  - yet reasonably easy to formalize
  - finding the optimal solution may be impractical
    - a solution that beats the opponent is “good enough”
  - unforgiving
    - a solution that is “not good enough” not only leads to higher costs, but to a loss to the opponent
- example: chess
  - size of the search space
    - branching factor around 35
    - about 50 moves per player
    - about 35^{100} or 10^{154} nodes
    - about 10^{40} distinct nodes (size of the search graph)

Single-Person Game

- conventional search problem
  - identify a sequence of moves that leads to a winning state
  - examples: Solitaire, dragons and dungeons, Rubik's cube
  - little attention in AI
- some games can be quite challenging
  - some versions of solitaire
  - a heuristic for Rubik's cube was found by the Absolver program

Searching in a two player game

- Traditional (single agent) search methods only consider how close the agent is to the goal state (e.g. best first search).

- In two player games, decisions of both agents have to be taken into account: a decision made by one agent will affect the resulting search space that the other agent would need to explore.

- Question: Do we have randomness here since the decision made by the opponent is NOT known in advance?

  - No. Not if all the moves or choices that the opponent can make are finite and can be known in advance.

Searching in a two player game: Strategies

- Your Strategy for a move: you use the best strategy you can think of: depends on how “smart” you are
- What about opponent strategy?
- We don’t know exactly: could be a NOVICE, could be a MASTER
- Which is safer:
  - To assume that the opponent is a novice and may make dumb moves?
  - To assume that the opponent is very smart?
- Which is safer in a war:
  - To assume your opponent is weak
  - To assume your opponent is very strong?
- We assume that the opponent is as smart as possible, or as smart as we can think
- The opponent uses my own strategy for search (but in reverse):
  - If I try to maximize MY future choices in XO he tries to minimize MY chances.
  - I am MAX, he is MIN

Two Player Games: Evaluation Functions

- What an evaluation function could be: an assessment of my chances to win:
  - Chess: # of my figures – # of opponent figures (maybe weighted)
  - Tic_tac_Toe: number of open chances for me – number of opponent's chances
  - ***General***: Something that is good for me when higher and good for opponent when lower; recall: I am MAX and he is MIN and we have ONLY ONE Evaluation Function!
- Evaluation function is supposed to give an impression of how close MAX is to the goal: the higher the closer:

Two Player Games: Evaluation Functions

- The deeper you go: the more steps you imagine searching, the more accurate your evaluation function gets (getting closer to goal).

- So it is good to do the computation (of evaluation function) at the deepest possible level and then see how to act now to reach there: but that is costly and time consuming

- We need a compromise! Look ahead at a limited depth!: modest computation, modest knowledge about position:

Searching in a two player game

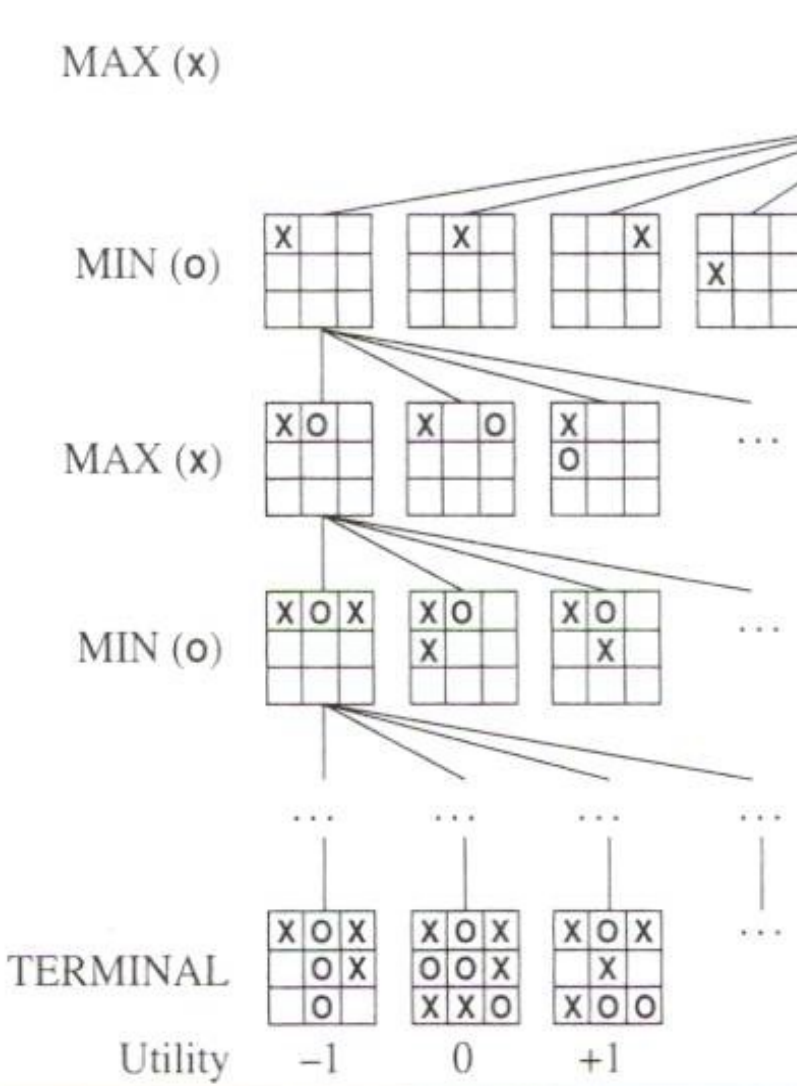
- To formalize a two player game as a search problem an agent can be called MAX and the opponent can be called MIN.

- **Problem Formulation**:
  - Initial state: board configurations and the player to move.
  - Successor function: list of pairs (move, state) specifying legal moves and their resulting states. (moves + initial state = game tree)
  - A terminal test: decide if the game has finished.
  - A utility function: produces a numerical value for (only) the terminal states. Example: In chess, outcome = win/loss/draw, with values +1, -1, 0 respectively.

    If you stop at non-terminal states, use an evaluation function to indicate the chances of winning

- Players need search tree to determine next move.

Partial game tree for Tic-Tac-Toe



- Root node represents the current board configuration; player must decide the best single move to make next
- Each level of search nodes in the tree corresponds to all possible board configurations for a particular player MAX or MIN.
- If it is my turn to move, then the root is labeled a "MAX" node; otherwise it is labeled a "MIN" node, indicating opponent's turn.
- Utility values found at the end can be returned back to their parent nodes.
- **Idea**: MAX chooses the board with the max utility value, MIN the minimum.

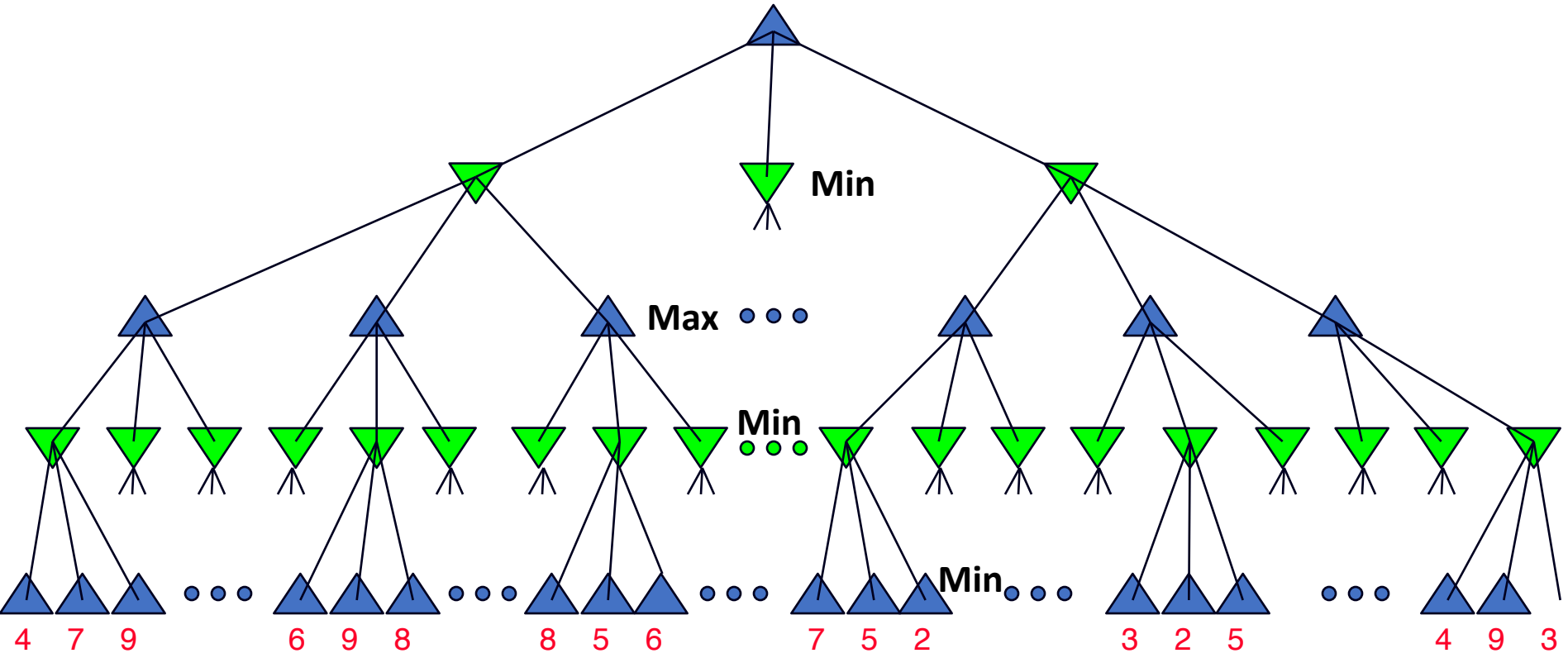
MiniMax (MinMax, MM) Algorithm

- An algorithm to search trees representing two-player zero-sum (my gain your loss) games.
- **Goal**: minimizing the possible loss for a worst case (maximum loss) scenario.
- Or maximize the minimum gain. Guaranteed; no matter what; how opponent plays; worst case scenario; gain can be MORE, never less
- Result: one move (one level down) then the process starts again.
- For this one move you may explore as many nodes as you have time for!
- MIN works in opposite direction to MAX
- Then work is repeated

MiniMax Algorithm

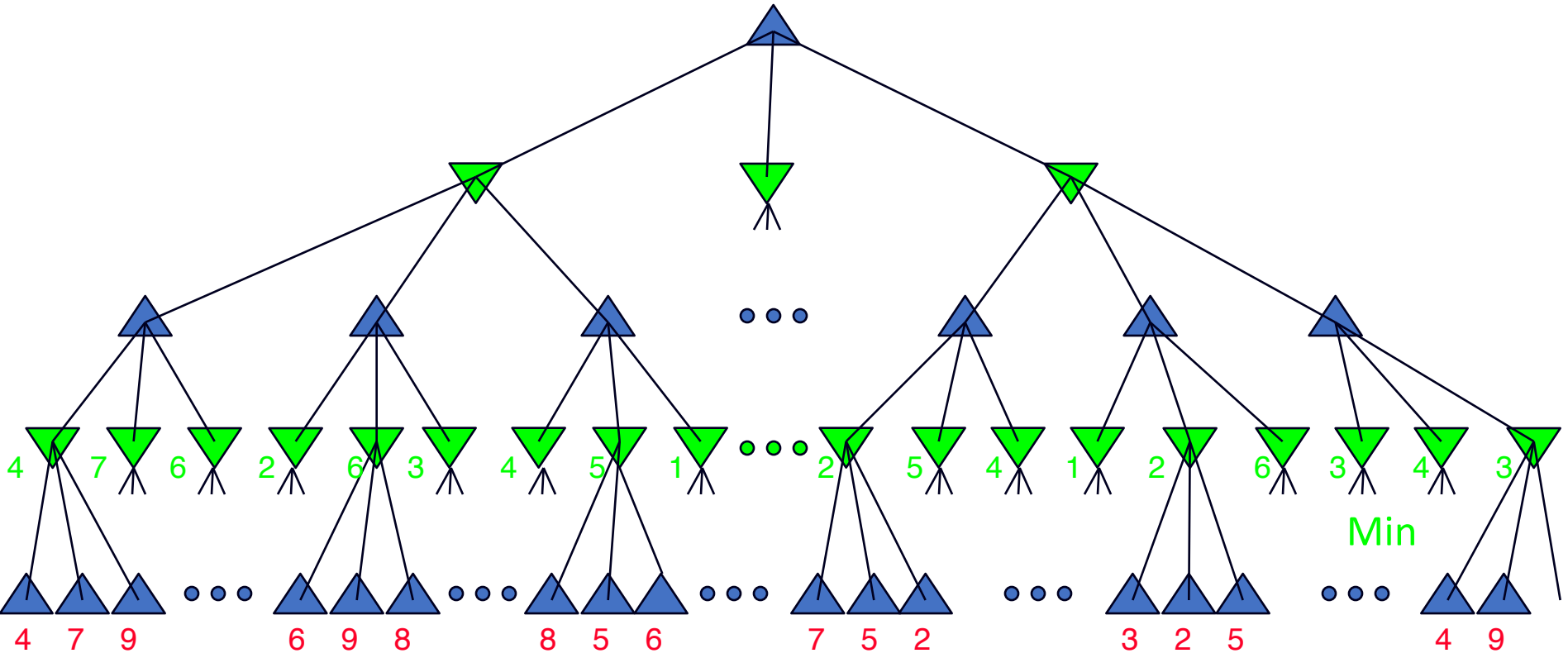
- Create start node as a MAX node with current board configuration
- Expand nodes down to some depth of lookahead in the game
- Apply the evaluation function at each of the leaf nodes
- "Back up" values for each of the non-leaf nodes until a value is computed for the root node
  - At MIN nodes, the backed-up value is the minimum of the values associated with its children
  - At MAX nodes, the backed-up value is the maximum of the values associated with its children
- Pick the operator associated with the child node whose backed-up value determined the value at the root

MiniMax Example



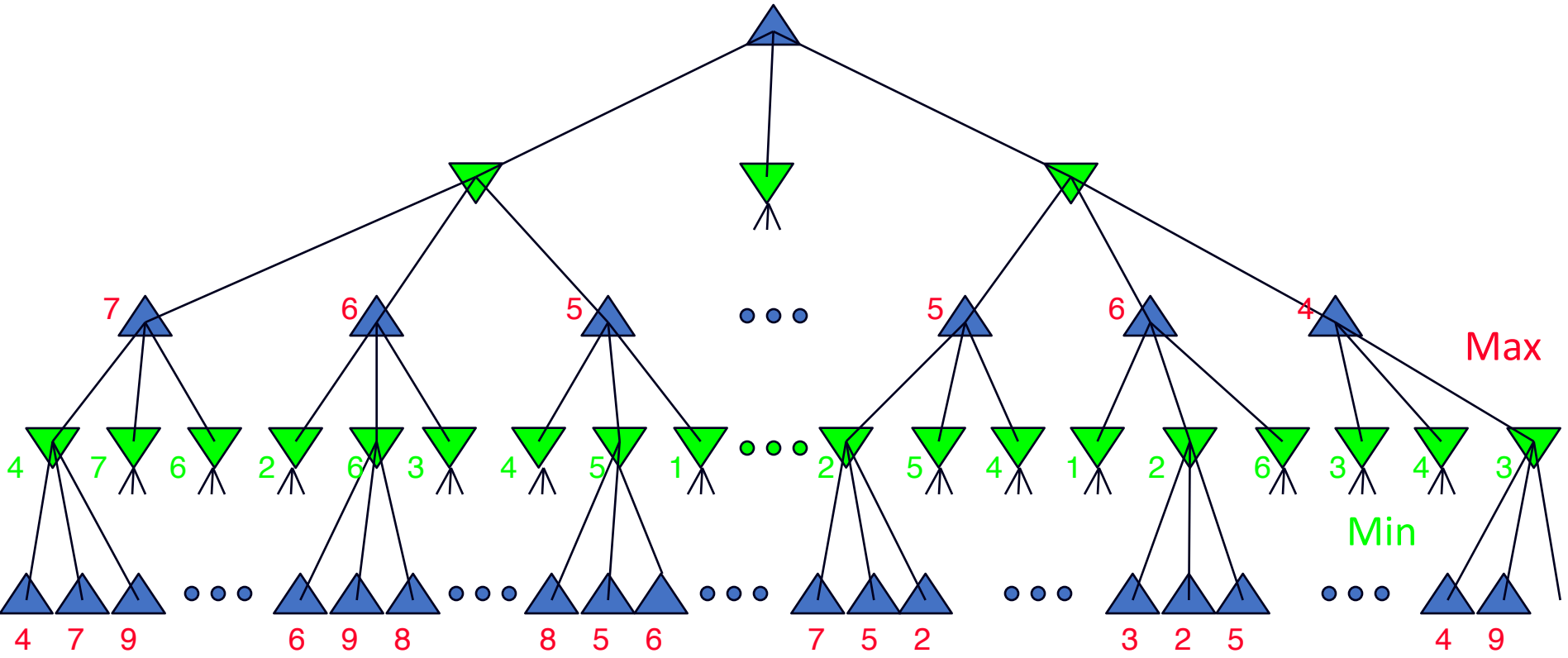
terminal nodes: values calculated from the utility function

MiniMax Example

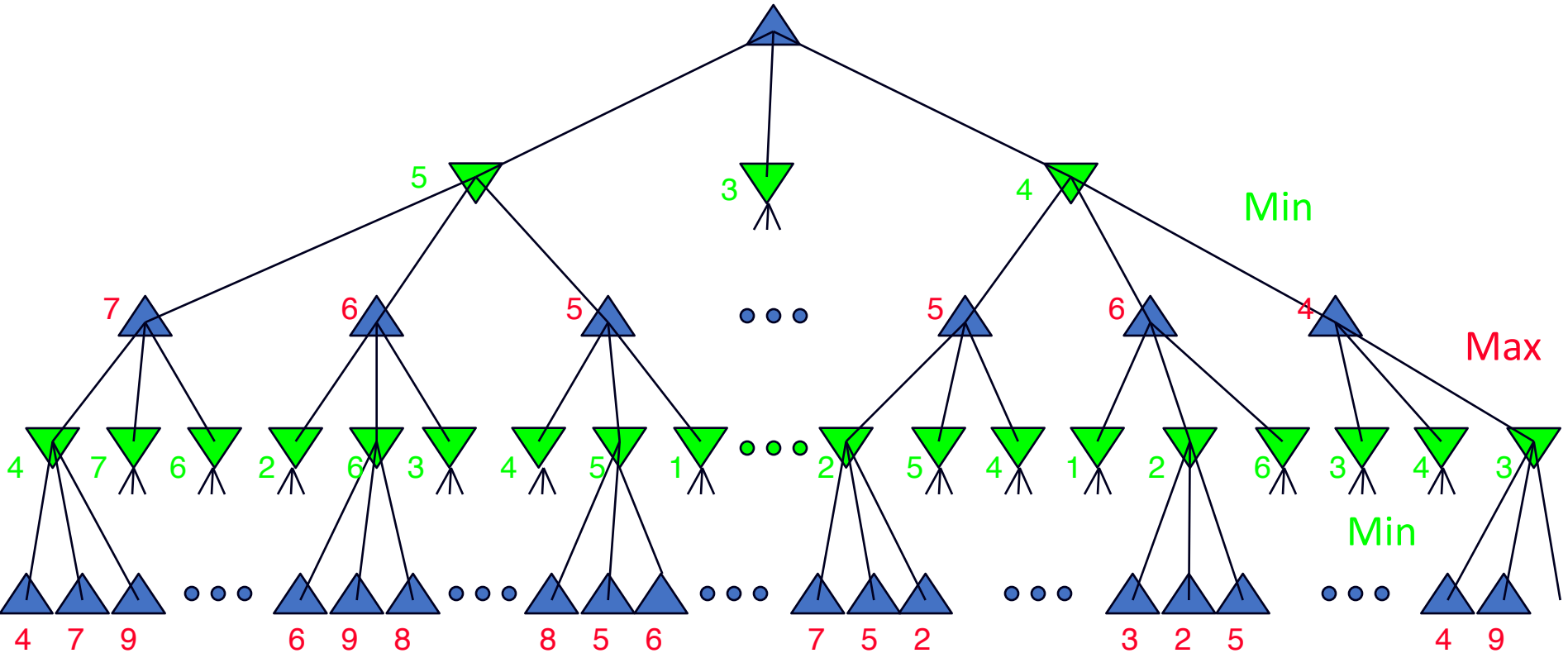


other nodes: values calculated via minimax algorithm

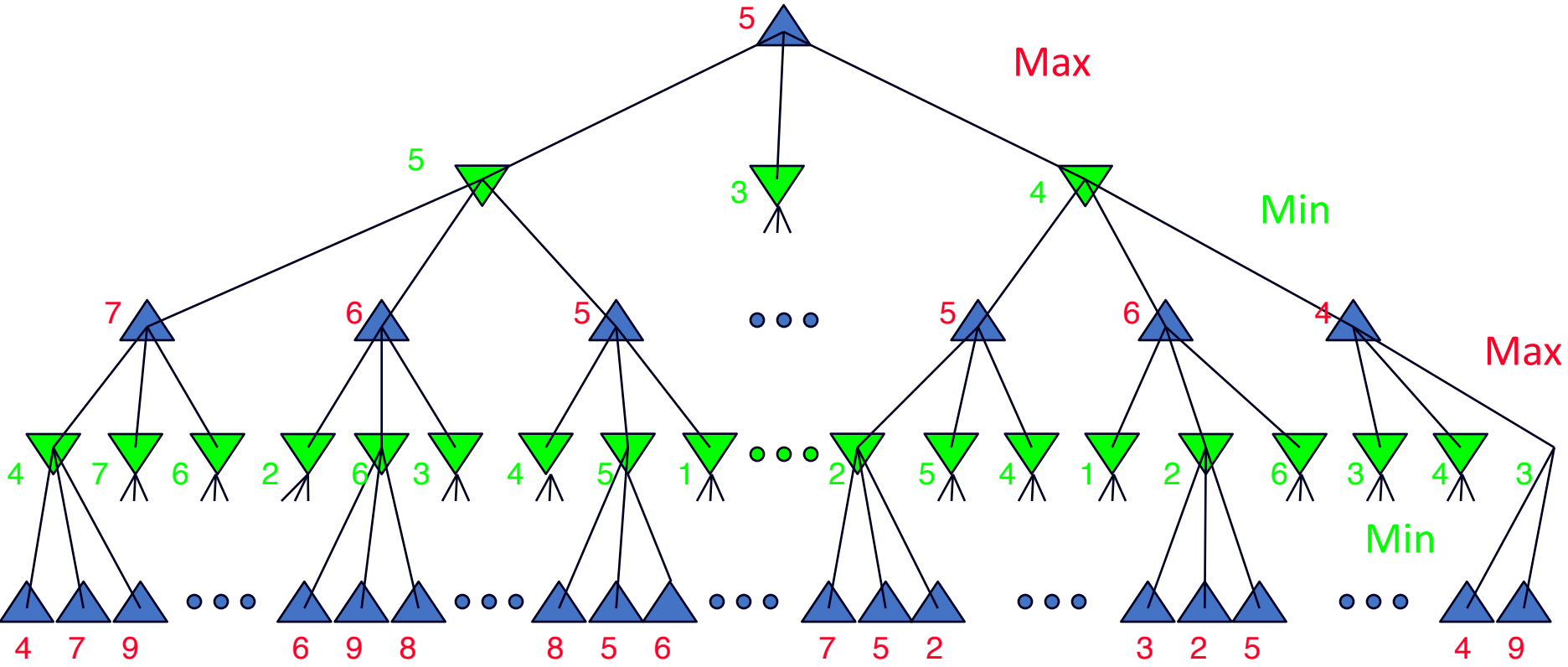
MiniMax Example



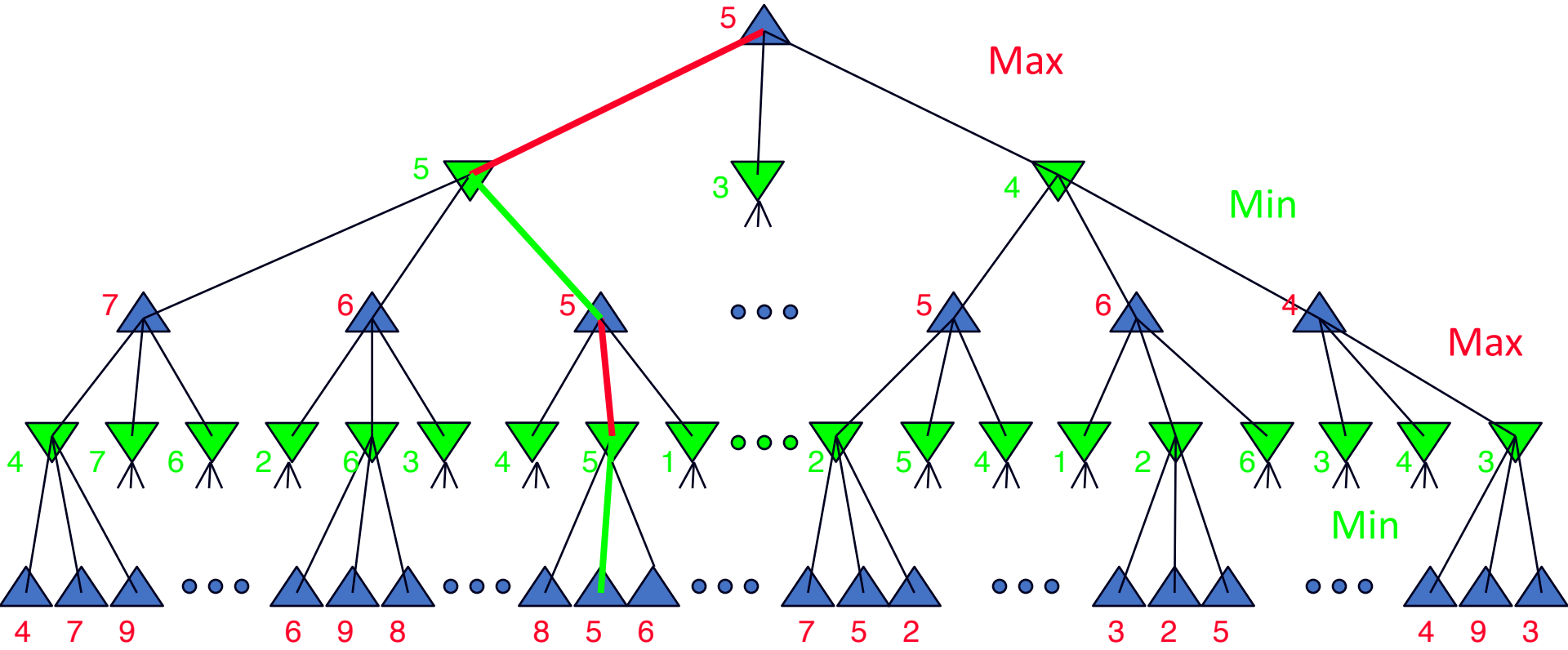
MiniMax Example



MiniMax Example



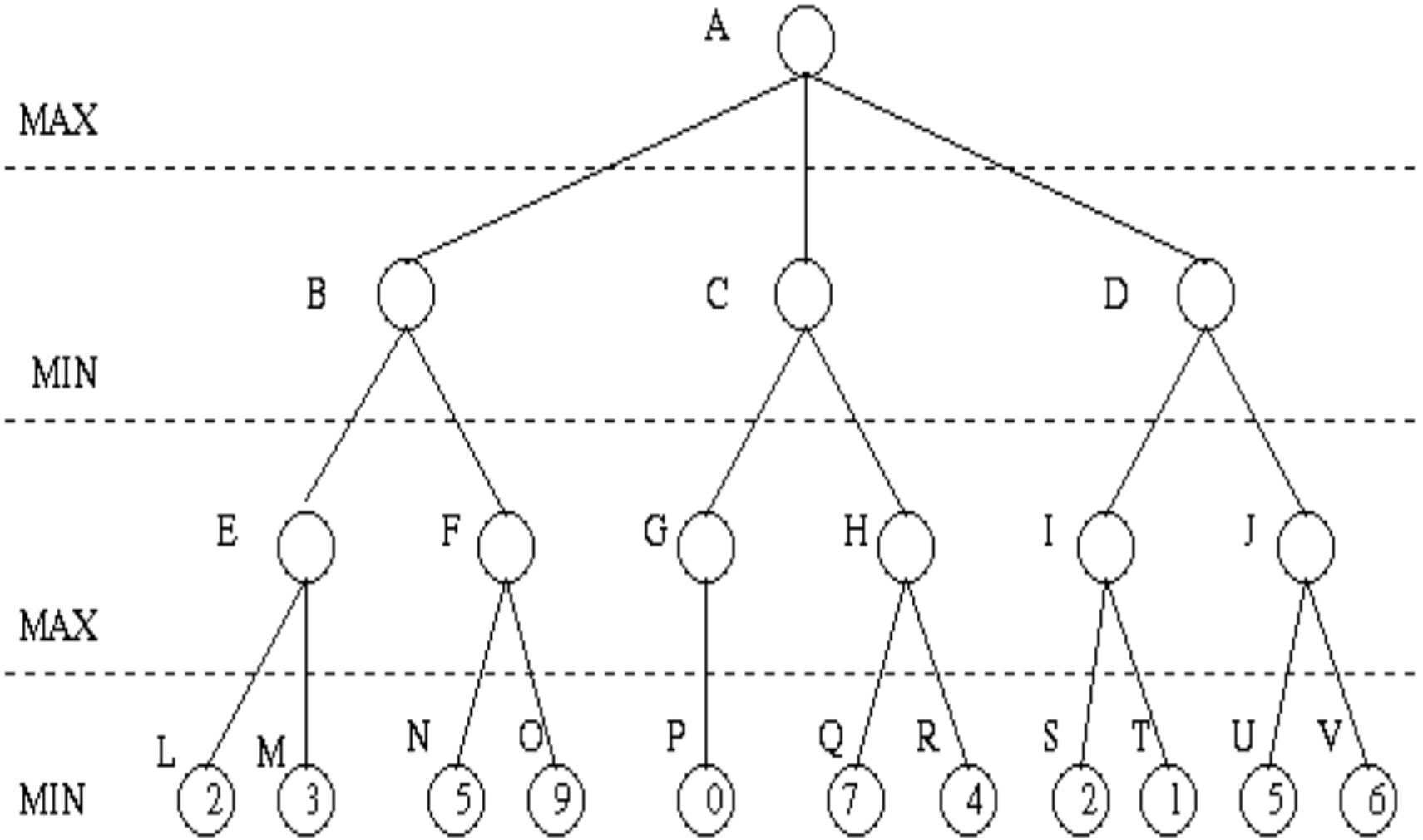
MiniMax Example



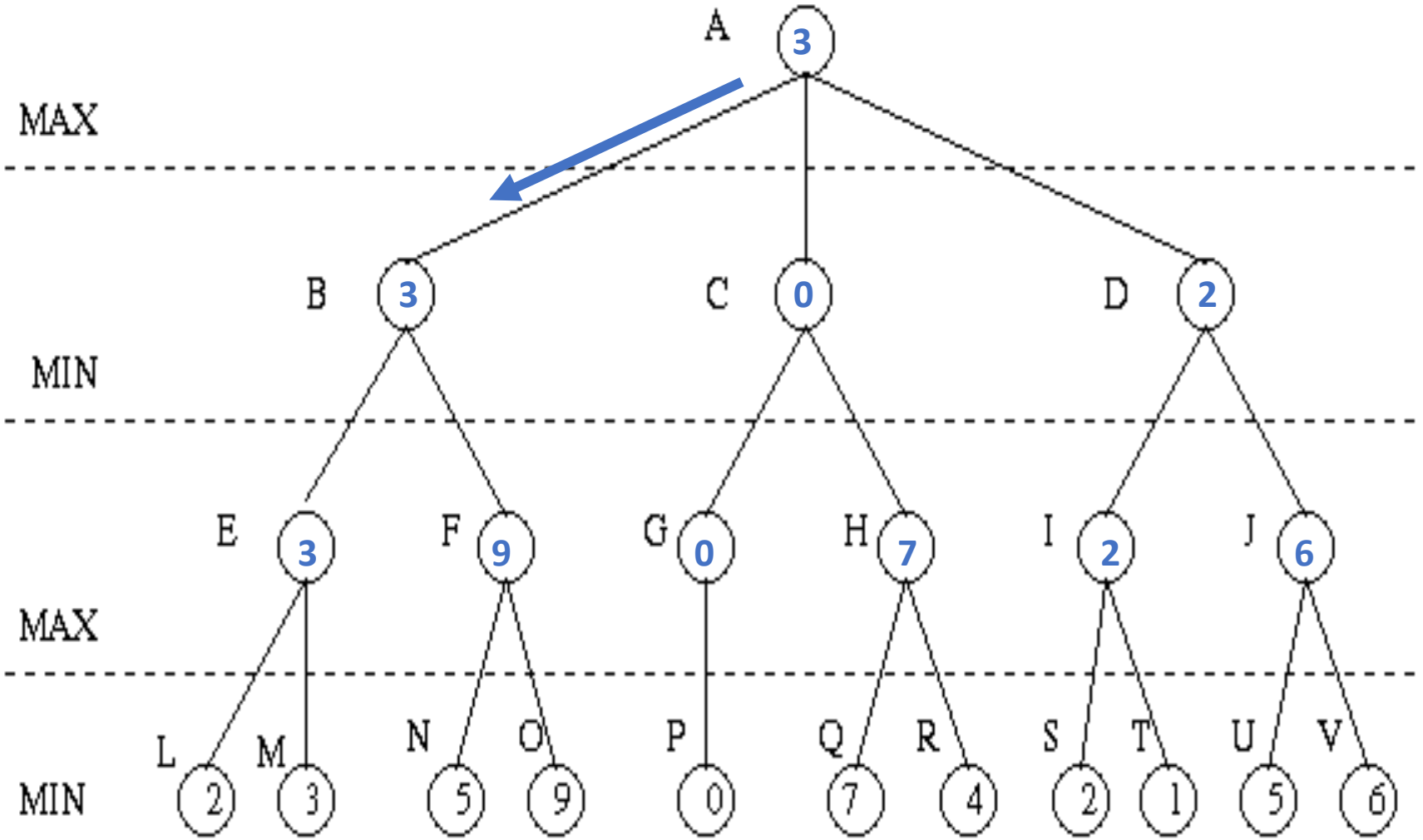
moves by Max and countermoves by Min

Question: can I gain less than 5 if I take a move?

MiniMax Exercise



MiniMax Exercise Solution



Evaluation Function

- Complete search until reaching terminal states is impractical for most games
- Alternative: search the tree only to a certain depth
  - Requires a cutoff-test to determine where to stop (e.g. # of levels)
    - Replaces the terminal test
    - The nodes at that level effectively become terminal leaf nodes
  - Uses a heuristics-based evaluation function to estimate the expected utility of the game from those leaf nodes (a measure of closeness to the goal)
    - Must be consistent with the utility function. (values for terminal nodes, or at least their order, must be the same)
    - Tradeoff between accuracy and time cost
    - Frequently weighted linear functions are used
      - $E = w_1f_1 + w_2f_2 + \ldots + w_nf_n$
      - Combination of features, weighted by their relevance

Example: Tic-Tac-Toe

- simple evaluation function

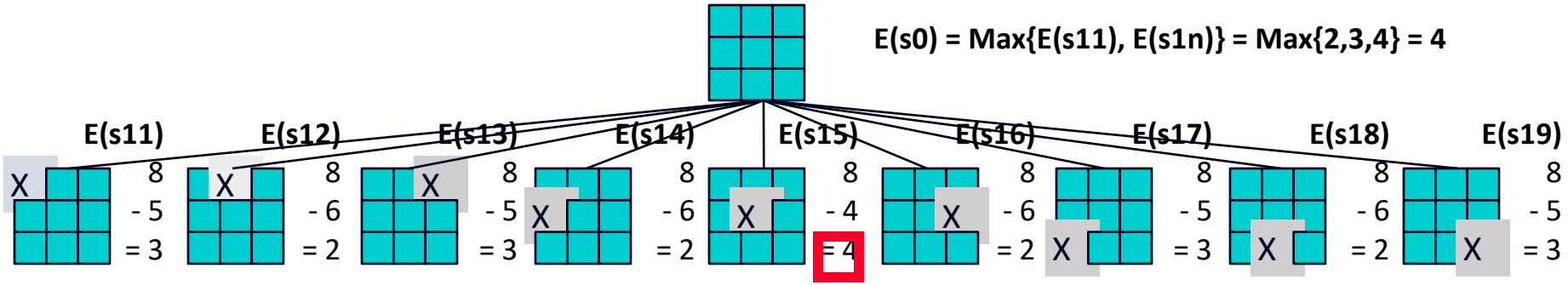
  E(s) = (rx + cx + dx) - (ro + co + do)

  (number of rows, columns, and diagonals open for MAX) - (number of rows, columns, and diagonals open for MIN)

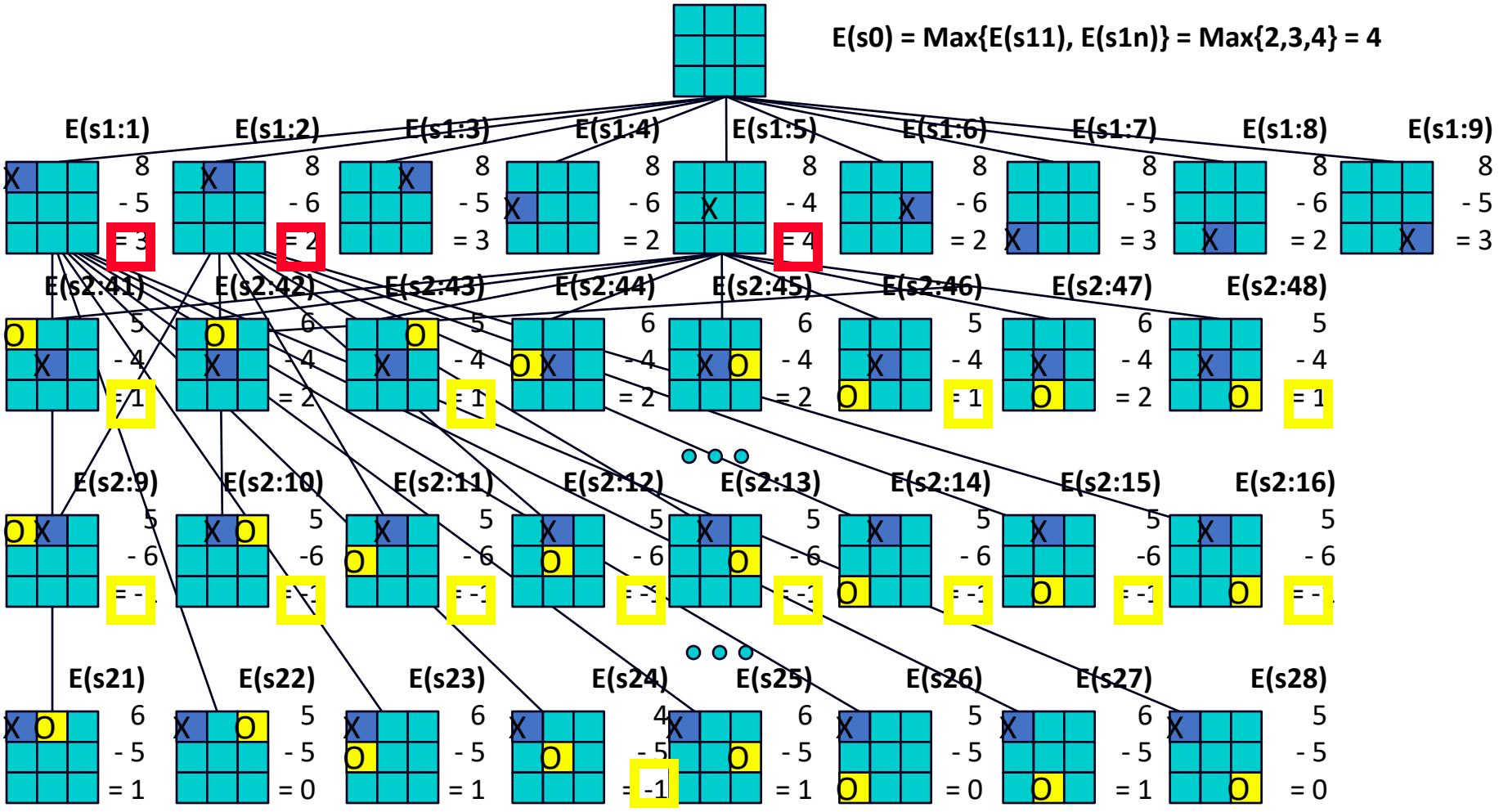
- 1-ply lookahead
  - start at the top of the tree
  - evaluate all 9 choices for player 1
  - pick the maximum E-value

- 2-ply lookahead
  - also looks at the opponents possible move
  - assuming that the opponents picks the minimum E-value

Tic-Tac-Toe 1-Ply



Tic-Tac-Toe 2-Ply



MiniMax Properties

Assume lookahead to depth d

- Space complexity
  - Depth-first search, so O(bd)
- Time complexity
  - Given branching factor b, so $O(b^d)$
- Time complexity is a major problem!
  - Computer typically only has a finite amount of time to make a move.

Pruning

- Discard parts of the search tree
  - Guaranteed not to contain good moves
  - Guarantee that the solution is not in that branch or sub-tree (if both players make optimal decisions, they will never end up in that part of the tree)
- Use pruning to ignore those branches
- Certain moves are not considered
  - Won't result in a better evaluation value than a move further up in the tree
  - They would lead to a less desirable outcome
- Applies to moves by both players
  - α (alpha) indicates the best choice for MAX so far, never decreases (initialize to -infinity)
  - β (beta) indicates the best choice for MIN so far, never increases (initialize to +infinity

Alpha-Beta Pruning

- Beta cutoff pruning occurs when maximizing

  if *child's alpha >= parent's beta*

  Stop expanding children. Why?

  - Opponent won't allow computer to take this move

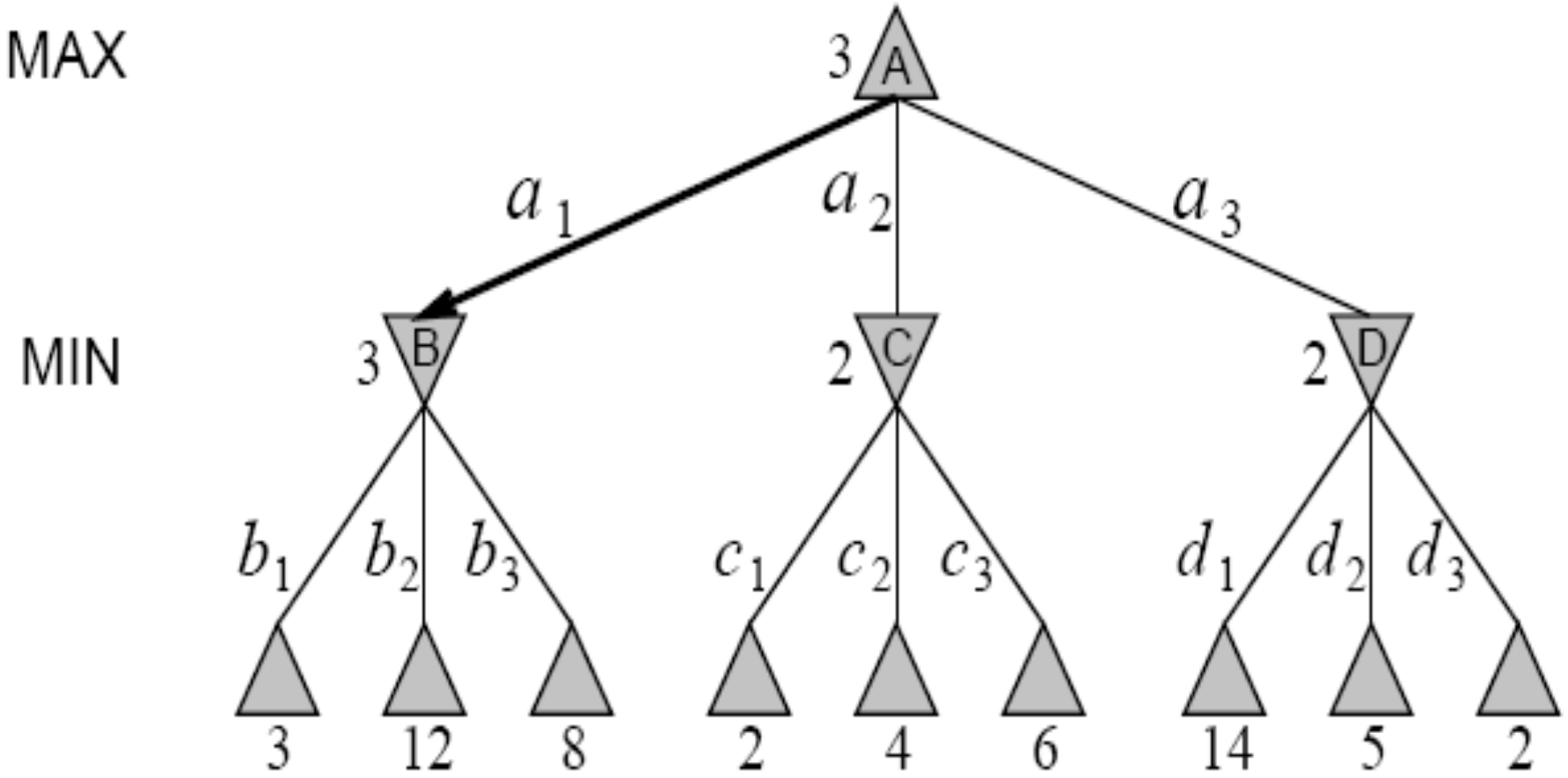
- Alpha cutoff pruning occurs when minimizing

  if *parent's alpha >= child's beta*

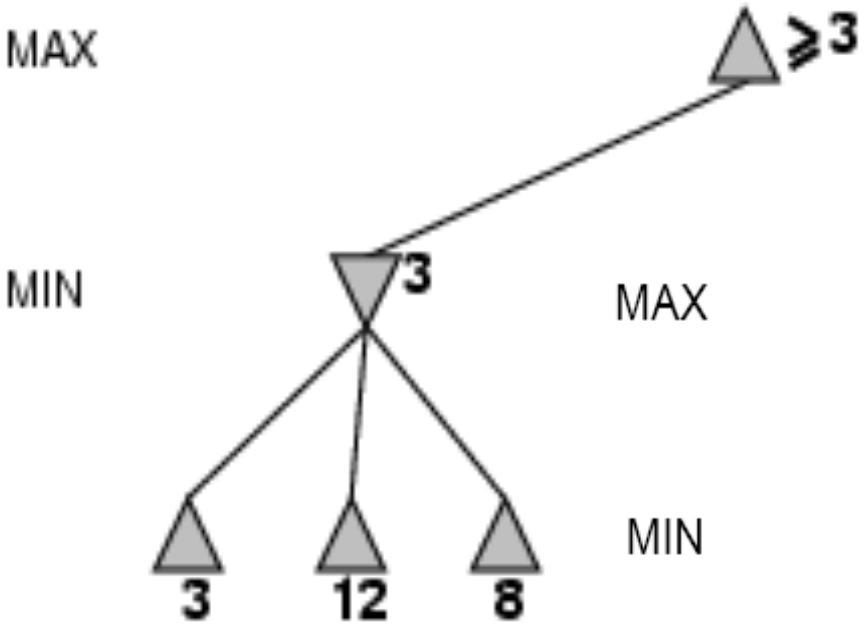
  Stop expanding children. Why?

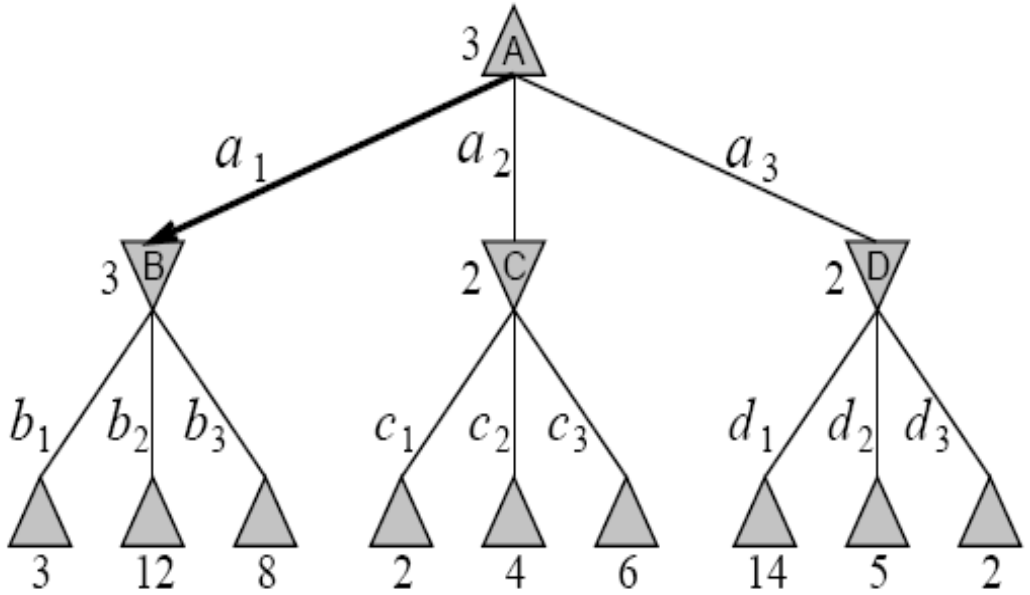
  - Computer has a better move than this

α-β Pruning Example

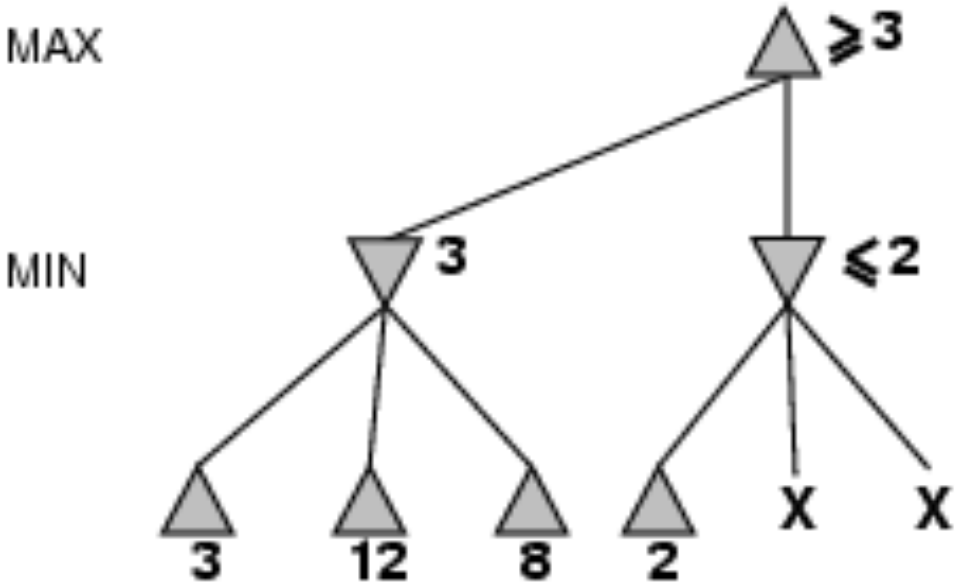


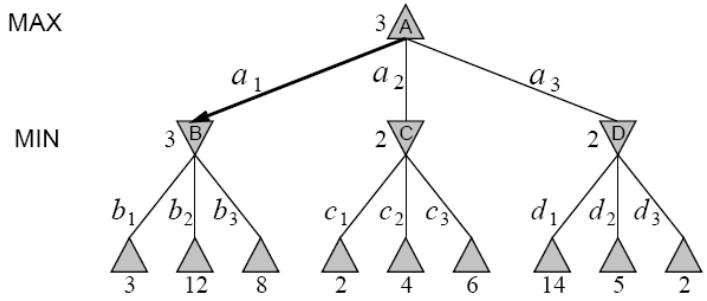
α-β Pruning Example



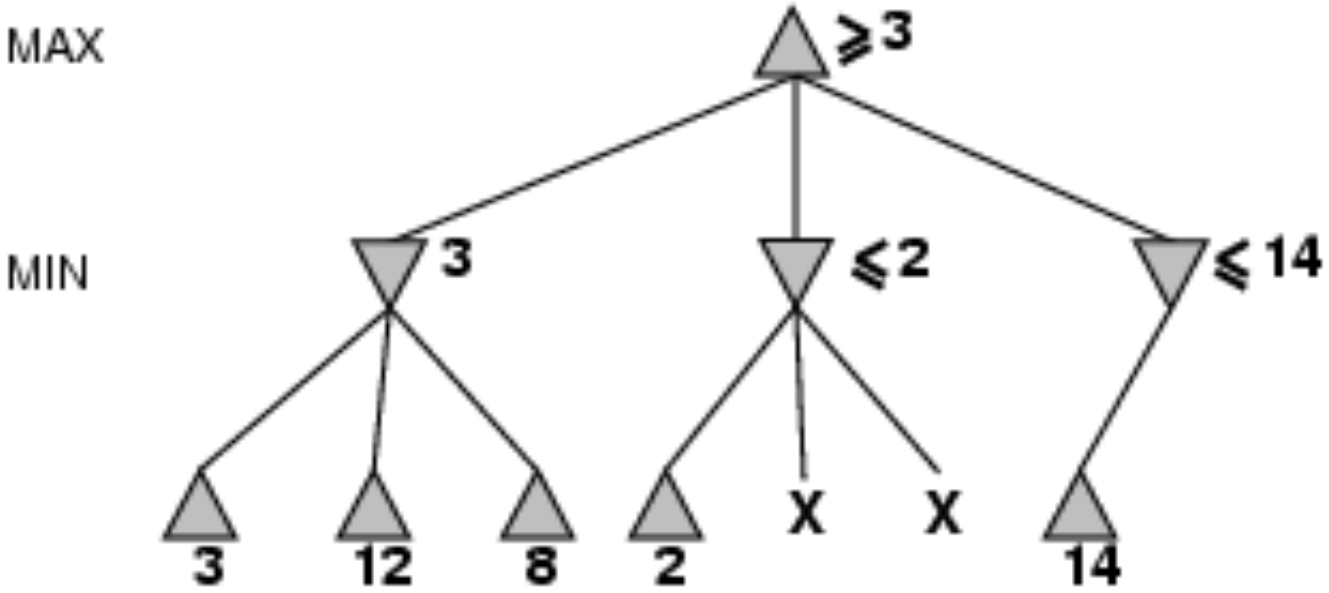


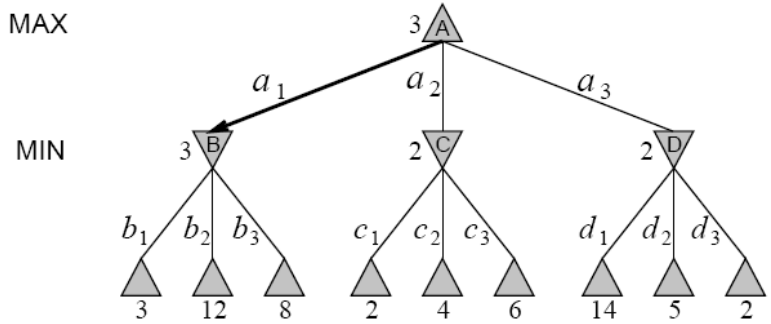
α-β Pruning Example



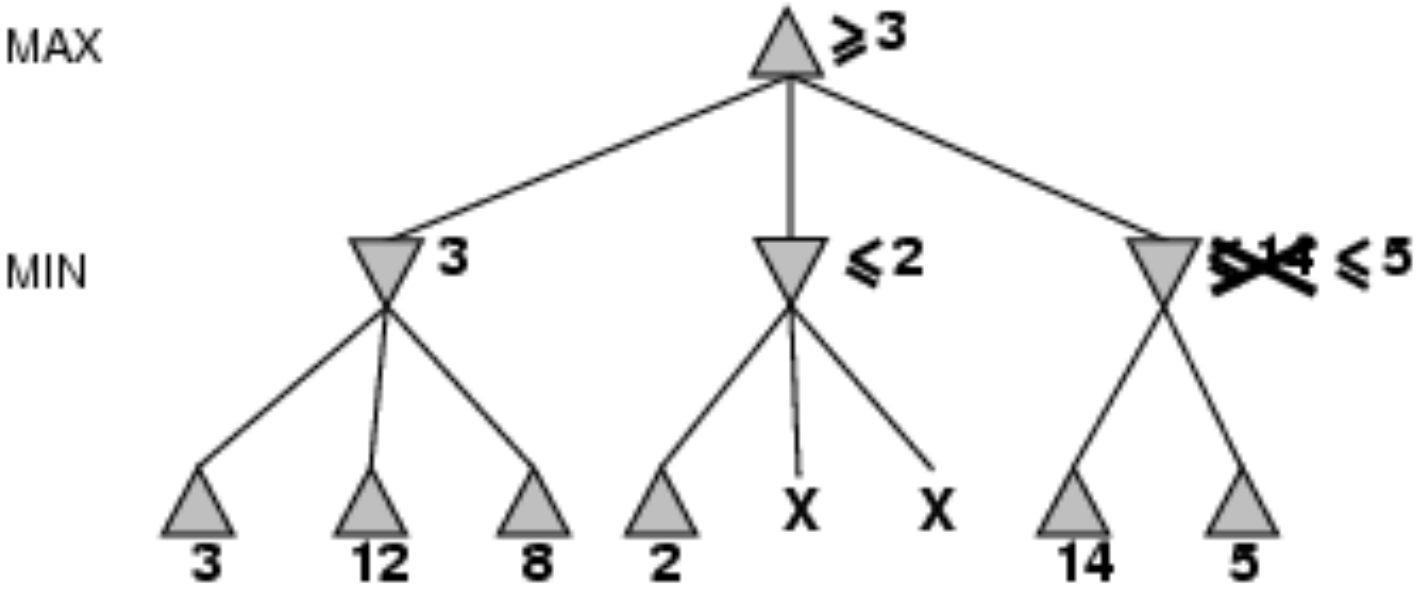


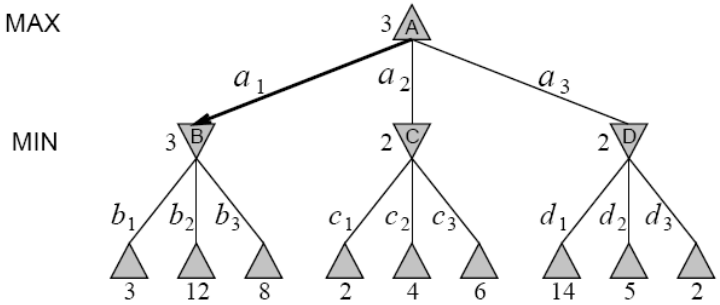
α-β Pruning Example



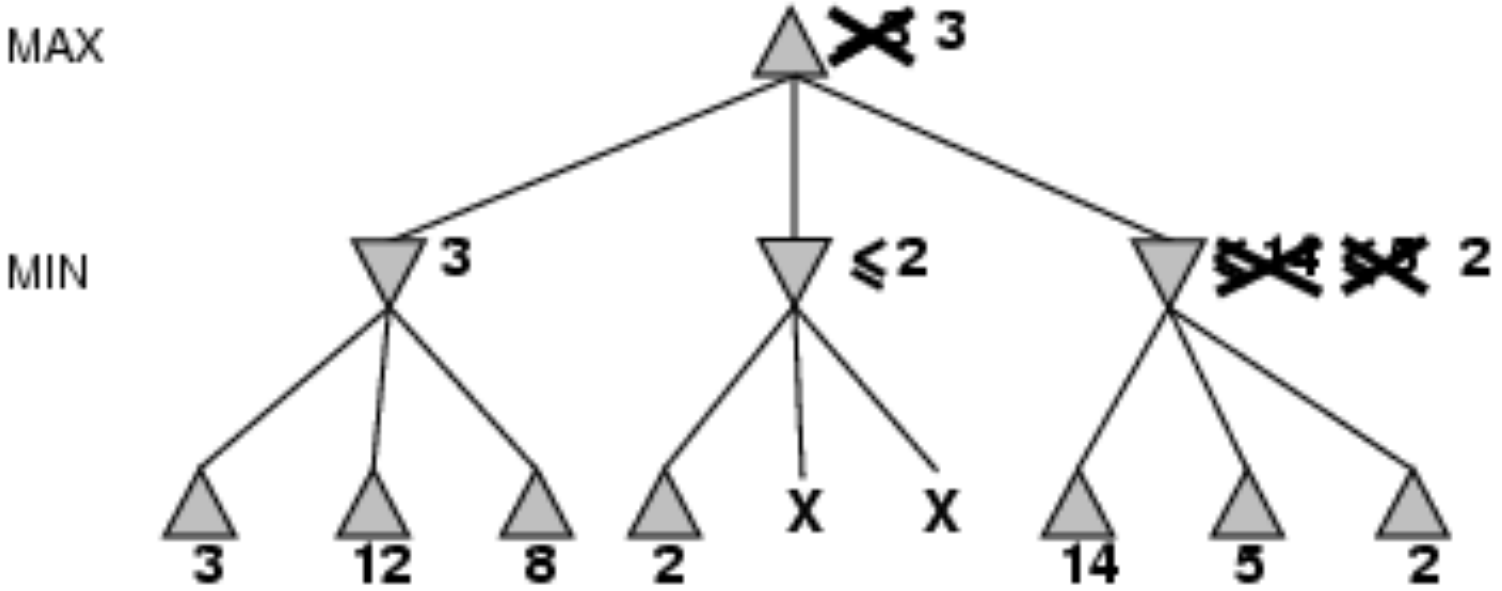


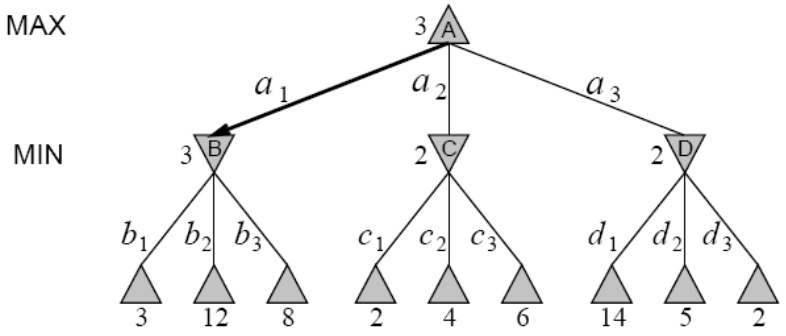
α-β Pruning Example





α-β Pruning Example





Alpha-Beta Pruning

Rules of Thumb

- α is the best (highest) found so far along the path for Max
- β is the best (lowest) found so far along the path for Min
- Search below a MIN node may be **alpha-pruned** if its $\beta \leq \alpha$ of some MAX ancestor
- Search below a MAX node may be **beta-pruned** if its $\alpha \geq \beta$ of some MIN ancestor

α -β Search Algorithm

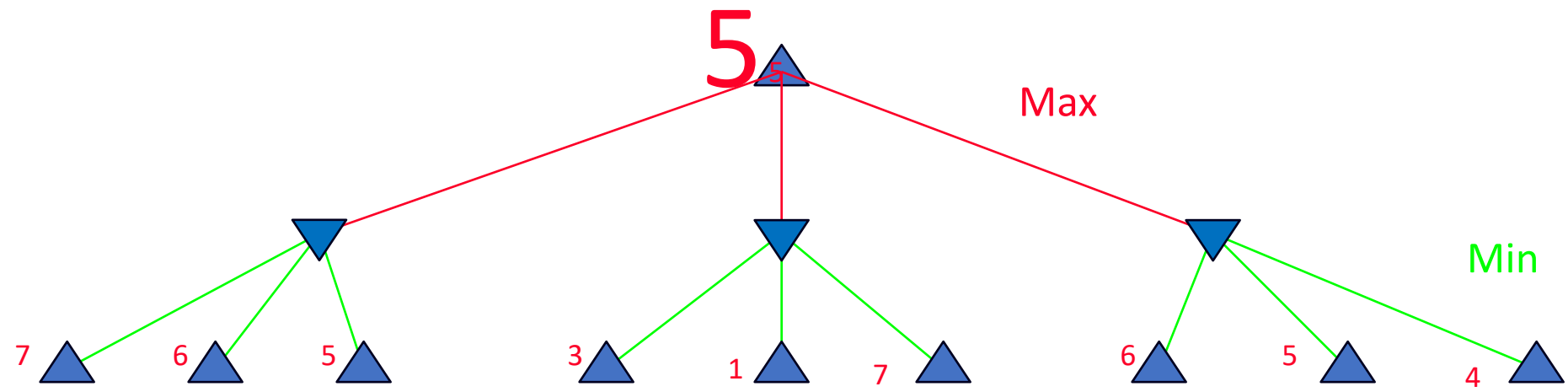
1. If terminal state, compute e(n) and return the result.
2. Otherwise, if the level is a **minimizing** level,
   - Until no more children or $\beta \le \alpha$
     - $\upsilon_i \leftarrow \alpha - \beta$ search on a child
     - If $\upsilon_i < \beta, \beta \leftarrow \upsilon_i$.
   - Return $min(\upsilon_i)$

pruning

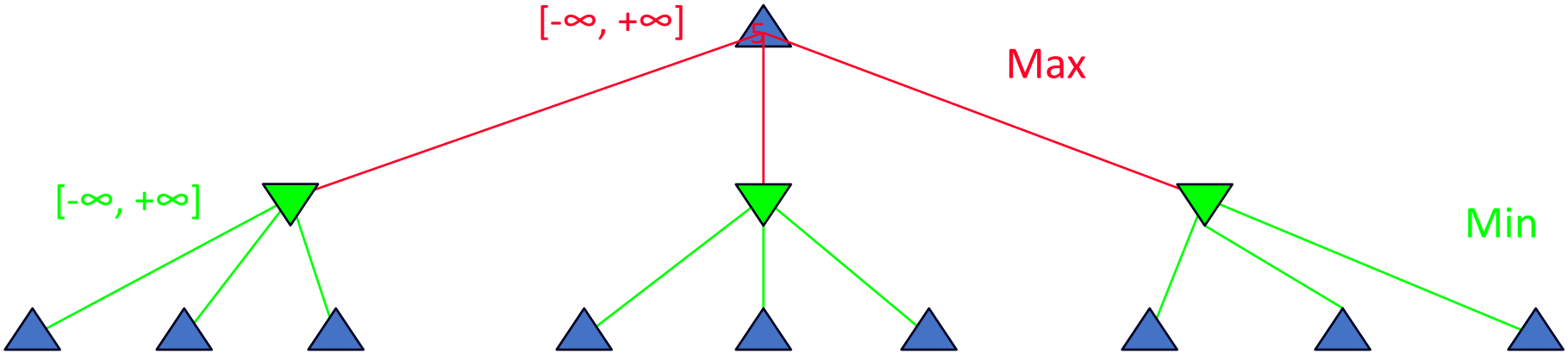
3. Otherwise, the level is a **maximizing** level:
   - Until no more children or $\alpha \ge \beta$,
     - $\upsilon_i \leftarrow \alpha - \beta$ search on a child.
     - If $\upsilon_i > \alpha$, set $\alpha \leftarrow \upsilon_i$
   - Return $max(\upsilon_i)$

pruning

Consider this Example



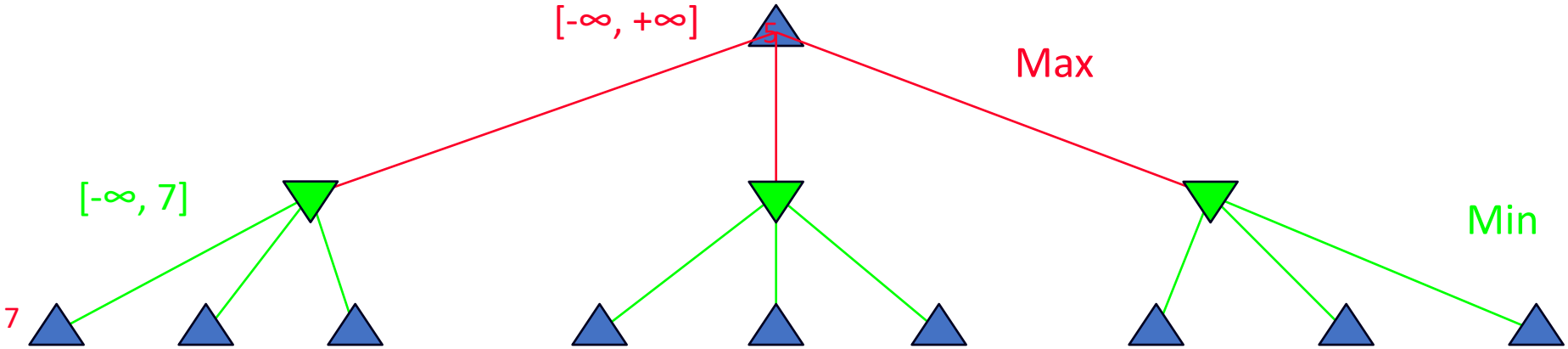
Alpha-Beta Example



α best choice for Max ?
β best choice for Min ?

- we assume a depth-first, left-to-right search as basic strategy
- the range of the possible values for each node are indicated
  - initially [-∞, +∞]
  - from Max's or Min's perspective
  - these *local* values reflect the values of the sub-trees in that node;
    the *global* values α and β are the best overall choices so far for Max or Min

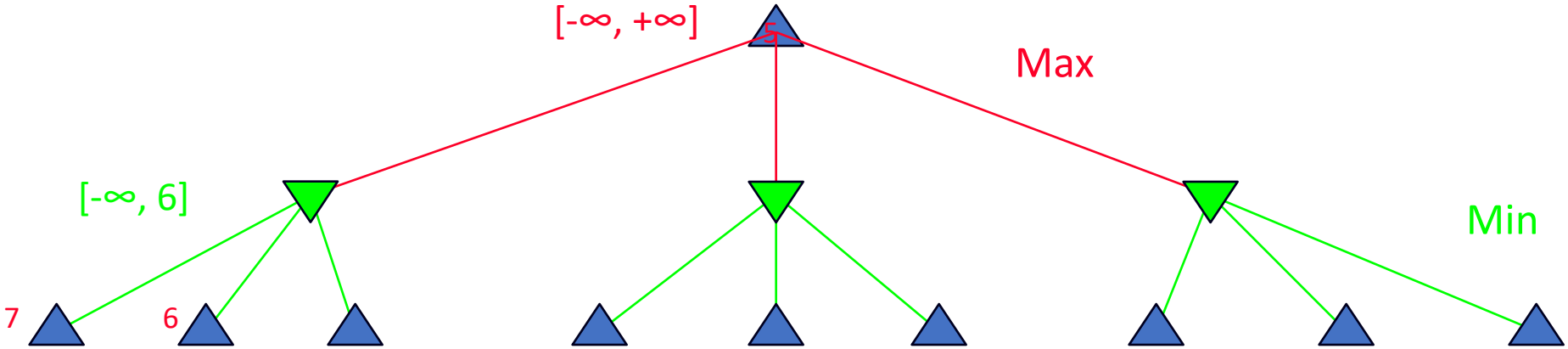
Alpha-Beta Example



α best choice for Max ?
β best choice for Min 7

- Min obtains the first value from a successor node

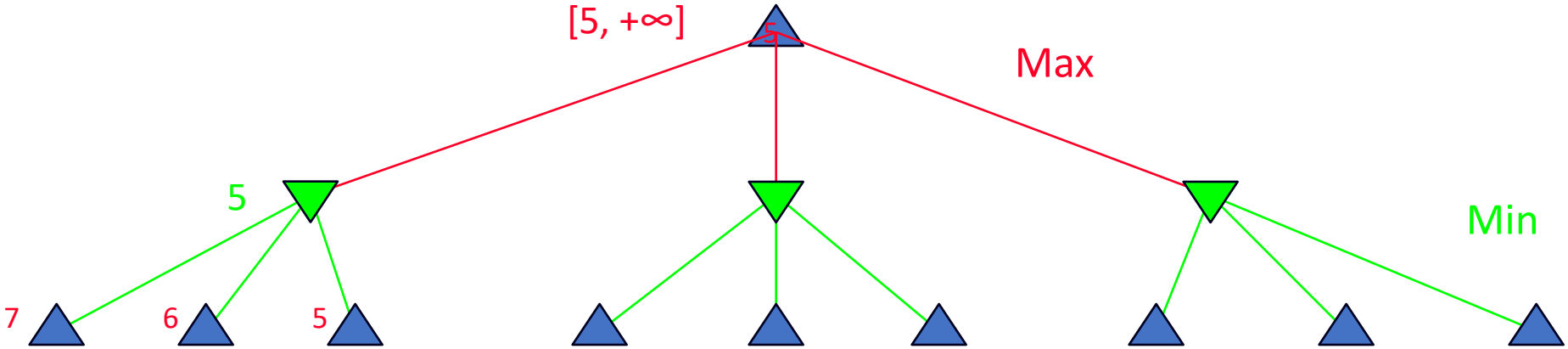
Alpha-Beta Example



α best choice for Max ?
β best choice for Min 6

- Min obtains the second value from a successor node

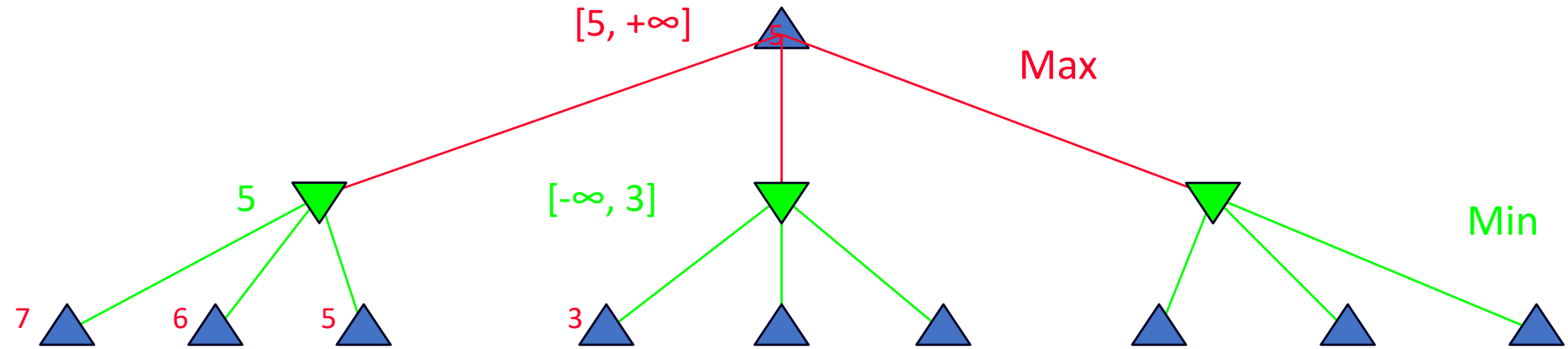
Alpha-Beta Example



α best choice for Max 5
β best choice for Min 5

- Min obtains the third value from a successor node
- this is the last value from this sub-tree, and the exact value is known
- Max now has a value for its first successor node, but hopes that something better might still come

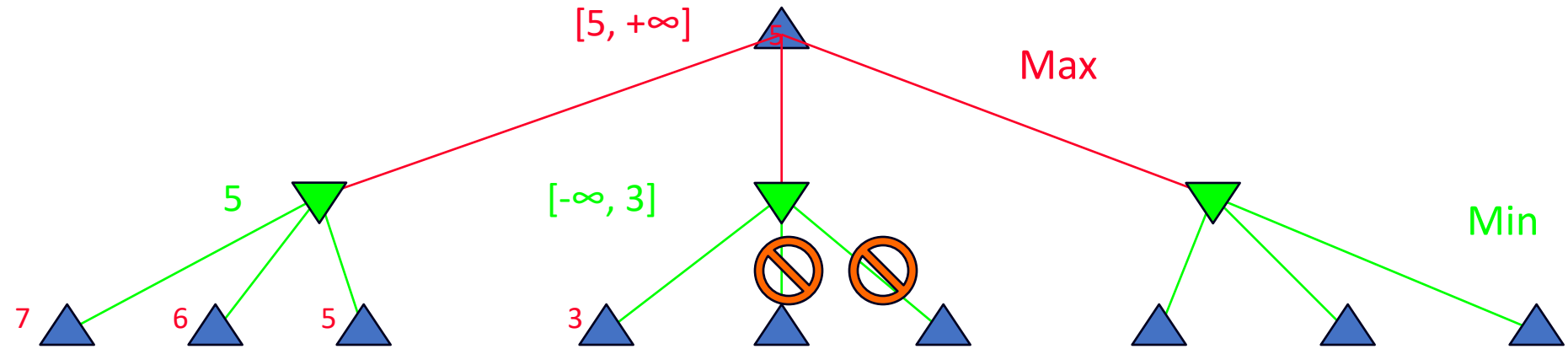
Alpha-Beta Example



α best choice for Max 5
β best choice for Min 3

- Min continues with the next sub-tree, and gets a better value
- Max has a better choice from its perspective, however, and will not consider a move in the sub-tree currently explored by Min
  - initially [-∞, +∞]

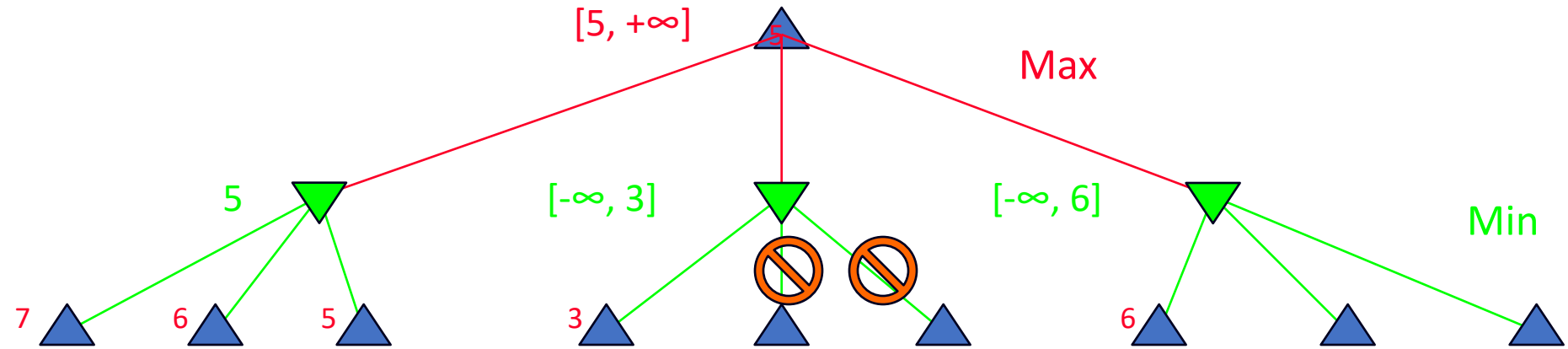
Alpha-Beta Example



α best choice for Max 5
β best choice for Min 3

- Min knows that Max won't consider a move to this sub-tree, and abandons it
- this is a case of *pruning*, indicated by 

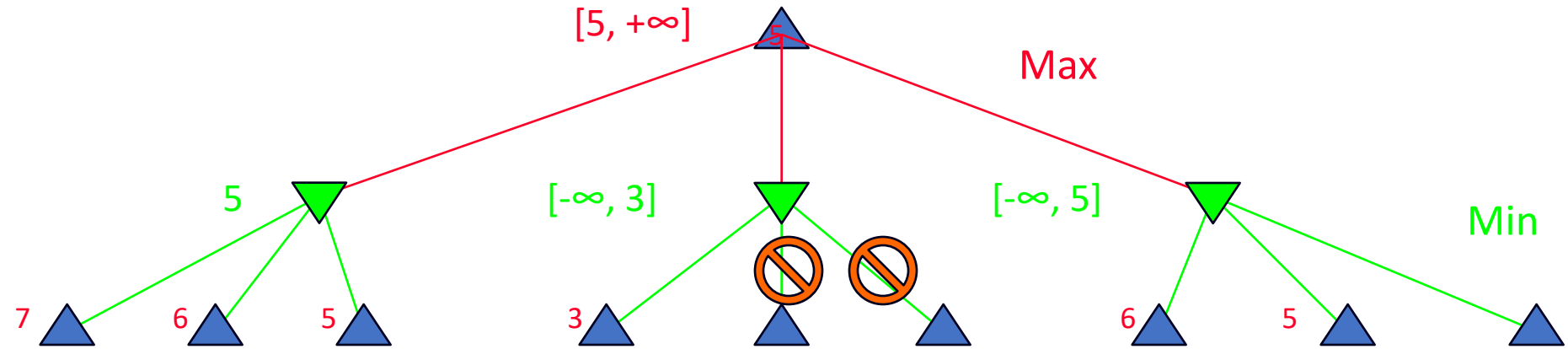
Alpha-Beta Example



α best choice for Max 5
β best choice for Min 3

- Min explores the next sub-tree, and finds a value that is worse than the other nodes at this level
- if Min is not able to find something lower, then Max will choose this branch, so Min must explore more successor nodes

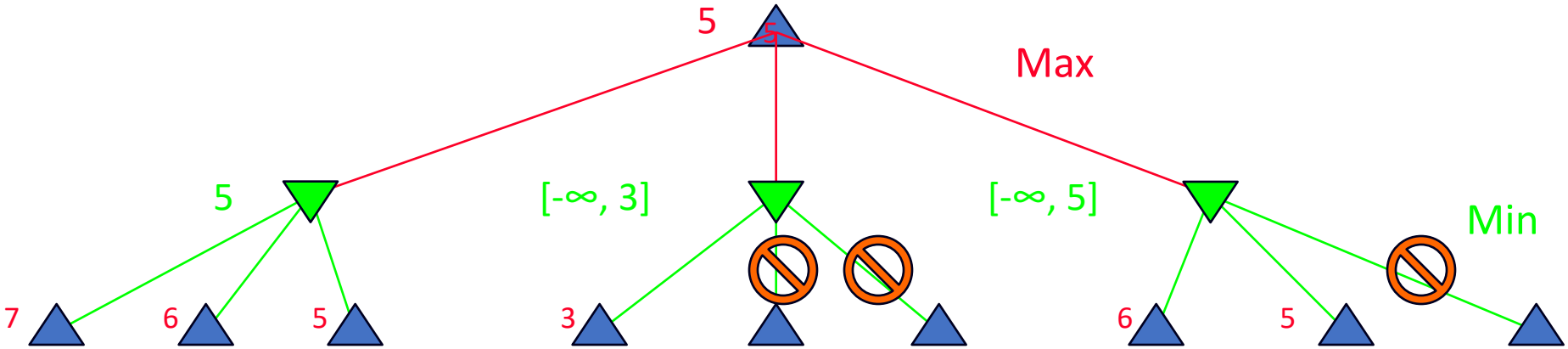
Alpha-Beta Example



α best choice for Max 5
β best choice for Min 3

- Min is lucky, and finds a value that is the same as the current worst value at this level
- Max can choose this branch, or the other branch with the same value

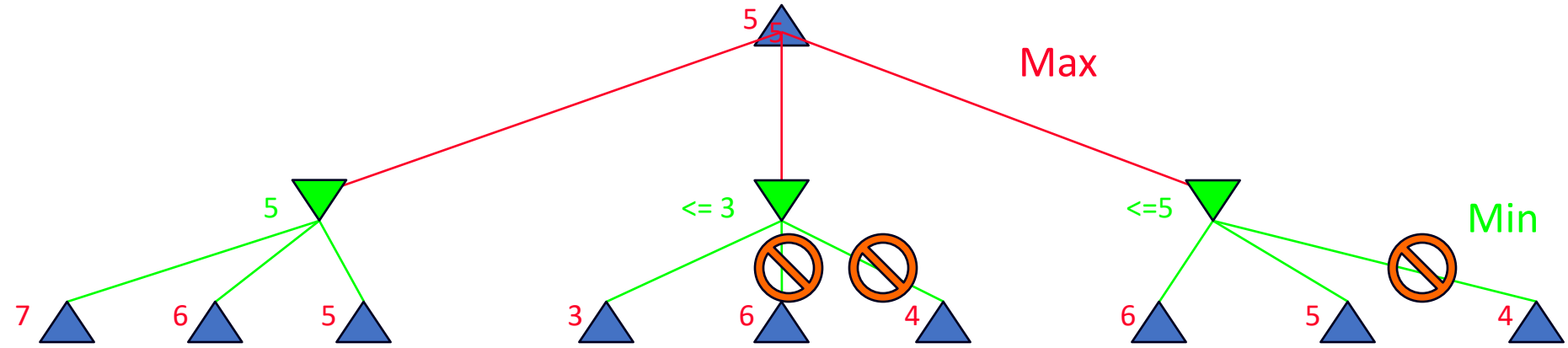
Alpha-Beta Example



α best choice for Max 5
β best choice for Min 3

- Min could continue searching this sub-tree to see if there is a value that is less than the current worst alternative in order to give Max as few choices as possible
  - this depends on the specific implementation
- Max knows the best value for its sub-tree

Alpha-Beta Example Overview



α best choice for Max 5

β best choice for Min 7 -> 6 -> 5 -> 3

- some branches can be pruned because they would never be considered
  - after looking at one branch, Max already knows that they will not be of interest since Min would choose a value that is less than what Max already has at its disposal

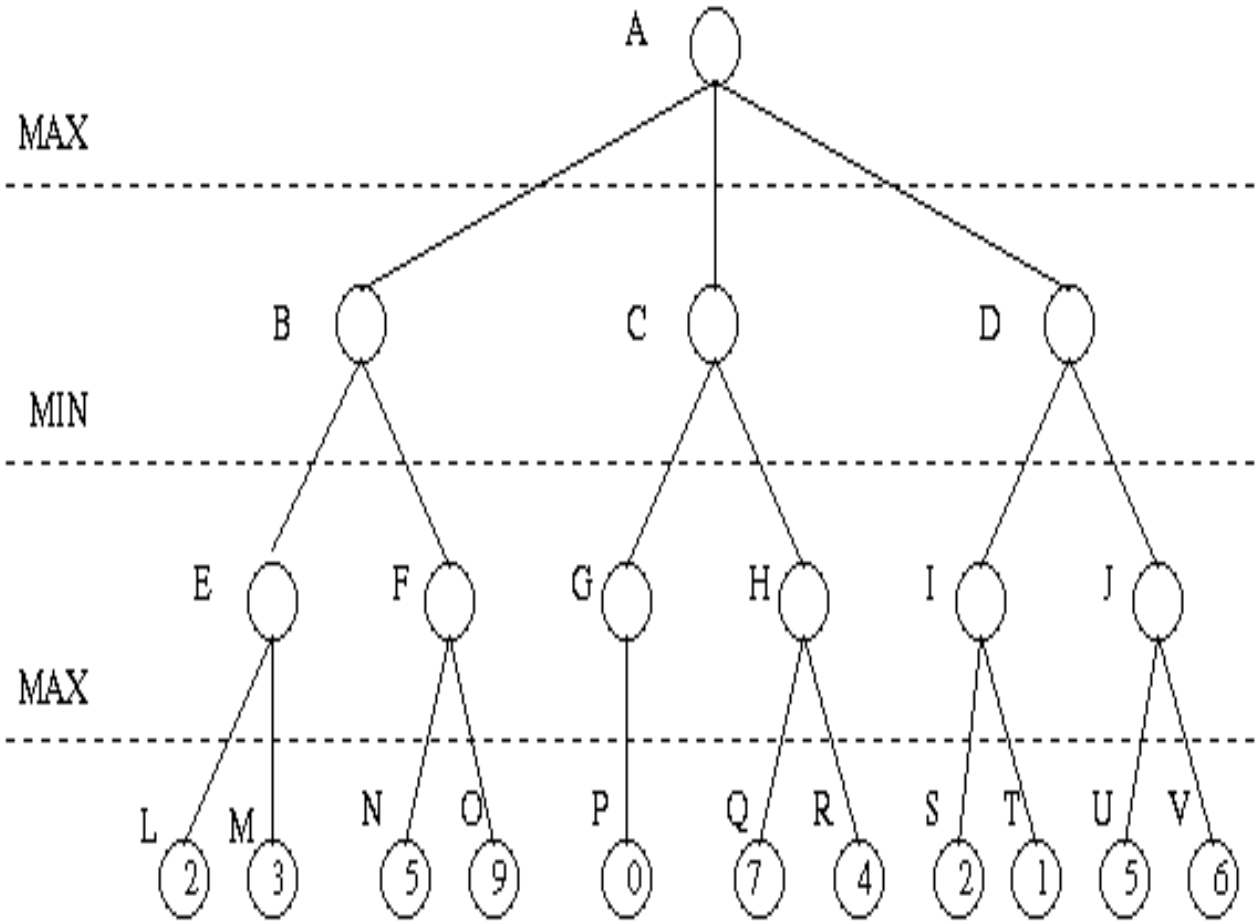
Properties of Alpha-Beta Pruning

- in the ideal case, the best successor node is examined first
  - results in $O(b^{d/2})$ nodes to be searched instead of $O(b^d)$
  - alpha-beta can look ahead twice as far as minimax
  - in practice, simple ordering functions are quite useful
- assumes an idealized tree model
  - uniform branching factor, path length
  - random distribution of leaf evaluation values
- transpositions tables can be used to store permutations
  - sequences of moves that lead to the same position
- requires additional information for good players
  - game-specific background knowledge
  - empirical data

Alpha-Beta Pruning Example

1. Search below a MIN node may be alpha-pruned if the beta value is <= to the alpha value of some MAX ancestor.

- 2. Search below a MAX node may be beta-pruned if the alpha value is >= to the beta value of some MIN ancestor.



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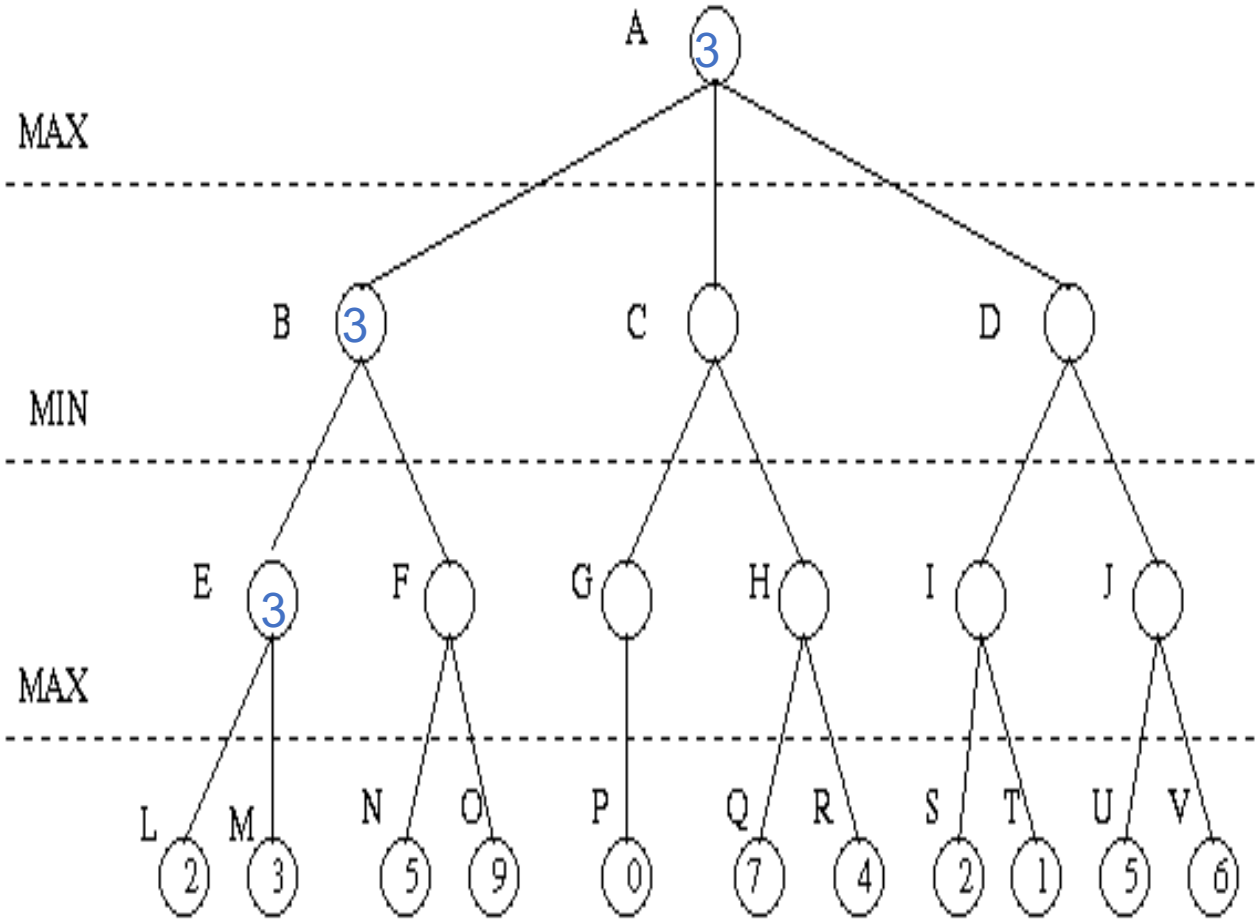

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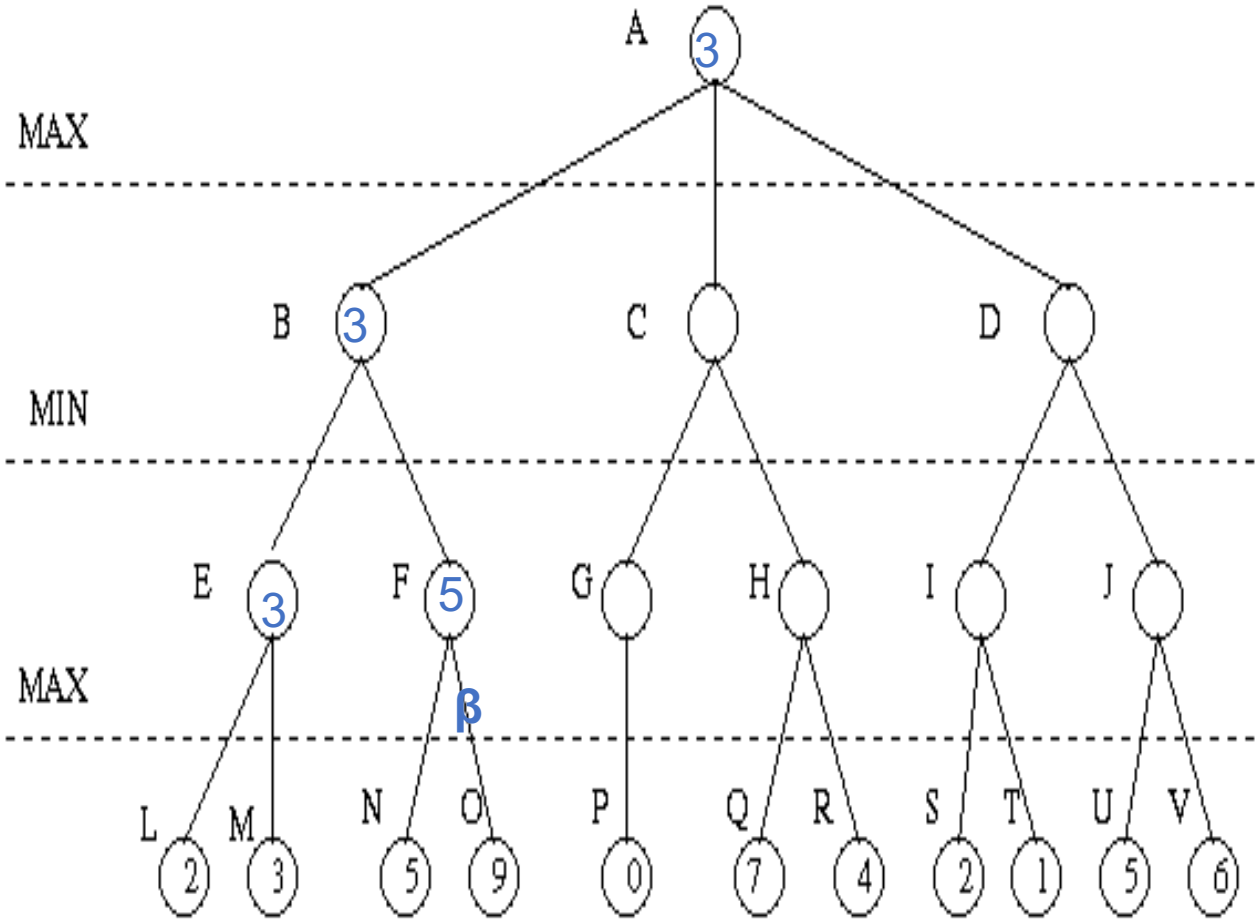

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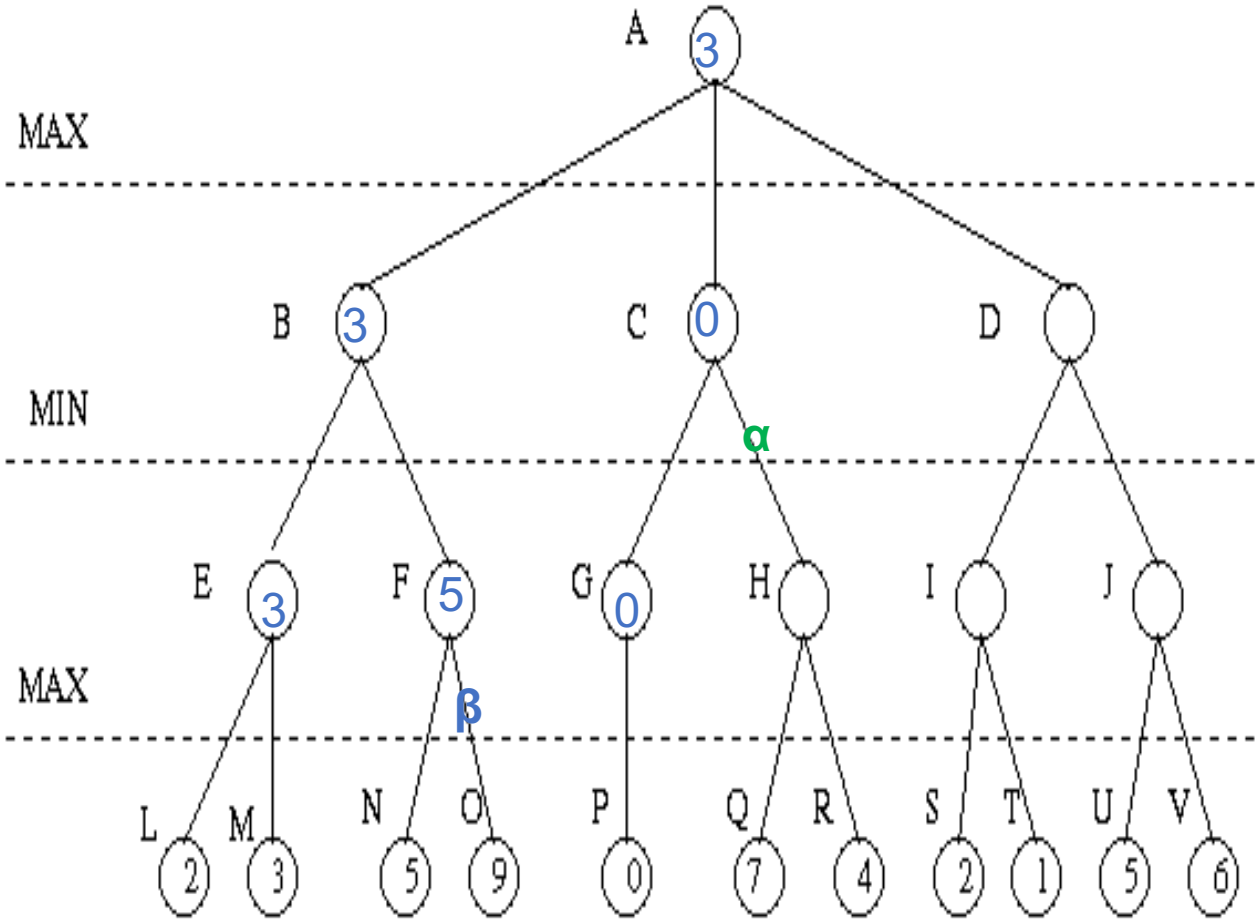

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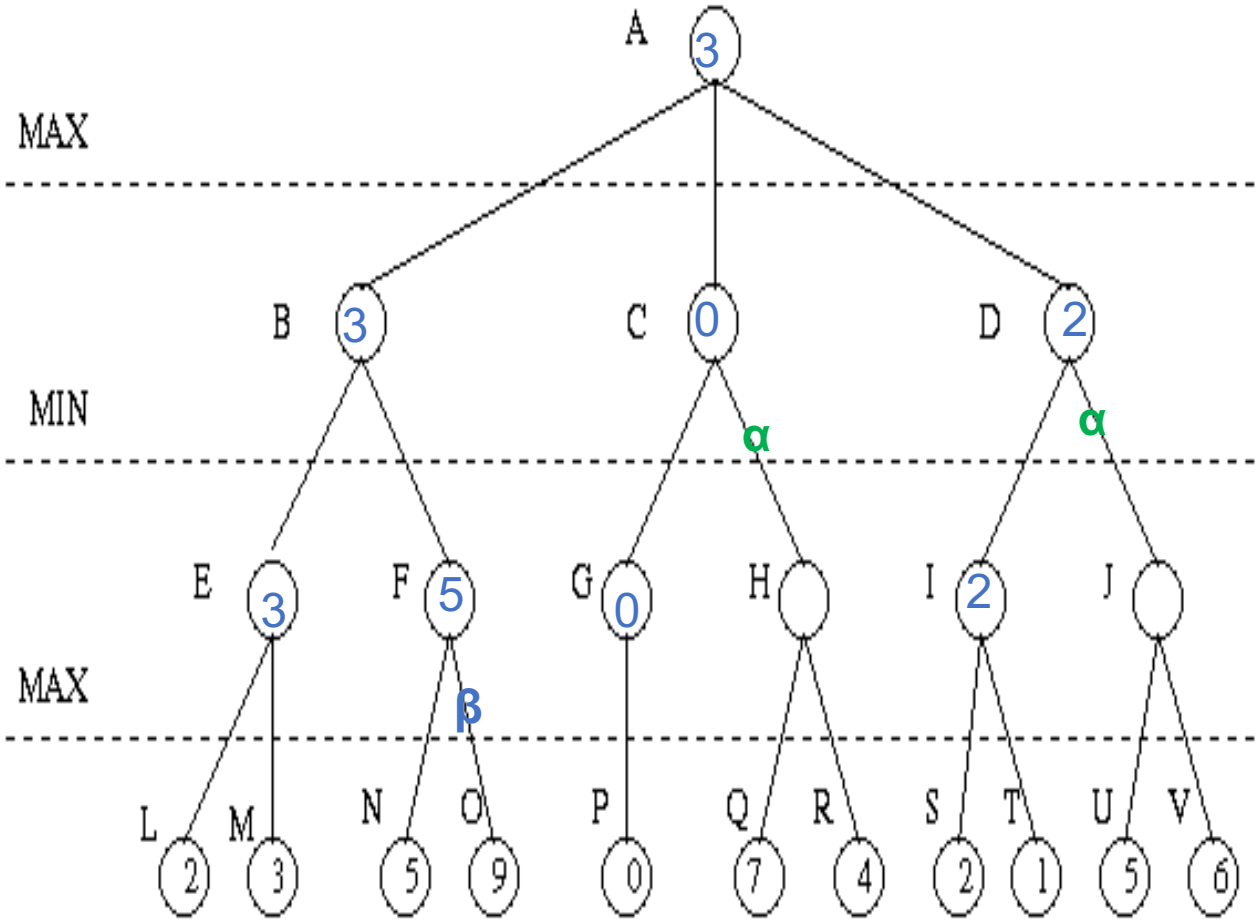


Checkers Case Study

- how to play: https://www.youtube.com/watch?v=yFrAN-LFZRU
- initial board configuration
  - Black — single on 20, single on 21, king on 31
  - Red — single on 23, king on 22
  - evaluation function

    $E(s) = (5\,x_1 + x_2) - (5r_1 + r_2)$

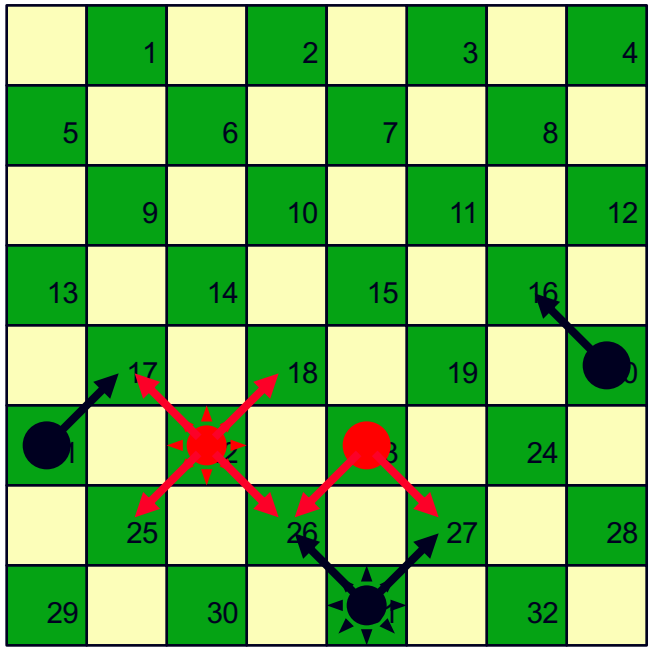
    where

    x_1 = black king advantage,

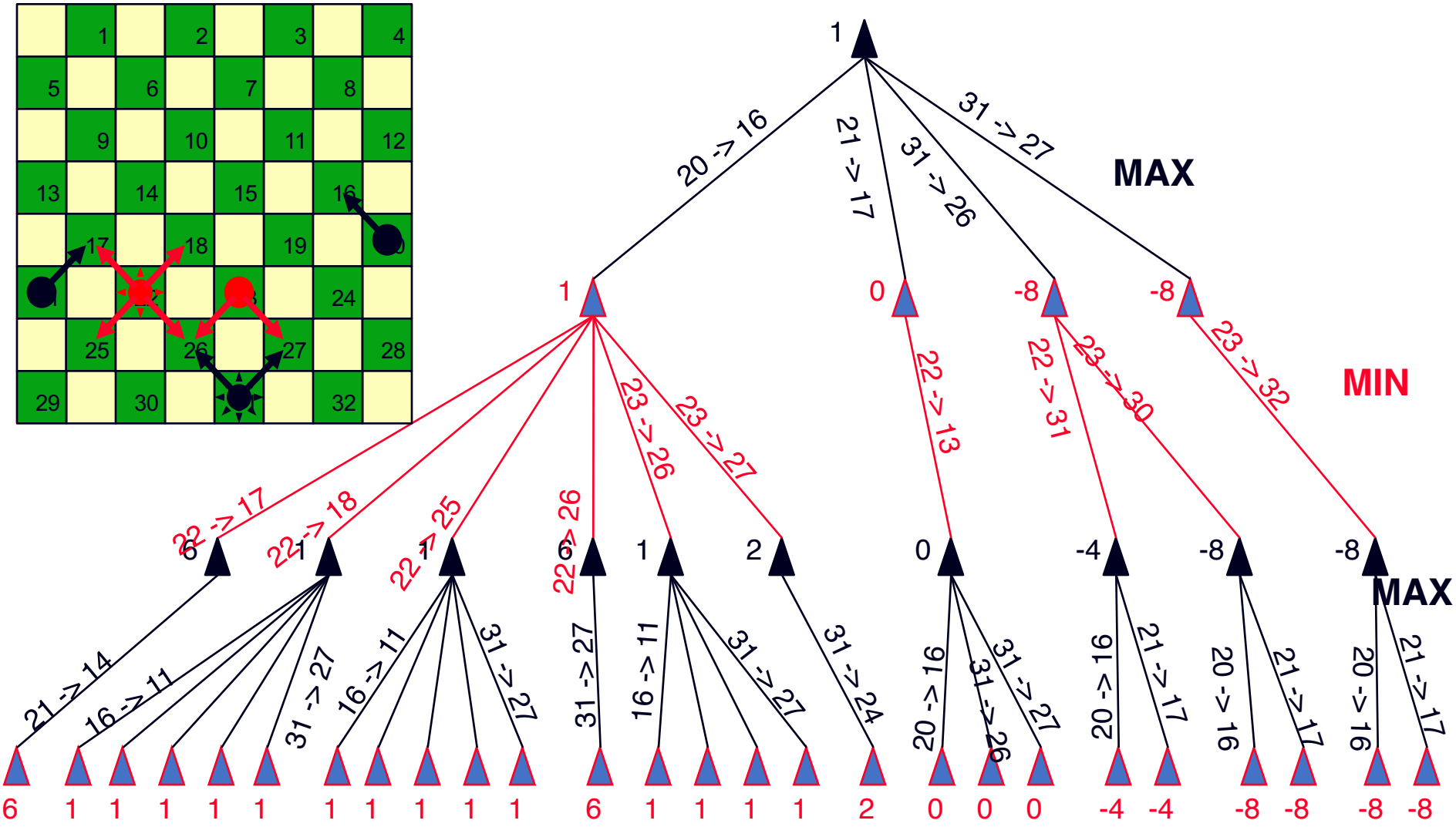
    x_2 = black single advantage,

    r_1 = red king advantage,

    r_2 = red single advantage



Checkers MiniMax Example



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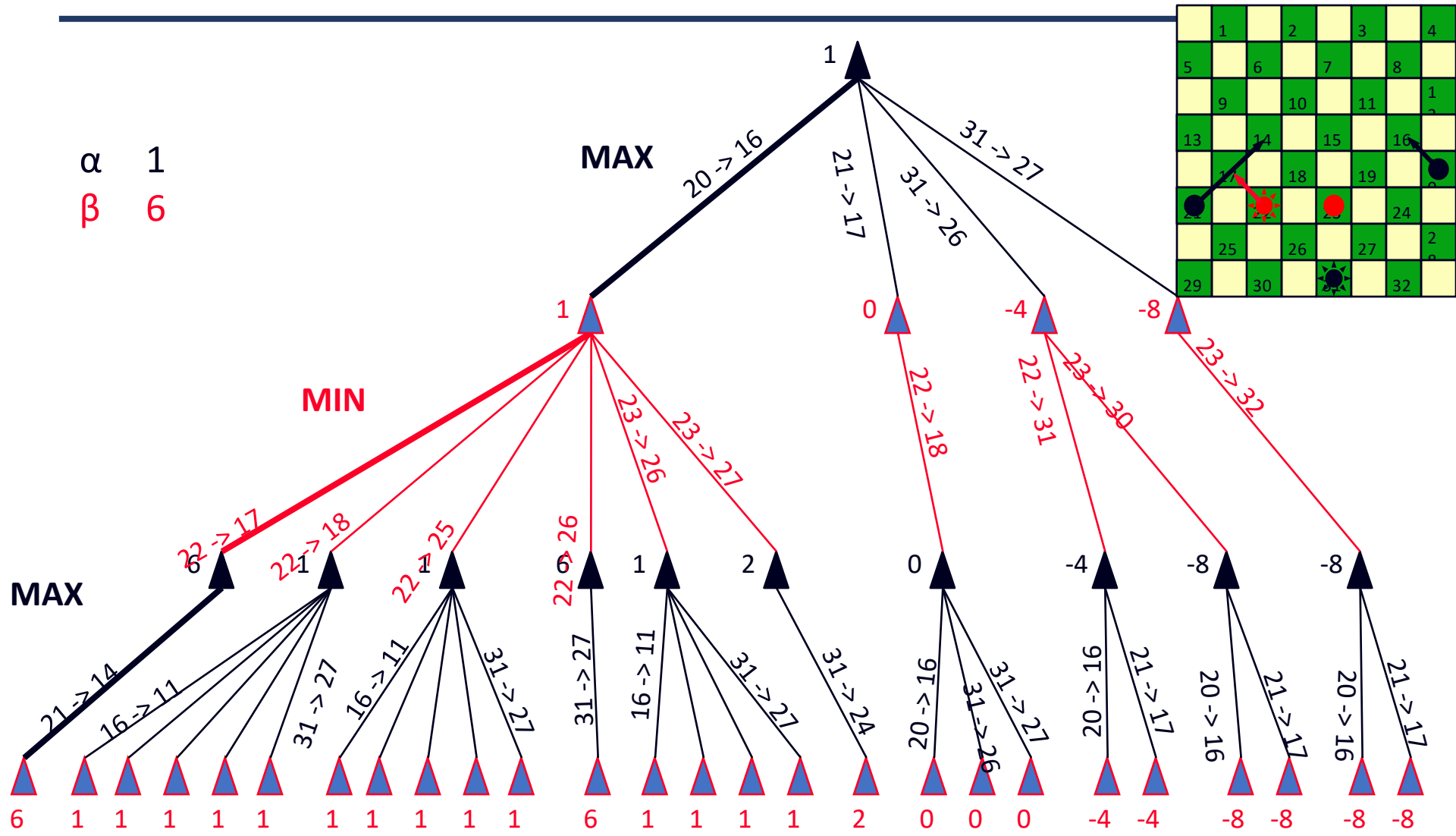

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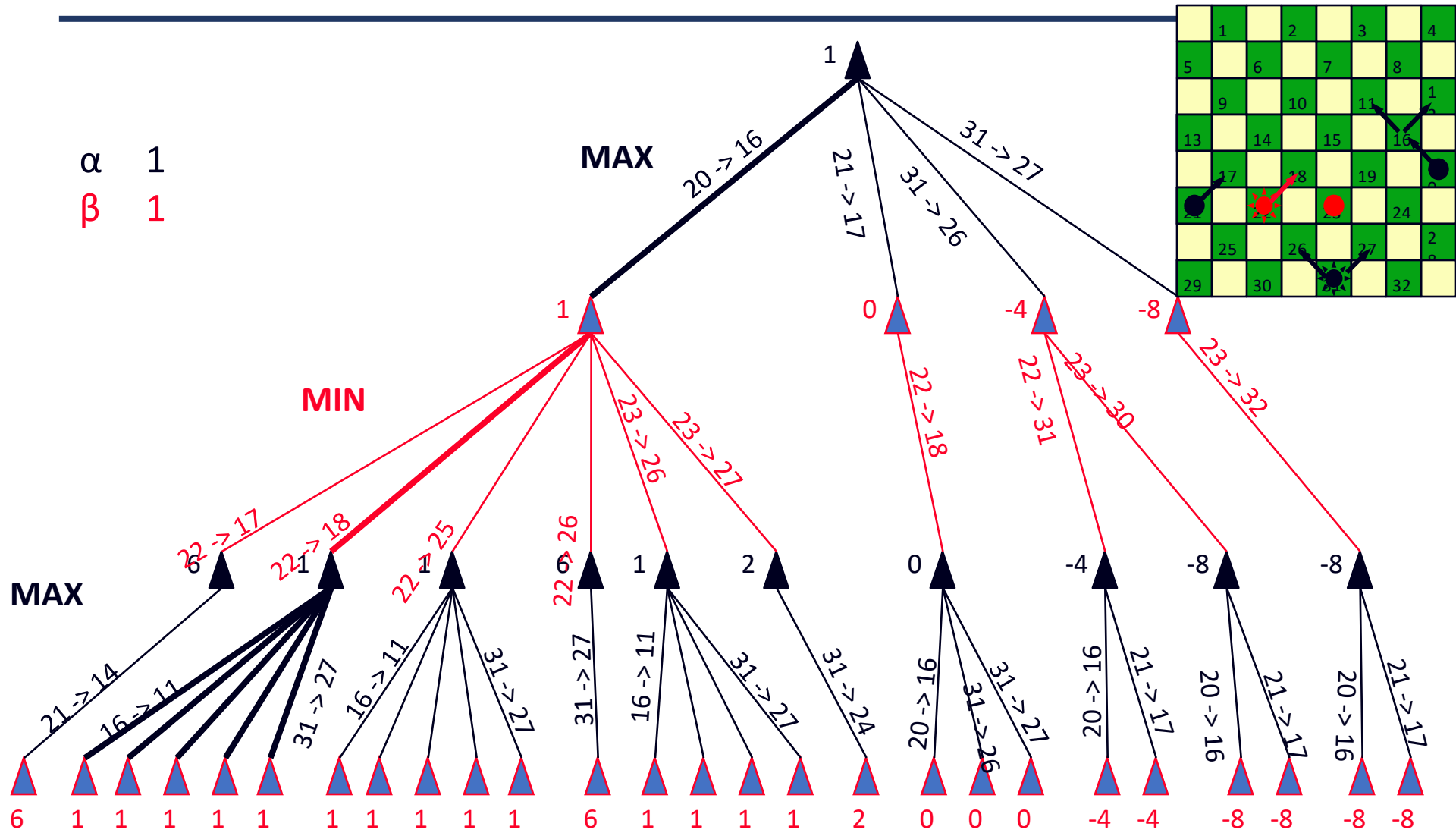
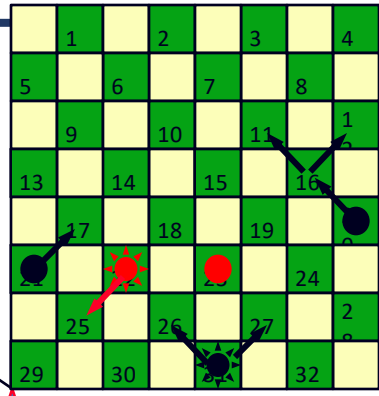


Checkers Alpha-Beta Example

α 1

β 1

β– **cutoff**: no need to examine further branches



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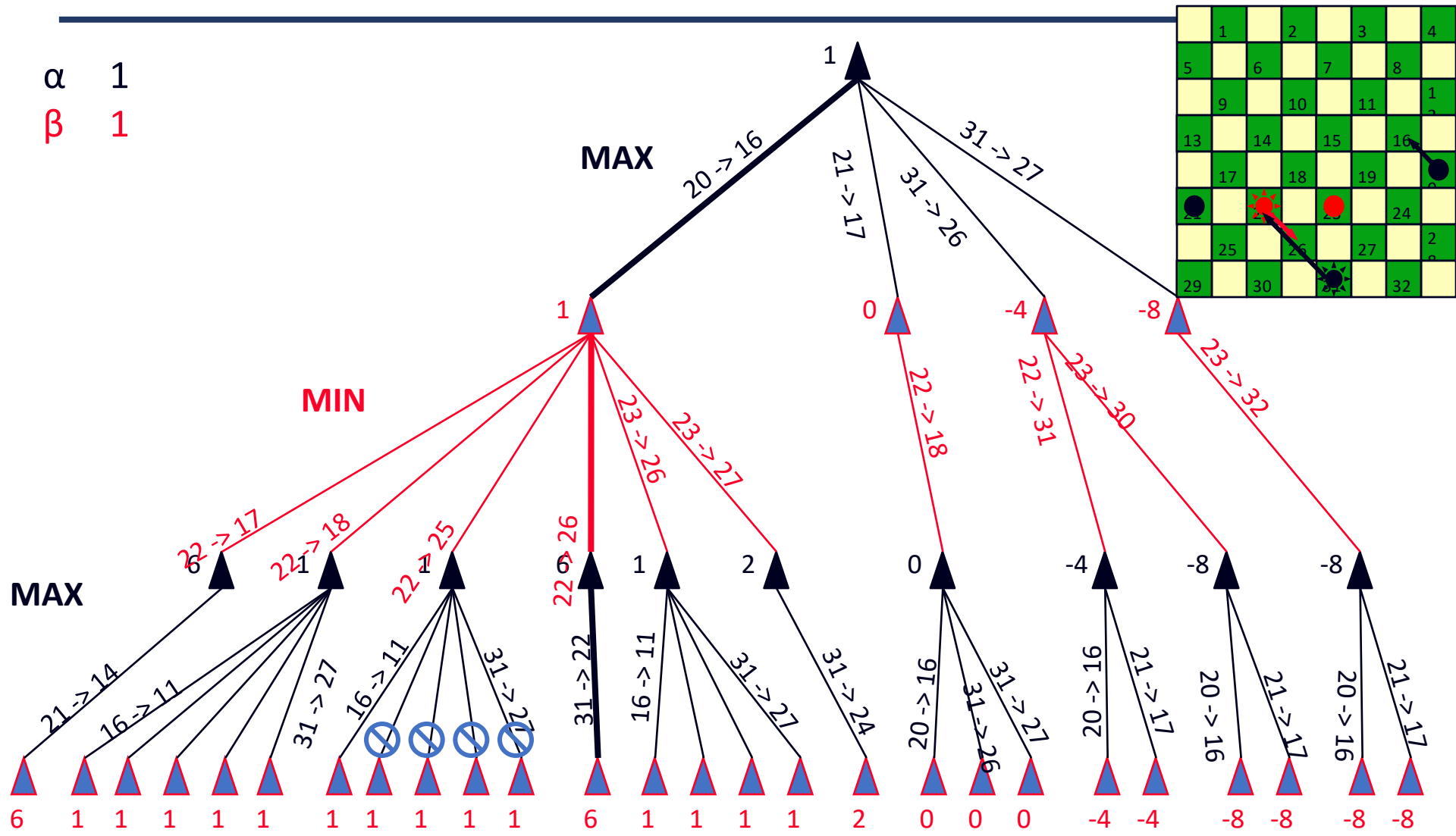

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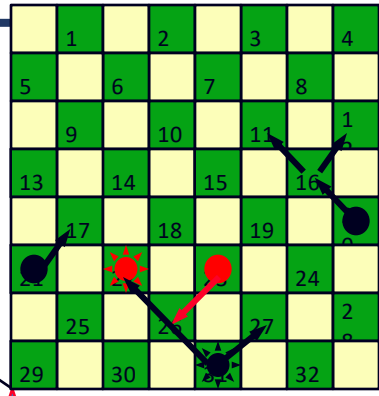

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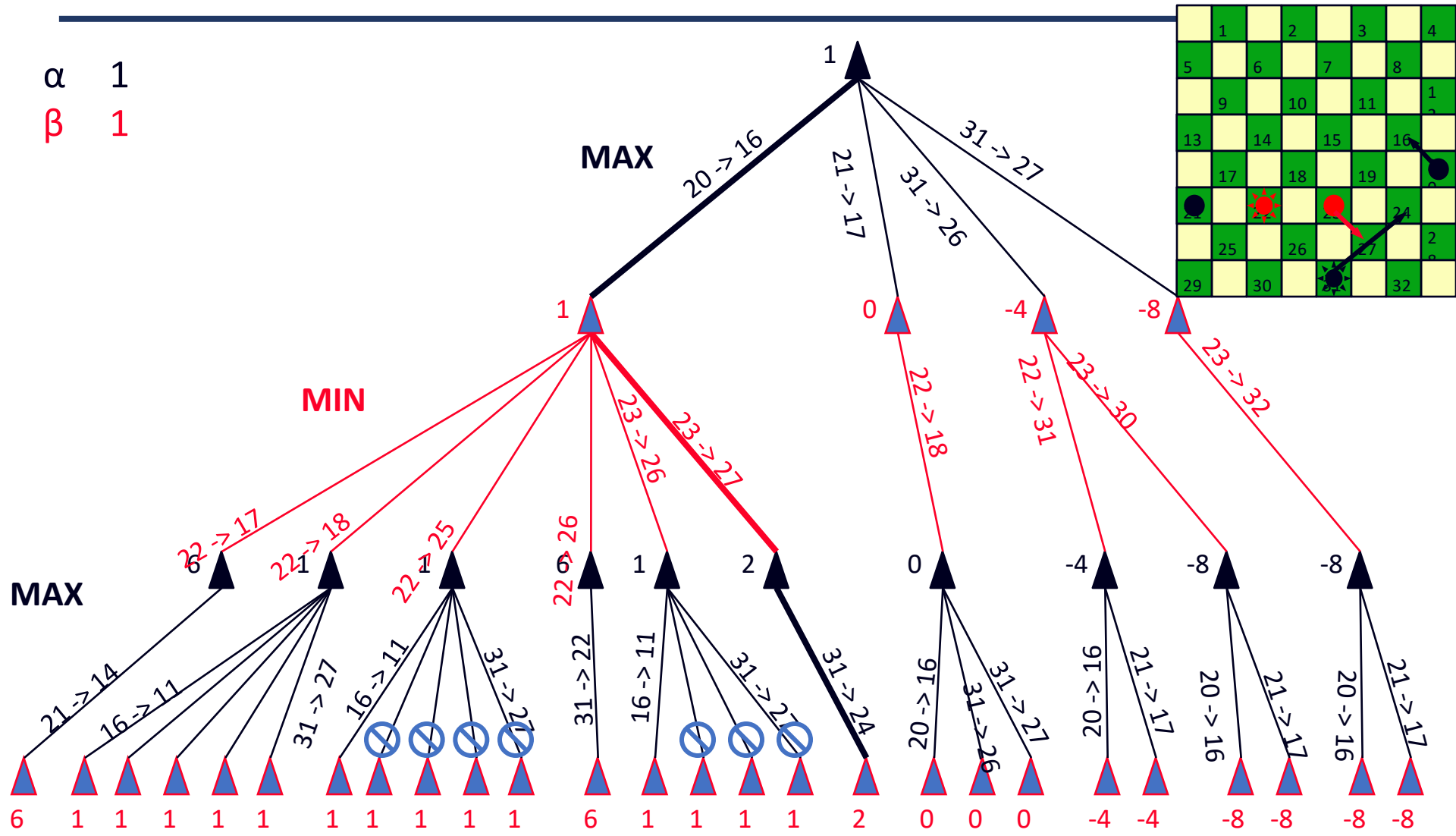

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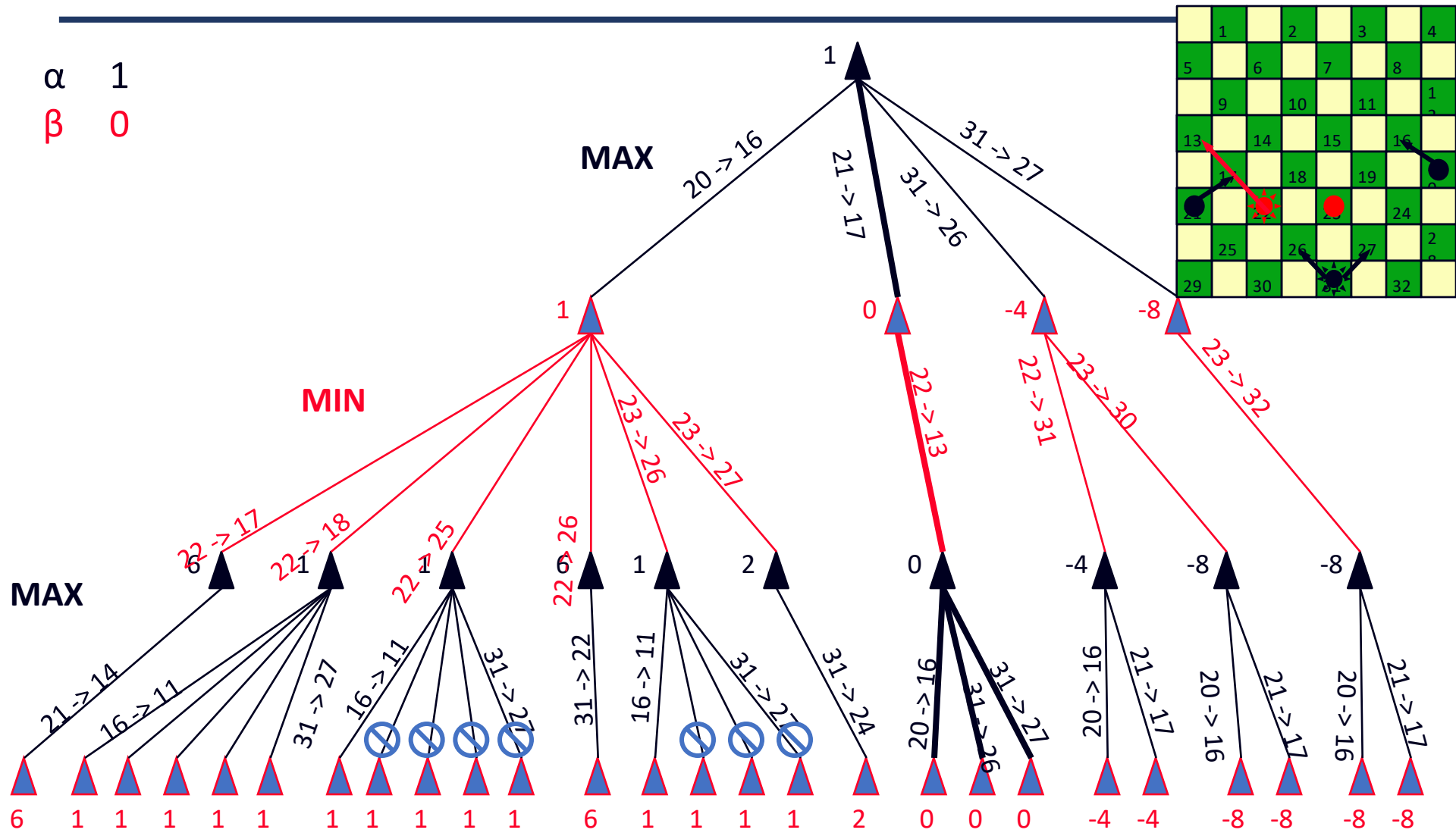

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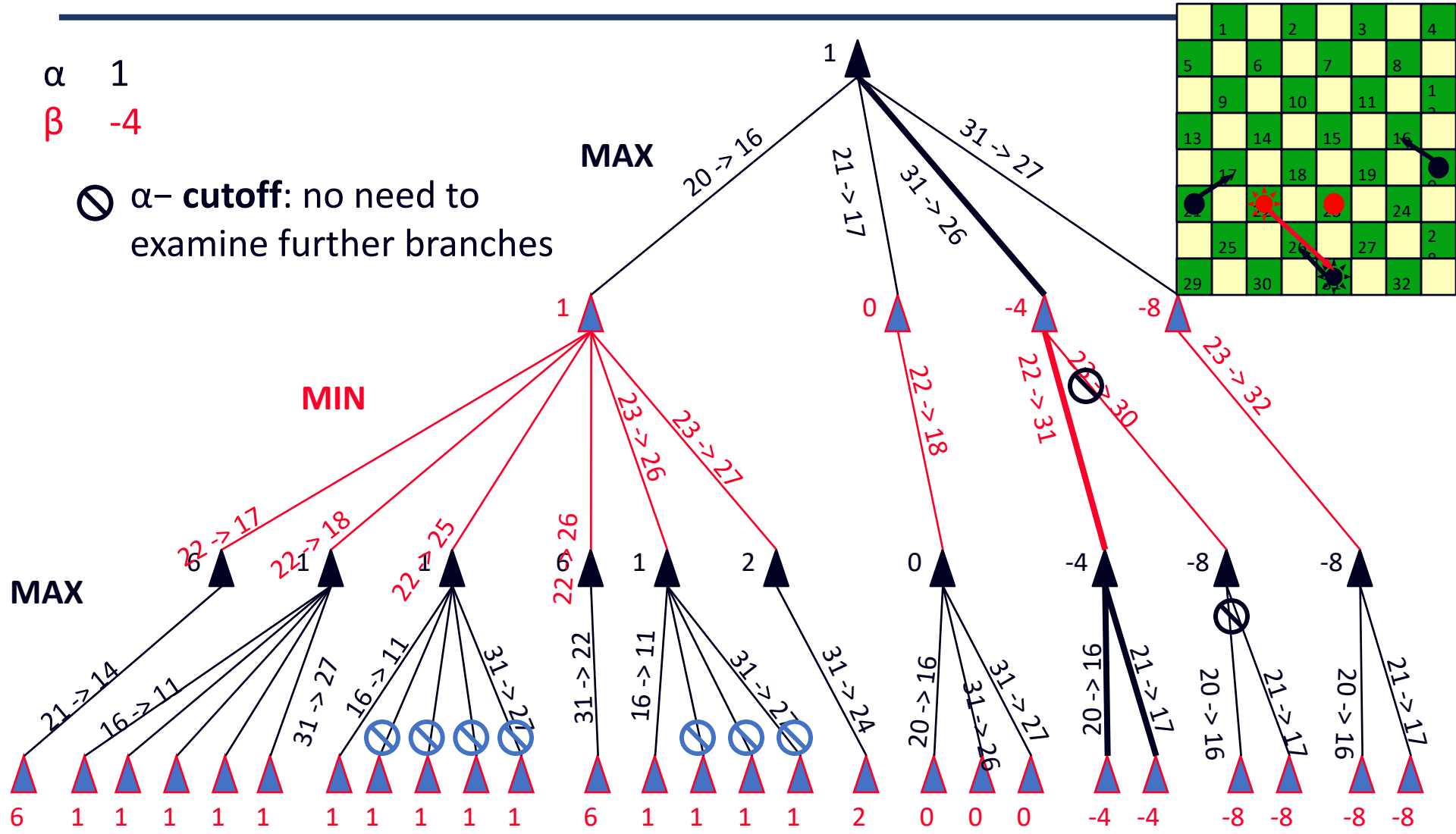

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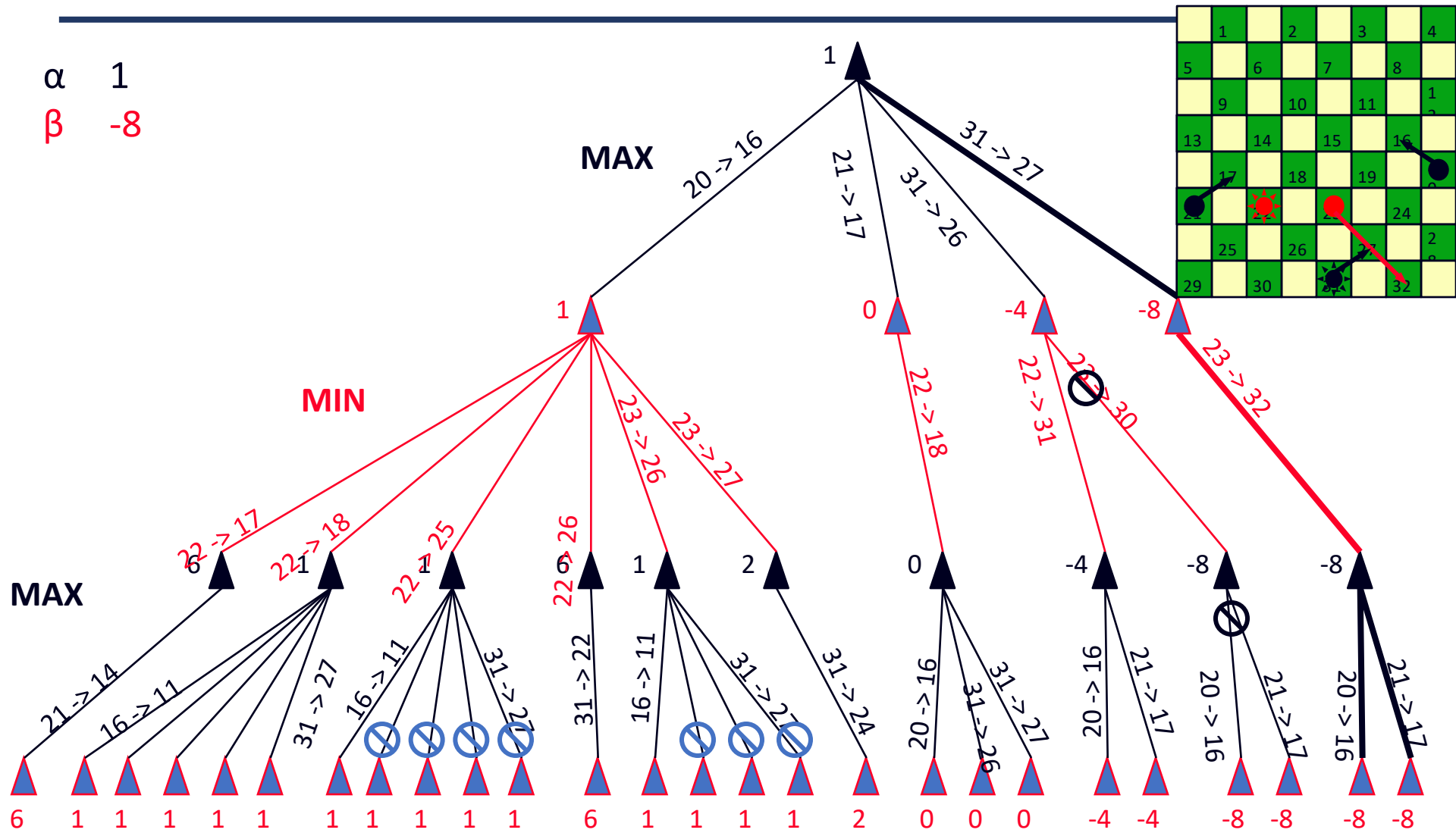


Search Limits

- search must be cut off because of time or space limitations
- strategies like depth-limited or iterative deepening search can be used
  - don’t take advantage of knowledge about the problem
- more refined strategies apply background knowledge
  - quiescent search
  - cut off only parts of the search space that don’t exhibit big changes in the evaluation function

Horizon Problem

- moves may have disastrous consequences in the future, but the consequences are not visible
  - the corresponding change in the evaluation function will only become evident at deeper levels
  - they are “beyond the horizon”
- determining the horizon is an open problem without a general solution
  - only some pragmatic approaches restricted to specific games or situation

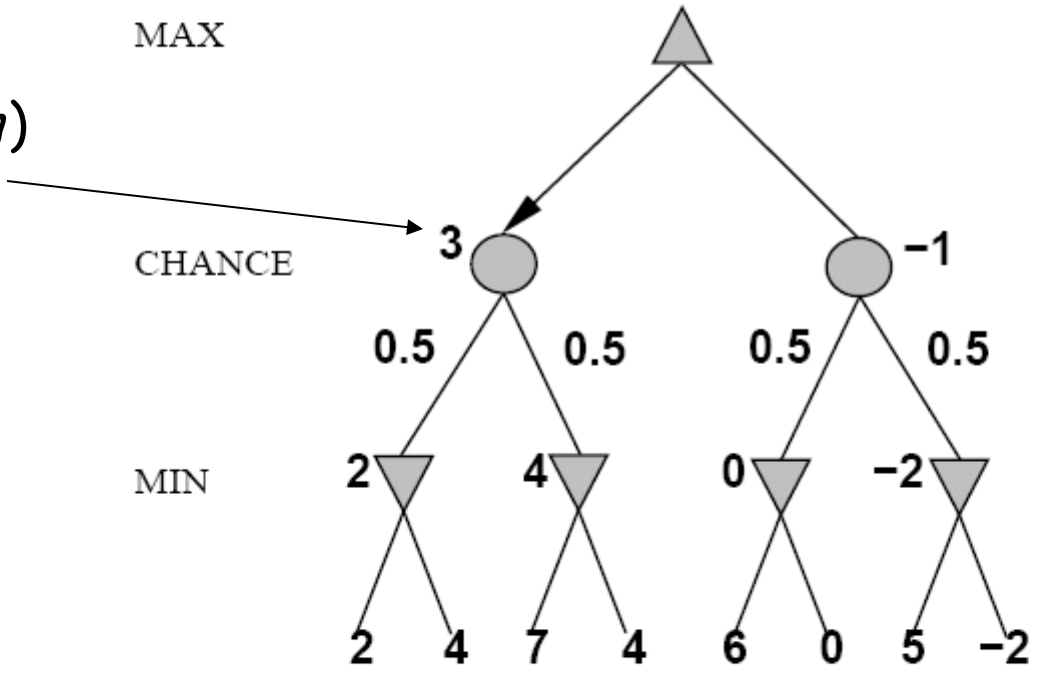
Games with Chance

- in many games, there is a degree of unpredictability through random elements
  - throwing dice, card distribution, roulette wheel, …
- this requires chance nodes in addition to the MAX and MIN nodes
  - branches indicate possible variations
  - each branch indicates the outcome and its likelihood

Expected Minimax

$$v = \sum_{\text{chance nodes}} P(n) \times \text{Minimax}(n)$$

$$3 = 0.5 \times 4 + 0.5 \times 2$$



Interleave chance nodes with min/max nodes

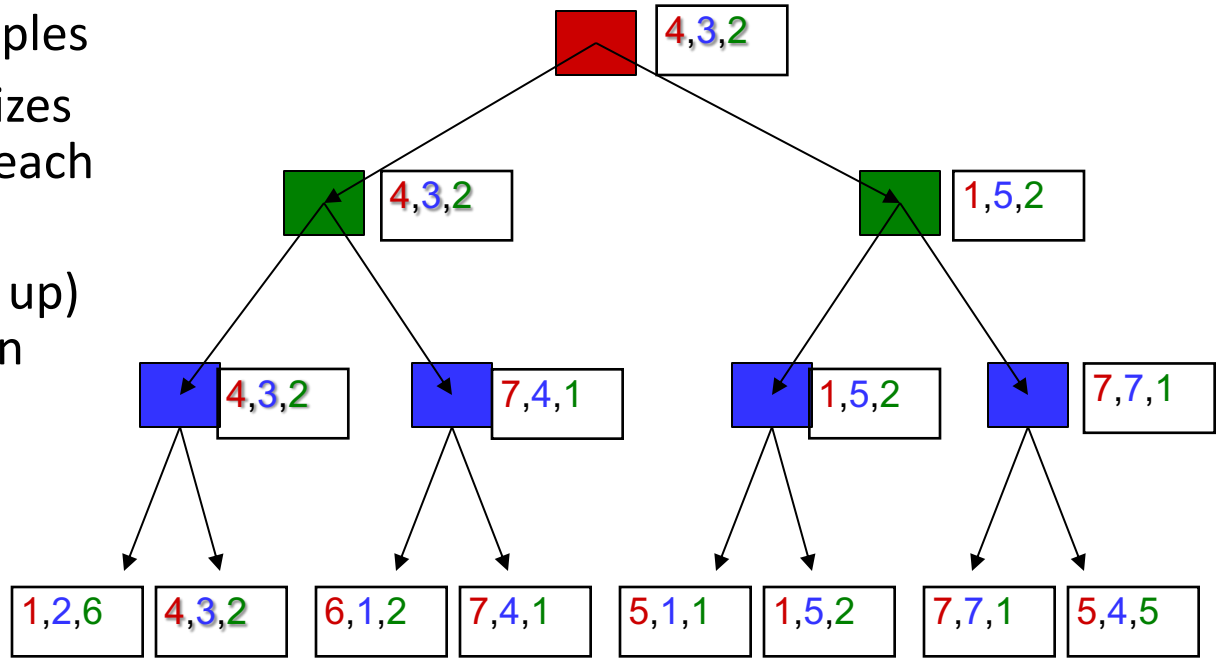
Again, the tree is constructed bottom-up

Decisions with Chance

- the utility value of a position depends on the random element
  - the definite minimax value must be replaced by an expected value
- calculation of expected values
  - utility function for terminal nodes
  - for all other nodes
    - calculate the utility for each chance event
    - weigh by the chance that the event occurs
    - add up the individual utilities

Multi-player Non-Zero-Sum Games

- Similar to minimax:
  - Utilities are now tuples
  - Each player maximizes their own entry at each node
  - Propagate (or back up) nodes from children
  - Can give rise to cooperation and competition dynamically…
  - Pruning?????



Chapter Summary

- many game techniques are derived from search methods
- the minimax algorithm determines the best move for a player by calculating the complete game tree
- alpha-beta pruning dismisses parts of the search tree that are provably irrelevant
- an evaluation function gives an estimate of the utility of a state when a complete search is impractical
- chance events can be incorporated into the minimax algorithm by considering the weighted probabilities of chance events