Engineering Electromagnetics

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Electromagnetics

Electromagnetics theory is a discipline concerned with the study of <u>CHARGES</u>, at <u>REST</u> and <u>MOTION</u>, that produce <u>CURRENT</u>, <u>ELICTRICAL</u>, and <u>MAGNATIC</u> fields.

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Electromagnetics

James Clerk Maxwell
1831-1879

- The study of EM includes:
  - ❑ Theoretical and applied concepts.

- The theoretical concepts are described by a set of:
  - ❑ Basic laws formulated through experiments.
  - ❑ These laws known as

Maxwell Equations

Maxwell's Equations

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Ampere's Circuital Law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday's Law of Induction

$$\nabla \cdot \mathbf{D} = \rho_v$$

Gauss' Law for the electric field

$$\nabla \cdot \mathbf{B} = 0$$

Gauss's Law for the magnetic field

where

D the electric flux density **Coulombs per meter squared**

B the magnetic flux density **Weber per meter squared**

E the electric field intensity **Volts per meter**

H the magnetic field intensity **Amperes per meter**

ρ_v **the volume charge density** **Quantity of charge per cubic meter**

J the current density **Ampere per meter squared**

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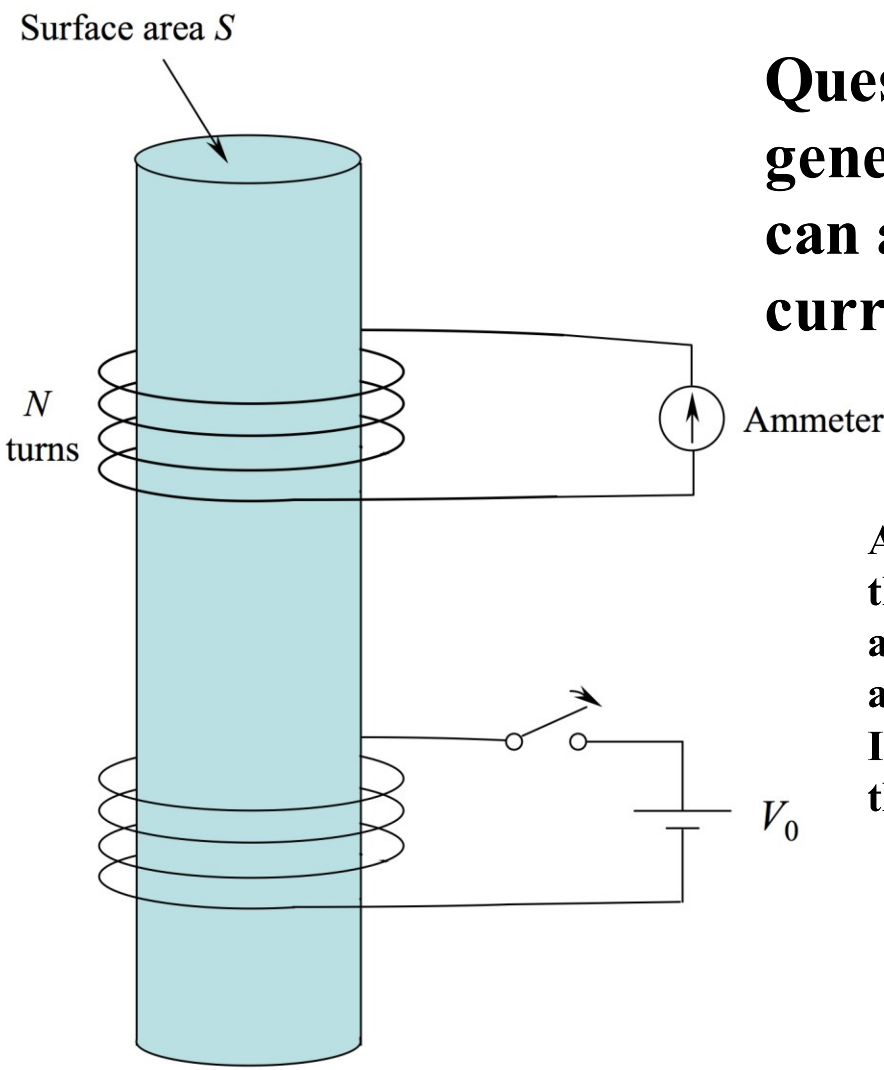
Faraday's Experiment



Question: If a current can generate a magnetic field, then can a magnetic field generate a current?

An experiment similar to that conducted to answer that question is shown here. Two sets of windings are placed on a shared iron core. In the lower set, a current is generated by closing the switch as shown. In the upper set, any induced current is registered by the ammeter.

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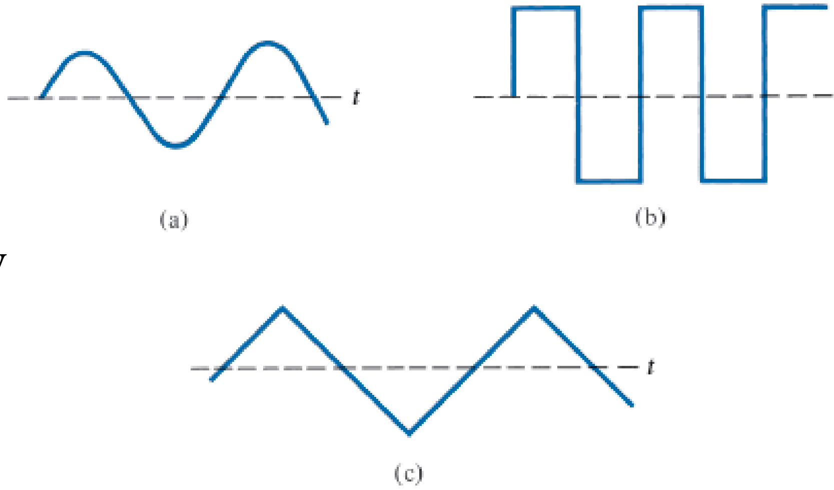
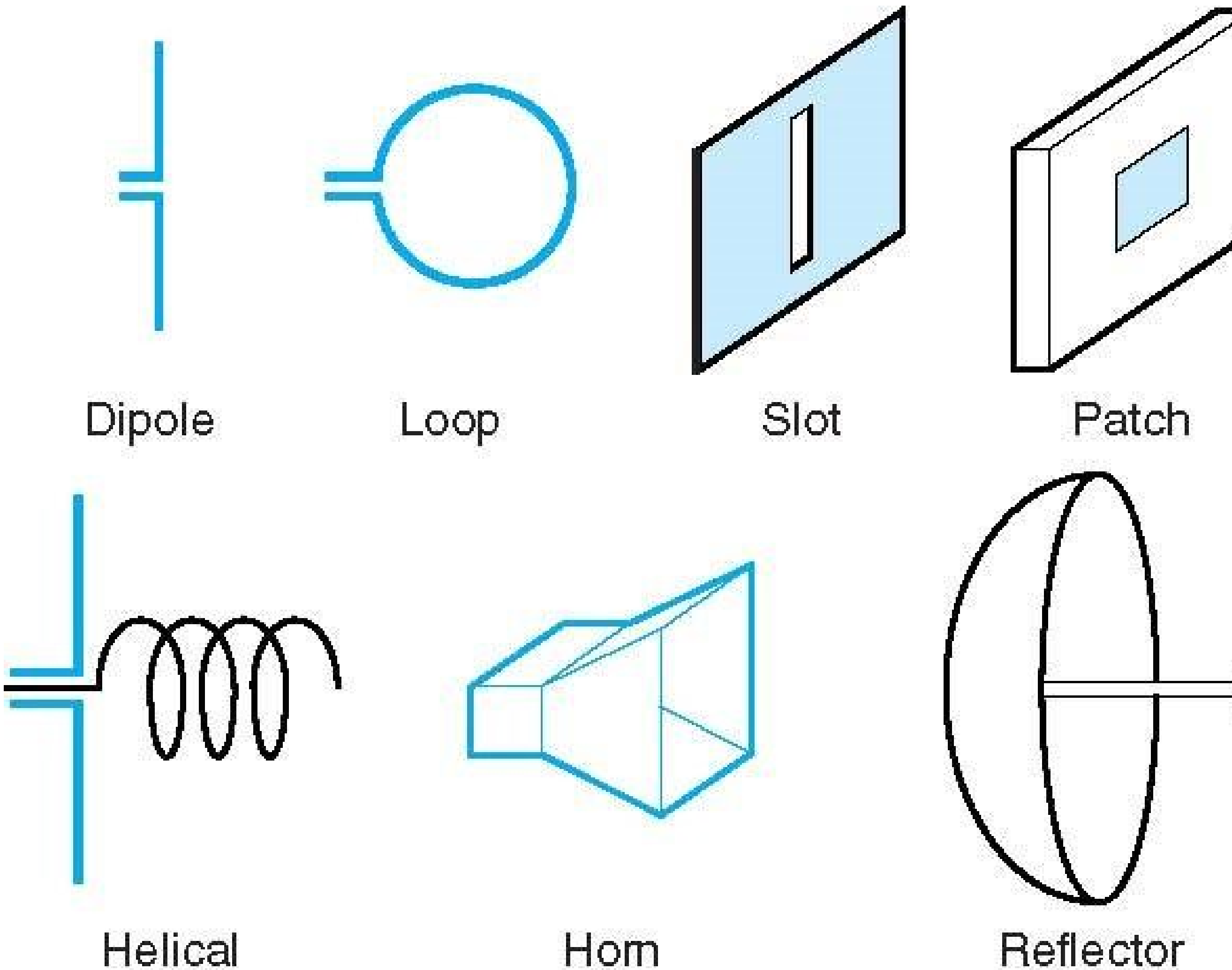
Some insights about EM fields

- In static EM fields, electric and magnetic fields are independent of each other, whereas in dynamic EM fields, the two fields are interdependent.

- Electrostatic fields are usually produced by static electric charges, whereas Magnetostatic fields are due to motion of electric charges with uniform velocity (direct current) or static magnetic charges (magnetic poles)



stationary charges	$\rightarrow$	electrostatic fields
steady currents	$\rightarrow$	magnetostatic fields
time-varying currents	$\rightarrow$	electromagnetic fields



Common single-element antennas.

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Vectors Analysis

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What is a Scaler quantity?

- The term *scalar* refers to a quantity whose value may be represented by a single (positive or negative) **real number**.
- **Examples:**

  Distance, temperature, mass, density, pressure, volume, volume resistivity, and voltage.

What is a Vector quantity

- A ***vector* quantity** has both a magnitude and a direction in space.
- **Examples**
  - Force, velocity, acceleration,

What is the field?

- **A field (scalar or vector) is a function that connects an arbitrary origin to a general point in space.**
- **The value of a field varies in general with both position and time.**

- Both ***scalar*** *fields* and ***vector*** *fields* exist.
  - The **temperature** and the **density** are examples of scalar fields.
  - The **gravitational** and **magnetic** fields of the earth, **voltage gradient**, and the **temperature gradient** are examples of vector fields.

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Vectors characteristics

- Vectors may be multiplied by scalars.
- When the scalar is positive, the magnitude of the vector changes, but its direction does not.
- It **reverses direction** when multiplied by a **negative scalar**.
- Multiplication of a vector by a scalar also obeys the **associative** and **distributive** laws of algebra.

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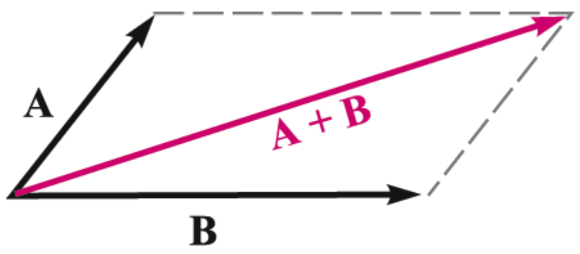
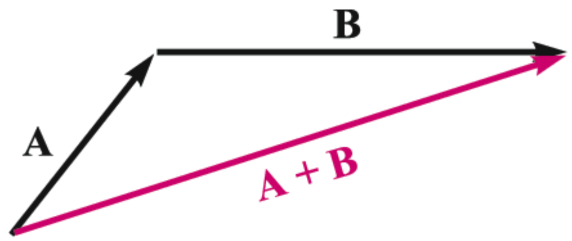
Vector Addition





Associative Law: $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$

Distributive Law: $(r + s)(\mathbf{A} + \mathbf{B}) = r(\mathbf{A} + \mathbf{B}) + s(\mathbf{A} + \mathbf{B})$

Describe a vector

To describe a vector accurately, some specific lengths, directions, angles, projections, or components must be given.

There are three simple methods of doing this,

- **Rectangular Cartesian coordinate system.**
- **cylindrical coordinate system and**
- **spherical coordinate system**

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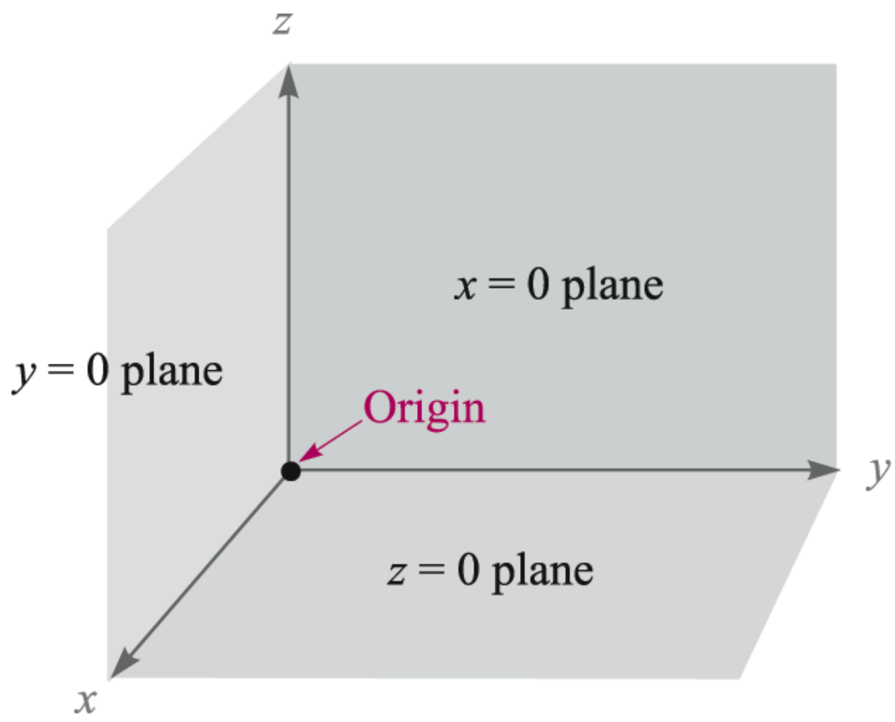
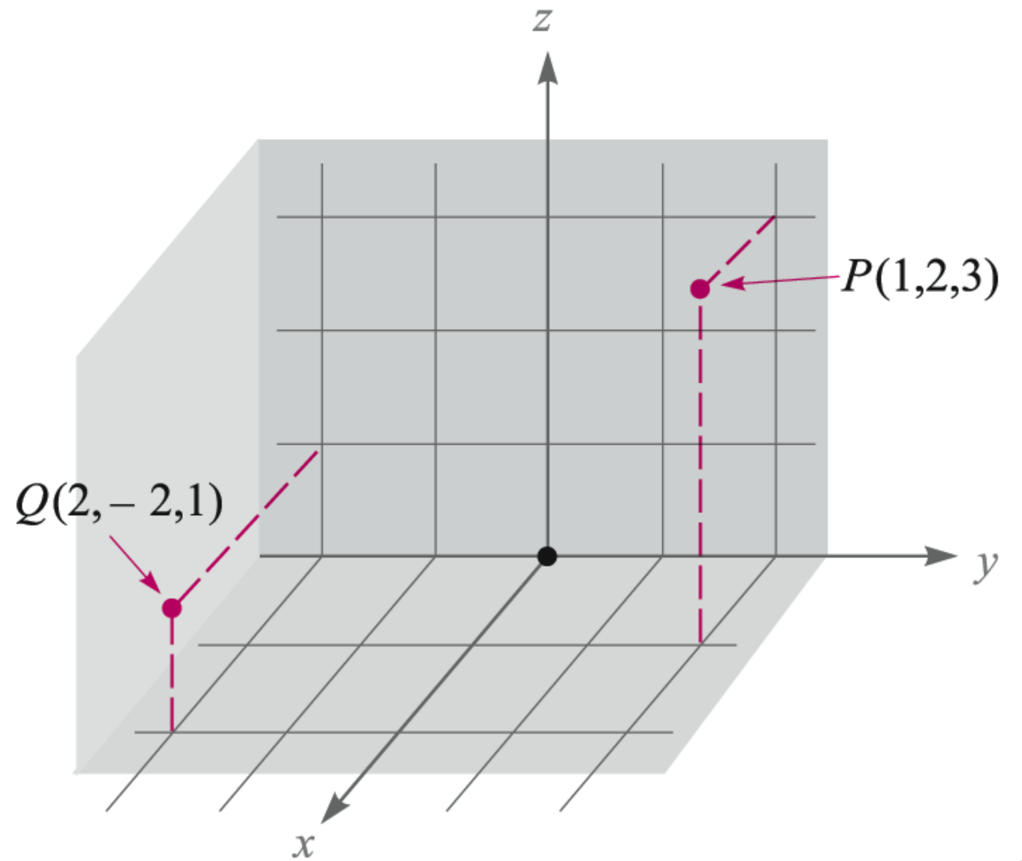
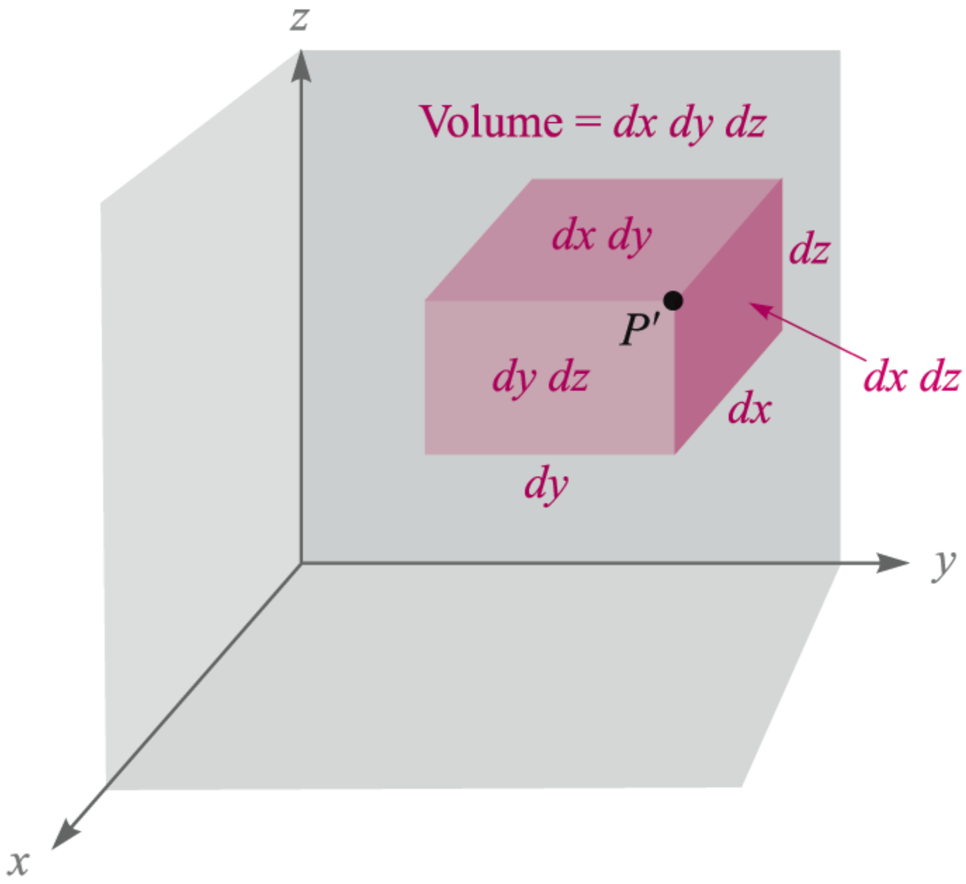
Rectangular Coordinate System



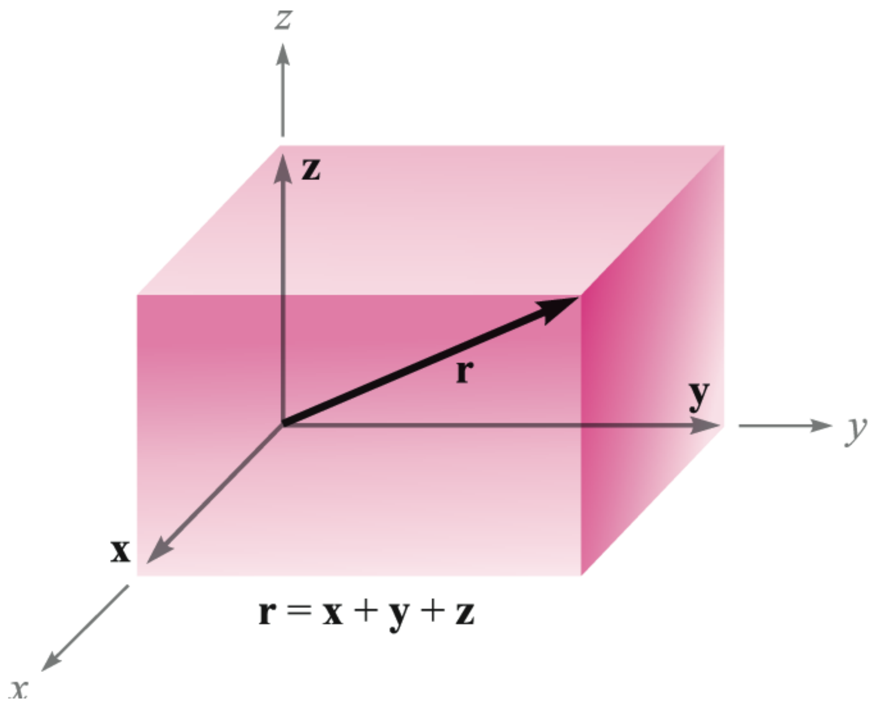
Point Locations in Rectangular Coordinates



Differential Volume Element

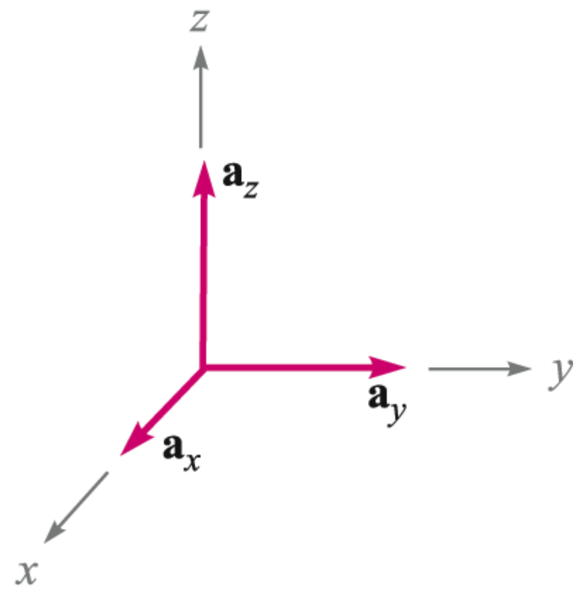


Orthogonal Vector Components

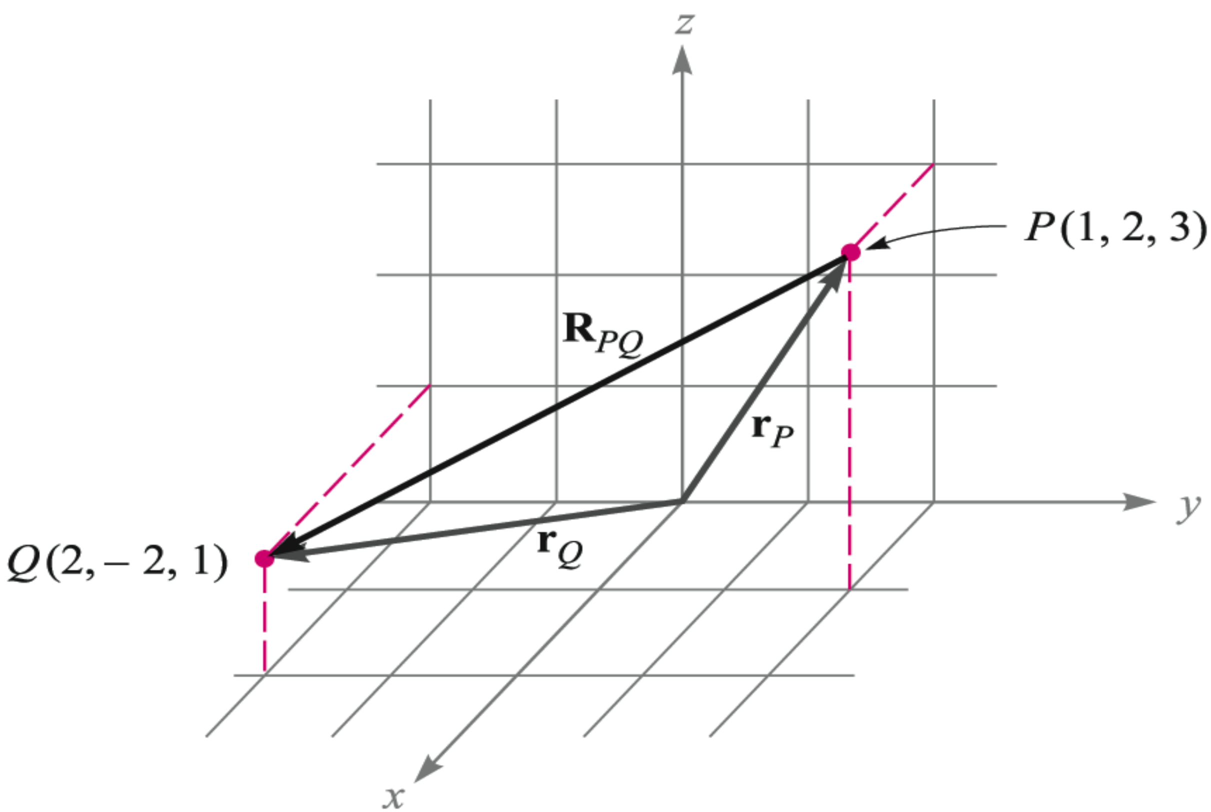


Orthogonal Unit Vectors

unit
vectors having unit magnitude by definition



Vector Representation in Terms of Orthogonal Rectangular Components



$$\mathbf{R}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P = (2-1)\mathbf{a}_x + (-2-2)\mathbf{a}_y + (1-3)\mathbf{a}_z$$
$$= \mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z$$

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Vector Expressions in Rectangular Coordinates

General Vector, **B**:

$$\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$$

Magnitude of **B**:

$$|\mathbf{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

Unit Vector in the Direction of **B:**

$$\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{B_x^2 + B_y^2 + B_z^2}} = \frac{\mathbf{B}}{|\mathbf{B}|}$$

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Example

Specify the unit vector extending from the origin toward the point $G(2, -2, -1)$

$$\mathbf{G} = 2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z$$

$$|\mathbf{G}| = \sqrt{(2)^2 + (-2)^2 + (-1)^2} = 3$$

$$\mathbf{a}_G = \frac{\mathbf{G}}{|\mathbf{G}|} = \frac{2}{3}\mathbf{a}_x - \frac{2}{3}\mathbf{a}_y - \frac{1}{3}\mathbf{a}_z = \underline{0.667\mathbf{a}_x - 0.667\mathbf{a}_y - 0.333\mathbf{a}_z}$$

Vector Field

We are accustomed to thinking of a specific vector:

$$\mathbf{v} = v_x \mathbf{a}_x + v_y \mathbf{a}_y + v_z \mathbf{a}_z$$

A vector field is a *function* defined in space that has magnitude and direction at all points:

$$\mathbf{v}(\mathbf{r}) = v_x(\mathbf{r})\mathbf{a}_x + v_y(\mathbf{r})\mathbf{a}_y + v_z(\mathbf{r})\mathbf{a}_z$$

where $\mathbf{r} = (x, y, z)$

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The Dot Product

Given two vectors **A** and **B**, the *dot product*, or *scalar product*, is defined as the product of the magnitude of **A**, the magnitude of **B**, and the cosine of the smaller angle between them,

$$\boxed{\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \cos \theta_{AB}}$$

Commutative Law: $\boxed{\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}}$

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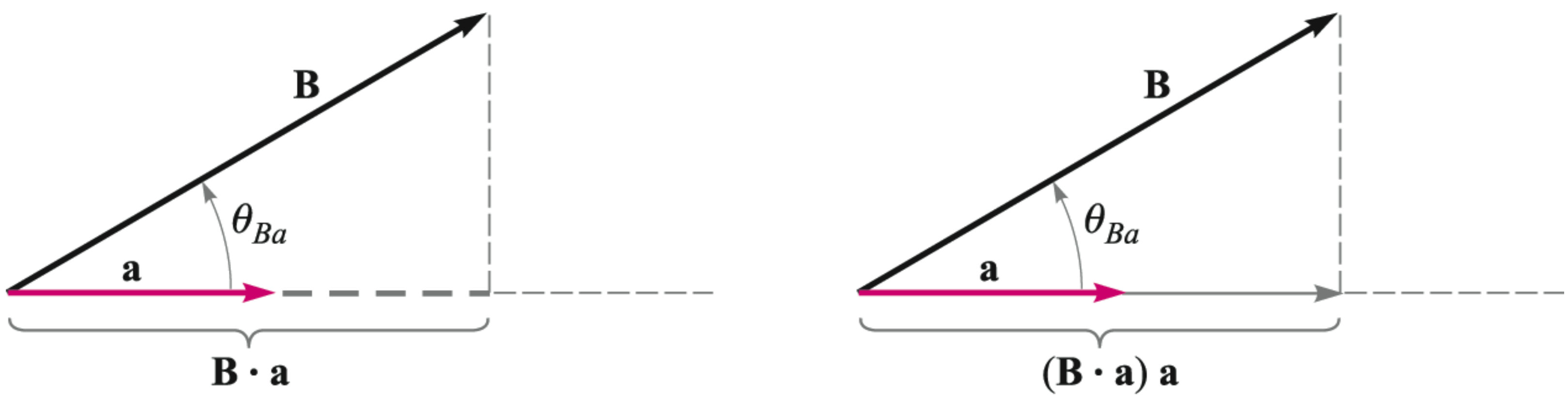
Vector Projections Using the Dot Product

One of the most important applications of the dot product is that of finding the component of a vector in a given direction



B • a gives the component of **B** in the horizontal direction

(**B • a**) **a** gives the *vector* component of **B** in the horizontal direction

B · a is the projection of B in the a direction.

Operational Use of the Dot Product

Given $\begin{cases} \mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z \\ \mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z \end{cases}$

Find $\boxed{\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z}$

where we have used: $\begin{cases} \mathbf{a}_x\cdot\mathbf{a}_y = \mathbf{a}_y\cdot\mathbf{a}_z = \mathbf{a}_x\cdot\mathbf{a}_z = 0 \\ \mathbf{a}_x\cdot\mathbf{a}_x = \mathbf{a}_y\cdot\mathbf{a}_y = \mathbf{a}_z\cdot\mathbf{a}_z = 1 \end{cases}$

Note also: $\boxed{\mathbf{A}\cdot\mathbf{A} = A^2 = |\mathbf{A}|^2}$

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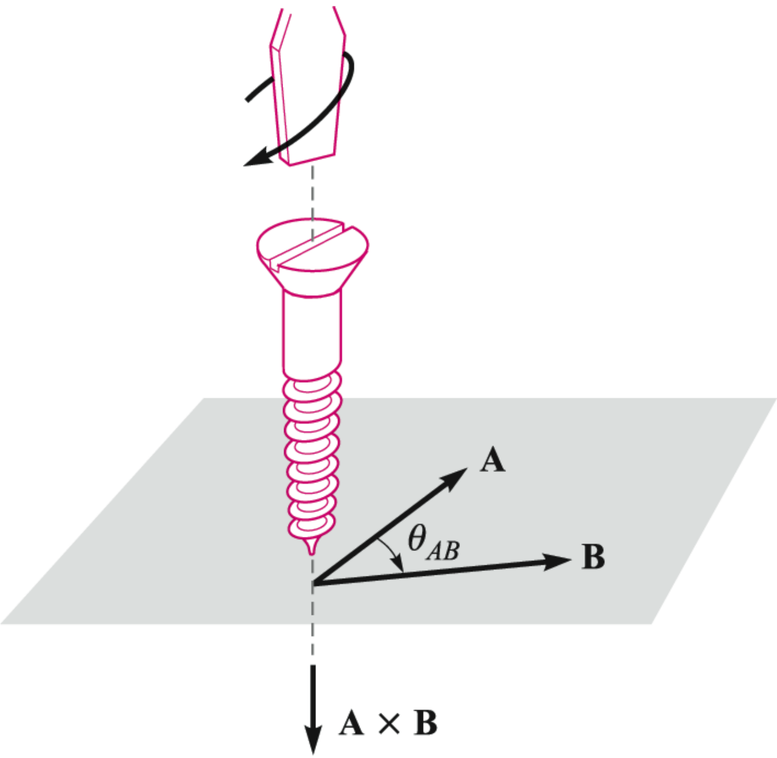
Cross Product

The cross product $\mathbf{A} \times \mathbf{B}$ is a vector; the magnitude of $\mathbf{A} \times \mathbf{B}$ is equal to the product of the magnitudes of $\mathbf{A}$, $\mathbf{B}$, and the sine of the smaller angle between $\mathbf{A}$ and $\mathbf{B}$; the direction of $\mathbf{A} \times \mathbf{B}$ is perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$ and is along that one of the two possible perpendiculars which is in the direction of advance of a right-handed screw as $\mathbf{A}$ is turned into $\mathbf{B}$.

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\, |\mathbf{B}| \sin\theta_{AB}}$$



Reversing the order of the vectors A and B results in a unit vector in the opposite direction, and we see that the cross product is not commutative, for

$$\mathbf{B} \times \mathbf{A} = -(\mathbf{A} \times \mathbf{B}).$$

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Operational Definition of the Cross Product in Rectangular Coordinates

Begin with:
$$\mathbf{A}\times\mathbf{B} = A_xB_x\mathbf{a}_x\times\mathbf{a}_x + A_xB_y\mathbf{a}_x\times\mathbf{a}_y + A_xB_z\mathbf{a}_x\times\mathbf{a}_z$$
$$+ A_yB_x\mathbf{a}_y\times\mathbf{a}_x + A_yB_y\mathbf{a}_y\times\mathbf{a}_y + A_yB_z\mathbf{a}_y\times\mathbf{a}_z$$
$$+ A_zB_x\mathbf{a}_z\times\mathbf{a}_x + A_zB_y\mathbf{a}_z\times\mathbf{a}_y + A_zB_z\mathbf{a}_z\times\mathbf{a}_z$$

$$\text{where}\ \begin{cases}\mathbf{a}_x\times\mathbf{a}_y = \mathbf{a}_z\\ \mathbf{a}_y\times\mathbf{a}_z = \mathbf{a}_x\\ \mathbf{a}_z\times\mathbf{a}_x = \mathbf{a}_y\end{cases}$$

Therefore:

$$\mathbf{A}\times\mathbf{B} = (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_x)\mathbf{a}_z$$

Or…
$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix}\mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

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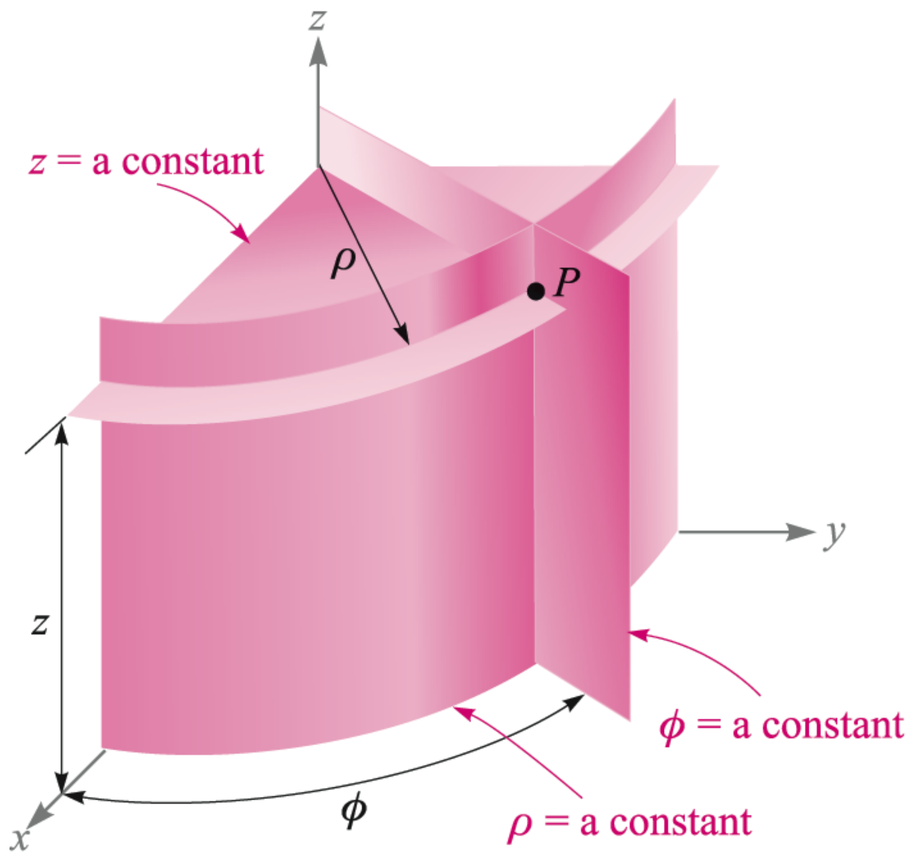
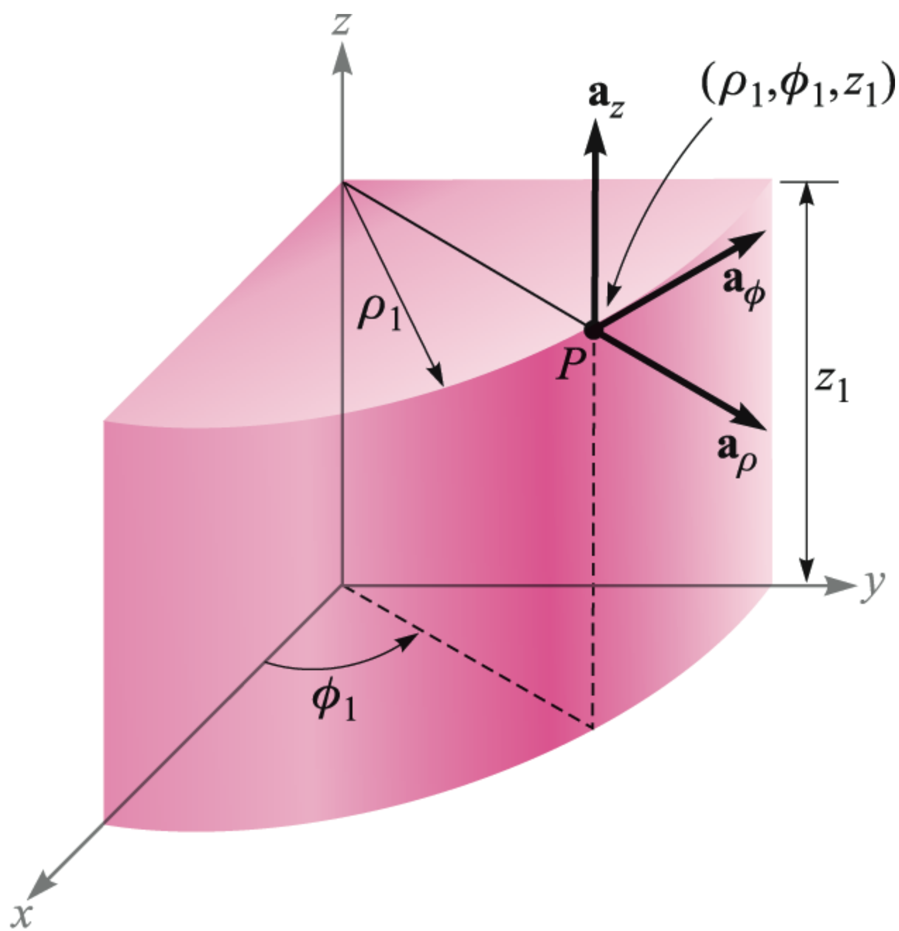
Circular Cylindrical Coordinates

Point P has coordinates
Specified by $P(\rho,\phi,z)$



Orthogonal Unit Vectors in Cylindrical Coordinates



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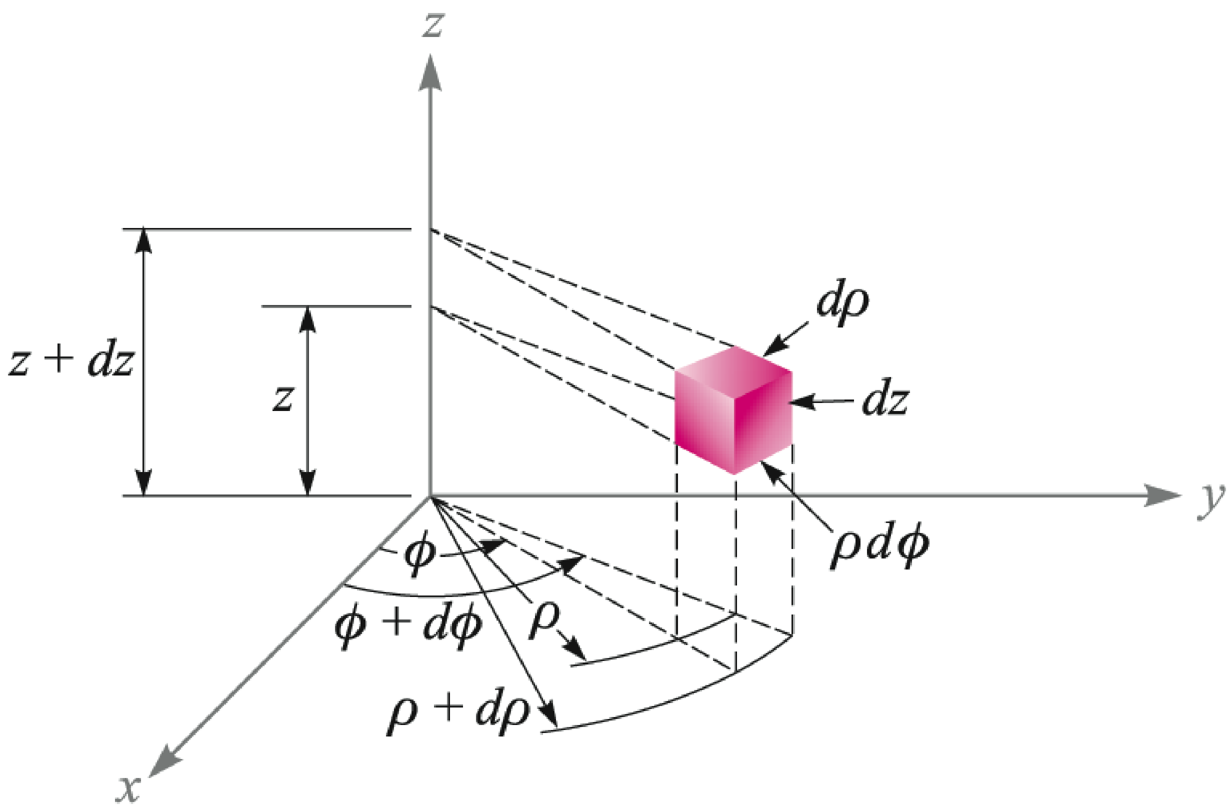
Differential Volume in Cylindrical Coordinates



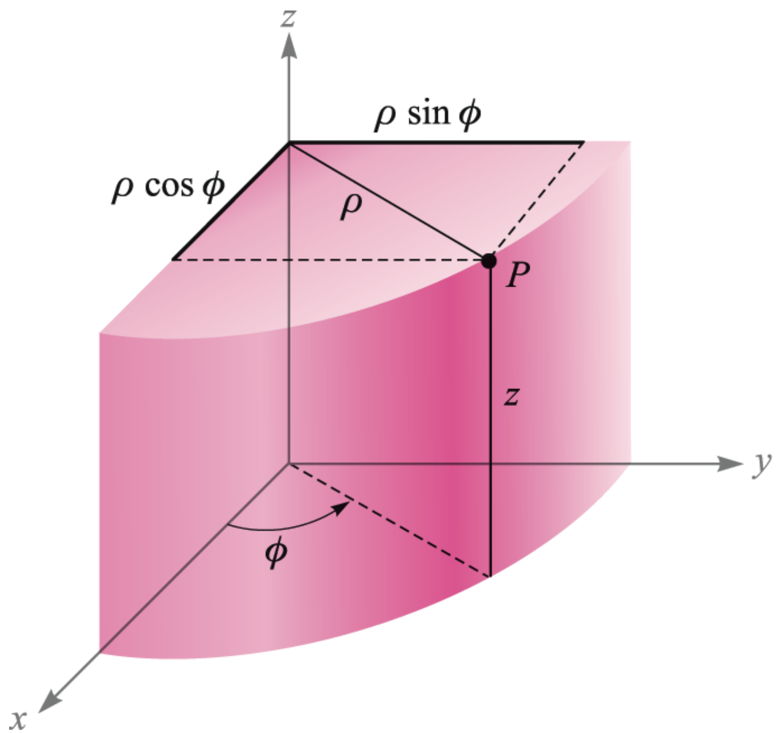
$$\boxed{dV = \rho \, d\rho \, d\phi \, dz}$$

Point Transformations in Cylindrical Coordinates

$$\rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$$

$$\phi = \tan^{-1} \frac{y}{x}$$

$$z = z$$



$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

$$z = z$$

Dot Products of Unit Vectors in Cylindrical and Rectangular Coordinate Systems

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	0

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Example

Transform the vector,

$$\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$$

into cylindrical coordinates:

Use these:

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	0

$$x = \rho\cos\phi$$

$$y = \rho\sin\phi$$

$$z = z$$

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Transform the vector,

$$\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$$

into cylindrical coordinates:

Start with:

$$B_\rho = \mathbf{B}\cdot\mathbf{a}_\rho = y(\mathbf{a}_x\cdot\mathbf{a}_\rho) - x(\mathbf{a}_y\cdot\mathbf{a}_\rho)$$

$$B_\phi = \mathbf{B}\cdot\mathbf{a}_\phi = y(\mathbf{a}_x\cdot\mathbf{a}_\phi) - x(\mathbf{a}_y\cdot\mathbf{a}_\phi)$$

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x\cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y\cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z\cdot$	0	0	0

$$\boxed{\begin{aligned} x &= \rho\cos\phi \\ y &= \rho\sin\phi \\ z &= z \end{aligned}}$$

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Transform the vector,

$$\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$$

into cylindrical coordinates:

Then:

$$B_\rho = \mathbf{B} \cdot \mathbf{a}_\rho = y(\mathbf{a}_x \cdot \mathbf{a}_\rho) - x(\mathbf{a}_y \cdot \mathbf{a}_\rho)$$

$$= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0$$

$$B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = y(\mathbf{a}_x \cdot \mathbf{a}_\phi) - x(\mathbf{a}_y \cdot \mathbf{a}_\phi)$$

$$= -y\sin\phi - x\cos\phi = -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$$

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	0

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

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Transform the vector,

$$\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$$

into cylindrical coordinates:

Finally:

$$B_\rho = \mathbf{B} \cdot \mathbf{a}_\rho = y(\mathbf{a}_x \cdot \mathbf{a}_\rho) - x(\mathbf{a}_y \cdot \mathbf{a}_\rho)$$

$$= y \cos\phi - x \sin\phi = \rho \sin\phi \cos\phi - \rho \cos\phi \sin\phi = 0$$

$$B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = y(\mathbf{a}_x \cdot \mathbf{a}_\phi) - x(\mathbf{a}_y \cdot \mathbf{a}_\phi)$$

$$= -y \sin\phi - x \cos\phi = -\rho \sin^2\phi - \rho \cos^2\phi = -\rho$$

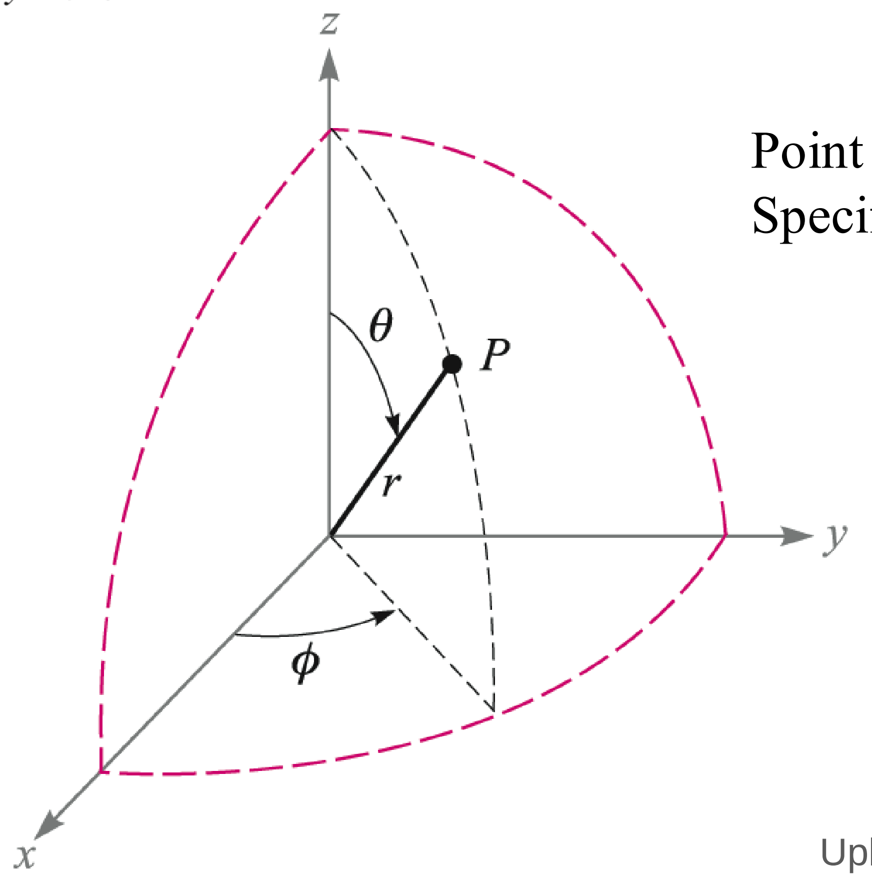
$$\boxed{\mathbf{B} = -\rho\mathbf{a}_\phi + z\mathbf{a}_z}$$

Spherical Coordinates

$$r = \sqrt{x^2 + y^2 + z^2} \qquad (r \geq 0)$$

$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad (0^\circ \leq \theta \leq 180^\circ)$$

$$\phi = \tan^{-1} \frac{y}{x}$$



Point P has coordinates
Specified by $P(r, \theta, \phi)$

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$

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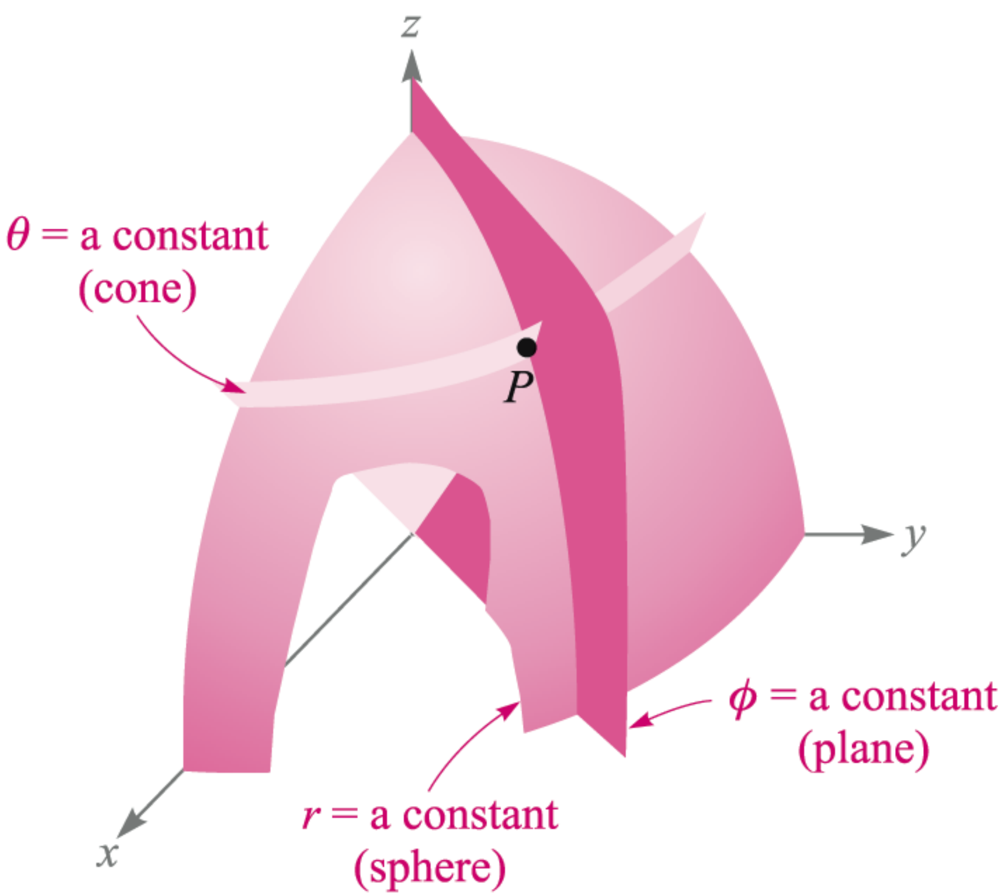
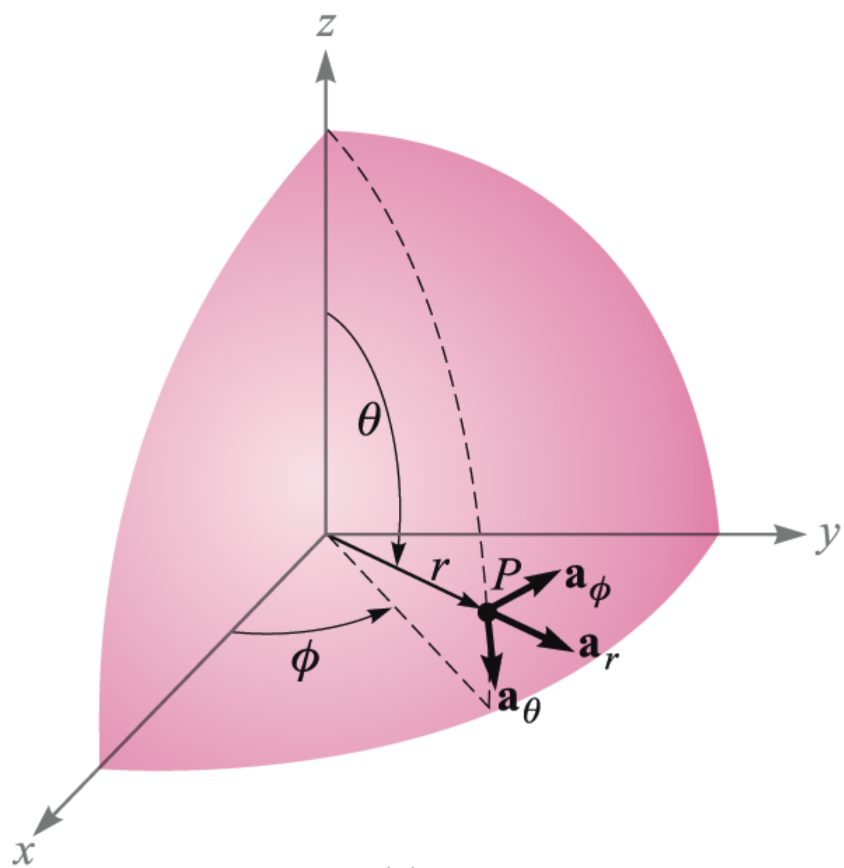
Constant Coordinate Surfaces in Spherical Coordinates



Unit Vector Components in Spherical Coordinates



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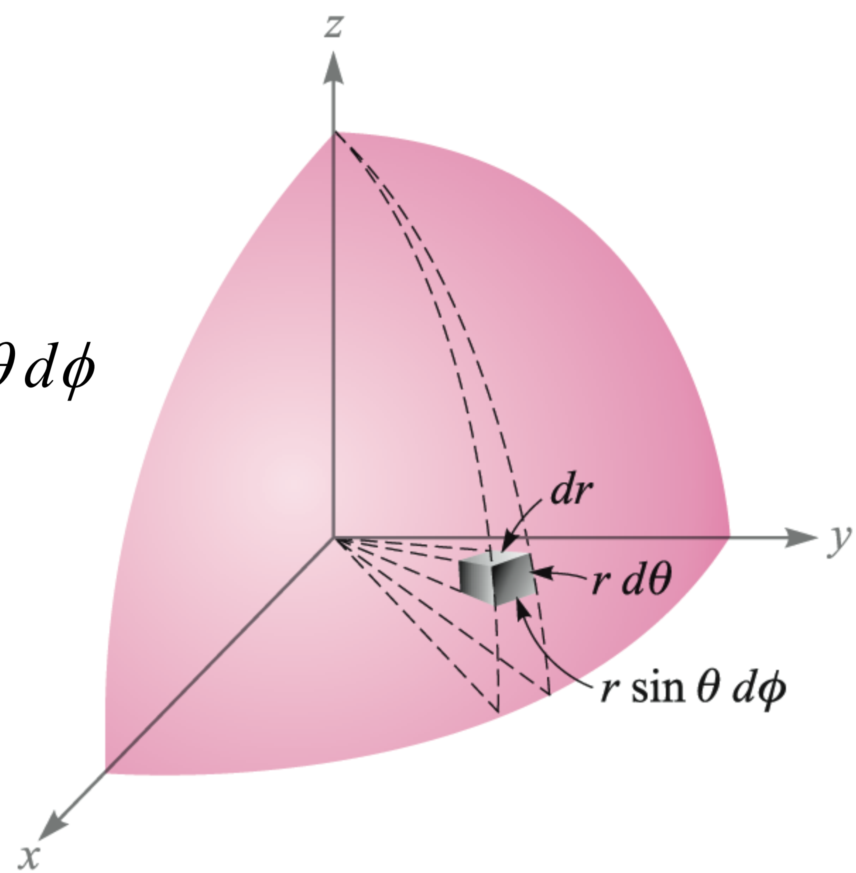
Differential Volume in Spherical Coordinates

$$dV = r^2 \sin\theta \; dr\, d\theta\, d\phi$$



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Dot Products of Unit Vectors in the Spherical and Rectangular Coordinate Systems

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

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Example: Vector Component Transformation

Transform the field, $\mathbf{G} = (xz/y)\mathbf{a}_x$, into spherical coordinates and components

$$G_r = \mathbf{G}\cdot\mathbf{a}_r = \frac{xz}{y}\mathbf{a}_x\cdot\mathbf{a}_r = \frac{xz}{y}\sin\theta\cos\phi$$

$$= r\sin\theta\cos\theta\frac{\cos^2\phi}{\sin\phi}$$

$$G_\theta = \mathbf{G}\cdot\mathbf{a}_\theta = \frac{xz}{y}\mathbf{a}_x\cdot\mathbf{a}_\theta = \frac{xz}{y}\cos\theta\cos\phi$$

$$= r\cos^2\theta\frac{\cos^2\phi}{\sin\phi}$$

$$G\phi = \mathbf{G}\cdot\mathbf{a}_\phi = \frac{xz}{y}\mathbf{a}_x\cdot\mathbf{a}_\phi = \frac{xz}{y}(-\sin\phi)$$

$$= -r\cos\theta\cos\phi$$

$$\boxed{\mathbf{G} = r\cos\theta\cos\phi\,(\sin\theta\cot\phi\,\mathbf{a}_r + \cos\theta\cot\phi\,\mathbf{a}_\theta - \mathbf{a}_\phi)}$$

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x\cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y\cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z\cdot$	$\cos\theta$	$-\sin\theta$	0

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

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