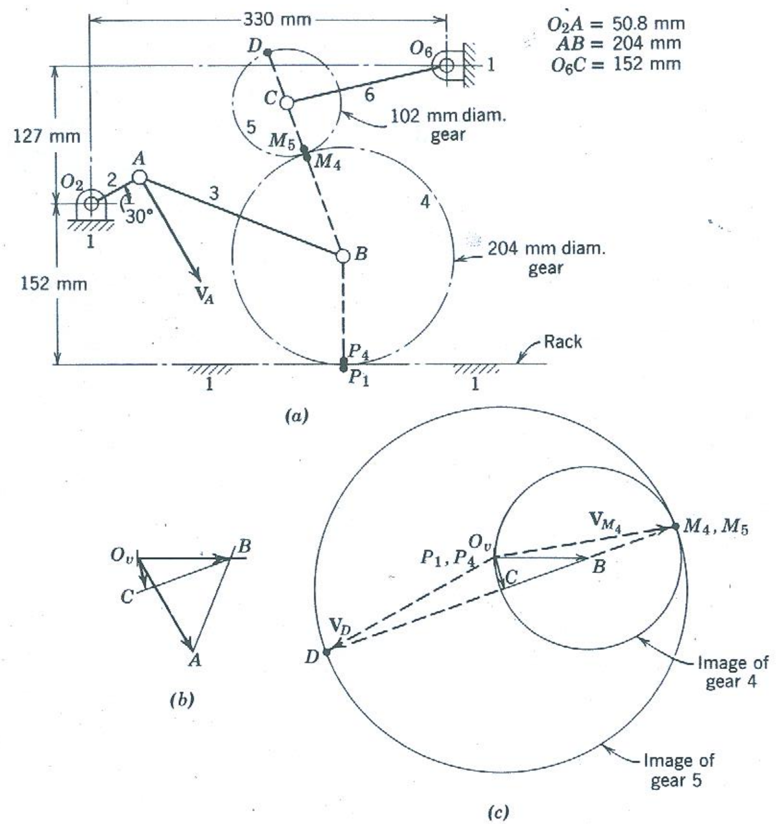

FIGURE 8.20

Since the distance BC between the centers of the two gears is constant in all phases of the mechanism, an equivalent link joining the two centers may be visualized. Therefore, a five-bar linkage is first analyzed to determine the velocities $\mathbf{V}_B$ and $\mathbf{V}_C$ of the centers of the gears. The velocity polygon of Fig. 8.20*b* shows the determination of $\mathbf{V}_B$ and $\mathbf{V}_{BA}$ from Eq. I. In a similar manner, $\mathbf{V}_C$ and $\mathbf{V}_{CB}$ are determined from Eq. II.

In Fig. 8.20*c*, the velocity vectors $\mathbf{V}_B$ and $\mathbf{V}_C$ of Fig. 10.20*b* are redrawn for the construction of the velocity images of gears 4 and 5. Because the velocity $\mathbf{V}_{P_1}$ of point P_1 is zero and $\mathbf{V}_{P_4} = \mathbf{V}_{P_1}(\mathbf{V}_{P_4P_1} = 0)$, the image of both points P_1 and P_4 is at the pole point O_v as shown. With point P_4 located on the polygon, the velocity image of gear 4 is drawn with B as a center and radius BP_4. The image of point M_4 on the circle is determined by drawing a line through B on the polygon perpendicular to the line M_4B on the configuration diagram. The image of point M_5 is the same as that of point M_4 because $\mathbf{V}_{M_5} = \mathbf{V}_{M_4}$. The image of gear 5 is, therefore, drawn with C as a center and radius CM_5. The image of point D is located on a diameter of the circle opposite point M_5.

The magnitudes and senses of ω_4 and ω_5 can now be determined from $\mathbf{V}_{BP_4}$ and $\mathbf{V}_{CM_5}$, respectively, as shown.

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8.12 INSTANTANEOUS CENTERS OF VELOCITY

In the foregoing paragraphs and examples, the velocity analyses of linkages were made from an understanding of relative velocity and the influence of motion constraint on relative velocity. In the following, another concept is utilized to determine the linear velocity of particles in mechanisms, namely, the concept of the instantaneous center of velocity. This concept is based on the fact that at a given instant a pair of coincident points on two links in motion will have identical velocities relative to a fixed link and, therefore, will have zero velocity relative to each other. At this instant either link will have pure rotation relative to the other link about the coincident points. A special case of this is where one link is moving and the other is fixed. A pair of coincident points on these two links will then have zero absolute velocity, and the moving link at this instant will be rotating relative to the fixed link about the coincident points. In both cases the coincident set of points is referred to as an *instantaneous center of velocity* (sometimes referred to as *instant center,* or *centro*). From the foregoing, it can be seen that an instantaneous center is (*a*) a point in both bodies, (*b*) a point at which the two bodies have no relative velocity, and (*c*) a point about which one body may be considered to rotate relative to the other body at a given instant. It is easily seen that when two links, either both moving or one moving and one fixed, are directly connected together, the center of the connecting joint is an instantaneous center for the two links. When two links, either both moving or one moving and one fixed, are not directly connected, however, an instant center for the two links will also exist for a given phase of the linkage as will be shown in the following section.

In the four-bar linkage of Fig. 8.21, it is obvious that relative to the fixed link, points O_2 and O_4 are locations of particles on links 2 and 4, respectively, which are at zero velocity. It is less obvious that on link 3, which has both translating and angular motion, a particle is also at zero velocity relative to the fixed link. Referring to the velocity polygon shown in Fig. 8.21, the velocity image of link 3 appears as the line AB and none of the particles on this line is

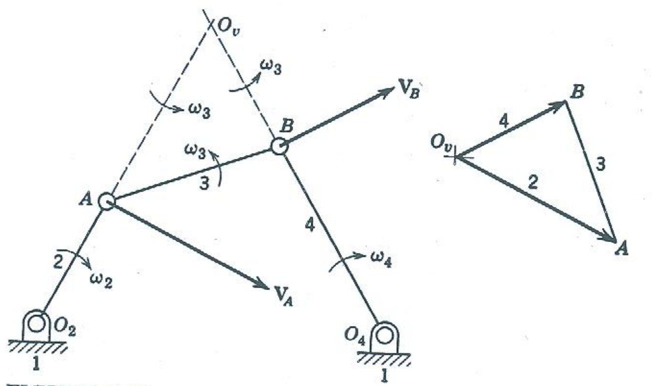

FIGURE 8.21

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at zero velocity. However, if link 3 is visualized large enough in extent as a rigid body to include O_v of the polygon, a particle of zero velocity is then included in the image. To determine the location of O_v, the instantaneous center of link 3 relative to link 1, on the mechanism, a triangle similar to O_vBA of the polygon is constructed on the mechanism so that the sides of the two similar triangles are mutually perpendicular. It is important to note that for the particles on link 3 at A and at B, the fixed vectors $\mathbf{V}_A$ and $\mathbf{V}_B$ on link 3 are normal to the lines drawn from the instantaneous center O_v to A and B.

Since A and the instantaneous center O_v are particles on a common rigid link, the magnitude of $\mathbf{V}_A$ may be determined from $V_A = \omega_3(O_vA)$. Similarly, $V_B = \omega_3(O_vB)$. The magnitude of the velocity of any particle on link 3 may be determined from the product of ω_3 and the radial distance from the instantaneous center to the particle, and the direction of the velocity vector is normal to the radial line.

It may also be seen that the instantaneous center of link 3 relative to link 1 changes position with respect to time because of the changes in the shape of the velocity polygon as the mechanism passes through a cycle of phases. However, for links in pure rotation, the instantaneous centers are fixed centers, such as O_2 and O_4 of links 2 and 4, respectively, of Fig. 8.21.

The determination of velocities by instantaneous centers does not require the velocity polygon of free vectors and is judged by many to be the quicker method. By the method of instantaneous centers, the velocity vectors are shown directly as fixed vectors.

In the solution of a problem, such as in Fig. 8.22, the locations of the instantaneous centers of the moving links relative to the fixed link are generally determined first. For links 2 and 4, O_2 and O_4 are obviously points of zero velocity. For links such as link 3, only the *directions* of the velocities of two particles on the link need to be known since the intersection of the normals to the velocity direction lines determines the instantaneous center.

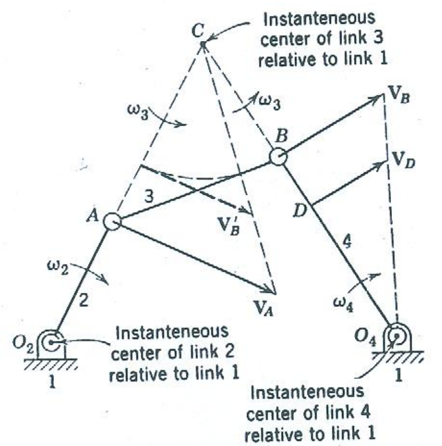

FIGURE 8.22

Fixed velocity vectors may be determined almost entirely by graphical cor struction. In Fig. 8.22, assuming ω_2 is the only information given, V_A may b computed from $\omega_2(O_2A)$ and $\mathbf{V}_A$ drawn normal to O_2A using the instantaneou center of link 2 relative to link 1. Considering particles A and B as part of lin 3, the magnitude of $\mathbf{V}_B$ may be determined from similar triangles, as shown b the graphical construction, since V_A and V_B are proportional to the distance c A and B from the instantaneous center of link 3 relative to link 1. The equatio which justifies the use of similar triangles in determining $\mathbf{V}_B$ may be written a $\omega_3 = V_A/(CA) = V_B/(CB)$. The velocity of any particle on link 4 such as D ma be determined graphically from similar triangles as shown using the instantaneou center of link 4 relative to link 1.

For links that are in pure translation, such as the slider in a slider-cranl mechanism, the direction lines of the velocities of all of its particles are parallel and the normals, also being parallel, intersect at infinity. Thus, the instantaneou center of a link in translation is at an infinite distance from the link, in a directior normal to the path of translation.

8.13 INSTANTANEOUS CENTER NOTATION

In the foregoing, instantaneous centers of velocity were determined for each o the moving links relative to the fixed link. The system of labeling these points is shown in Fig. 8.23, where the instantaneous center of link 3 relative to the fixed link is labeled 31 to indicate the motion of "3 relative to 1." Link 1 has the same instantaneous center relative to link 3 when link 3 is considered the fixed link, in which case link 1 appears to be rotating in the opposite sense ($\omega_{13} = -\omega_{31}$) relative to link 3. Since points 31 and 13 are the same point, either designation is acceptable although the simpler notation 13 is preferred. The instantaneous center of link 2 relative to link 1 is labeled 21 or 12, and that of link 4 relative to link 1 is labeled 41 or 14 as shown.

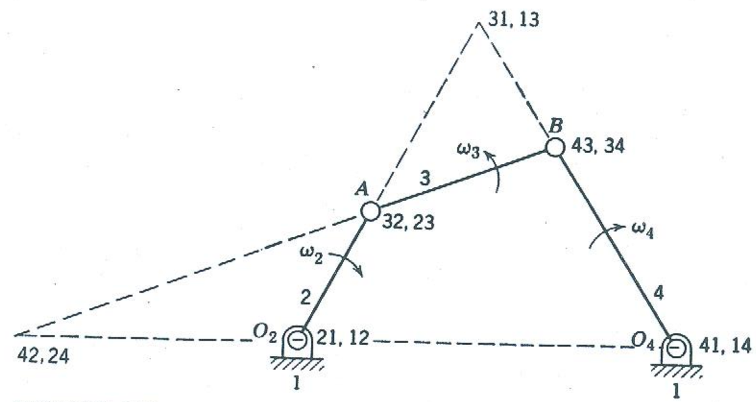

FIGURE 8.23

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Also of interest is the instantaneous center of one link relative to another where both links are moving relative to the fixed link. Such a center is shown at point A in Fig. 8.23, where both A_2 and A_3 have a common absolute velocity $\mathbf{V}_A$ because of the pinned joint so that the relative velocities $\mathbf{V}_{A_3A_2}$ and $\mathbf{V}_{A_2A_3}$ are zero. It is obvious that point A is the instantaneous center 32 about which link 3 is rotating relative to link 2 at an angular velocity ω_{32}. Point A is also the instantaneous center 23. In a similar manner point B is the instantaneous center 43 or 34. The instantaneous center 42 or 24 is also shown in Fig. 8.23. However, the method of determining its location will not be presented until the next section.

8.14 KENNEDY'S THEOREM

For three independent bodies in general plane motion, Kennedy's theorem states that the three instantaneous centers lie on a common straight line. In Fig. 8.24, three independent links (1, 2, and 3) are shown in motion relative to each other. There are three instantaneous centers (12, 13, and 23), whose instantaneous locations are to be determined.

If link 1 is regarded as a fixed link, or datum link, the velocities of particles A_2 and B_2 on link 2 and the velocities of D_3 and E_3 on link 3 may be regarded as absolute velocities relative to link 1. The instantaneous center 12 may be located from the intersection of the normals to the velocity direction lines drawn from A_2 and B_2. Similarly, the center 13 is located from normals drawn from particles D_3 and E_3. The instantaneous centers 12 and 13 are relative to link 1.

The third instantaneous center 23 remains to be determined. On a line drawn through the centers 12 and 13, there exists a particle C_2 on link 2 at an absolute velocity $\mathbf{V}_{C_2}$ having the same direction as the absolute velocity $\mathbf{V}_{C_3}$ of a particle C_3 on link 3. Since $\mathbf{V}_{C_2}$ is proportional to the distance of C_2 from 12, the magnitude of $\mathbf{V}_{C_2}$ is determined from the graphical construction shown, and

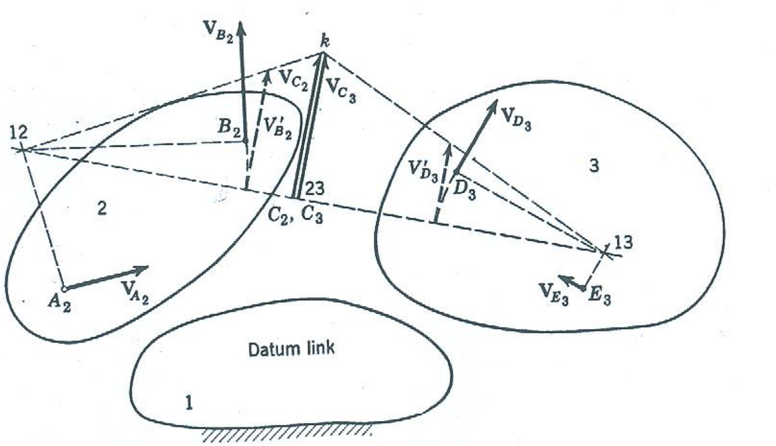

FIGURE 8.24

$\mathbf{V}_{C_3}$ is determined in a similar manner. From the intersection of the construction lines at k, a common location of C_2 and C_3 is determined such that the absolute velocities $\mathbf{V}_{C_2}$ and $\mathbf{V}_{C_3}$ are identical. This location is the instantaneous center 23, since the absolute velocities of the coincident particles are common and the relative velocities $\mathbf{V}_{C_2C_3}$ and $\mathbf{V}_{C_3C_2}$ are zero. It should be obvious that 23 is on a straight line with 12 and 13 in order for the directions of $\mathbf{V}_{C_2}$ and $\mathbf{V}_{C_3}$ to be common.

Kennedy's theorem is extemely useful in determining the locations of instantaneous centers in mechanisms having a large number of links, many of which are in general plane motion.

8.15 DETERMINATION OF INSTANTANEOUS CENTERS BY KENNEDY'S THEOREM

In a mechanism consisting of n links, there are $n - 1$ instantaneous centers relative to any given link. For n number of links, there is a total of $n(n-1)$ instantaneous centers. However, since for each location of instantaneous centers there are two centers, the total number N of locations is given by

$$N = \frac{n(n-1)}{2}$$

The number of locations of centers increases rapidly with numbers of links as shown below.

n LINKS	N CENTERS
4	6
5	10
6	15
7	21

Example 8.6. For the Whitworth mechanism shown in Fig. 8.25, determine the 15 locations of instantaneous centers of zero velocity.

Solution. Because of the large number of locations to be determined, it is desirable to use a system of accounting for the centers as they are determined. The circle diagram shown in Fig. 8.25 is one of the simplest means of accounting. The numbers of the links are designated on the periphery of the circle, and the chord linking any two numbers represents an instantaneous center. In the upper circle are shown eight centers which may be determined by inspection. Five of the centers (12, 14, 23, 45, and 56) are at pin-jointed connections as shown. Two centers (16 and 34) are at infinity, since link 6 is in translation relative to link 1, and link 3 is in translation relative to link 4. Because the absolute velocity directions of points B and C of link 5 are known, the intersection of the normals locates 15. Thus, eight centers are located by inspection, as shown by the solid lines on the circle diagram.

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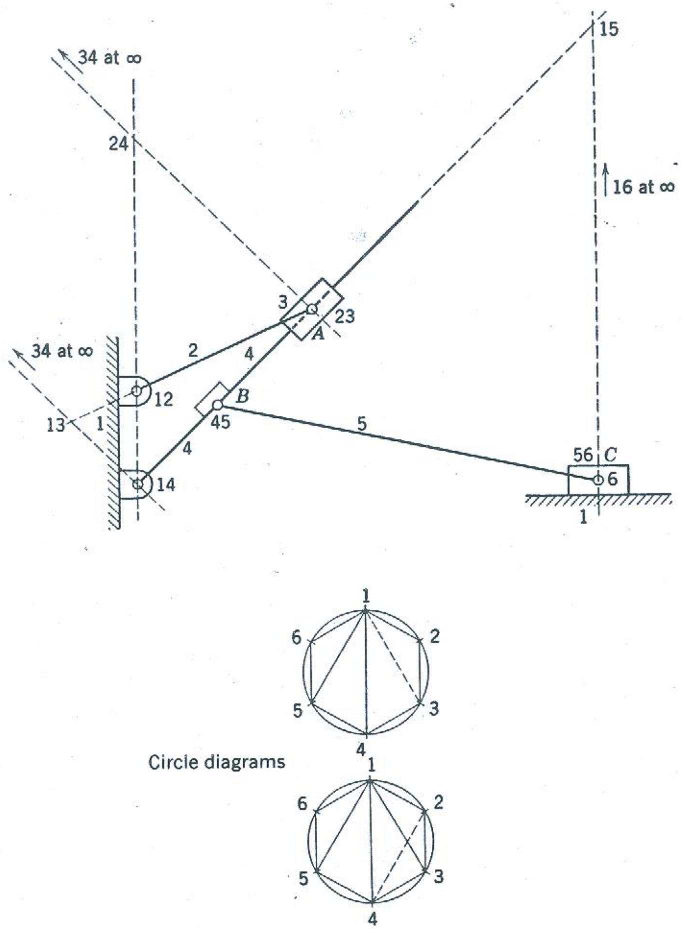

FIGURE 8.25

For centers less obviously determined, Kennedy's theorem may be used. In the upper circle, to locate center 13, a dashed line is drawn such that it closes two triangles. The triangle 1–2–3 represents the three centers (12, 23, and 13) of links 1, 2, and 3, which according to Kennedy's theorem lie on a straight line. Similarly, triangle 1–3–4 represents the centers 13, 34, and 14, which also lie on a straight line. The intersection of the two lines on the mechanism locates the center 13, which must lie on both lines. The dashed line may be made solid to indicate that the unknown center has been located. The lower circle shows the next step in which the center 24 is located using triangles 2–3–4 and 1–2–4. It may be seen that 24 is the logical center to determine rather than 25 or 26, which cannot be drawn as common to two triangles until other centers have been determined.

In Fig. 8.25, 10 of the 15 centers are shown. Figure 8.26 shows the same mechanism with all 15 centers located.

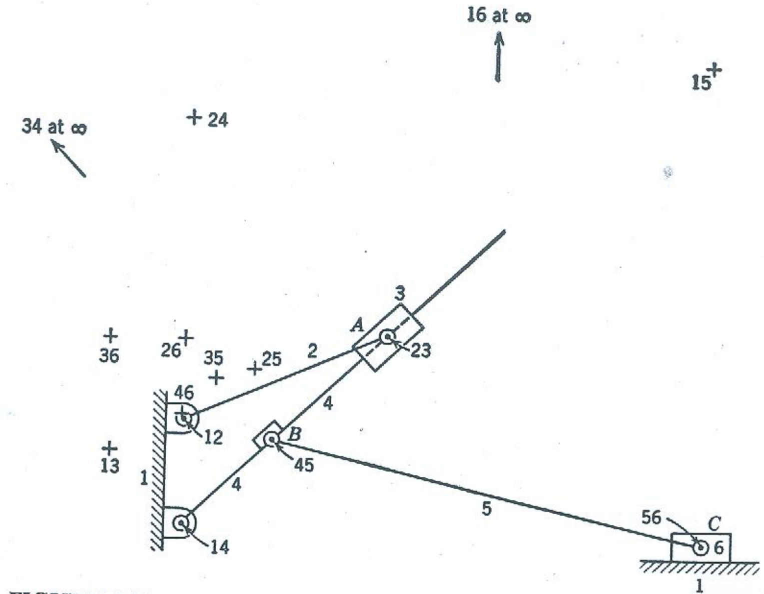

FIGURE 8.26

8.16 DETERMINATION OF VELOCITY BY INSTANTANEOUS CENTERS

Kennedy's theorem may be used to great advantage in determining directly the absolute velocity of any given particle of a mechanism without necessarily determining the velocities of intermediate particles as required by the vector polygon method. In connection with the Whitworth mechanism of Fig. 8.25, for example, the velocity of the tool support (link 6) may be determined from the known speed of the driving link 2 without first determining the velocities of points on the connecting links 3, 4, and 5.

Example 8.7. For the Whitworth mechanism shown in Fig. 8.27, determine the absolute velocity $\mathbf{V}_C$ of the tool support when the driving link 2 rotates at a speed such that $\mathbf{V}_A$ = 30 ft/s as shown.

Solution. Two solutions for $\mathbf{V}_C$ are shown in Fig. 8.27. In the first of these (Fig. 8.27*a*), links 1, 3, and 5 are involved such that instantaneous centers 13, 15, and 35 are used. $\mathbf{V}_A$ is the known absolute velocity of a particle on link 3 relative to link 1; thus, links 3 and 1 are involved. The absolute velocity $\mathbf{V}_C$ is to be determined for a particle on link 5 also relative to link 1, thus involving links 5 and 1. According to Kennedy's theorem, the instantaneous centers 13, 15, and 35 are on a common straight line as shown in Fig. 8.27*a*. Using center 13, the absolute velocity $\mathbf{V}_{P_3}$ for a particle P_3 located at 35 on link 3 may be determined graphically from similar triangles by swinging $\mathbf{V}_A$ to position $\mathbf{V}'_A$ using center 13 as a pivot point. Point 35 represents the location of coincident particles P_3 on link 3 and P_5 on link 5, for which the absolute velocities are common (see Fig. 8.24). Thus,

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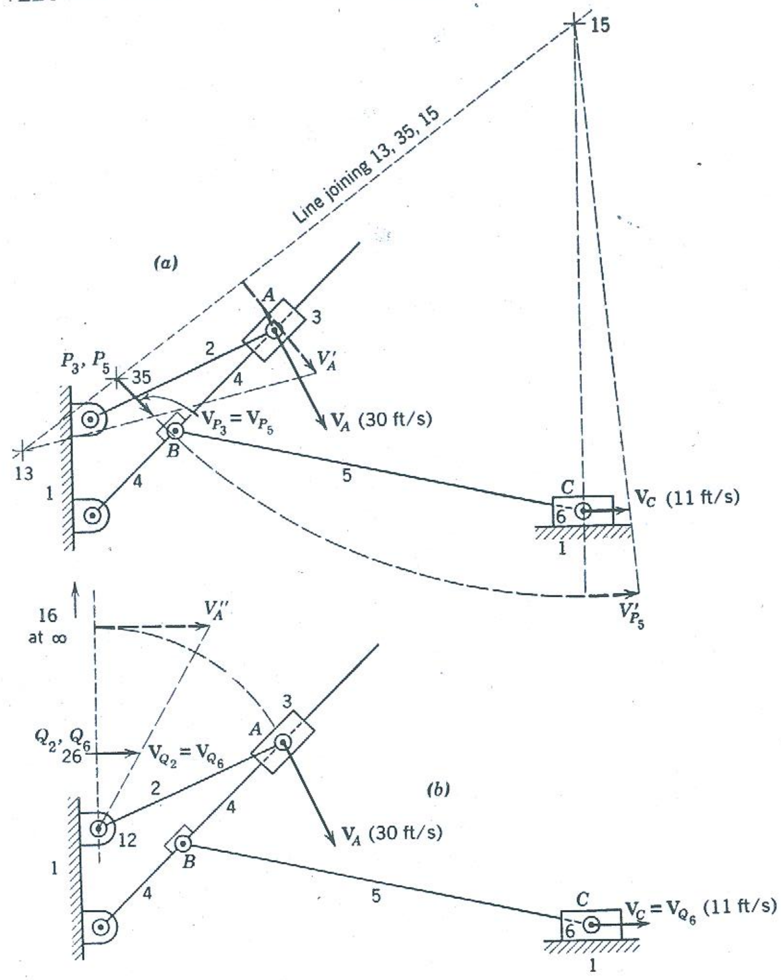

FIGURE 8.27

V_{P_3} is also the absolute velocity V_{P_5} of a particle on link 5. Since both P_5 and C are points on link 5, the absolute velocity V_C may be determined from similar triangles by swinging V_{P_5} to position V'_{P_5} using center 15 as a pivot point. The length of V_C is measured to determine magnitude of velocity.

In the above solution, the centers 13 and 15 relative to the fixed link are *pivot points,* and the center 35 of the moving links is the *transfer point.* By properly identifying these points, the determination of velocities becomes systematic.

The second solution (Fig. 8.27*b*) for V_C is similar to the first, in which pivot points 12 and 16 are used because V_A represents the absolute velocity of a particle on link 2 and V_C is the absolute velocity of a particle on link 6. Center 26 is the transfer point representing the location of coincident particles Q_2 and Q_6 on links 2 and 6, for which the absolute

velocities V_{Q_2} and V_{Q_6} are common. V_{Q_2} is determined graphically from V_A using cent 12 as a pivot point. Since pivot point 16 is at infinity, link 6 is in pure translation relati to link 1 so that V_C is the same in magnitude and direction as V_{Q_2} and V_{Q_6}, as shown.

8.17 ROLLING ELEMENTS

The method of instantaneous centers is frequently applied to mechanisms cor sisting of rolling elements as in epicyclic gear trains (Fig. 8.28). As shown pre viously, the relative velocity of the coincident particles at the point of contact c two rolling links is zero. Thus, an instantaneous center exists at the point o contact.

For the reduction drive shown in Fig. 8.28, the instantaneous centers ar as shown. The speed reduction ratio ω_{31}/ω_{41} (the internal gear speed to carrie speed when the sun gear is fixed) may be determined from linear velocities o particles as shown. Assuming that the absolute angular velocity ω_{41} of the carrie is known, V_A may be determined considering A as a particle on link 4. V_A is alsc the absolute velocity of a particle on link 2; therefore, using the center 12, the absolute velocity V_{P_2} of P_2 on link 2 may be determined graphically from similar triangles. Since center 23 is the location of coincident particles on links 2 and 3 having a common absolute velocity, ω_{31} may be calculated from V_{P_3}.

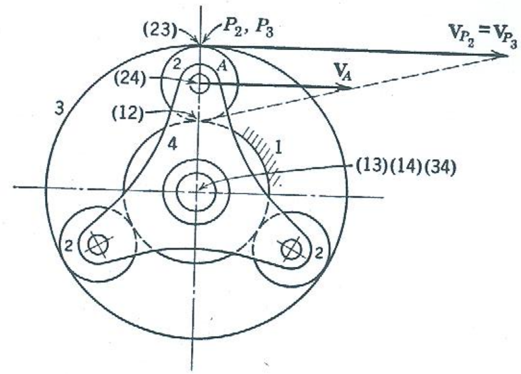

FIGURE 8.28

8.18 GRAPHICAL DETERMINATION OF ACCELERATION IN MECHANISMS BY VECTOR POLYGONS

As in the determination of velocities of particles in a mechanism, the linear accelerations of particles may also be determined by graphical construction of acceleration polygons and acceleration images. It is important that the relative acceleration of pairs of particles be understood.

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8.19 RELATIVE ACCELERATION OF PARTICLES IN MECHANISMS

If the acceleration $\mathbf{A}_Q$ of a particle Q is known, the acceleration of another particle $\mathbf{A}_P$ may be determined by adding the relative acceleration vector $\mathbf{A}_{PQ}$ as shown in the following vector equation:

$$\mathbf{A}_P = \mathbf{A}_Q + \mathbf{A}_{PQ} \tag{8.26}$$

As discussed in the sections on relative velocity, it is shown that the relative velocity of a pair of particles depends on the type of constraint used in a given mechanism. Similarly, the relative acceleration $\mathbf{A}_{PQ}$ in mechanisms depends on the type of built-in constraint.

8.20 RELATIVE ACCELERATION OF PARTICLES IN A COMMON LINK

As shown in Fig. 8.29*a*, when two particles P and Q in the same rigid link are considered, the fixed distance PQ constrains particle P to move on a circular arc relative to Q regardless of the absolute linear motion of Q. Therefore, since the path of P relative to Q is circular, the acceleration vector $\mathbf{A}_{PQ}$ may be represented by the perpendicular components of acceleration $\mathbf{A}^n_{PQ}$ and $\mathbf{A}^t_{PQ}$ normal and tangent, respectively, to the relative path at P. Regardless of the linear absolute acceleration of Q, the angular motions of the link relative to Q are the same as relative to the fixed link because a particle such as Q has no angular motion. For the circular path of P relative to Q, the angular velocity ω_r of the radius of curvature PQ is the same as the absolute angular velocity ω_3 of the link. Also, the angular acceleration α_r of the radius of curvature is the same as the absolute angular acceleration α_3 of the link.

The magnitude of the normal relative acceleration $\mathbf{A}^n_{PQ}$ may be determined from Eq. 8.4*a*:

$$|\mathbf{A}^n_{PQ}| = (PQ)\omega_3^2 = \frac{V^2_{PQ}}{PQ} \tag{8.27}$$

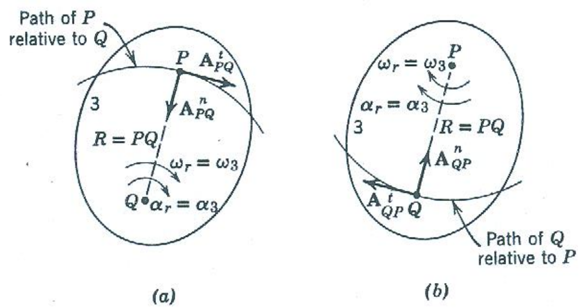

FIGURE 8.29

The magnitude of the tangential relative acceleration $\mathbf{A}^t_{PQ}$ may be determined from Eq. 8.4*b*:

$$|\mathbf{A}^t_{PQ}| = (PQ)\alpha_3 \tag{8.28}$$

Because the relative path is circular, dR/dt is zero.

Observe that the direction of $\mathbf{A}^n_{PQ}$ is normal to the relative path and that its sense is toward the center of curvature Q so that the vector is directed from P toward Q as shown in Fig. 8.29*a*. The direction of $\mathbf{A}^t_{PQ}$ is tangent to the relative path (normal to line PQ), and the sense of the vector depends on the sense of α_r. In Fig. 8.29*b*, the relative acceleration vectors $\mathbf{A}^n_{QP}$ and $\mathbf{A}^t_{QP}$ of Q relative to P are shown where the magnitudes and senses of ω_3 and α_3 are the same as in Fig. 8.29*a*. The relative path shown is that of Q observed at P. It is to be noted that $\mathbf{A}^n_{QP} = -\mathbf{A}^n_{PQ}$ and $\mathbf{A}^t_{QP} = -\mathbf{A}^t_{PQ}$, where the minus signs indicate "opposite in sense."

Example 8.8. When the mechanism is in the phase shown in Fig. 8.30*a*, link 2 rotates with the angular velocity ω_2 of 30 rad/s and an angular acceleration α_2 of 240 rad/s² in the directions given. Determine the acceleration $\mathbf{A}_B$ of point B, the acceleration $\mathbf{A}_C$ of point C, the angular acceleration α_3 of link 3, the angular acceleration α_4 of link 4, and

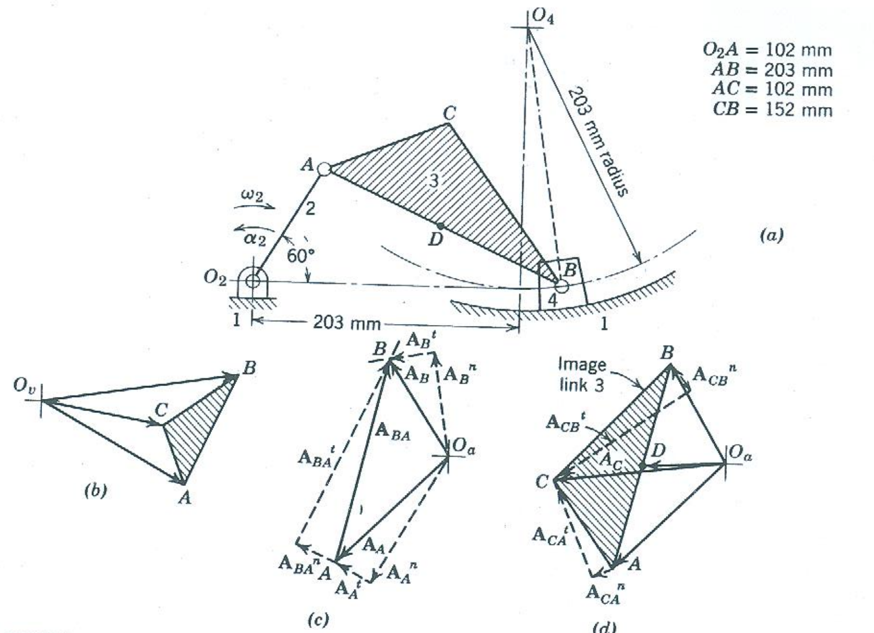

FIGURE 8.30

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the relative acceleration α_{34}. Velocity and acceleration equations can be written as follows:

I. $\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$

II. $\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$

III. $\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$

where

$\mathbf{V}_B$ = direction perpendicular to O_4B, magnitude unknown

$\mathbf{V}_A = (O_2A)\omega_2 = (102)30 = 3060$ mm/s, direction perpendicular to O_2A

$\mathbf{V}_{BA}$ = direction perpendicular to BA, magnitude unknown

$\mathbf{V}_C$ = direction unknown, magnitude unknown

$\mathbf{V}_{CA}$ = direction perpendicular to CA, magnitude unknown

$\mathbf{V}_{CB}$ = direction perpendicular to CB, magnitude unknown

Measured on the polygon of Fig. 8.30*b*, $V_B = 3660$ mm/s, $V_{BA} = 2300$ mm/s, $V_{CA} = 1130$ mm/s, and $V_{CB} = 1750$ mm/s.

IV. $\mathbf{A}_B = \mathbf{A}_A + \mathbf{A}_{BA}$

$$\mathbf{A}_B^n + \mathbf{A}_B^t = \mathbf{A}_A^n + \mathbf{A}_A^t + \mathbf{A}_{BA}^n + \mathbf{A}_{BA}^t$$

where

$\mathbf{A}_B^n = \dfrac{V_B^2}{O_4B} = \dfrac{3660^2}{203} = 66{,}000$ mm/s², direction from B toward O_4

$\mathbf{A}_B^t$ = direction perpendicular to $\mathbf{A}_B^n$, magnitude unknown

$\mathbf{A}_A^n = \dfrac{V_A^2}{O_2A} = \dfrac{3060^2}{102} = 91{,}800$ mm/s², direction from A toward O_2

$\mathbf{A}_A^t = (O_2A)\alpha_2 = (102)240 = 24{,}500$ mm/s, direction perpendicular to $\mathbf{A}_A^n$

$\mathbf{A}_{BA}^n = \dfrac{V_{BA}^2}{BA} = \dfrac{2300^2}{203} = 26{,}100$ mm/s², direction from B toward A

$\mathbf{A}_{BA}^t$ = direction perpendicular to $\mathbf{A}_{BA}^n$, magnitude unknown

Measured on the polygon of Fig. 8.30*c*, $A_B = 70{,}400$ mm/s², $A_B^t = 24{,}700$ mm/s², $A_{BA}^t = 129{,}000$ mm/s², and

$$\alpha_3 = \frac{A_{BA}^t}{BA} = \frac{129{,}000}{203} = 635 \text{ rad/s}^2 \qquad \text{(ccw)}$$

$$\alpha_4 = \frac{A_B^t}{O_4B} = \frac{24{,}700}{203} = 122 \text{ rad/s}^2 \qquad \text{(cw)}$$

$$\alpha_{34} = \alpha_3 - \alpha_4 = 635 - (-122) = 757 \text{ rad/s}^2 \qquad \text{(ccw)}$$

V. $\mathbf{A}_C = \mathbf{A}_A + \mathbf{A}_{CA}^n + \mathbf{A}_{CA}^t$

VI. $\mathbf{A}_C = \mathbf{A}_B + \mathbf{A}_{CB}^n + \mathbf{A}_{CB}^t$

where

$\mathbf{A}_C$ = direction unknown, magnitude unknown

$\mathbf{A}_{CA}^n = \dfrac{V_{CA}^2}{CA} = \dfrac{1130^2}{102} = 12{,}500$ mm/s², direction from C toward A

$\mathbf{A}_{CA}^t$ = direction perpendicular to $\mathbf{A}_{CA}^n$, magnitude unknown

$\mathbf{A}_{CB}^n = \dfrac{V_{CB}^2}{CB} = \dfrac{1750^2}{152} = 20{,}100$ mm/s², direction from C toward B

$\mathbf{A}_{CB}^t$ = direction perpendicular to $\mathbf{A}_{CB}^n$, magnitude unknown

Measured on the polygon of Fig. 8.30*d*, $A_C = 104{,}000$ mm/s².

The velocity polygon of Fig. 8.30*b* shows the determination of $\mathbf{V}_B$ and $\mathbf{V}_{BA}$ from Eq. I. In a similar manner $\mathbf{V}_C$, $\mathbf{V}_{CA}$, and $\mathbf{V}_{CB}$ are determined from Eqs. II and III. The shaded triangle ABC of the velocity polygon is the velocity image of link 3.

Equation IV expresses $\mathbf{A}_B$ in terms of $\mathbf{A}_A$ and $\mathbf{A}_{BA}$, and all of the components of this equation are known as indicated in magnitude, sense, and direction or in direction. In constructing the acceleration polygon Fig. 8.30*c* starting with the right side of Eq. IV, the vector $\mathbf{A}_A^n$ is drawn from pole O_a to which is added $\mathbf{A}_A^t$. This gives the vector $\mathbf{A}_A$ whose tip is labeled "A." Next, add the vector $\mathbf{A}_{BA}^n$ starting at point A, and to it add the direction of $\mathbf{A}_{BA}^t$. As can be seen, it is impossible to complete the solution using only the components on the right side of Eq. IV. Therefore, consider the left side of the equation and draw vector $\mathbf{A}_B^n$ from O_a and to it add the direction of $\mathbf{A}_B^t$. The intersection of the direction of $\mathbf{A}_{BA}^t$ and the direction of $\mathbf{A}_B^t$ completes the polygon. Arrowheads are now added to the vectors $\mathbf{A}_{BA}^t$ and $\mathbf{A}_B^t$ so that the addition of the vectors of the polygon agrees with the addition of the terms of Eq. IV. The resultant of the vectors $\mathbf{A}_B^n$ and $\mathbf{A}_B^t$ gives $\mathbf{A}_B$ whose tip is labeled "*B*." The resultant of $\mathbf{A}_{BA}^n$ and $\mathbf{A}_{BA}^t$ is also shown on the polygon.

The magnitudes and senses of α_3 and α_4 can now be determined from $\mathbf{A}_{BA}^t$ and $\mathbf{A}_B^t$, respectively, as shown.

To determine $\mathbf{A}_C$, it is necessary to use Eqs. V and VI, which give the relations between $\mathbf{A}_C$ and $\mathbf{A}_A$ and $\mathbf{A}_B$. The components of these equations are known as indicated. For clarity, the acceleration vectors $\mathbf{A}_A$ and $\mathbf{A}_B$ are redrawn in Fig. 8.30*d* from Fig. 8.30*c* without their normal and tangential components. Use Eq. V, and draw the vector $\mathbf{A}_{CA}^n$ from point A in Fig. 8.30*d* and to it add the direction of $\mathbf{A}_{CA}^t$. Consider next Eq. VI and draw the vector $\mathbf{A}_{CB}^n$ from point B and to it add the direction of $\mathbf{A}_{CB}^t$. The intersection of the direction of $\mathbf{A}_{CA}^t$ and the direction of $\mathbf{A}_{CB}^t$ completes the polygon. This intersection is point C, which gives $\mathbf{A}_C$. Arrowheads are now added to the vectors $\mathbf{A}_{CA}^t$ and $\mathbf{A}_{CB}^t$ so that the vector addition checks with Eqs. V and VI. The shaded triangle ABC of Fig. 8.30*d* is the acceleration image of link 3.

The acceleration of any point D as shown on link 3 can be determined by locating its corresponding position on the acceleration image of link 3. The vector from O_a to D is $\mathbf{A}_D$ as shown in Fig. 8.30*d*.

8.21 RELATIVE ACCELERATION OF COINCIDENT PARTICLES ON SEPARATE LINKS. CORIOLIS COMPONENT OF ACCELERATION

The next mechanism to be considered is one in which there is relative sliding between two links, as between links 3 and 4 as shown in Fig. 8.31, and it is required to determine ω_4 and α_4 given ω_2 and α_2. In this mechanism, points A_2

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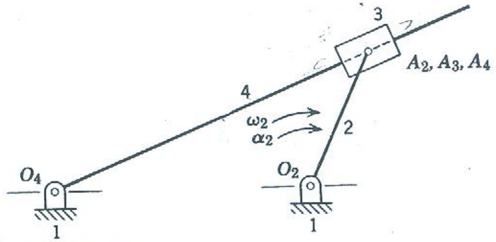

FIGURE 8.31

and A_3 are the same point, and point A_4 is their projection on link 4. To find ω_4 and α_4, the velocity and acceleration of the two coincident points A_2 and A_4, each on separate links, must be analyzed.[3]

The equation for the velocity of point A_4 can be written as follows:

$$\mathbf{V}_{A_4} = \mathbf{V}_{A_2} + \mathbf{V}_{A_4A_2} \tag{8.29}$$

In this equation, $\mathbf{V}_{A_2}$ is known in magnitude, sense, and direction and $\mathbf{V}_{A_4}$ and $\mathbf{V}_{A_4A_2}$ are known in direction. The velocity polygon can easily be drawn, and $\mathbf{V}_{A_4}$ determined, from which ω_4 can be calculated.

The acceleration of point A_4 can be determined from the following equation:

$$\mathbf{A}_{A_4} = \mathbf{A}_{A_2} + \mathbf{A}_{A_4A_2} \tag{8.30}$$

which can be expanded as follows:

$$\mathbf{A}^n_{A_4} + \mathbf{A}^t_{A_4} = \mathbf{A}^n_{A_2} + \mathbf{A}^t_{A_2} + \mathbf{A}^n_{A_4A_2} + \mathbf{A}^t_{A_4A_2} + 2\boldsymbol{\omega}_2 \times \mathbf{V}_{A_4A_2} \tag{8.31}$$

In going from Eq. 8.30 to Eq. 8.31, the following substitution was made:

$$\mathbf{A}_{A_4A_2} = \mathbf{A}^n_{A_4A_2} + \mathbf{A}^t_{A_4A_2} + 2\boldsymbol{\omega}_2 \times \mathbf{V}_{A_4A_2}$$

To determine the relative acceleration between two moving coincident points, it is necessary to add a third component as shown. This component is known as *Coriolis component,* which was developed using vector mathematics in section 8.6. Also, because points A_4 and A_2 are coincident, the terms $\mathbf{A}^n_{A_4A_2}$ and $\mathbf{A}^t_{A_4A_2}$ do not represent the usual normal and tangential components of two points on the same rigid body as previously considered. For this reason, they often appear in the literature written with a capital script $\mathcal{A}$. The magnitude of $\mathbf{A}^n_{A_4A_2}$ can be calculated from the relation

$$|\mathbf{A}^n_{A_4A_2}| = \frac{V^2_{A_4A_2}}{R} \tag{8.32}$$

[3]Point A_3 could have been used instead of A_2 as the point coincident with A_4. However, point A_2 is generally preferred because it is on a link directly connected to the ground and its motion can be easily visualized.

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where R is the radius of curvature of the path of point A_4 relative to point A_2. This component is directed from the coincident points along the radius toward the center of curvature. The tangential component $\mathbf{A}^t_{A_4A_2}$ is known in direction and is tangent to the path of $\mathbf{A}_4$ relative to $\mathbf{A}_2$ at the coincident points. The magnitude of the Coriolis component $2\boldsymbol{\omega}_2 \times \mathbf{V}_{A_4A_2}$ is easily calculated because $\boldsymbol{\omega}_2$ is given data and $\mathbf{V}_{A_4A_2}$ can be determined from the velocity polygon. The direction of this component is normal to the path of A_4 relative to A_2, and its sense is the same as that of $\mathbf{V}_{A_4A_2}$ rotated about its origin 90° in the direction of ω_2. An example of this method of determining the direction will be given in a later section.

In Eq. 8.31, all of the components can easily be determined in magnitude, sense, and direction or in direction except $\mathbf{A}^n_{A_4A_2}$. This component calculated from $V^2_{A_4A_2}/R$ can only be determined if the instantaneous radius of curvature R of the path of A_4 relative to A_2 is known. Unfortunately, because this path is not easily determined for the mechanism shown in Fig. 8.31, it is necessary to rewrite Eq. 8.31 in the following form:

$$\mathbf{A}^n_{A_2} + \mathbf{A}^t_{A_2} = \mathbf{A}^n_{A_4} + \mathbf{A}^t_{A_4} + \mathbf{A}^n_{A_2A_4} + \mathbf{A}^t_{A_2A_4} + 2\boldsymbol{\omega}_4 \times \mathbf{V}_{A_2A_4} \tag{8.33}$$

With Eq. 8.31 written in this form, $\mathbf{A}^n_{A_2A_4}$ can easily be evaluated as zero because the path of A_2 relative to link 4 (which contains point A_4) is a straight line and R is infinite. The acceleration polygon can now be drawn and $\mathbf{A}^t_{A_4}$ determined, from which α_4 is calculated.

While it is easy to see in Fig. 8.31 that the path of point A_2 relative to point A_4 is a straight line by inverting the mechanism and letting link 4 be the fixed link, it is very difficult to visualize the path of A_4 relative to A_2. As a means of determining this path, consider Fig. 8.32, where link 2 is now the fixed link. In this figure, link 1 is placed in a number of angular positions relative to link 2, and the relative position of A_4 is determined for each position of link 1. It may be seen that the position of link 4 is always in a direction from O_4 through A_2 and that A_4 is a fixed distance from O_4. As shown, the path of A_4 on link 2 is curvilinear and tangent to link 4 at point A_2. Unfortunately, the path is not circular so that the radius of curvature is difficult to determine.

Consider next the case where link 4 of Fig. 8.31 has been replaced by a curved link of circular form as shown in Fig. 8.33. In this linkage, the path of A_2 relative to A_4 is a circular arc of known radius and center of curvature. The magnitude of $\mathbf{A}^n_{A_2A_4}$ is therefore not zero, and the vector representing this component will be directed from point A toward the center of curvature C.

The Coriolis component is always in the same direction as the $\mathbf{A}^n_{A_2A_4}$ component, if one exists, but its sense may or may not be the same. Considering the Coriolis term $2\boldsymbol{\omega}_4 \times \mathbf{V}_{A_2A_4}$ for the linkage of Fig. 8.33, its direction and sense can easily be determined as follows. Draw the vector representing the relative velocity $\mathbf{V}_{A_2A_4}$ in its correct direction and sense. Rotate this vector 90° about its origin in the same sense as ω_4. This will give the direction and sense of the Coriolis component as shown in Fig. 8.34. As can be seen, the terms $\mathbf{A}^n_{A_2A_4}$ and $2\boldsymbol{\omega}_4 \times \mathbf{V}_{A_2A_4}$ have the same sense for this case and will therefore add together. Obviously, this method of determining the direction and sense of Coriolis applies even if the $\mathbf{A}^n_{A_2A_4}$ component is zero.

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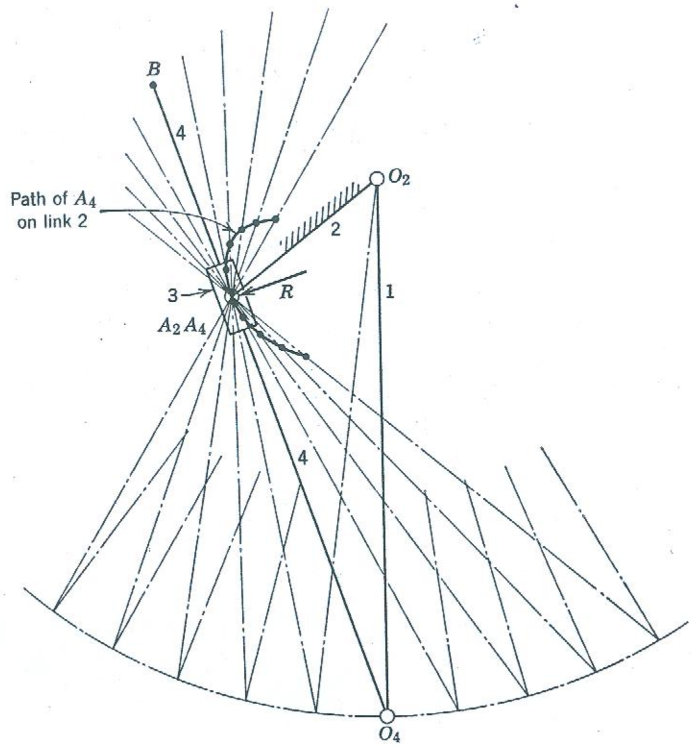

FIGURE 8.32

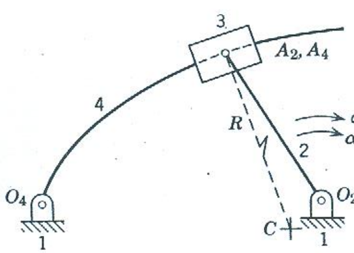

FIGURE 8.33

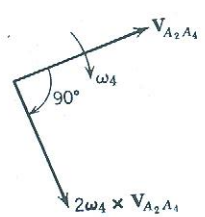

FIGURE 8.34

Example 8.9. In the crank-shaper mechanism shown in Fig. 8.35*a* link 2 rotates at a constant angular velocity ω_2 of 10 rad/s. Determine the acceleration $\mathbf{A}_{A_4}$ of point A_4 on link 4 and the angular acceleration α_4 when the mechanism is in the phase shown. Velocity and acceleration equations can be written as follows:

I. $\mathbf{V}_{A_4} = \mathbf{V}_{A_2} + \mathbf{V}_{A_4A_2}$

where

$\mathbf{V}_{A_4}$ = direction perpendicular to O_4A_4, magnitude unknown

$\mathbf{V}_{A_2} = (O_2A_2)\omega_2 = (4)10 = 40$ in./s, direction perpendicular to O_2A_2

$\mathbf{V}_{A_4A_2}$ = direction parallel to O_4A_4, magnitude unknown

Measured on the polygon of Fig. 8.35*b*, $V_{A_4} = 13$ in./s, $V_{A_4A_2} = 38$ in./s, and

$$\omega_4 = \frac{V_{A_4}}{O_4A_4} = \frac{13}{10} = 1.3 \text{ rad/s} \qquad \text{(ccw)}$$

II. $\mathbf{A}_{A_4} = \mathbf{A}_{A_2} + \mathbf{A}_{A_4A_2}$

III. $\mathbf{A}_{A_2} = \mathbf{A}_{A_4} + \mathbf{A}_{A_2A_4}$

$$\mathbf{A}^n_{A_2} + \mathbf{A}^t_{A_2} = \mathbf{A}^n_{A_4} + \mathbf{A}^t_{A_4} + \mathbf{A}^n_{A_2A_4} + \mathbf{A}^t_{A_2A_4} + 2\boldsymbol{\omega}_4 \times \mathbf{V}_{A_2A_4}$$

where

$$\mathbf{A}^n_{A_2} = \frac{V^2_{A_2}}{O_2A_2} = \frac{40^2}{4} = 400 \text{ in./s}^2\text{, direction from } A_2 \text{ toward } O_2$$

$$\mathbf{A}^t_{A_2} = 0 \qquad (\alpha_2 = 0)$$

$$\mathbf{A}^n_{A_4} = \frac{V^2_{A_4}}{O_4A_4} = \frac{13^2}{10} = 16.9 \text{ in./s}^2\text{, direction from } A_4 \text{ toward } O_4$$

$\mathbf{A}^t_{A_4}$ = direction perpendicular to $\mathbf{A}^n_{A_4}$, magnitude unknown

$$\mathbf{A}^n_{A_2A_4} = \frac{V^2_{A_2A_4}}{R} = 0 \qquad (R = \infty)$$

$2\boldsymbol{\omega}_4 \times \mathbf{V}_{A_2A_4} = 2(1.3)38 = 98.8$ in./s², direction perpendicular to $\mathbf{V}_{A_2A_4}$

$\mathbf{A}^t_{A_2A_4}$ = direction perpendicular to $2\boldsymbol{\omega}_4 \times \mathbf{V}_{A_2A_4}$, magnitude unknown

Measured on the polygon of Fig. 8.35*c*, $A_{A_4} = 475$ in./s², $A^t_{A_4} = 474$ in./s², and

$$\alpha_4 = \frac{A^t_{A_4}}{O_4A_4} = \frac{474}{10} = 47.4 \text{ rad/s}^2 \qquad \text{(cw)}$$

Link 4 is a guide link which constrains points A_2 and A_3 to follow a straight-line path on link 4. Two pairs of coincident points may be considered, either A_2 and A_4 or A_3 and A_4. For this illustration, A_2 and A_4 are chosen, and the straight guide path is the relative path of A_2 on link 4. Thus, the vectors $\mathbf{V}_{A_2A_4}$ and $\mathbf{A}_{A_2A_4}$ are involved, and the $\mathbf{A}^n_{A_2A_4}$ component of $\mathbf{A}_{A_2A_4}$ can easily be determined because $R = \infty$.

The velocity polygon of Fig. 8.35*b* shows the determination of $\mathbf{V}_{A_4}$ and $\mathbf{V}_{A_4A_2}$ from Eq. I. The calculation for ω_4 is also shown.

Equation II expresses $\mathbf{A}_{A_4}$ in terms of $\mathbf{A}_{A_2}$ and $\mathbf{A}_{A_4A_2}$. However, because the path of point A_4 relative to point A_2 is not easily determined, Eq. II is rewritten in the form of Eq. III so as to use the component $\mathbf{A}_{A_2A_4}$ as discussed above.

All of the components of Eq. III are known as indicated in magnitude, sense, and direction or in direction only. In constructing the acceleration polygon of Fig. 8.35*c* starting with the right side of Eq. III, the vector $\mathbf{A}^n_{A_4}$ is drawn first, followed by the direction of

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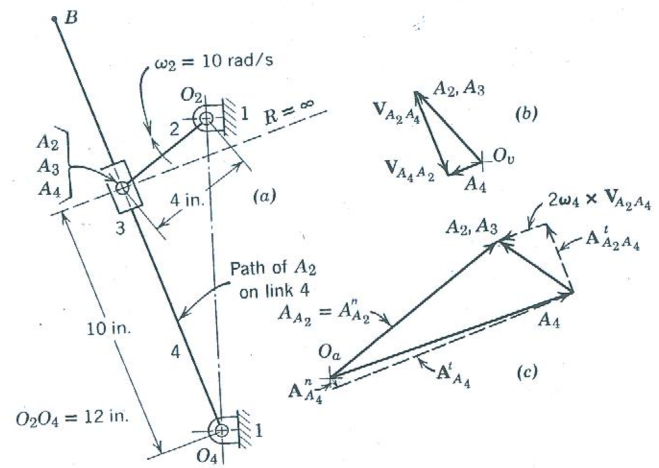

FIGURE 8.35

$\mathbf{A}^t_{A_4}$. This is all that can be laid off from the right side of Eq. III at present. Therefore, consider the left side of Eq. III and draw the vector $\mathbf{A}_{A_2}$. Next, draw the vector $2\boldsymbol{\omega}_4 \times \mathbf{V}_{A_2A_4}$ so that its tip meets the tip of vector $\mathbf{A}_{A_2}$. Draw $\mathbf{A}^t_{A_2A_4}$ perpendicular to the Coriolis component until it intersects the direction of the vector representing $\mathbf{A}^t_{A_4}$; this completes the polygon. Arrowheads are now added to the vectors $\mathbf{A}^t_{A_4}$ and $\mathbf{A}^t_{A_2A_4}$ so that the addition of the vectors of the polygon agrees with the addition of the terms of Eq. III. The magnitude and sense of α_4 can now be determined from $\mathbf{A}^t_{A_4}$ as shown.

Example 8.10. In the mechanism shown in Fig. 8.36*a*, link 2 drives link 3 through a pin at point B. Link 2 rotates at a uniform angular velocity ω_2 of 50 rad/s, and the radius of curvature R of the slot in link 3 is 305 mm. Determine the acceleration $\mathbf{A}_{B_3}$ of point B_3 on link 3 and the angular acceleration α_3 for the position shown. Velocity and acceleration equations can be written as follows:

I. $\mathbf{V}_{B_3} = \mathbf{V}_{B_2} + \mathbf{V}_{B_3B_2}$

where

$\mathbf{V}_{B_3}$ = direction perpendicular to O_3B_3, magnitude unknown

$\mathbf{V}_{B_2} = (O_2B_2)\omega_2 = (50.8)50 = 2540$ mm/s, direction perpendicular to O_2B_2

$\mathbf{V}_{B_3B_2}$ = direction perpendicular to $\mathbf{R}$, magnitude unknown

Measured on the polygon of Fig. 8.36*b*, $V_{B_3} = 1650$ mm/s, $V_{B_3B_2} = 2540$ mm/s, and

$$\omega_3 = \frac{V_{B_3}}{O_3B_3} = \frac{1650}{208} = 7.93 \text{ rad/s} \qquad \text{(ccw)}$$

II. $\mathbf{A}_{B_3} = \mathbf{A}_{B_2} + \mathbf{A}_{B_3B_2}$

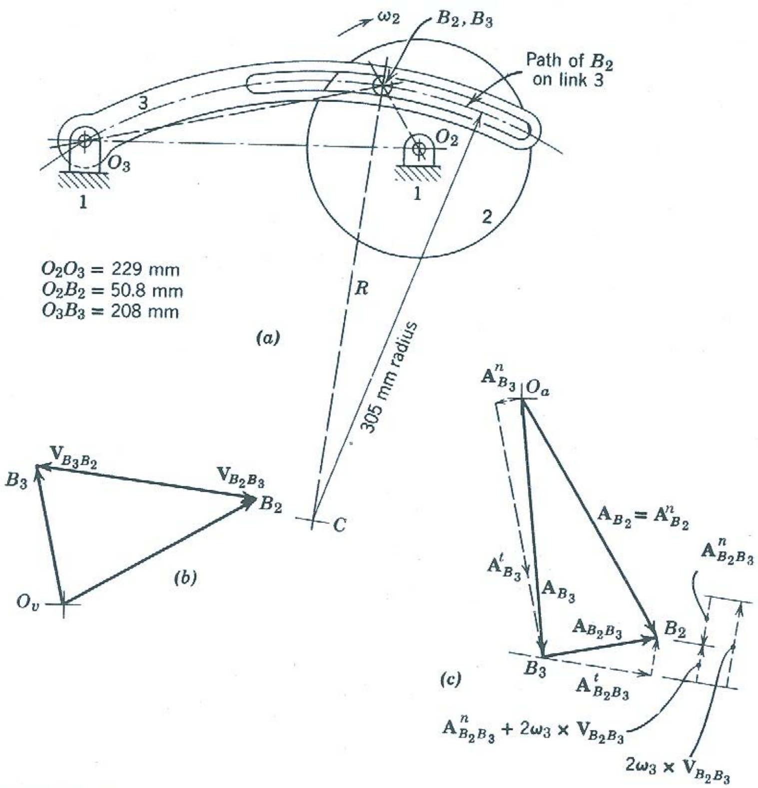

FIGURE 8.36

III. $\mathbf{A}_{B_2} = \mathbf{A}_{B_3} + \mathbf{A}_{B_2B_3}$

$$\mathbf{A}^n_{B_2} + \mathbf{A}^t_{B_2} = \mathbf{A}^n_{B_3} + \mathbf{A}^t_{B_3} + \mathbf{A}^n_{B_2B_3} + \mathbf{A}^t_{B_2B_3} + 2\boldsymbol{\omega}_3 \times \mathbf{V}_{B_2B_3}$$

where

$$\mathbf{A}^n_{B_2} = \frac{V^2_{B_2}}{O_2B_2} = \frac{2540^2}{50.8} = 127{,}000 \text{ mm/s}^2, \text{ direction from } B_2 \text{ toward } O_2$$

$$\mathbf{A}^t_{B_2} = 0 \qquad (\alpha_2 = 0)$$

$$\mathbf{A}^n_{B_3} = \frac{V^2_{B_3}}{O_3B_3} = \frac{1650^2}{208} = 13{,}100 \text{ mm/s}^2, \text{ direction from } B_3 \text{ toward } O_3$$

$\mathbf{A}^t_{B_3}$ = direction perpendicular to $\mathbf{A}^n_{B_3}$, magnitude unknown

$$\mathbf{A}^n_{B_2B_3} = \frac{V^2_{B_2B_3}}{R} = \frac{2540^2}{305} = 21{,}200 \text{ mm/s}^2, \text{ direction from } B_2 \text{ toward } C$$

$2\boldsymbol{\omega}_3 \times \mathbf{V}_{B_2B_3} = 2(7.93)2540 = 40{,}300$ mm/s², direction perpendicular to $\mathbf{V}_{B_2B_3}$

$\mathbf{A}^t_{B_2B_3}$ = direction perpendicular to $2\boldsymbol{\omega}_3 \times \mathbf{V}_{B_2B_3}$, magnitude unknown

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Measure on the polygon of Fig. 8.36c, $\mathbf{A}_{B_3} = 122{,}000$ mm/s², $\mathbf{A}^t_{B_3} = 120{,}000$ mm/s², and

$$\alpha_3 = \frac{A^t_{B_3}}{O_3B_3} = \frac{120{,}000}{208} = 577 \text{ rad/s}^2 \qquad \text{(cw)}$$

Link 3 is a guide link which constrains point B_2 on link 2 to follow a circular path on link 3. Points B_2 and B_3 on link 3 are coincident, and the circular guide path is the relative path of B_2 on link 3. Therefore, the vectors $\mathbf{V}_{B_2B_3}$ and $\mathbf{A}_{B_2B_3}$ are involved in the analysis.

The velocity polygon of Fig. 8.36b shows the determination of $\mathbf{V}_{B_3}$ and $\mathbf{V}_{B_3B_2}$ from Eq. I. The calculation for ω_3 is also shown.

Equation II gives $\mathbf{A}_{B_3}$ in terms of $\mathbf{A}_{B_2}$ and $\mathbf{A}_{B_3B_2}$. Because the path of B_2 relative to B_3 is known to be a circular arc and the path of B_3 relative to B_2 is not easily determined, Eq. II is rewritten in the form of Eq. III so as to use the component $\mathbf{A}_{B_2B_3}$.

All of the components of Eq. III are known as indicated in magnitude, sense, and direction, or in direction only. The acceleration polygon of Fig. 8.36c is started with the right side of Eq. III by drawing the vector $\mathbf{A}^n_{B_3}$ followed by the direction of $\mathbf{A}^t_{B_3}$. This is all that can be laid off on the right side of Eq. III at the moment, so consider the left side of the equation and draw the vector $\mathbf{A}_{B_2}$. The vectors $\mathbf{A}^n_{B_2B_3}$ and $2\boldsymbol{\omega}_3 \times \mathbf{V}_{B_2B_3}$ have opposite sense. Determine the resultant of these two vectors, and add it to the polygon so that its tip meets the tip of vector $\mathbf{A}_{B_2}$. Draw $\mathbf{A}^t_{B_2B_3}$ perpendicular to $\mathbf{A}^n_{B_2B_3}$ until it intersects the direction of the vector representing $\mathbf{A}^t_{B_3}$; this completes the polygon. Arrowheads are now added to the vectors $\mathbf{A}^t_{B_3}$ and $\mathbf{A}^t_{B_2B_3}$ so that the addition of the vectors of the polygon agrees with the addition of the terms of Eq. III. The magnitude and sense of α_3 can now be determined from $\mathbf{A}^t_{B_3}$ as shown.

8.22 RELATIVE ACCELERATION OF COINCIDENT PARTICLES AT THE POINT OF CONTACT OF ROLLING ELEMENTS

An important type of constraint in mechanisms is that which occurs because one link is constrained to roll on another link without relative sliding of the two surfaces at the point of contact. In Fig. 8.37 are shown the rolling pitch circles of a pair of gears in mesh with particles P_3 on link 3 and P_2 on link 2 coincident in position at the point of contact of the rolling circles. As concluded in an earlier paragraph, the relative velocity $\mathbf{V}_{P_3P_2}$ of the coincident particles is zero, and the absolute velocities $\mathbf{V}_{P_3}$ and $\mathbf{V}_{P_2}$ are identical.

The relative acceleration $\mathbf{A}_{P_3P_2}$ of the coincident particles may be represented by component accelerations, a component $\mathbf{A}^t_{P_3P_2}$ in the t–t direction of the common tangent to the surfaces at the point of contact, and a component $\mathbf{A}^n_{P_3P_2}$ in a direction normal to the surfaces at the point of contact. The tangential component of relative acceleration $\mathbf{A}^t_{P_3P_2}$ is the vector difference of the absolute tangential accelerations $\mathbf{A}^t_{P_3}$ and $\mathbf{A}^t_{P_2}$ shown in Fig. 8.37. Like the tangential velocities $\mathbf{V}_{P_3}$ and $\mathbf{V}_{P_2}$, the tangential accelerations $\mathbf{A}^t_{P_3}$ and $\mathbf{A}^t_{P_2}$ are identical because of the condition of no slipping of the surfaces at the point of contact. No slipping requires that there be no relative motion of the two particles in the direction of possible sliding, which is the tangent direction. Thus, because $\mathbf{A}^t_{P_3}$ and $\mathbf{A}^t_{P_2}$ are identical, the tangential component of acceleration of P_3 relative to P_2 is zero.

The normal component of relative acceleration $\mathbf{A}^n_{P_3P_2}$ is the vector difference

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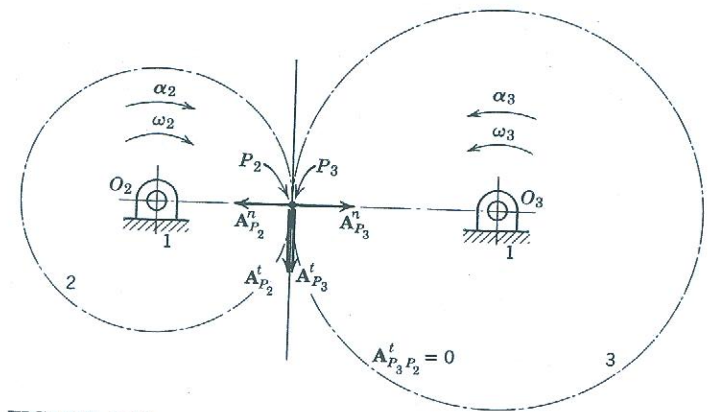

FIGURE 8.37

of the absolute accelerations $\mathbf{A}^n_{P_3}$ and $\mathbf{A}^n_{P_2}$ shown in Fig. 8.37 in the normal direction. It may be seen that the absolute normal acceleration of P_3 is toward O_3 and that of P_2 is toward O_2. These are parallel vectors, but the senses of the vectors are opposite so that the magnitude of $\mathbf{A}^n_{P_3P_2}$ is the sum of the magnitudes of $\mathbf{A}^n_{P_3}$ and $\mathbf{A}^n_{P_2}$. Thus, it is important to observe that a normal relative acceleration $\mathbf{A}^n_{P_3P_2}$ exists although the tangential relative acceleration is zero.

In a mechanism such as is shown in Fig. 8.37 where the centers of the gears are fixed, it is not necessary to draw an acceleration polygon to determine $\mathbf{A}_{P_3}$ and α_3. The angular acceleration α_3 can easily be determined from α_2 and from the ratio of the gear radii using the fact that $\mathbf{A}^t_{P_3} = \mathbf{A}^t_{P_2}$. After α_3 and ω_3 have been found, the components $\mathbf{A}^n_{P_3}$ and $\mathbf{A}^t_{P_3}$ can be calculated and combined to give $\mathbf{A}_{P_3}$. In more complex cases where gear centers are in motion, as in the following example, it is recommended that solutions be undertaken using polygon construction.

Example 8.11. In the mechanism shown in Fig. 8.38a, gear 2 rotates about O_2 with a constant angular velocity ω_2 of 10 rad/s, and gear 3 rolls on gear 2. Determine the acceleration $\mathbf{A}_{P_3}$ of point P_3 on gear 3 and the velocity and acceleration images of gears 2 and 3. Velocity and acceleration equations can be written as follows:

I. $\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$

II. $\mathbf{V}_{P_2} = \mathbf{V}_A + \mathbf{V}_{P_2A}$

where

$\mathbf{V}_B$ = direction perpendicular to O_4B, magnitude unknown

$\mathbf{V}_A$ = $(O_2A)\omega_2 = (2)10 = 20$ in./s, direction perpendicular to O_2A

$\mathbf{V}_{BA}$ = direction perpendicular to line joining points B and A, magnitude unknown

$\mathbf{V}_{P_2}$ = direction perpendicular to O_2P_2, magnitude unknown

$\mathbf{V}_{P_2A}$ = direction perpendicular to P_2A, magnitude unknown

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