Thermal Fluid Engineering
ENMC4411
Chapter 5 Viscous flow in ducts

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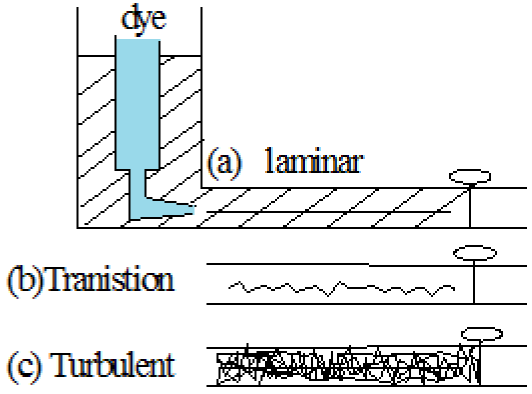
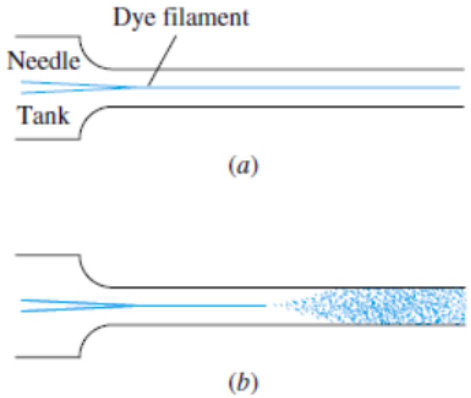
Outline

- Laminar and Turbulent flow
- Entrance effect
- Friction losses in smooth pipes
- Friction losses in rough pipes
- Minor losses
- Energy equation for pipe flow
- Pipe flow problems.

laminar & turbulent flow

- laminar flow : in laminar flow a well – ordered pattern prevails , where by fluid layers are assumed to slide over each other.
- Reynold experiment;
  - when the valve is opened slightly the dye will move through p intact forming a thread , in this case flow is laminar.
  - when valve is progressively opened the dye assumed fluctuating motion as it flows through the pipe here transitions is taking place
  - further opening of valve results in a condition where by an irregular fluctuation is developed in the flow . When the dye is completely dispersed in the pipe . This irregular flow is turbulent flow .





Laminar & Turbulent Flow

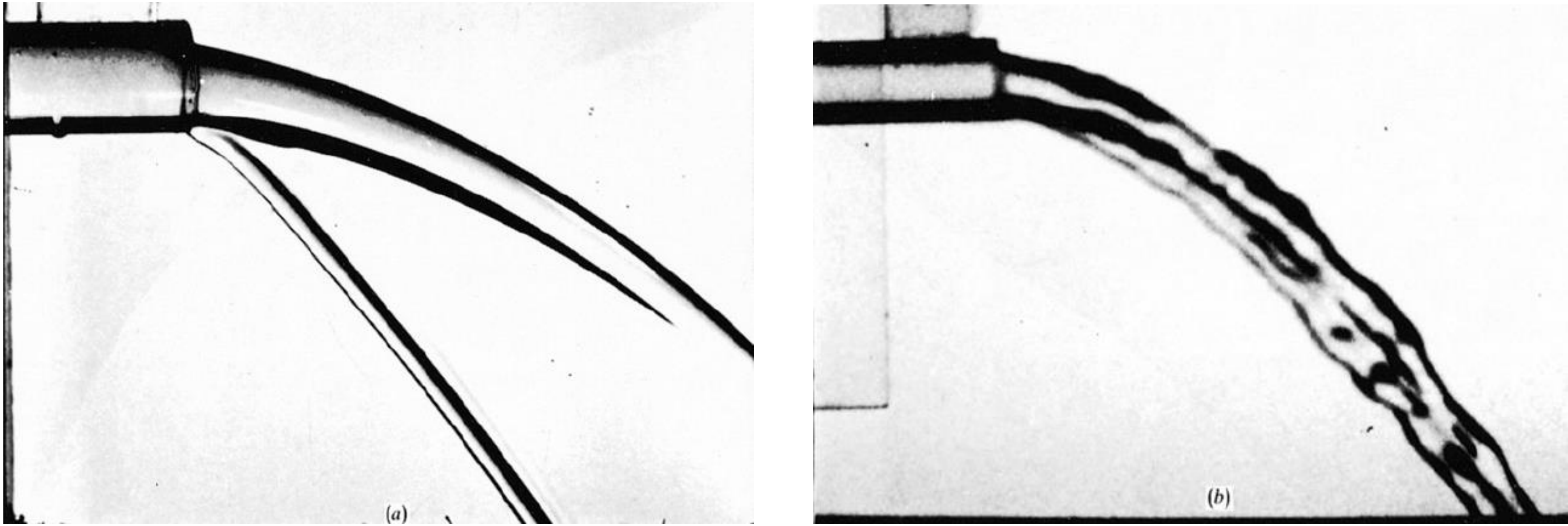


Fig. 6.2 Flow issuing at constant speed from a pipe: (*a*) high viscosity, low-Reynolds-number, laminar flow; (*b*) low-viscosity, high-Reynolds-number, turbulent flow.

Laminar & Turbulent Flow

- Re is the criteria of transition from laminar to Turbulent

  $Re = (\rho VD/\mu) = (VD/\nu) =$ (Inertial force / Viscous force)

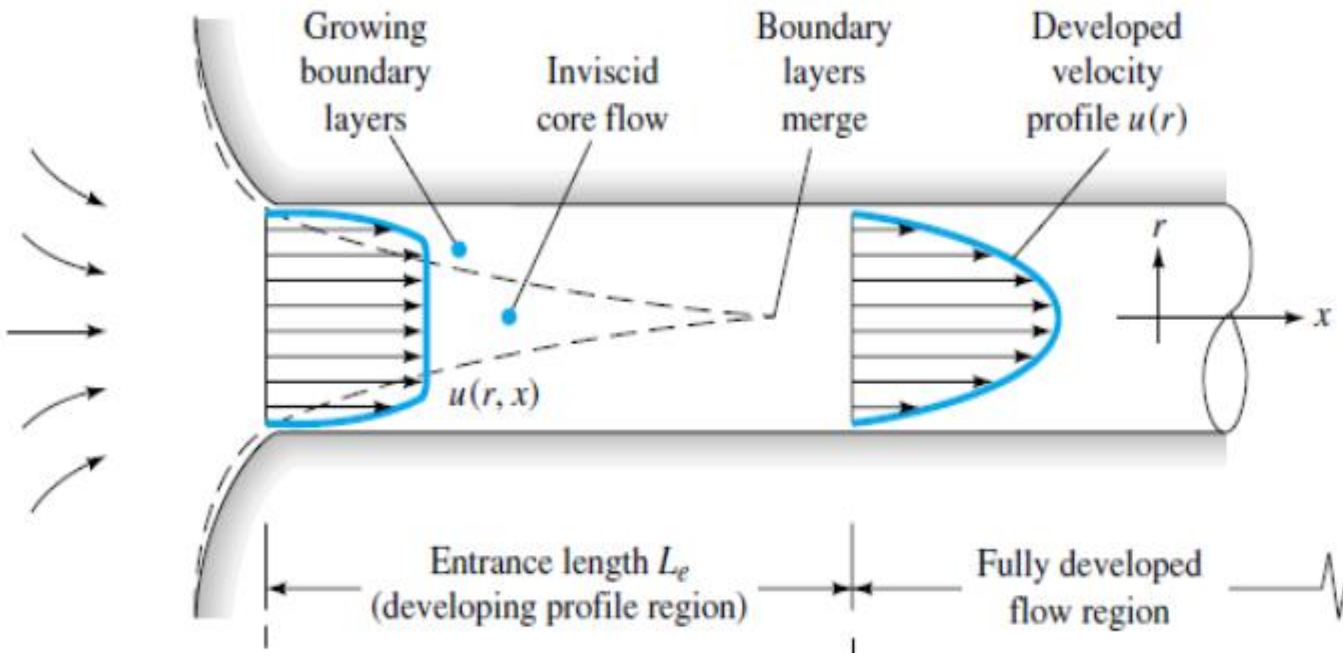
- For pipe flow $Re_{critical} = 2300$

  200<Re <4000 transition

  Re > 4000 well developed turbulent flow .

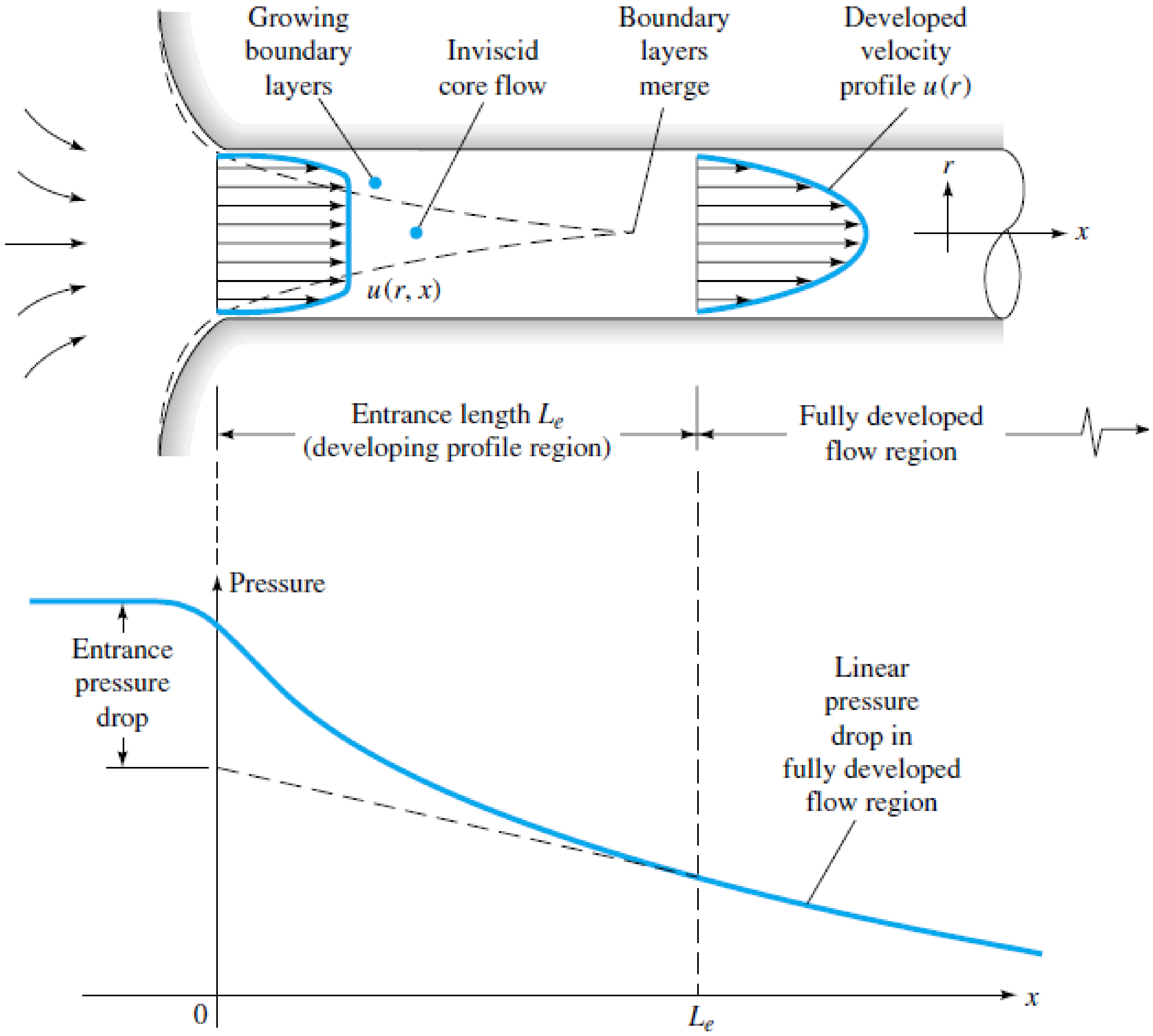
Entrance effect

- An internal flow is constrained by the bounding walls, and the viscous effects will grow and meet and permeate the entire flow.
- Viscous boundary layers grow downstream, retarding the axial flow $u(r, x)$ at the wall and thereby accelerating the center-core flow to maintain the incompressible continuity requirement.
- At a finite distance from the entrance, the boundary layers merge and the inviscid core disappears.
- The tube flow is then entirely viscous, and the axial velocity adjusts
- slightly further until at $x = L_e$ it no longer changes with x and is said to be *fully developed,* $u = u(r)$ only.



Entrance effect

- Downstream of $x \geq Le$ the velocity profile is constant, the wall shear is constant, and the pressure drops linearly with x, for either laminar or turbulent flow.



Entrance length

- Dimensional analysis shows that the Reynolds number is the only parameter affecting entrance length.

$$L_e = f(d, V, \rho, \mu)$$

$$\frac{L_e}{d} = g\left(\frac{\rho V d}{\mu}\right) = g(\text{Re})$$

- For laminar flow the accepted correlation is $\frac{L_e}{d} \approx 0.06\ \text{Re}$ laminar
- In turbulent flow the boundary layers grow faster, and *Le* is relatively shorter, according to the approximation for smooth walls

$$\frac{L_e}{d} \approx 4.4\ \text{Re}_d^{1/6} \qquad \text{turbulent}$$

EXAMPLE 6.2

- A ½ in-diameter water pipe is 60 ft long and delivers water at 5 gal/min at 20°C. What fraction of this pipe is taken up by the entrance region?

$$Q = (5\ \text{gal/min})\frac{0.00223\ \text{ft}^3/\text{s}}{1\ \text{gal/min}} = 0.0111\ \text{ft}^3/\text{s}$$

$$V = \frac{Q}{A} = \frac{0.0111\ \text{ft}^3/\text{s}}{(\pi/4)[(\frac{1}{2}/12)\ \text{ft}]^2} = 8.17\ \text{ft/s}$$

$$\text{Re}_d = \frac{Vd}{\nu} = \frac{(8.17\ \text{ft/s})[(\frac{1}{2}/12)\ \text{ft}]}{1.09 \times 10^{-5}\ \text{ft}^2/\text{s}} = 31{,}300$$

This is greater than 2300; hence the flow is fully turbulent

$$\frac{L_e}{d} \approx 4.4\ \text{Re}_d^{1/6} = (4.4)(31{,}300)^{1/6} = 25 \qquad \frac{L_e}{L} = \frac{25}{1440} = 0.017 = 1.7\%$$

Flow in a Circular Pipe, h_f

- The steady-flow energy equation for pipe section 1 to 2

$$\frac{P_1}{\gamma} + \frac{{V_1}^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{{V_2}^2}{2g} + Z_2 + hf$$

It can be shown theoretically, derived from mass, linear momentum and energy equation that friction head loss hf

$$h_f = 128\mu QL/\pi\gamma d^4$$

- This formula is known as Hagen- Poiseuille equation and used for friction losses for laminar flow in smooth pipes.

EXAMPLE 6.4

- An oil with 900 kg/m^3 and 0.0002 m^2/s flows upward through an inclined pipe as shown in Fig. E6.4. The pressure and elevation are known at sections 1 and 2, 10 m apart. Assuming steady laminar flow, (*a*) verify that the flow is up, (*b*) compute *hf* between 1 and 2, and compute (*c*) *Q*, (*d*) *V*, and (*e*) Re*d*. Is the flow really laminar?

$$\mu = \rho\nu = (900\ \text{kg/m}^3)(0.0002\ \text{m}^2/\text{s}) = 0.18\ \text{kg/(m}\cdot\text{s)}$$

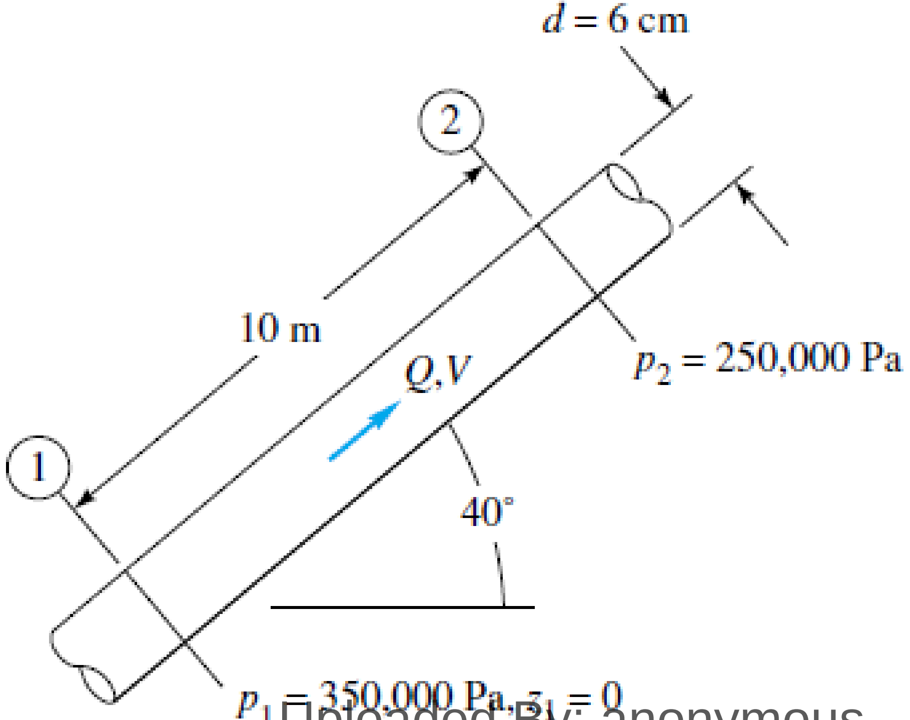
$$z_2 = \Delta L \sin 40° = (10\ \text{m})(0.643) = 6.43\ \text{m}$$

$$\text{HGL}_1 = z_1 + \frac{p_1}{\rho g} = 0 + \frac{350{,}000}{900(9.807)} = 39.65\ \text{m}$$

$$\text{HGL}_2 = z_2 + \frac{p_2}{\rho g} = 6.43 + \frac{250{,}000}{900(9.807)} = 34.75\ \text{m}$$

$$hf = \frac{P_1}{\gamma} - \frac{P_2}{\gamma} + Z_1 - Z_2$$

$$h_f = \text{HGL}_1 - \text{HGL}_2 = 39.65\ \text{m} - 34.75\ \text{m} = 4.9\ \text{m}$$



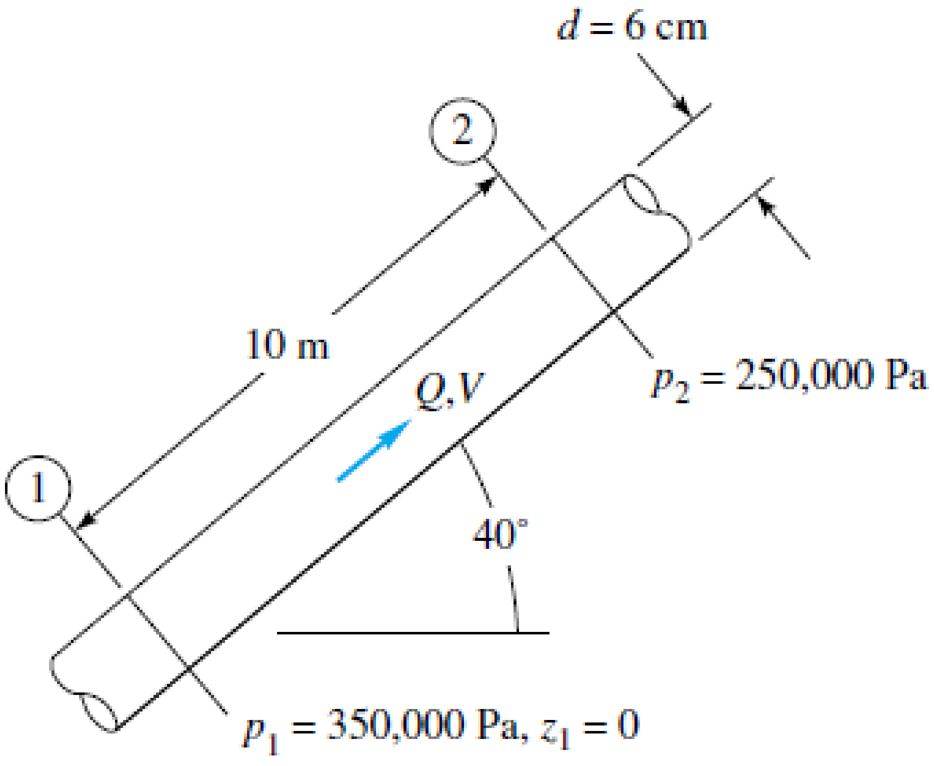
EXAMPLE 6.4

$$h_f = 128\mu QL/\pi\gamma d^4$$

$$Q = \frac{\pi\rho g d^4 h_f}{128\mu L} = \frac{\pi(900)(9.807)(0.06)^4(4.9)}{128(0.18)(10)} = 0.00$$

$$V = \frac{Q}{\pi R^2} = \frac{0.0076}{\pi(0.03)^2} = 2.7 \text{ m/s}$$

$$\mathrm{Re}_d = \frac{Vd}{\nu} = \frac{2.7(0.06)}{0.0002} = 810$$



This is well below the transition value Re*d* 2300, and so we are fairly certain the flow is laminar.

Viscometers

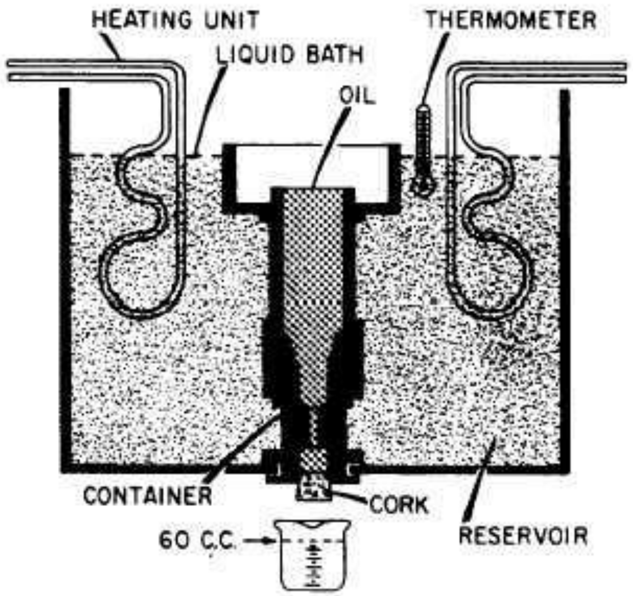
- Saybolt viscometer

$$h_f \approx z_1 - z_2 - \frac{V_2^2}{2g}$$

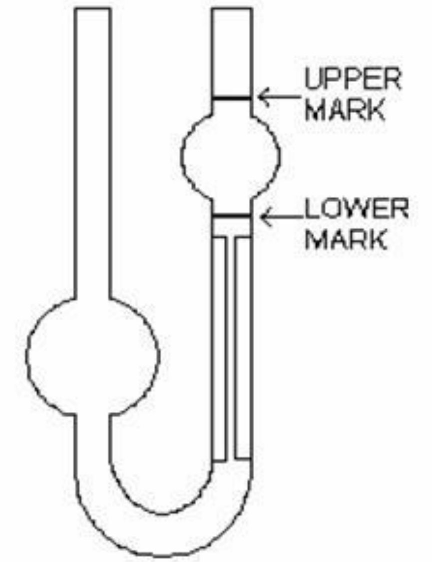
$$hf = 32\mu VL/\gamma d^2$$

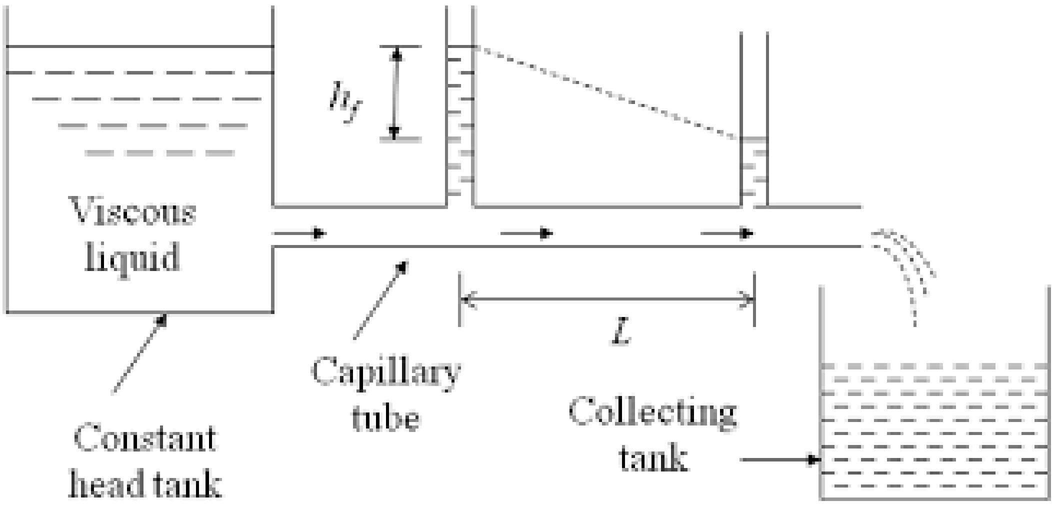
$$h_f = 128\mu QL/\pi\gamma d^4$$

Viscosity in centipoise from seconds Saybot



$$cST = 0.226 \times SUS - \frac{195}{SUS}$$





Friction losses

- For smooth pipes use Hagen- Poiseuille formula if laminar flow.

$$h_f = 128\mu QL/\pi\gamma d^4$$

- For rough and turbulent use correlations based on dimensional analysis.

$$\Delta P = f(d, l, \mu, V, \rho, \varepsilon)$$

$$\Delta P/\rho V^2 = f(\frac{l}{d}, \frac{\varepsilon}{d}, \frac{\rho V d}{\mu})$$

Friction head loss

$$hf = \frac{V^2 l}{2gd} f\left(\frac{\varepsilon}{d}, \frac{\rho V d}{\mu}\right) = (\frac{l}{d})\frac{V^2}{2g} f$$

Where f is known as friction factor and it is a function of Reynold number and pipe roughness. This formula is known as Darcy – Weisback formula. Friction factor is given as

$$f = F(\frac{\varepsilon}{d}, \frac{\rho V d}{\mu}) = \mathrm{F}(\frac{\varepsilon}{d}, \mathrm{Re})$$

Various correlations exist for friction factor including;

for smooth pipe

$$f = \begin{cases} 0.316\,\mathrm{Re}_d^{-1/4} & 4000 < \mathrm{Re}_d < 10^5 \quad \text{H. Blasius (1911)} \\ \left(1.8 \log \dfrac{\mathrm{Re}_d}{6.9}\right)^{-2} & \text{Ref. 9} \end{cases}$$

$$\frac{1}{f^{1/2}} = 2.0 \log(\mathrm{Re}_d f^{1/2}) - 0.8$$

Friction head loss

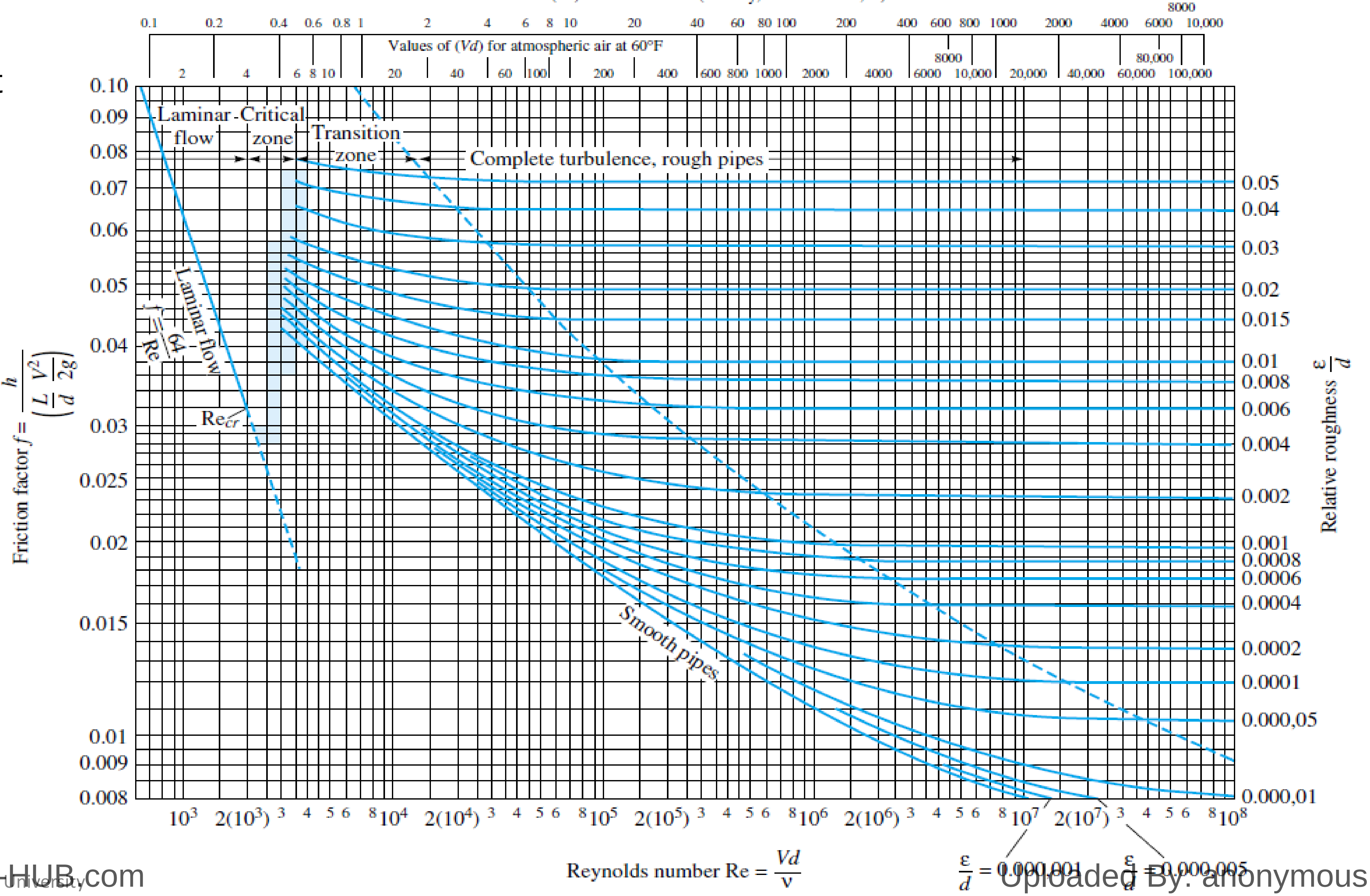
- For rough pipes
    - Colebrook equation

$$\frac{1}{f^{1/2}} = -2.0 \log \left(\frac{\epsilon/d}{3.7} + \frac{2.51}{\mathrm{Re}_d f^{1/2}} \right)$$

- It was plotted in 1944 by Moody into what is now called the *Moody chart* for pipe friction
- An alternate explicit formula given by Haaland

$$\frac{1}{f^{1/2}} \approx -1.8 \log \left[\frac{6.9}{\mathrm{Re}_d} + \left(\frac{\epsilon/d}{3.7} \right)^{1.11} \right]$$

Moody chart for pipe friction with smooth and rough walls.



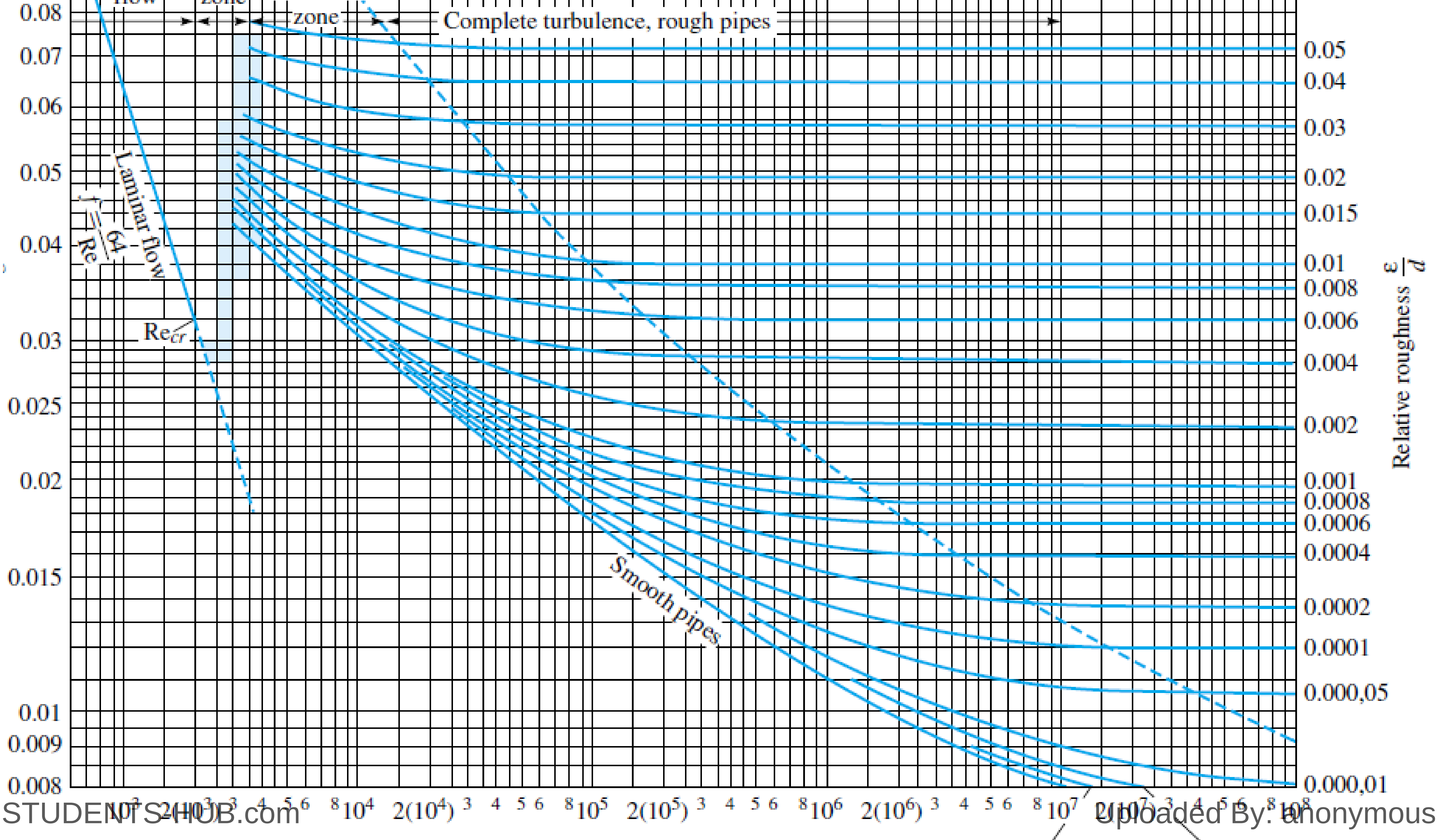
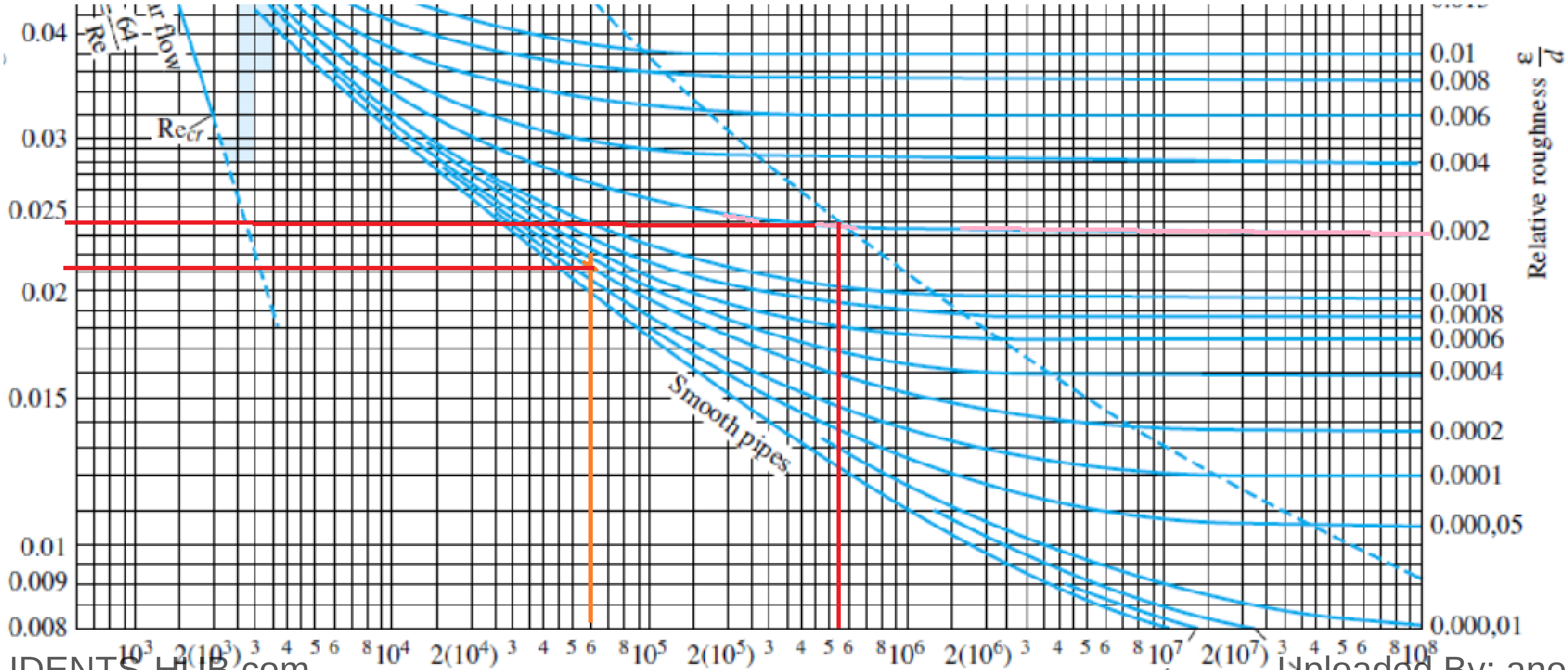

flow
zone
zone
Complete turbulence, rough pipes
Laminar flow
$f = \frac{64}{Re}$
Re_{cr}
Smooth pipes
Relative roughness $\frac{\varepsilon}{d}$
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.008
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.000,05
0.000,01
10^3 $2(10^3)$ 3 4 5 6 8 10^4 $2(10^4)$ 3 4 5 6 8 10^5 $2(10^5)$ 3 4 5 6 8 10^6 $2(10^6)$ 3 4 5 6 8 10^7 $2(10^7)$ 3 4 5 6 8 10^8

Table 6.1 Recommended Roughness Values for Commercial Ducts

Material	Condition	ϵ ft	ϵ mm
Steel	Sheet metal, new	0.00016	0.05
	Stainless, new	0.000007	0.002
	Commercial, new	0.00015	0.046
	Riveted	0.01	3.0
	Rusted	0.007	2.0
Iron	Cast, new	0.00085	0.26
	Wrought, new	0.00015	0.046
	Galvanized, new	0.0005	0.15
	Asphalted cast	0.0004	0.12
Brass	Drawn, new	0.000007	0.002
Plastic	Drawn tubing	0.000005	0.0015
Glass	—	Smooth	Smooth
Concrete	Smoothed	0.00013	0.04
	Rough	0.007	2.0
Rubber	Smoothed	0.000033	0.01
Wood	Stave	0.0016	0.5

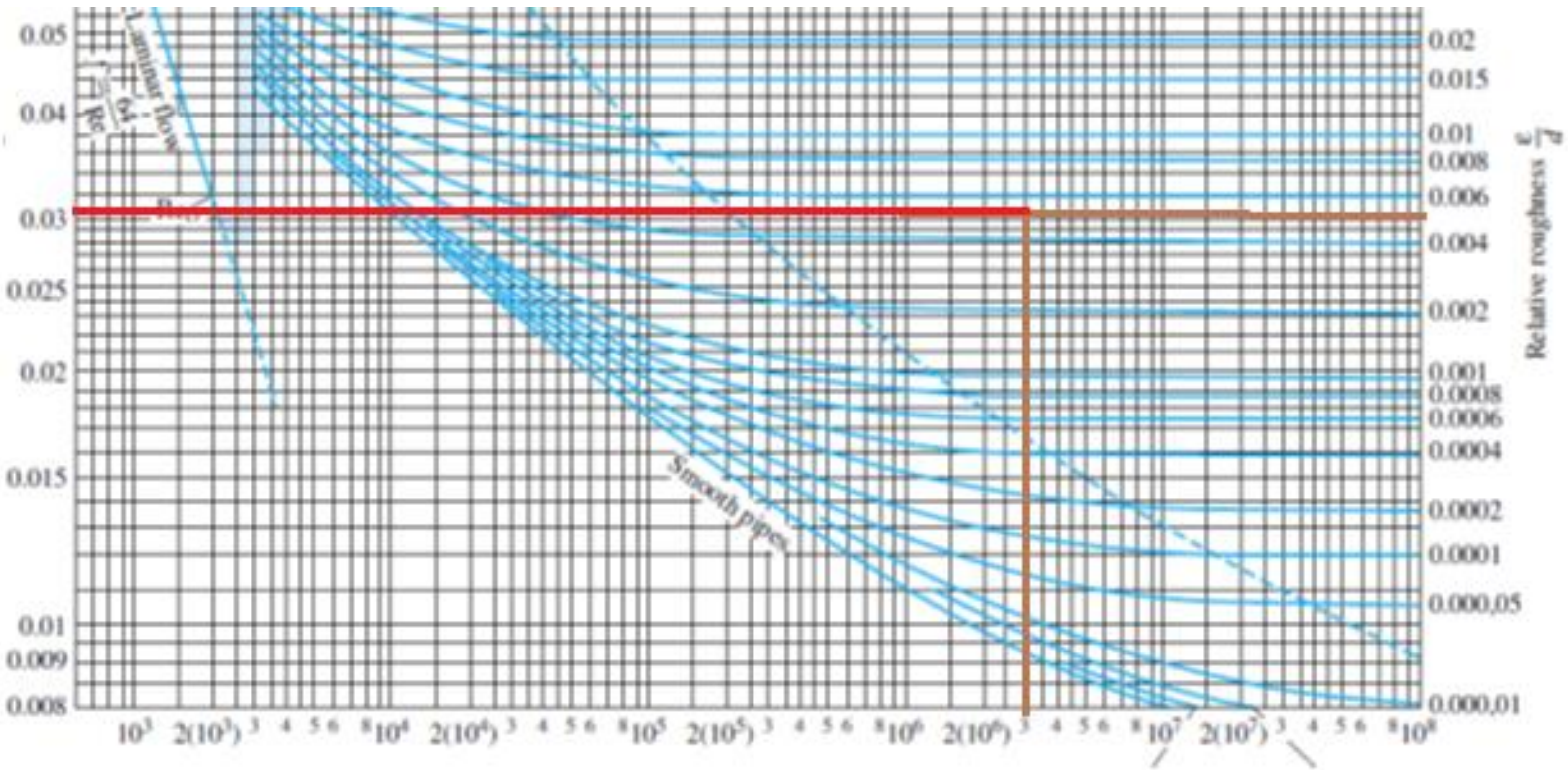
Examples Moody chart

- (a) Re = 550000=5.5x10^5 , ϵ/d =0. 002, and (b) for Re = 6x10^4 , ϵ/d =0.0002
- From chart (a) f =0.021, (b) f =0.024 . What is f for (b) if smooth pipe?



Examples Moody chart

- Cast iron pipe with d = 5 cm and $Re=3x10^6$
- For cast iron roughness ϵ= 0.26 mm then $\frac{\epsilon}{d} = \frac{0.26}{50} = 0.0052$
- From chart f = 0.03



EXAMPLE 6.6

- Compute the loss of head and pressure drop in 200 ft of horizontal 6-in-diameter asphalted cast-iron pipe carrying water with a mean velocity of 6 ft/s.
- Re = ($\rho VD/\mu$) = (VD/ν)= 2.7x 10^5
- ϵ/d =0.0004/(6/12) = 0.0008
- ϵ/d = 0.0008, and Re 2.7x 10^5 approximately, f = 0.02

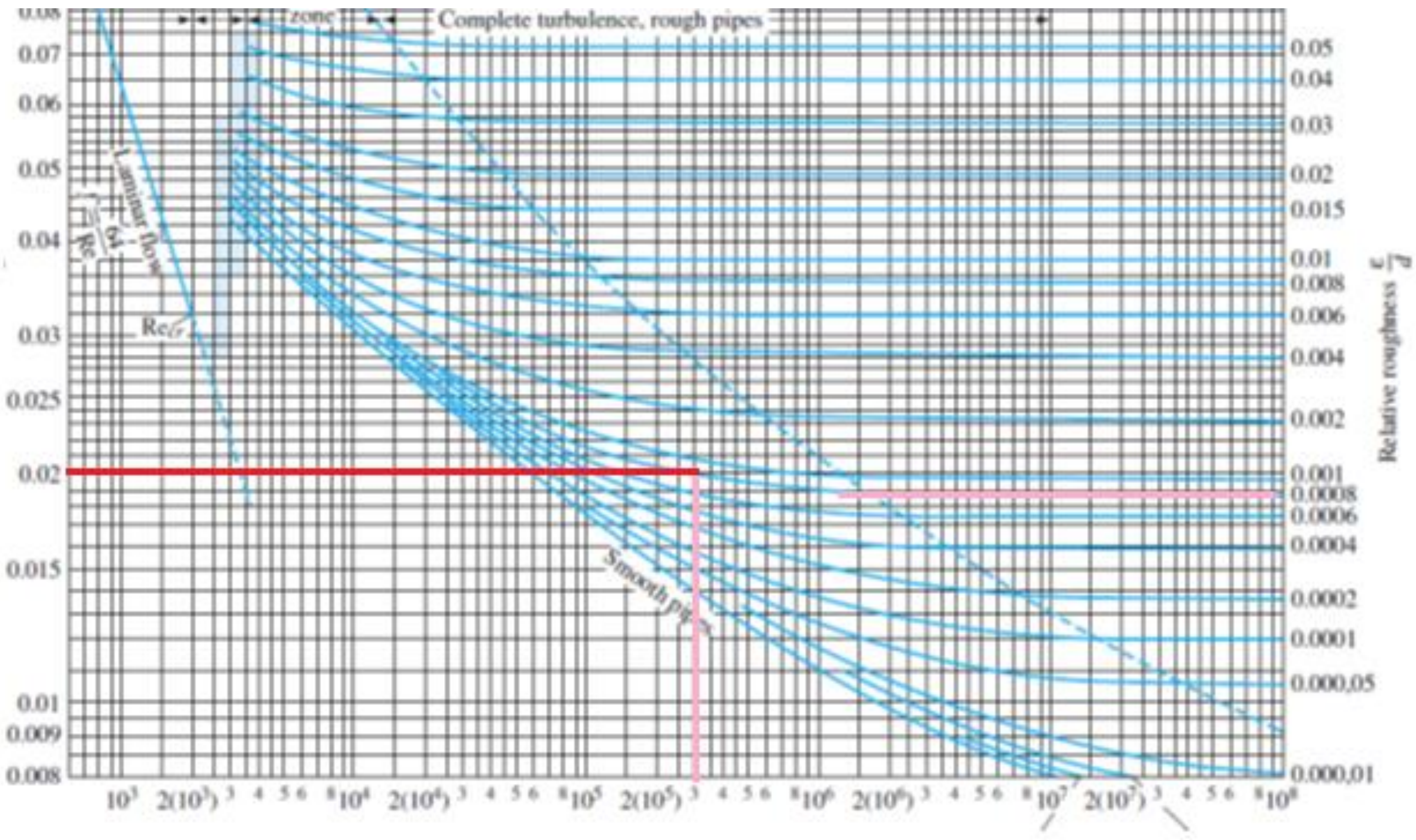
$$h_f = f\frac{L}{d}\frac{V^2}{2g} = (0.02)\frac{200}{0.5}\frac{(6\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = 4.5\ \text{ft}$$

- The pressure drop for a horizontal pipe ($z1= z2$) is

$$\Delta p = \rho g h_f = (62.4\ \text{lbf/ft}^3)(4.5\ \text{ft}) = 280\ \text{lbf/ft}^2$$

EXAMPLE 6.6

- $\epsilon / d = 0.0008$, and Re 2.7x 10^5 approximately, $f = 0.02$



EXAMPLE 6.7

- Oil, with 900 kg/m3 and ν= 0.00001 m^2/s, flows at 0.2 m^3/s through 500 m of 200-mmdiameter cast-iron pipe. Determine (*a*) the head loss and (*b*) the pressure drop if the pipe slopes down at 10° in the flow direction.

$$V = \frac{Q}{\pi R^2} = \frac{0.2\ \text{m}^3/\text{s}}{\pi(0.1\ \text{m})^2} = 6.4\ \text{m/s}$$

$$\text{Re}_d = \frac{Vd}{\nu} = \frac{(6.4\ \text{m/s})(0.2\ \text{m})}{0.00001\ \text{m}^2/\text{s}} = 128{,}000$$

$$\frac{\epsilon}{d} = \frac{0.26\ \text{mm}}{200\ \text{mm}} = 0.0013$$

EXAMPLE 6.7

Enter the Moody chart on the right at ϵ /*d*= 0.0013 with Re= 128,000,
f = 0.0225

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = (0.0225)\frac{500\text{ m}}{0.2\text{ m}}\frac{(6.4\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 117\text{ m}$$

$$h_f = \frac{\Delta p}{\rho g} + z_1 - z_2 = \frac{\Delta p}{\rho g} + L\sin 10°$$

$$\Delta p = \rho g[h_f - (500\text{ m})\sin 10°] = \rho g(117\text{ m} - 87\text{ m})$$

$$= (900\text{ kg/m}^3)(9.81\text{ m/s}^2)(30\text{ m}) = 265{,}000\text{ kg/(m}\cdot\text{s}^2) = 265{,}000\text{ Pa}$$

Minor losses

- Head loss :

    1) Friction losses (Major losses)

    2) Minor losses : (i) entrance and exit .

    (ii) enlargement and contraction .

    (iii) fittings .

- Minor losses : $h_{minor} = K (V^2/2g) = h_m$

    K : coefficient is given for various fitting

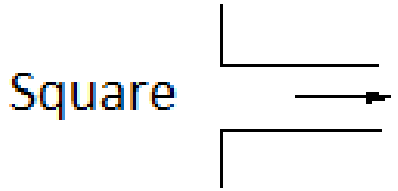
Total head losses are

$$\Delta h_{tot} = h_f + \sum h_m = \frac{V^2}{2g}\left(\frac{fL}{d} + \sum K\right)$$

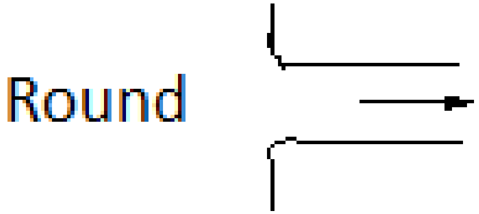
Exit and entrance

- For all types of exits : inward , rounded , K = 1
- <u>Entrance losses</u>

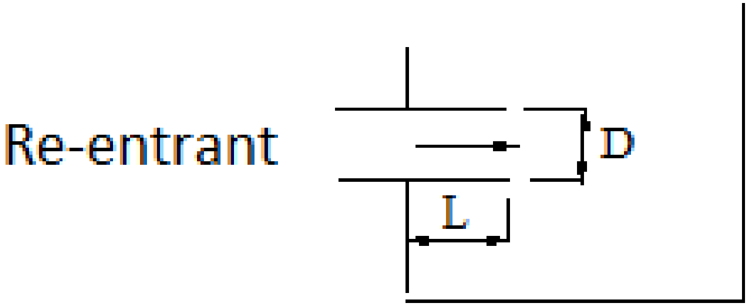
(i)square entrance K = 0.5



(ii)Rounded K = 0.01 –0.05 or F. White p.372 fig 6.21 (b)



(iii)Re-entrant (inward) K = 0.8-1.0 see fig 6.24(a) P.372 “F. White “



Sudden expansion/ Contraction

- It can be shown using continuity , Bernoulli's ,and moment equations that for a sudden expansion ,

  $h_m = K_{SE}(V^2 /2g)$

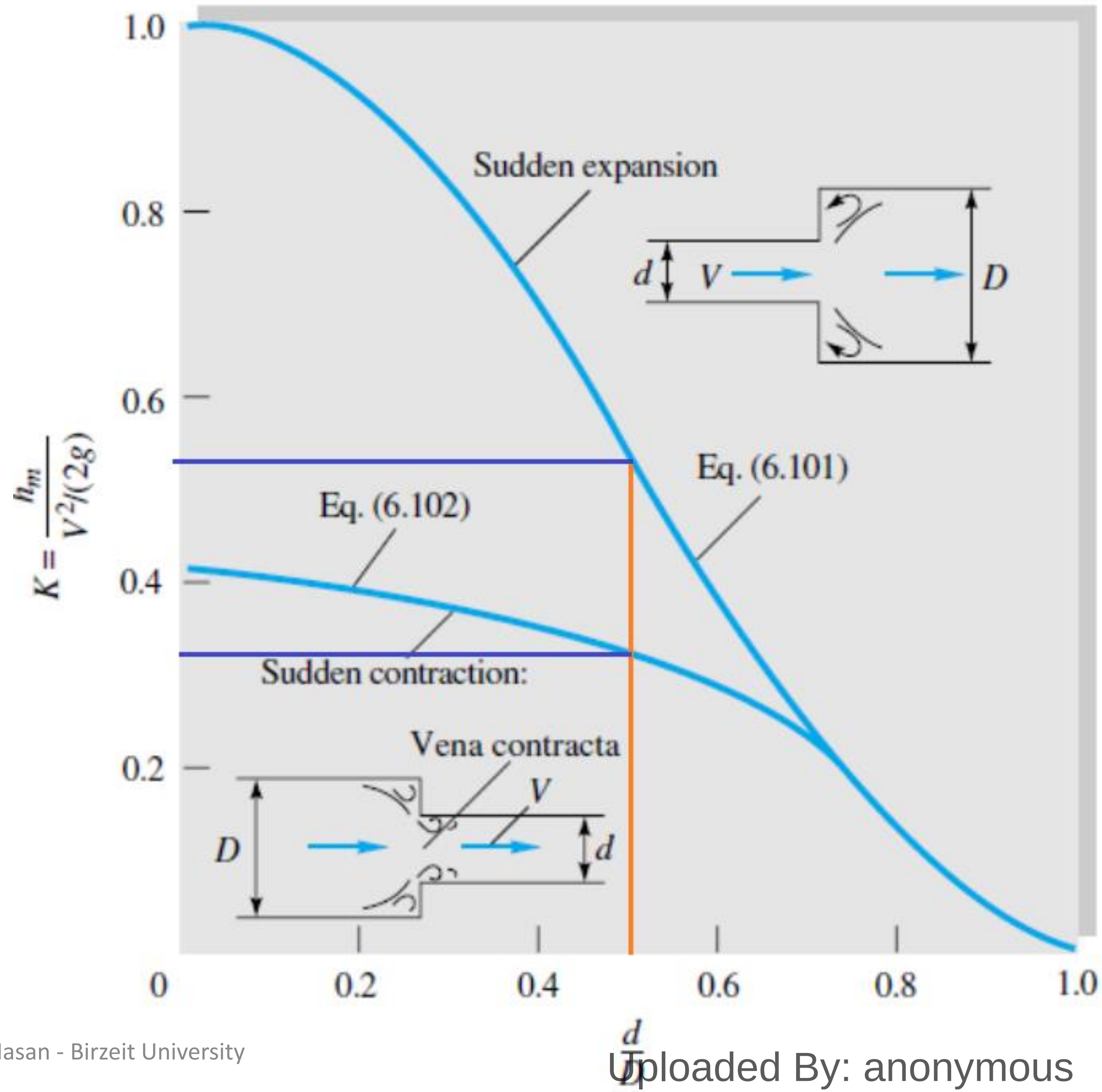
  where $$K_{SE} = \left(1 - \frac{d^2}{D^2}\right)^2$$

- Sudden contraction may use ,

  $$K_{SC} \approx 0.42\left(1 - \frac{d^2}{D^2}\right)$$

- See (fig 6.22 P.372 F. White)
- Example if d=1 in and D=2 in: find K_{SE} and K_{SC} Then d/D=0.5 and

  K_{SE} =0.55 while K_{SC}= 0.32

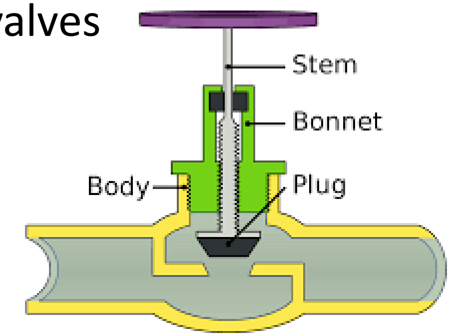


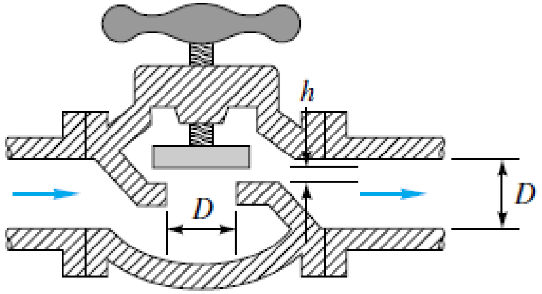
Pipe fittings

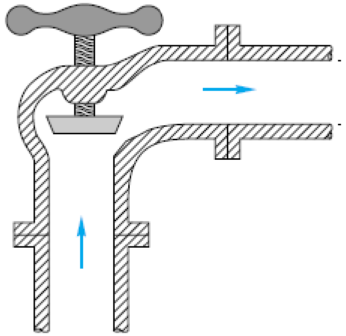
- $h_{minor} = K (V^2/2g)$
- K depends on type of fitting, pipe diameter, type of connection.
- Pipe connections : screwed or flanged
- Type of fittings include; valves, elbows, tees.

Valves

Globe valves

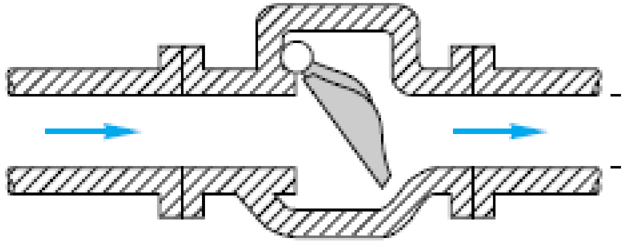


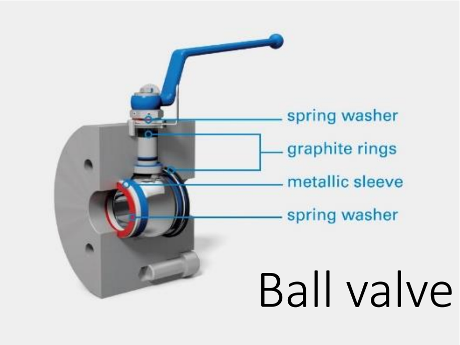




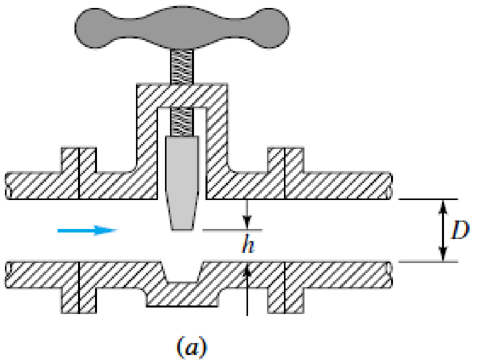
Angle valves

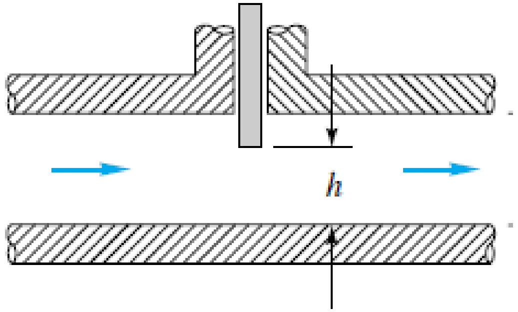
Non-return valves

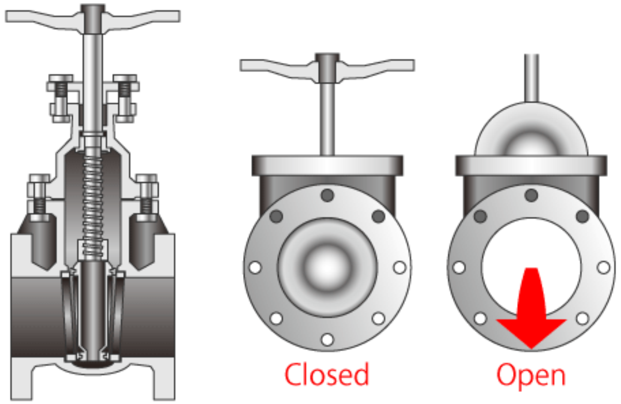




Gate valves







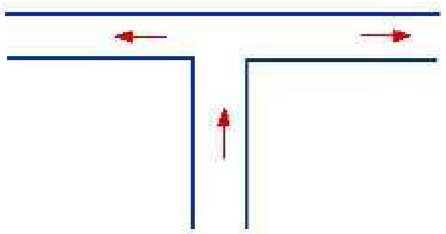
Pipe fittings

Resistance Coefficients -fittings

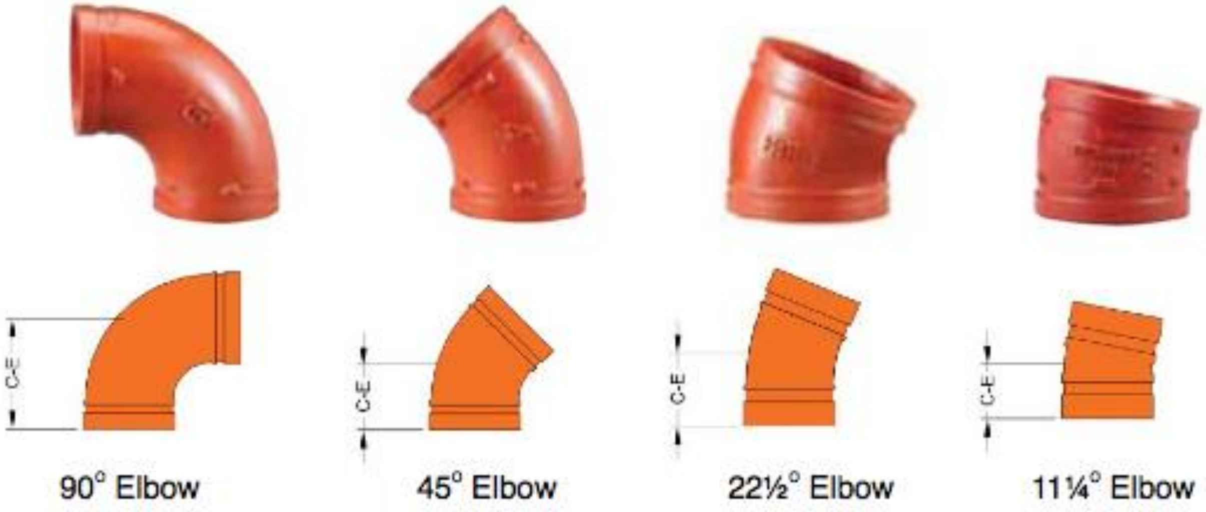
$h_{minor} = K (V^2/2g)$

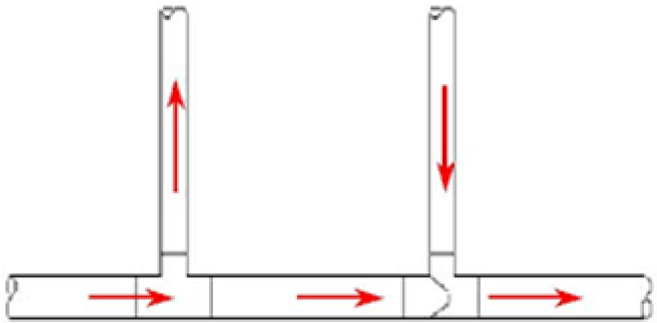
	Nominal diameter, in								
	Screwed				Flanged				
	½	1	2	4	1	2	4	8	20
Valves (fully open):									
Globe	14	8.2	6.9	5.7	13	8.5	6.0	5.8	5.5
Gate	0.30	0.24	0.16	0.11	0.80	0.35	0.16	0.07	0.03
Swing check	5.1	2.9	2.1	2.0	2.0	2.0	2.0	2.0	2.0
Angle	9.0	4.7	2.0	1.0	4.5	2.4	2.0	2.0	2.0
Elbows:									
45° regular	0.39	0.32	0.30	0.29					
45° long radius					0.21	0.20	0.19	0.16	0.14
90° regular	2.0	1.5	0.95	0.64	0.50	0.39	0.30	0.26	0.21
90° long radius	1.0	0.72	0.41	0.23	0.40	0.30	0.19	0.15	0.10
180° regular	2.0	1.5	0.95	0.64	0.41	0.35	0.30	0.25	0.20
180° long radius					0.40	0.30	0.21	0.15	0.10
Tees:									
Line flow	0.90	0.90	0.90	0.90	0.24	0.19	0.14	0.10	0.07
Branch flow	2.4	1.8	1.4	1.1	1.0	0.80	0.64	0.58	0.41

Tees & elbows

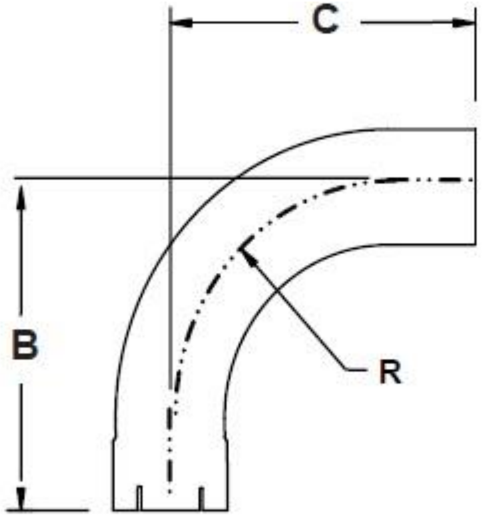


Branched flow





In flow



Examples

- Find K for 2 inch screwed 90 regular elbow? K=0.95
- Find K for 2 inch flanged 90 regular elbow? K=0.39
- Find K for 4 inch gate valve fully open if flanged, if screwed?

  Flanged K=0.16, screwed K=0.11

	Nominal diameter, in							
	Screwed				Flanged			
	1/2	1	2	4	1	2	4	8
Valves (fully open):								
Globe	14	8.2	6.9	5.7	13	8.5	6.0	5.8
Gate	0.30	0.24	0.16	0.11	0.80	0.35	0.16	0.07
Swing check	5.1	2.9	2.1	2.0	2.0	2.0	2.0	2.0
Angle	9.0	4.7	2.0	1.0	4.5	2.4	2.0	2.0
Elbows:								
45° regular	0.39	0.32	0.30	0.29				
45° long radius					0.21	0.20	0.19	0.16
90° regular	2.0	1.5	0.95	0.64	0.50	0.39	0.30	0.26
90° long radius	1.0	0.72	0.41	0.23	0.40	0.30	0.19	0.15
180° regular	2.0	1.5	0.95	0.64	0.41	0.35	0.30	0.25
180° long radius					0.40	0.30	0.21	0.15

Equivalent length

- Minor losses expressed in terms of equivalent length L_e of pipe that has the same head losses for the same flow.
- $f (L_e /D) (V^2 /2g) = K (V^2 /2g)$

then $L_e = KD/f$

- L_e is added to pipe length in $L_l = [(L_e + L) /D] (V^2 /2g) f$
- Example if K = 20 for a pipe where f = 0.02 and D = 300 mm then

  $L_e = (20*(0.3)/0.02) = 300$ m
- In general $L_e = (D/f) \sum K_i$

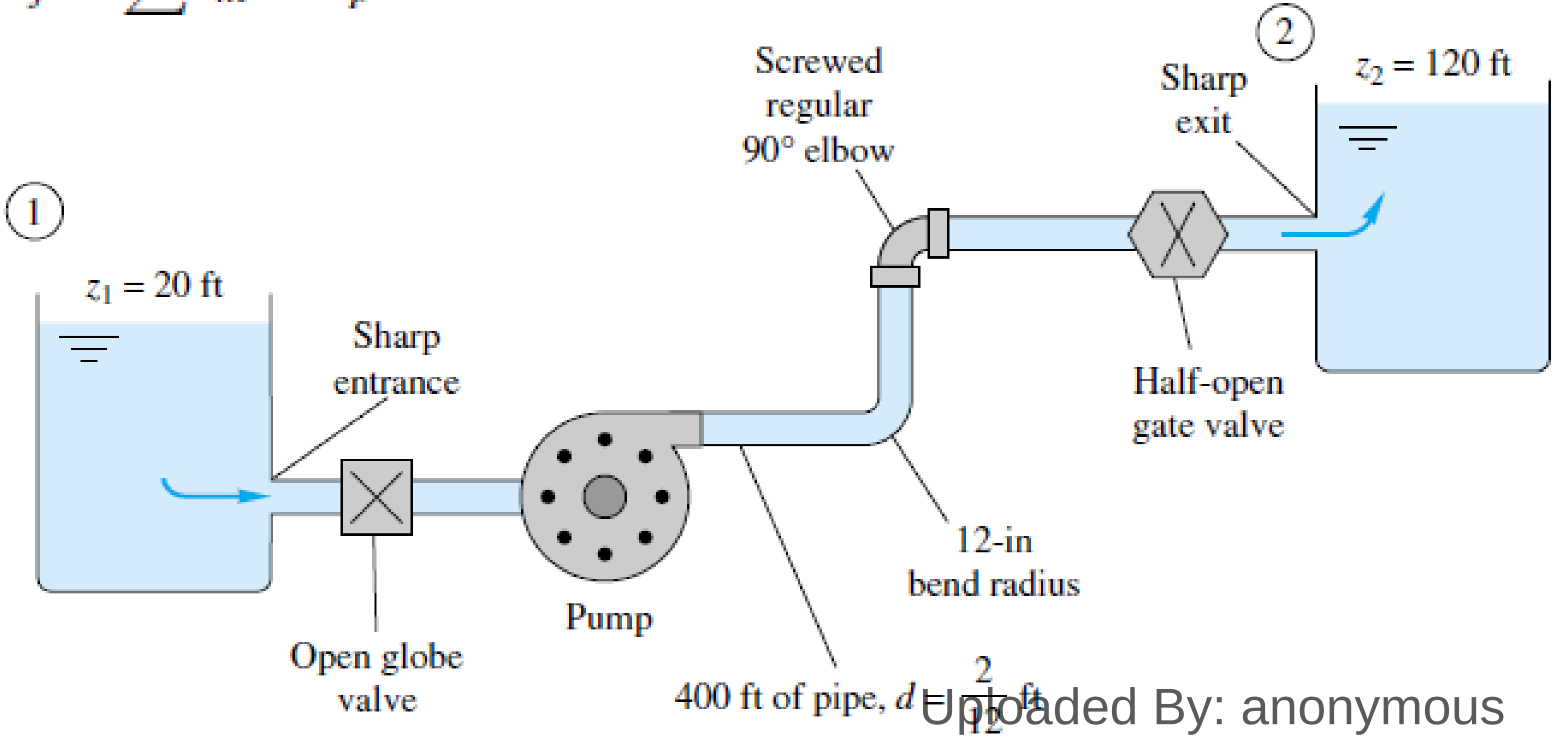
EXAMPLE 6.16

- Water, ρ=1.94 slugs/ft3 and ν=0.000011 ft2/s, is pumped between two reservoirs at 0.2 ft3/s through 400 ft of 2-in-diameter pipe and several minor losses, as shown in Fig. E6.16. The roughness ratio is ϵ/d = 0.001. Compute the pump horsepower required.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) + h_f + \sum h_m - h_p$$

$$V = \frac{Q}{A} = \frac{0.2\ \text{ft}^3/\text{s}}{\frac{1}{4}\pi(\frac{2}{12}\ \text{ft})^2} = 9.17\ \text{ft/s}$$

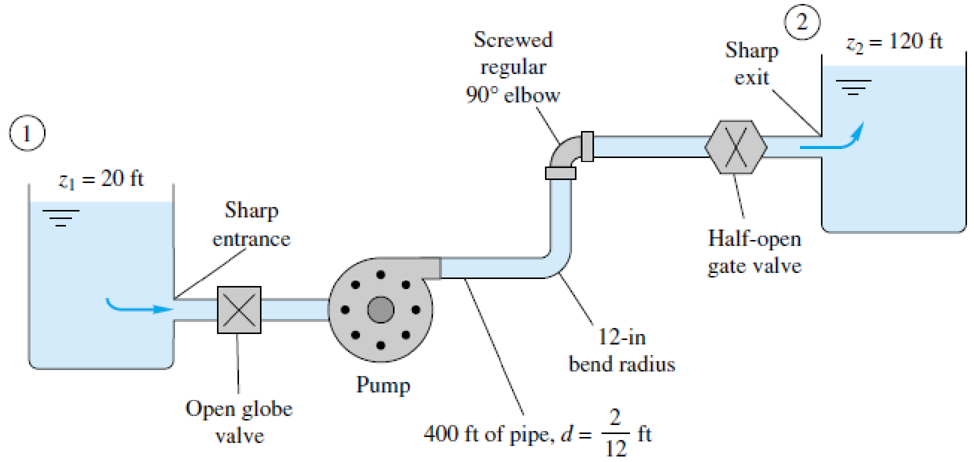
$$\text{Re}_d = \frac{Vd}{\nu} = \frac{9.17(\frac{2}{12})}{0.000011} = 139{,}000$$



EXAMPLE 6.16

$$h_p = z_2 - z_1 + h_f + \sum h_m = 120 \text{ ft} - 20 \text{ ft} + \frac{V^2}{2g}\left(\frac{fL}{d} + \sum K\right)$$

Loss	K
Sharp entrance (Fig. 6.21)	0.5
Open globe valve (2 in, Table 6.5)	6.9
12-in bend (Fig. 6.20)	0.15
Regular 90° elbow (Table 6.5)	0.95
Half-closed gate valve (from Fig. 6.18*b*)	2.7
Sharp exit (Fig. 6.21)	1.0
	$\sum K = 12.2$



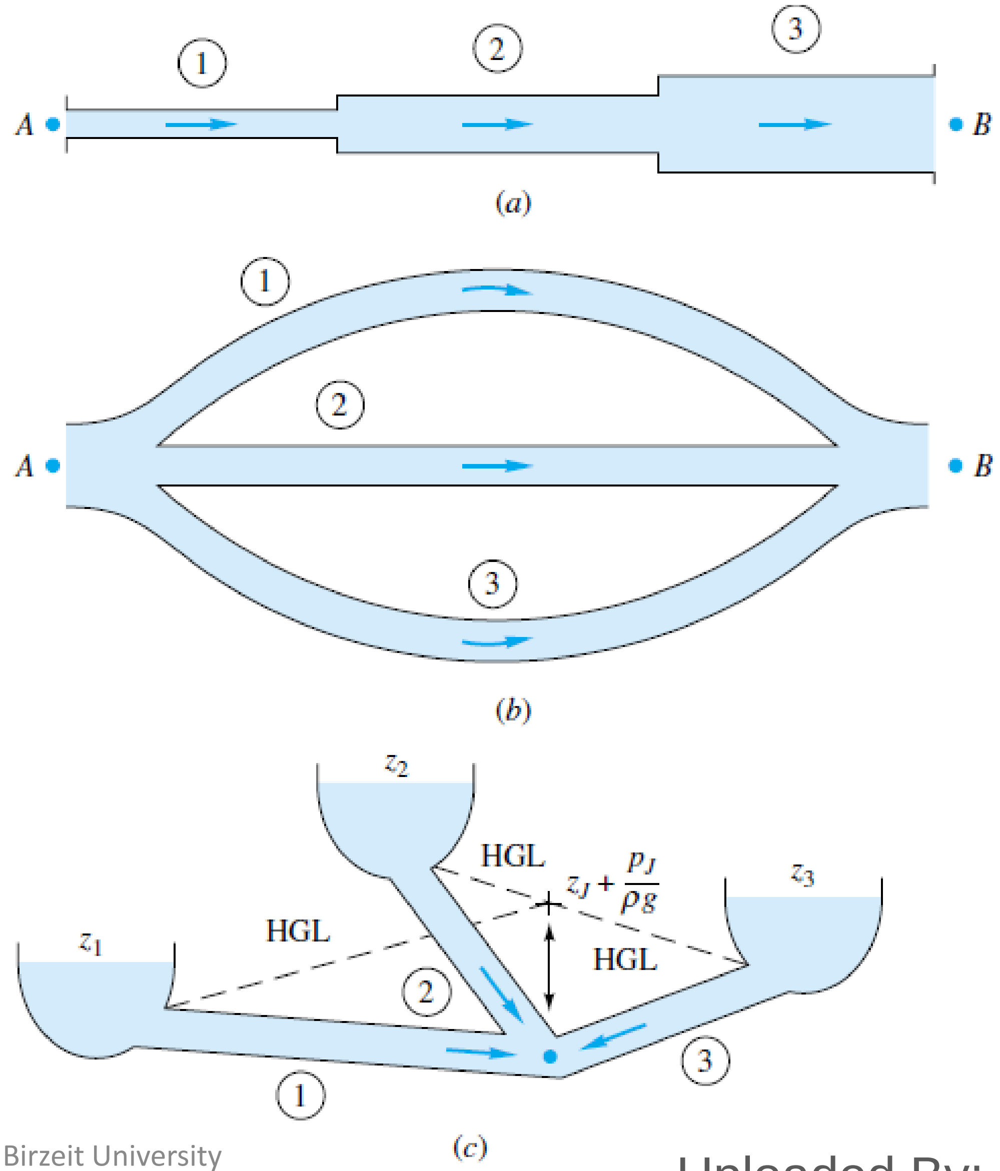
For $\epsilon/d = 0.001$, from the Moody chart read $f = 0.0216$

$$h_p = 100 \text{ ft} + \frac{(9.17 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)}\left[\frac{0.0216(400)}{\frac{2}{12}} + 12.2\right] = 100 \text{ ft} + 84 \text{ ft} = 184 \text{ ft} \quad \text{pump head}$$

$$P = \rho g Q h_p = [1.94(32.2) \text{ lbf/ft}^3](0.2 \text{ ft}^3/\text{s})(184 \text{ ft}) = 2300 \text{ ft} \cdot \text{lbf/s} \qquad P = \frac{2300}{550} = 4.2 \text{ hp}$$

Multiple pipe problems

Fig. 6.24 Examples of multiple-pipe systems: (*a*) pipes in series; (*b*) pipes in parallel; (*c*) the three-reservoir junction problem



Only single path pipe problems are covered in this course.

Pipe problems

- Three Types of Pipe-Flow Problems; only type 1 is required
  - Pressure drop, power unknown Type 1

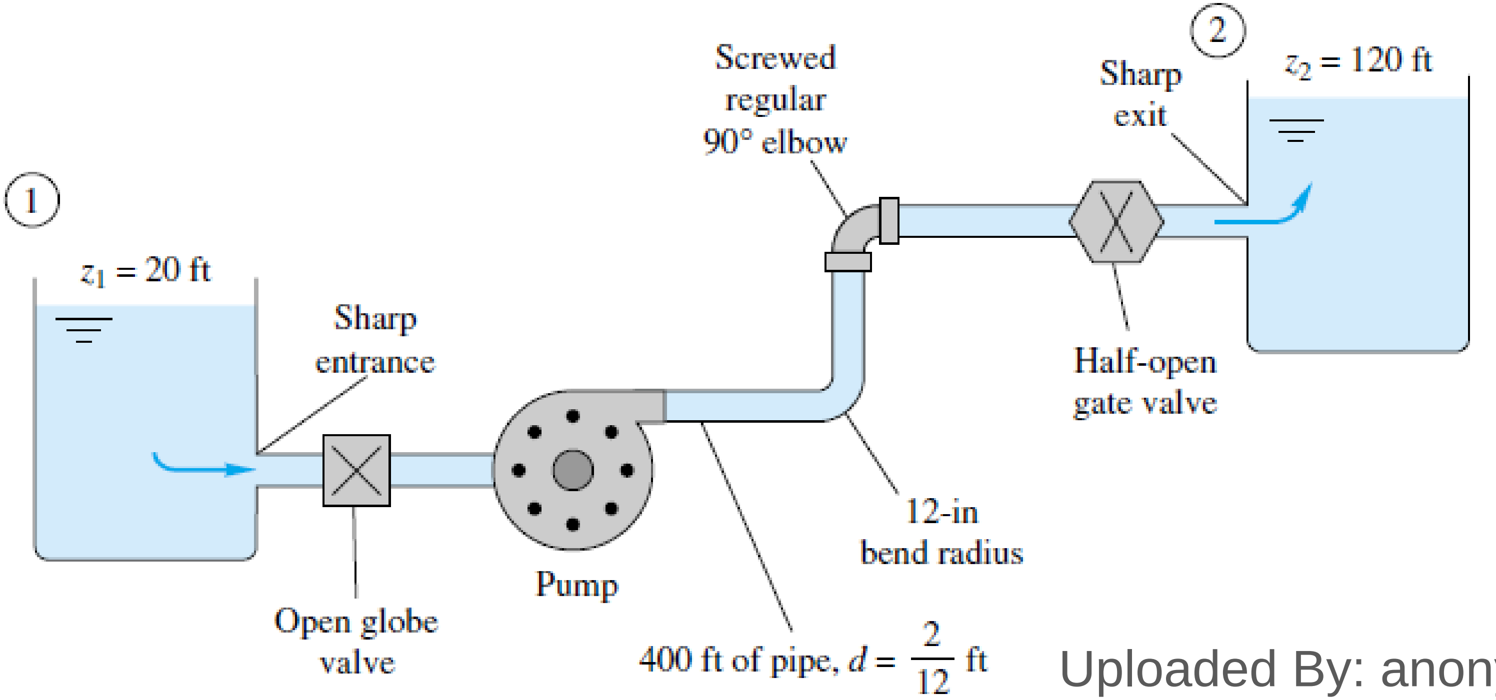
    Given *d*, *L*, and *V* or *Q*,ρ ,μ , and *g*, compute the head loss *hf* (head-loss problem).
  - Flow rate is unknown Type 2

    Given *d*, *L*, h_f ,ρ ,μ , and *g*, compute the velocity *V* or flow rate *Q* (flow- rate problem).
  - Pipe diameter is unknown Type 3

    Given *Q*, *L*, *hf* , ρ ,μ and *g*, compute the diameter *d* of the pipe (sizing problem).

Type 1 Pressure is unknown

- Straight forward solution . Conditions at one section , l, ϵ , geometry ,flow are all given and the condition at other section is desired (P_2 or ΔP).
- See example 6.16 pump head and power are required.



Non circular Conduits

- For non circular cross section ,you may solve using all previous relation and method by replacing the diameter with hydraulic diameter.
- D_h = 4 (cross section Area / wetted perimeter) = $4A/P_w$
- Example:

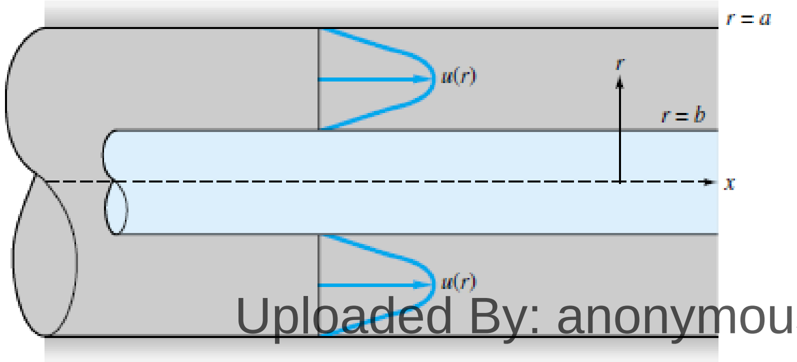
(i) Circular $D_h = 4\pi (D^2/4)/ \pi D = D$

(ii) Rectangular $D_h = 4ba/(2b+2a) = 2 ba/(b+a)$

For two parallel sheets where length **b** is much larger than separation **a** between the sheets $D_h = 2 ba/(b+a) = 2a$

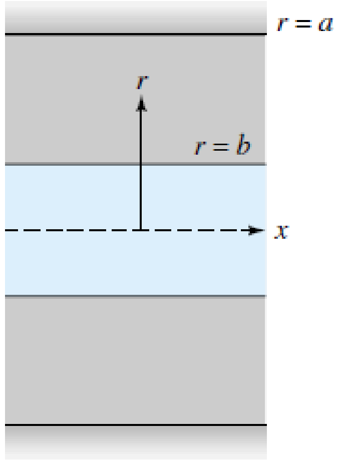
(iii) Circular annulus

$D_h = 4[\pi(D_1^2/4) - \pi(D_2^2/4)]/ [\pi D_1 + \pi D_2] = [(D_1^2-D_2^2)/(D_1+D_2)] = (D_1-D_2)$



hydraulic diameter

$$\mathrm{Re}_{D_h} = \frac{VD_h}{\nu} \qquad f = F\left(\frac{VD_h}{\nu}, \frac{\epsilon}{D_h}\right) \qquad h_f \approx f\,\frac{L}{D_h}\,\frac{V^2}{2g}$$



$$D_{\text{eff}} = D_h = \frac{4A}{\mathscr{P}}$$ reasonable accuracy

EXAMPLE 6.13

- Fluid flows at an average velocity of 6 ft/s between horizontal parallel plates a distance of 2.4 in apart. Find the head loss and pressure drop for each 100 ft of length for ρ= 1.9 slugs/ft3 and (*a*) ν=0.00002 ft3/s and (*b*) ν=0.002 ft3/s. Assume smooth walls.

The spacing is 2*h* = 2.4 in= 0.2 ft, and *Dh* =4*h*= 0.4 ft.

separation **a** between the sheets

D_h = 2a

$$\mathrm{Re}_{D_h} = \frac{VD_h}{\nu} = \frac{(6.0\ \text{ft/s})(0.4\ \text{ft})}{0.00002\ \text{ft}^2/\text{s}} = 120{,}000$$

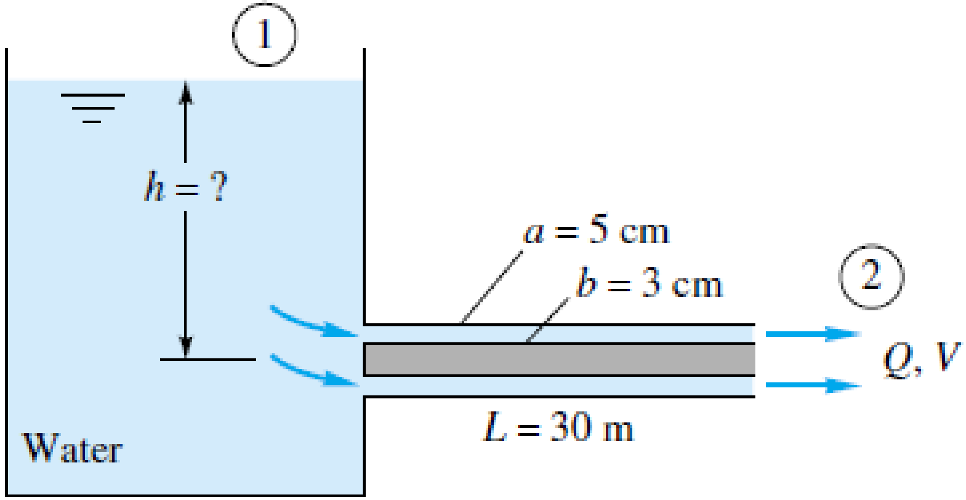
For reasonable accuracy, simply look on the Moody chart for smooth walls

$$f \approx 0.0173 \quad h_f \approx f\frac{L}{D_h}\frac{V^2}{2g} = 0.0173\,\frac{100}{0.4}\,\frac{(6.0)^2}{2(32.2)} \approx 2.42\ \text{ft}$$

$$\Delta p = \rho g h_f = 1.9(32.2)(2.42) = 148\ \text{lbf/ft}^2$$

EXAMPLE 6.14

- What should the reservoir level *h* be to maintain a flow of 0.01 m3/s through the commercial steel annulus 30 m long shown in Fig. E6.14? Neglect entrance effects and take ρ=1000 kg/m3 and ν=1.02x10^6 m2/s for water.



$$V = \frac{Q}{A} = \frac{0.01\ \text{m}^3/\text{s}}{\pi[(0.05\ \text{m})^2 - (0.03\ \text{m})^2]} = 1.99\ \text{m/s}$$

$$D_h = 2(a - b) = 2(0.05 - 0.03)\ \text{m} = 0.04\ \text{m}$$

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \left(\frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2\right) + gh_f$$

But $p_1 = p_2 = p_a$, $V_1 \approx 0$, and $V_2 = V$ in the pipe.

$$h_f = f\frac{L}{D_h}\frac{V^2}{2g} = z_1 - z_2 - \frac{V^2}{2g} \qquad h = \frac{V^2}{2g}\left(1 + f\frac{L}{D_h}\right)$$

EXAMPLE 6.14

$$\mathrm{Re}_{D_h} = \frac{VD_h}{\nu} = \frac{1.99(0.04)}{1.02 \times 10^{-6}} = 78{,}000$$

$$\frac{\epsilon}{D_h} = \frac{0.046 \text{ mm}}{40 \text{ mm}} = 0.00115$$

where 0.046 mm has been read from Table 6.1 for commercial steel surfaces. From the Moody chart, read *f* = 0.0232.

$$h \approx \frac{(1.99 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)}\left(1 + 0.0232\,\frac{30 \text{ m}}{0.04 \text{ m}}\right) = 3.71 \text{ m}$$

End of pipe flow