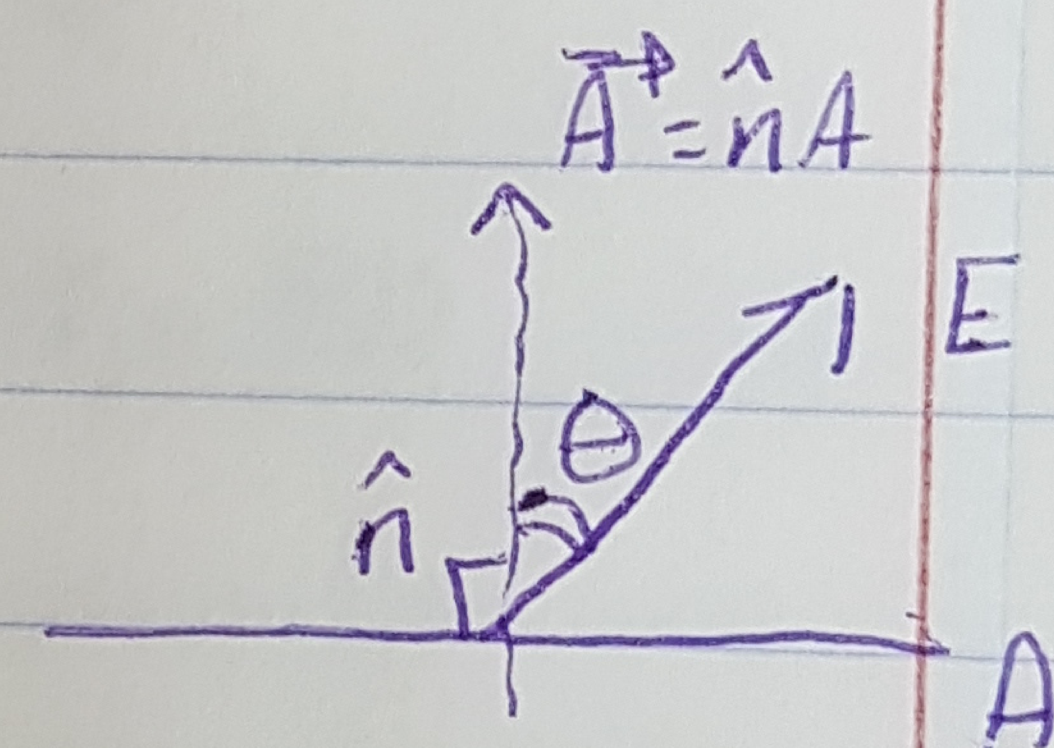
Chapter 23 - Gauss' law

Electric flux (Φ): The number of electric field lined crossing perpendicularly the surface.

$$\boxed{\Phi = \vec{E} \cdot \vec{A}} = E A \cos\theta \quad (Nm^2/C)$$

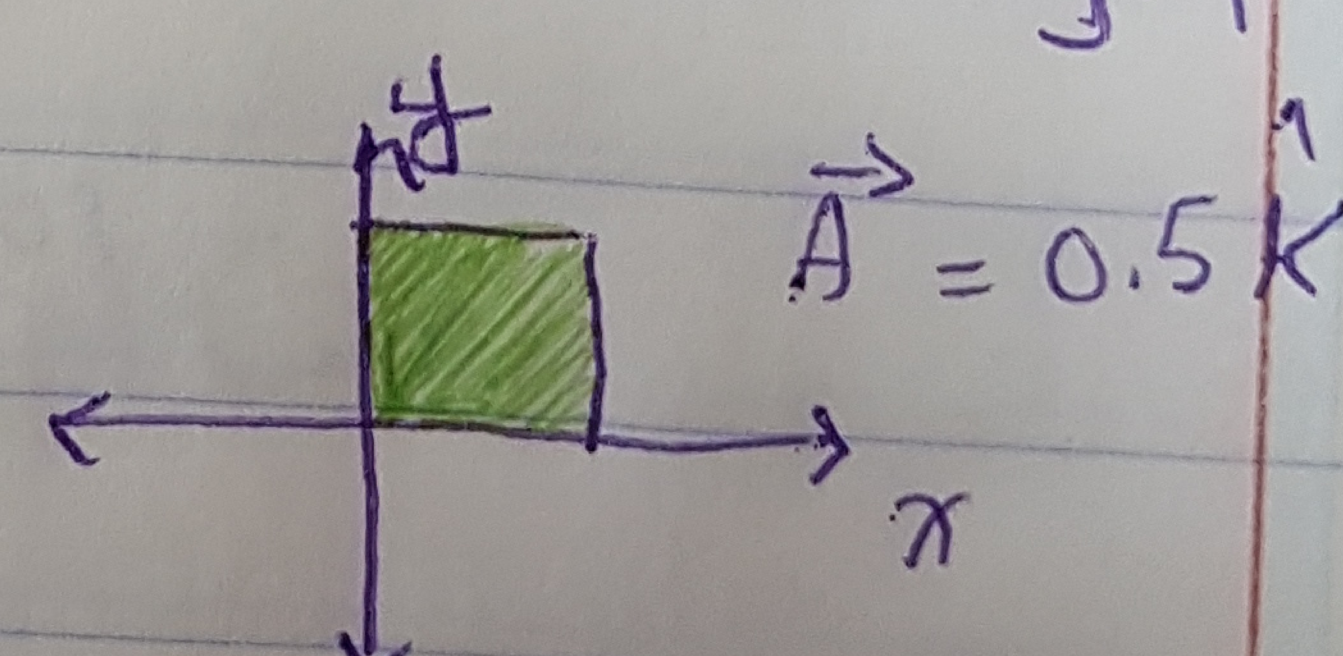
For variable $\vec{E} \Rightarrow \Phi = \int \vec{E}\, d\vec{A}$



eg) $\vec{E} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ N/C

(a) find $\vec{E}$ through a square of $A = 0.5\,m^2$ in $(x-y)$ plane

$\Rightarrow \Phi = \vec{E} \cdot \vec{A} = 2\,Nm^2/C$



(b) Find Φ? if $A = 0.5\,m^2$ in $(x-y)$ plane

$\vec{A} = 0.5$

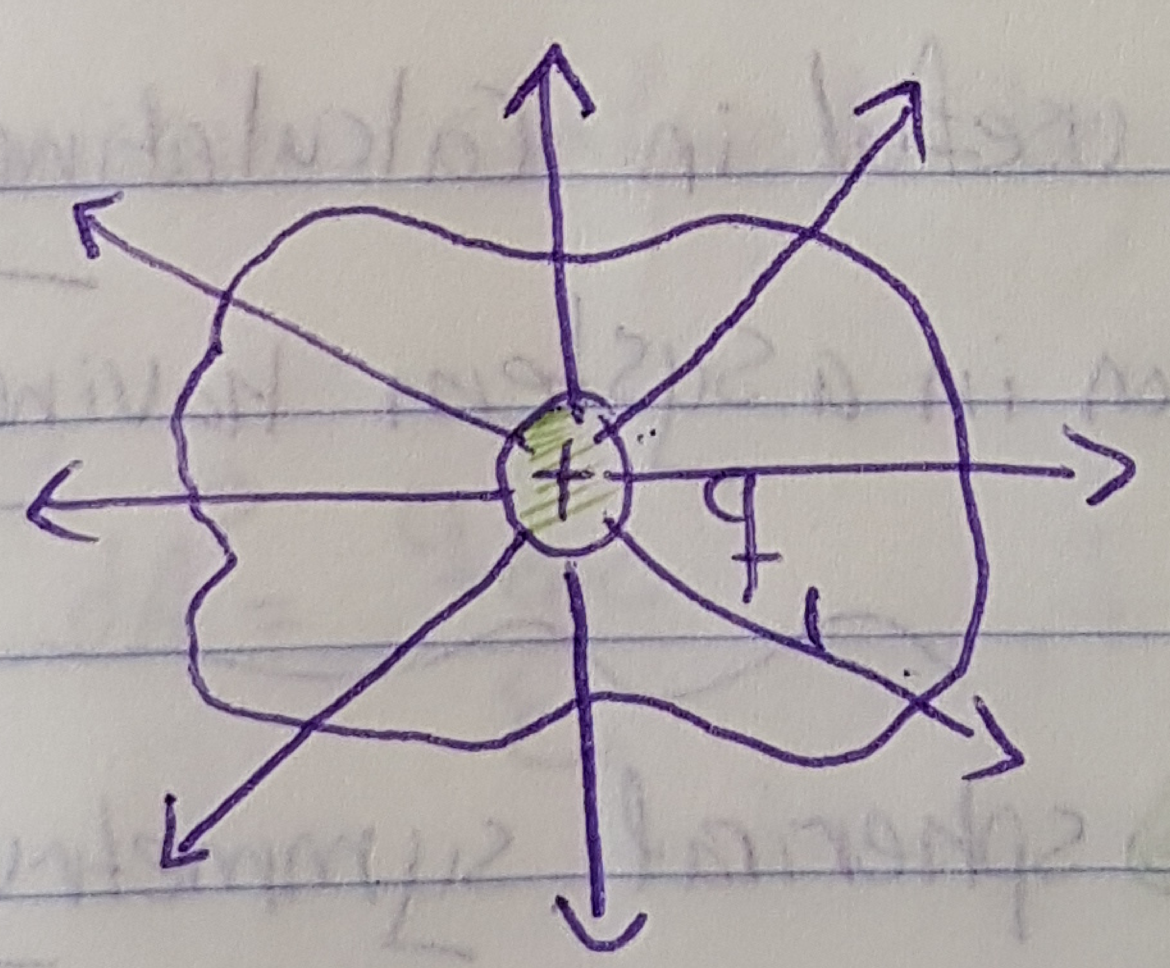
$\Phi = 1.5\,Nm^2/C$

Gauss' law:

* Electric flux through a closed surface = q inside the surface over ε_0. $\Phi_{\text{closed surface}} = \frac{q_{enc}}{\varepsilon_0}$

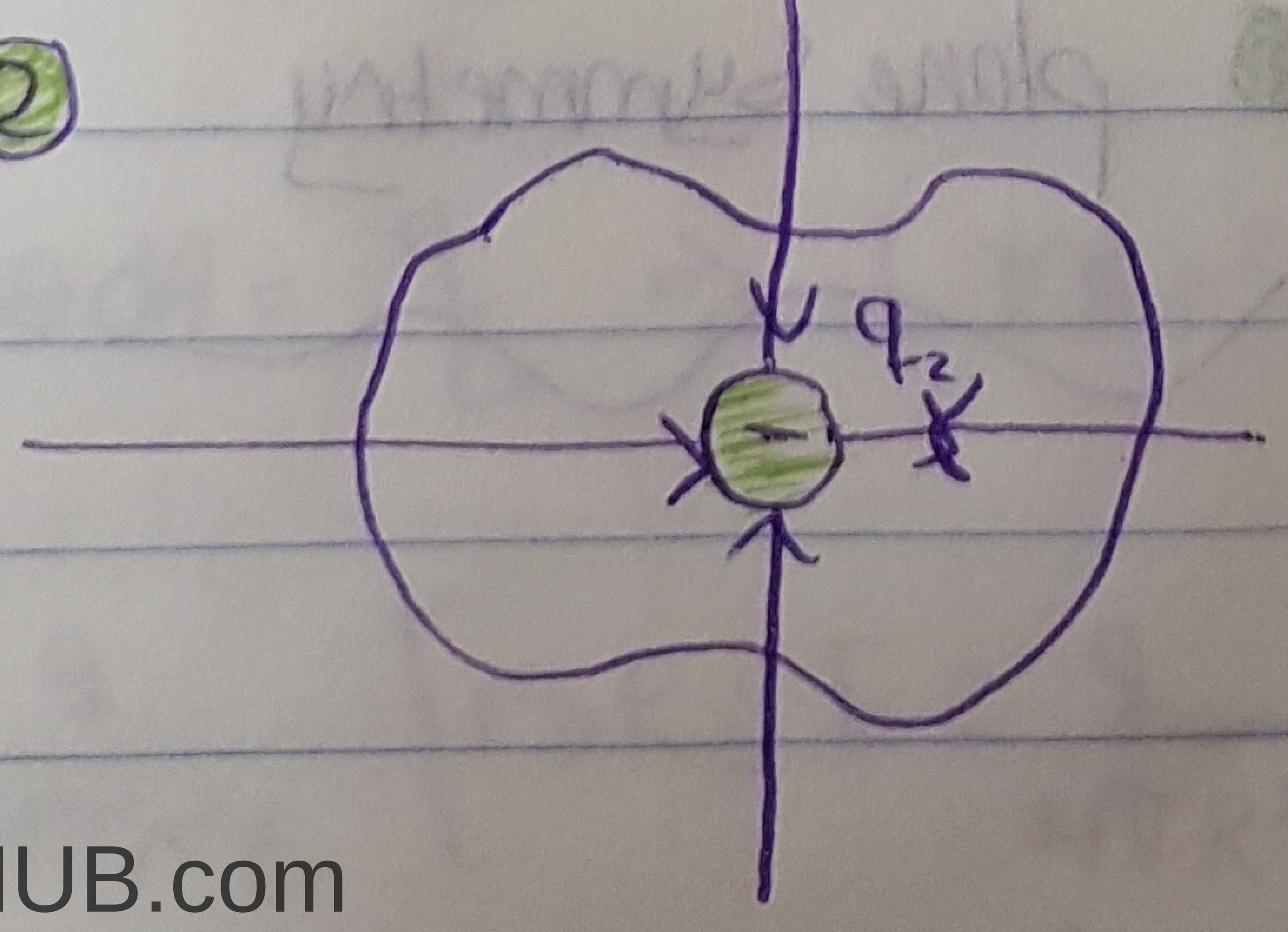
$$\Rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \Rightarrow \text{Gauss's law}$$

$\oint$ this means "closed surface"

e.g ① 

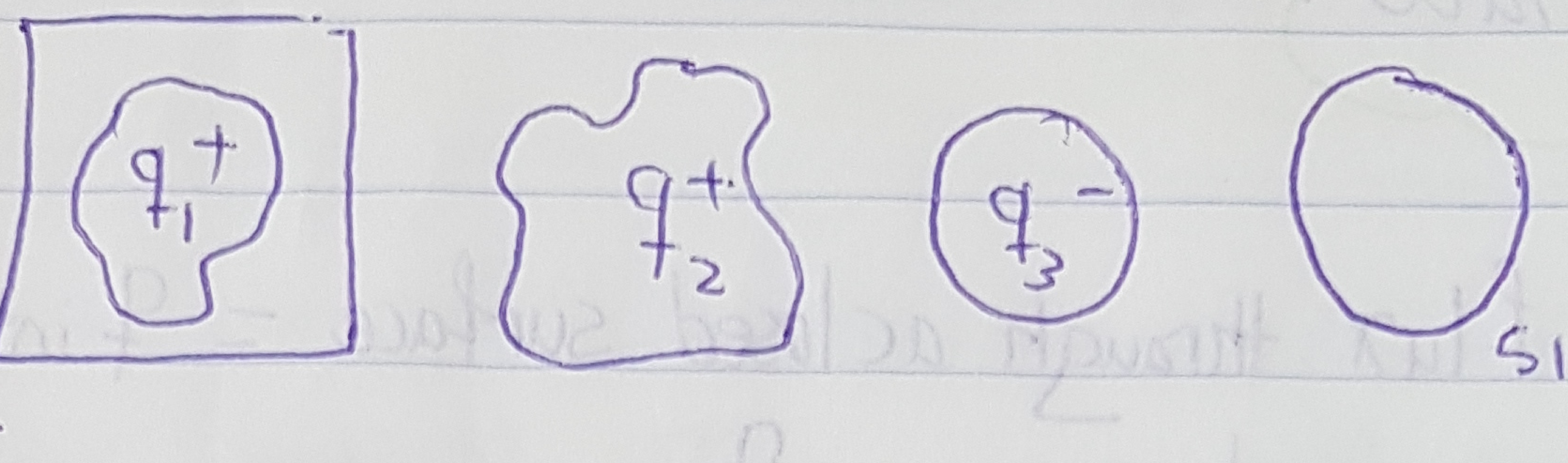
Φ is outward

$\Phi = \frac{+q_1}{\varepsilon_0}$

② 

Φ is inward

$\Phi = \frac{-q_2}{\varepsilon_0}$



$$\Rightarrow \oint \vec{E} \cdot d\vec{A} = 0$$

$$q_{enc_{S_1}} = 0$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{+q_1}{\varepsilon_0}$$

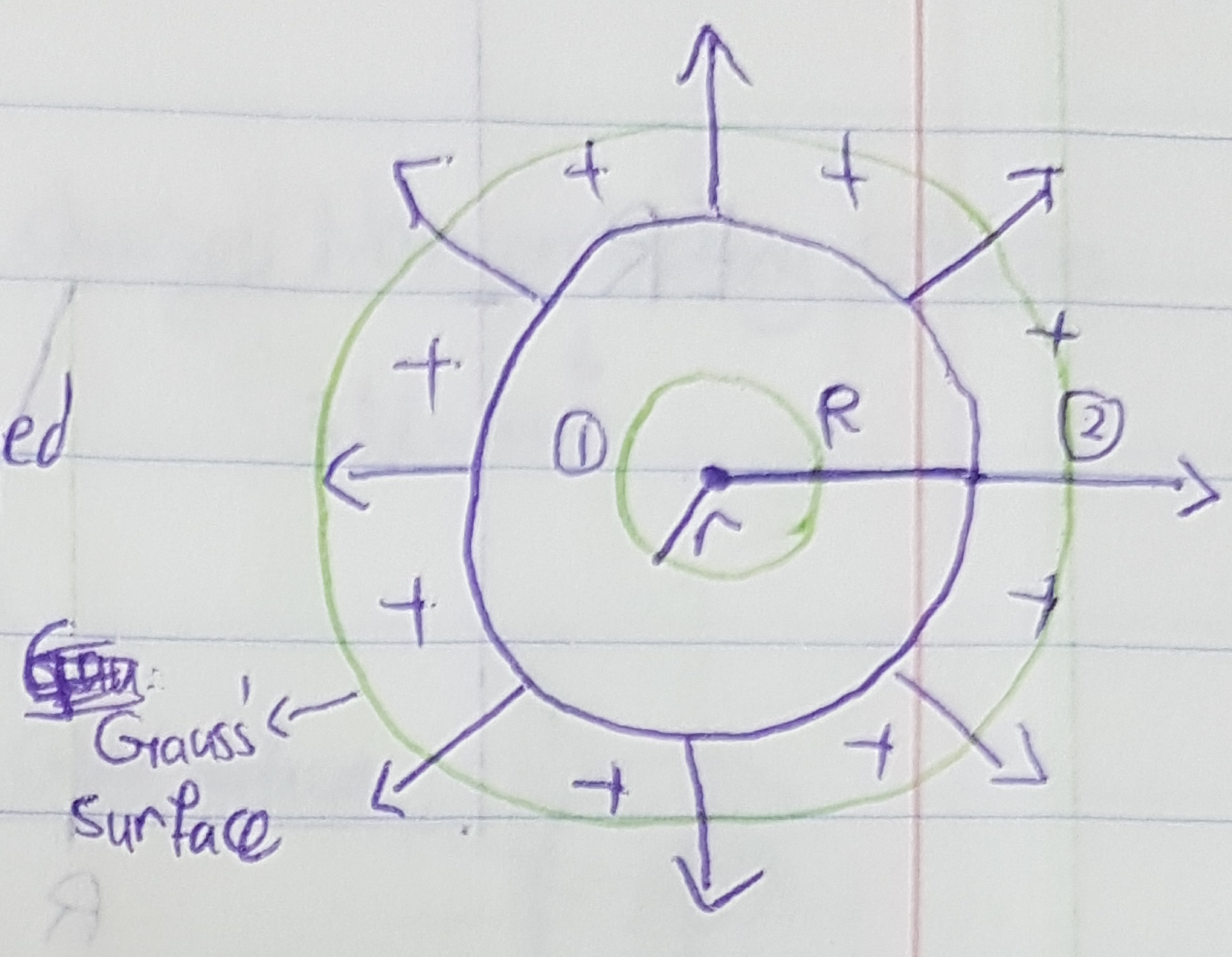
* Gauss' law is useful in Calculating $\vec{E}$ due to continous Charged distribution in a system having high symmetry

* Symmetry ⇒ ① spherical symmetry
② Line symmetry (symmetrical symmetry)
③ plane symmetry

* Applications on Gauss' law

① Spherical symmetry

ⓐ $\vec{E}$ due to uniformly charged Spherical Shell



e.g. radius = R

Charge = q

$$\sigma = \frac{q}{4\pi R^2}$$

Find $\vec{E}$ inside the sphere, outside the sphere

① Draw a Gaussian surface to be a sphere of radius $= r < R$

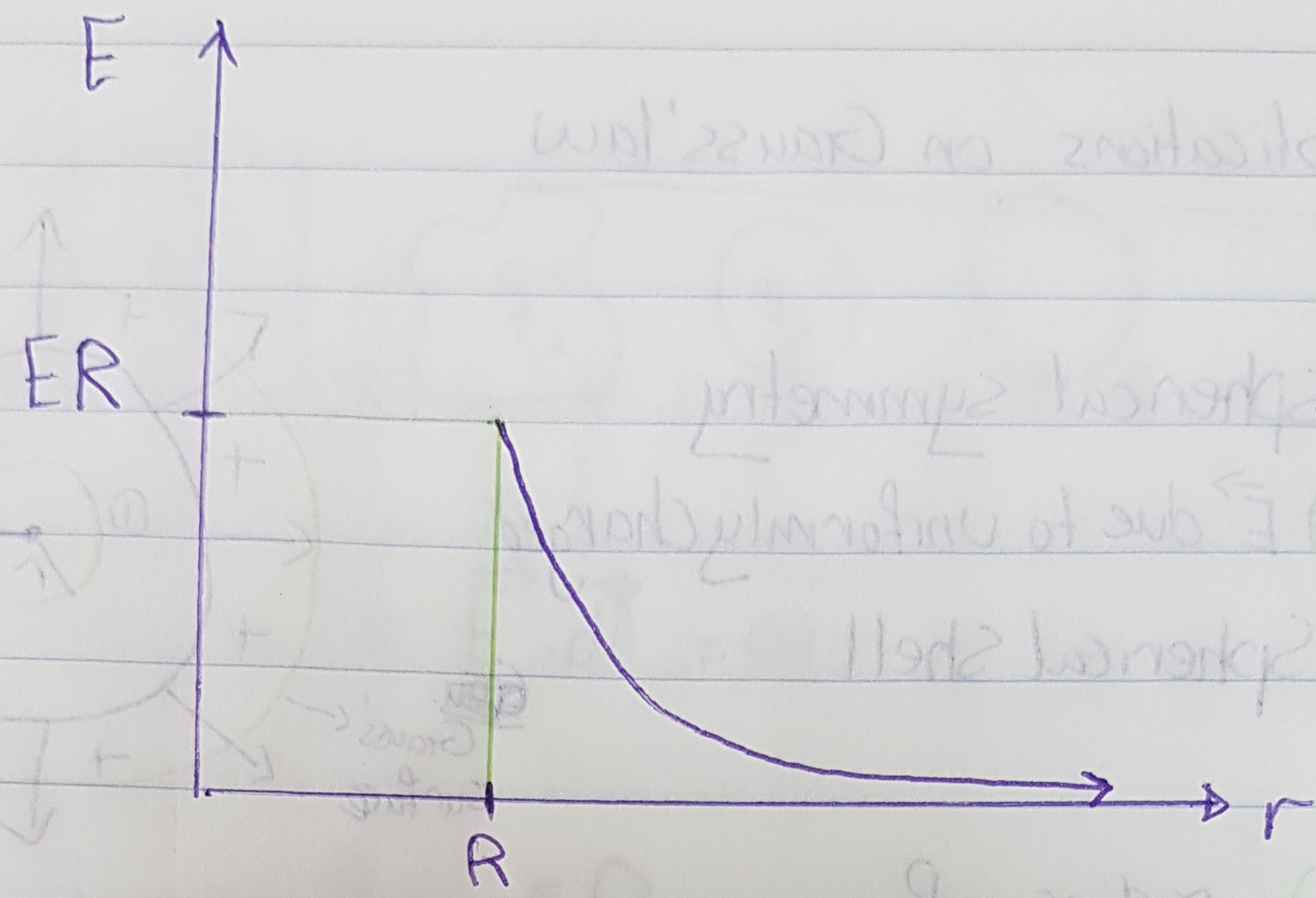
① $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$, $E = 0$, $r < R$

② 〃 〃 〃 $r > R$

$$E \int \cos\theta \, dA = \frac{q}{\varepsilon_0} \Rightarrow E \pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi r^2 \varepsilon_0} \quad \left[r \geq R , \; E = \frac{q}{4\pi R^2} \right]$$

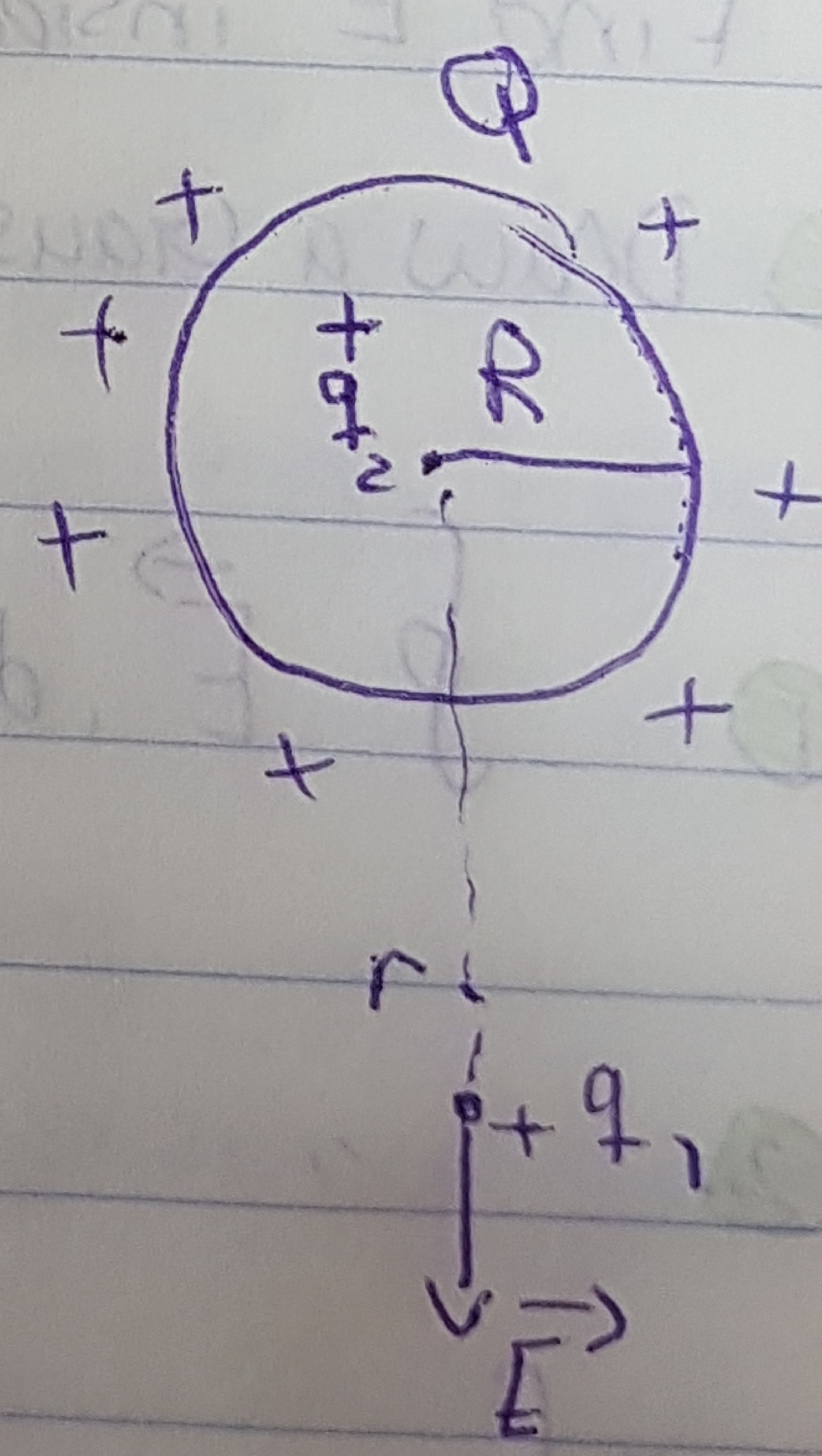
E_{max}



e.g

(a) F on $q_1 = qE$

$F_{on\,q} = q_1 \left(\frac{Q}{4\pi \varepsilon_0 r^2} \right)$



(b) $F_{on\,q_2} = q_2 E = 0$

spherical symmetry

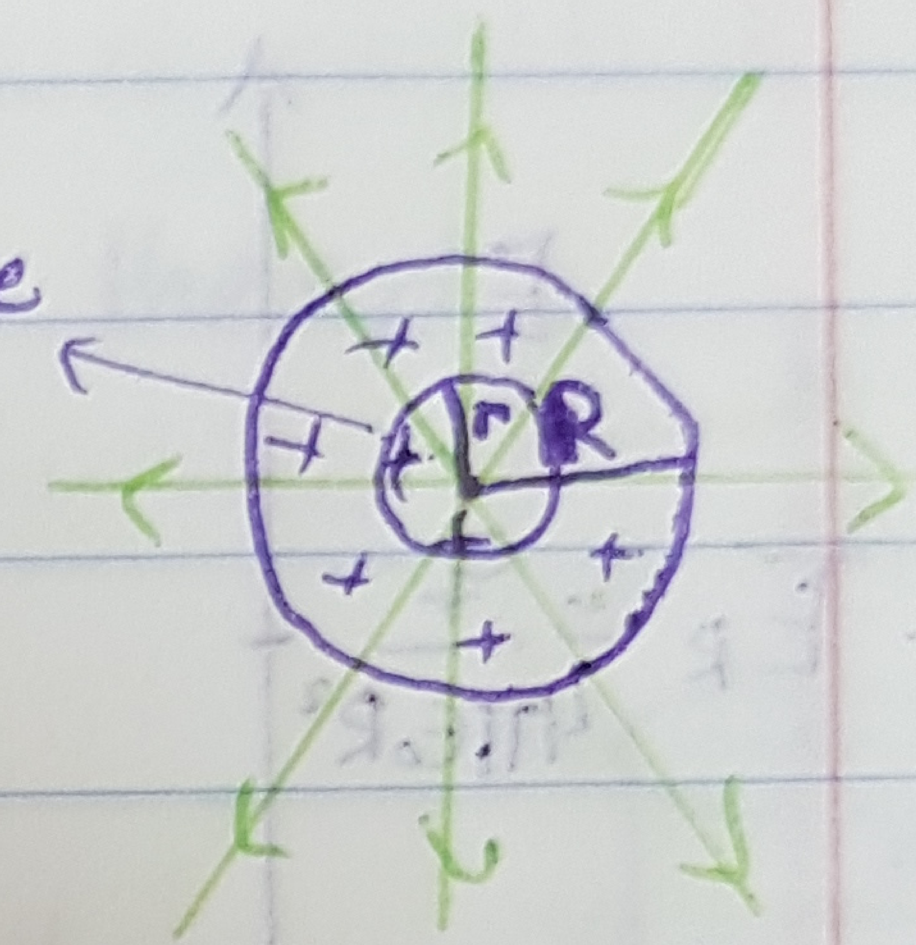
Case 2 $\vec{E}$ due to a uniformly charge Nonconducting sphere:

مادة عازلة

Radius = R

Charge = q

Gauss' surface



$$\rho = \frac{q}{\text{volume}} = \frac{q}{\frac{4}{3}\pi R^2} \quad (C/m^3)$$

* find $\vec{E}$ ① at $r<R$

② at $r>R$

① draw G.S to be a sphere of raduis $(r) < R$

$$\oint \vec{E}\, d\vec{A} = \frac{q_{enc}}{\epsilon_0} \Rightarrow E \oint \cos\theta\, dA = \frac{\rho \frac{4}{3}\pi r^3}{\epsilon_0}$$

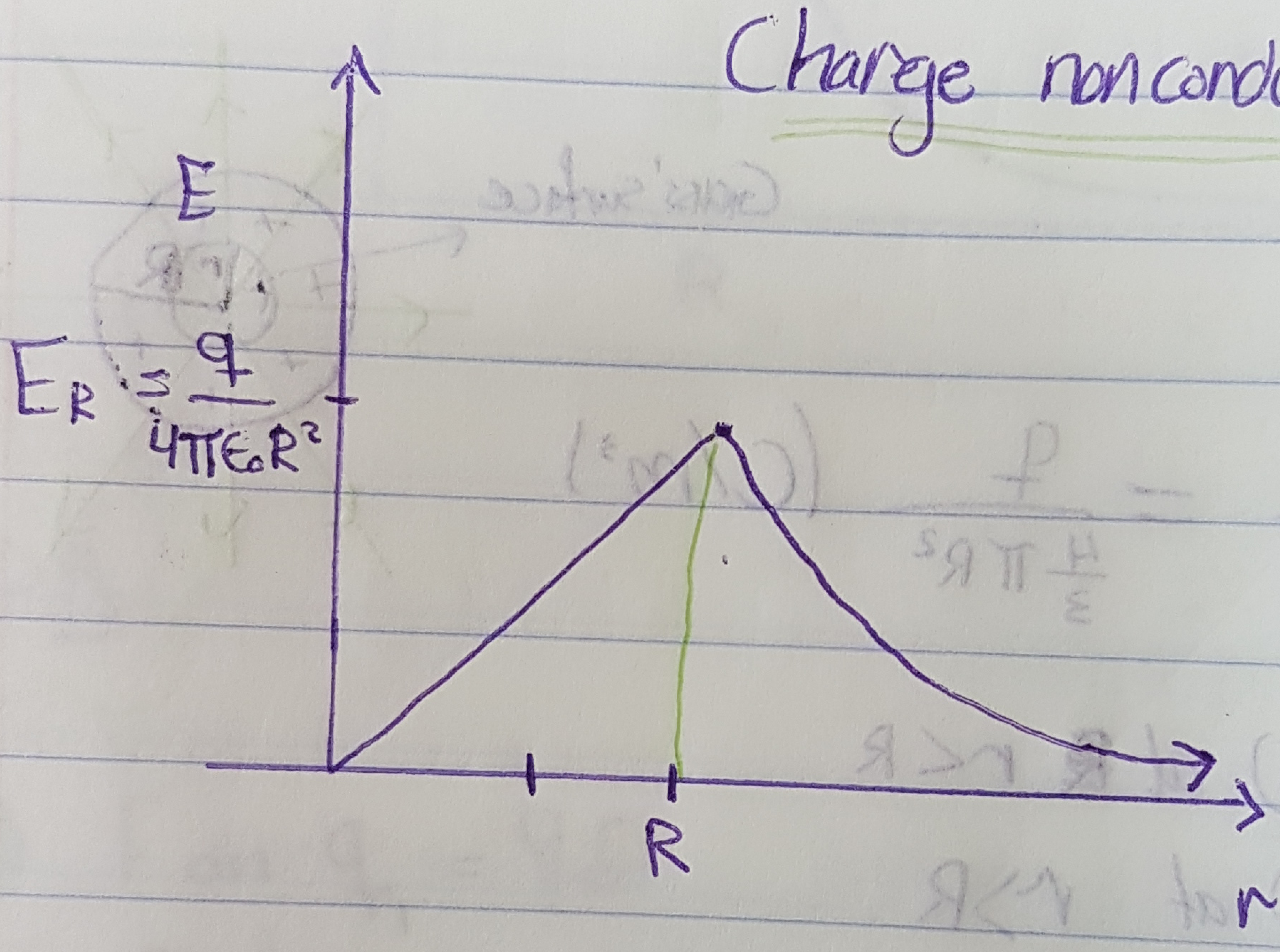
$$E\, \cancel{4\pi} r^2 = \frac{\cancel{4\pi} r^3 \rho}{3\epsilon_0} \Rightarrow \boxed{E = \frac{\rho}{3\epsilon_0} r}, \quad r \leq R$$

$$\boxed{E = \left(\frac{q}{4\pi\epsilon_0 R^3}\right) r}$$

② Draw G.S to be a sphere of radius r > R

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \Rightarrow E\, 4\pi r^2 = \frac{q}{\epsilon_0}$

Charge nonconducting sphere



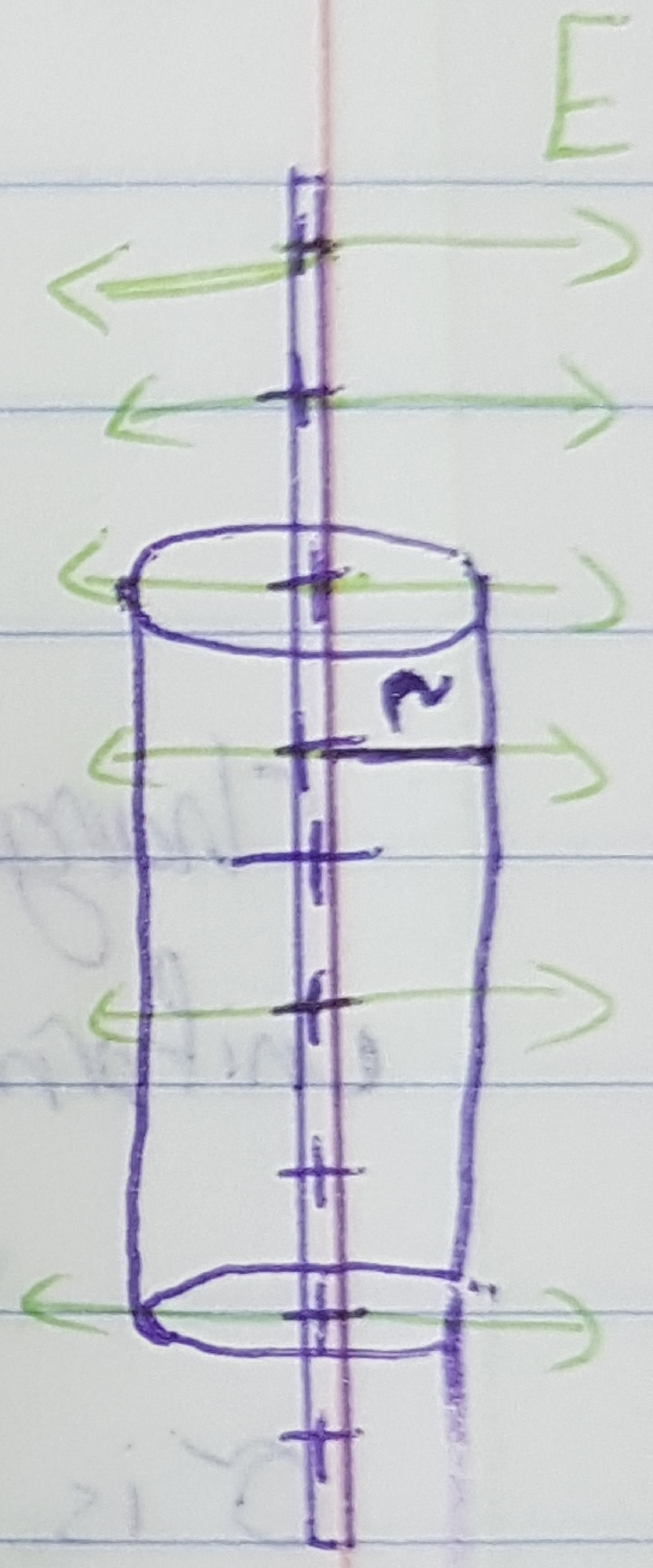
Cylindrical symmetry

⇒ line symmetry

$\vec{E}$ due to a uniformly charged infinite rod "very long rod"

Charged rod has λ . find E at r from the rod

$\lambda \rightarrow$ C/m



+ Draw G.S to be a cylinder.

length = h

radius = r

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\cancel{\cos\theta\, E \oint} \quad E\cos\theta\,(2\pi r h) = \frac{\lambda h}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

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planer Symmetry

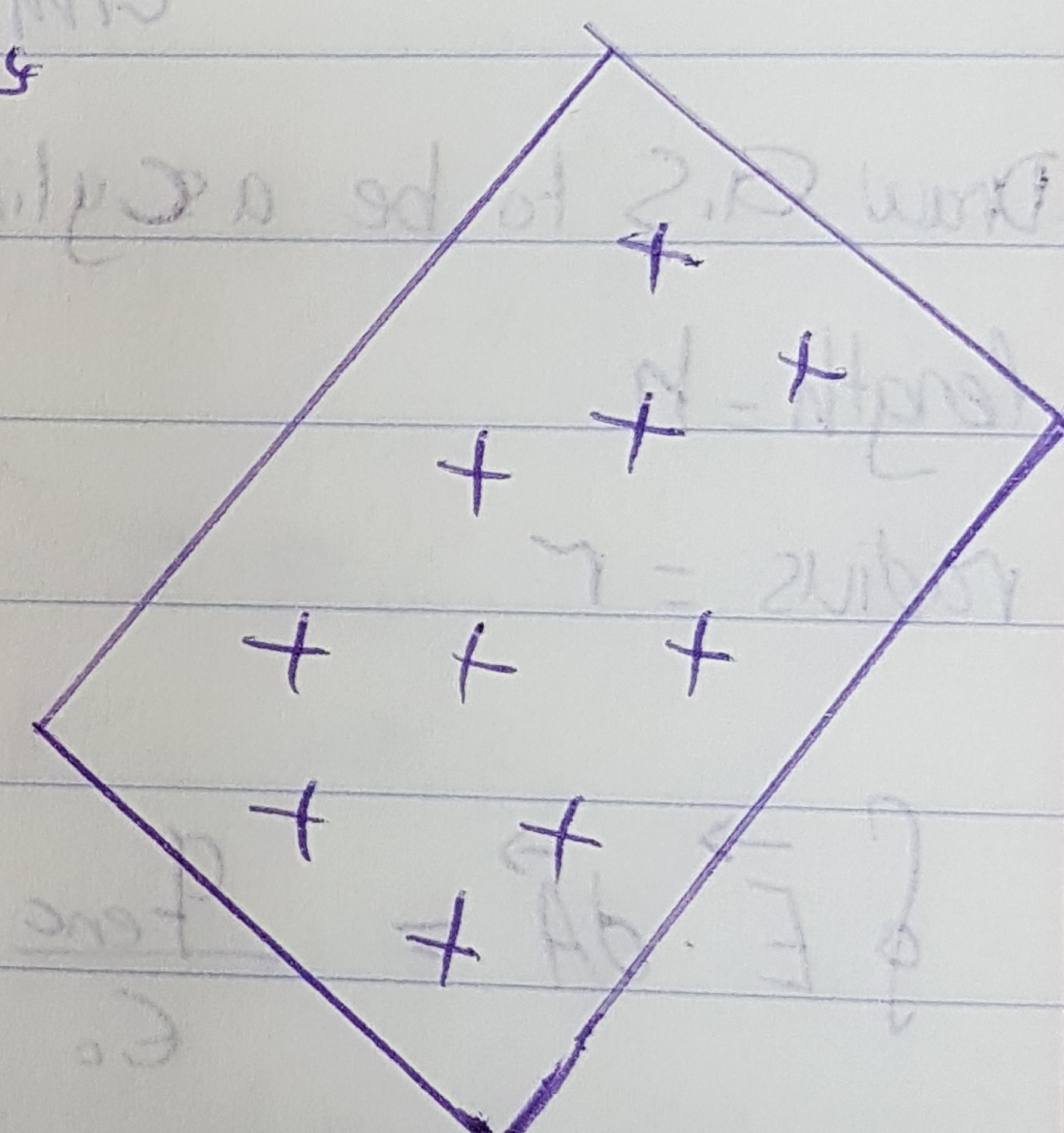
$\vec{E}$ due to a uniformly charged infinite thin sheet

non conducting

لأنها لولم تكن كذلك ستكون الشحنة

على الوجهين

Charge is distributed uniformly in one face.

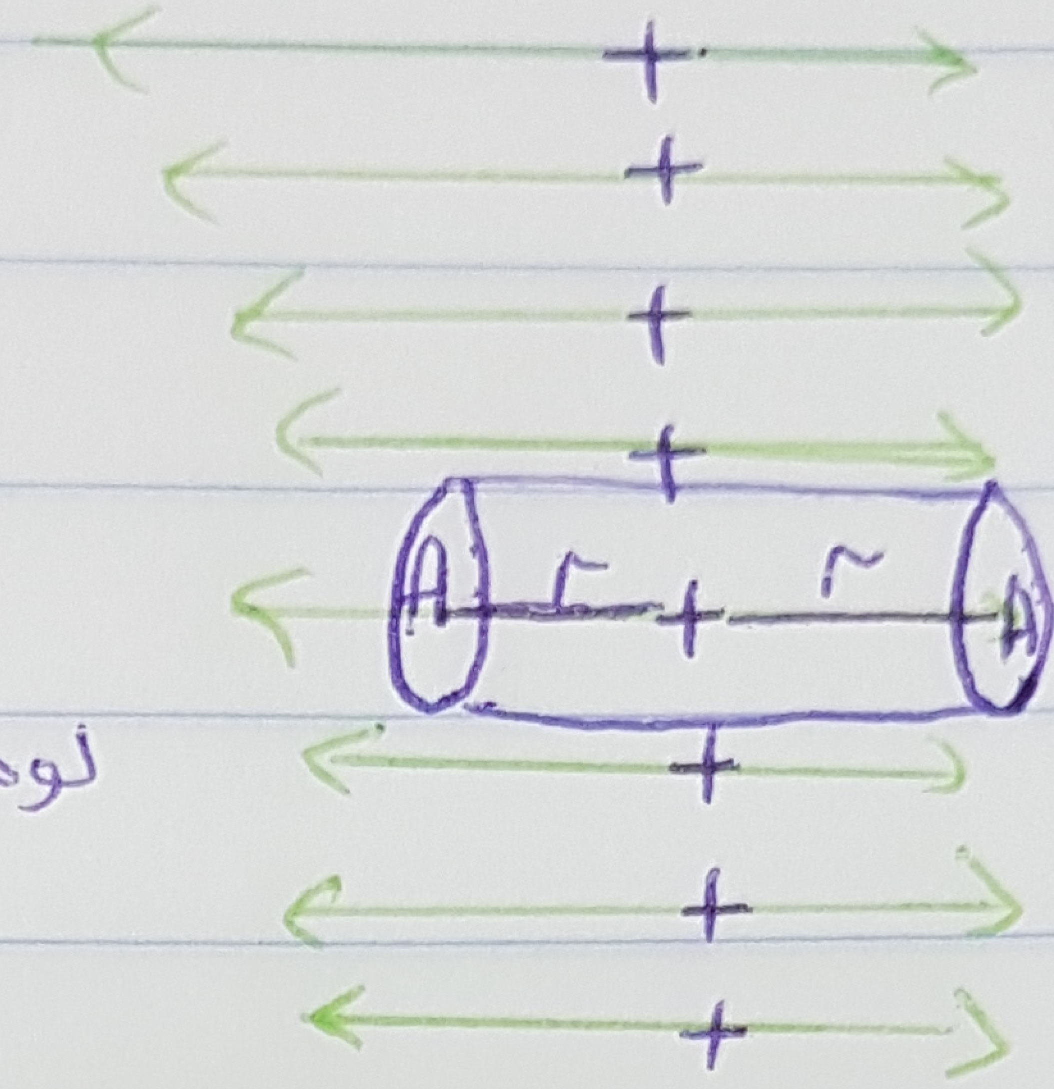


σ is constant.

find E a distance r from the sheet.

* Draw G.S to be a cylinder of length = 2r "r from each side"

$$\oint \vec{E}\, d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$



$$EA + EA = \frac{\sigma \vec{A}}{\epsilon_0}$$

لأنو جه فقط المشحون

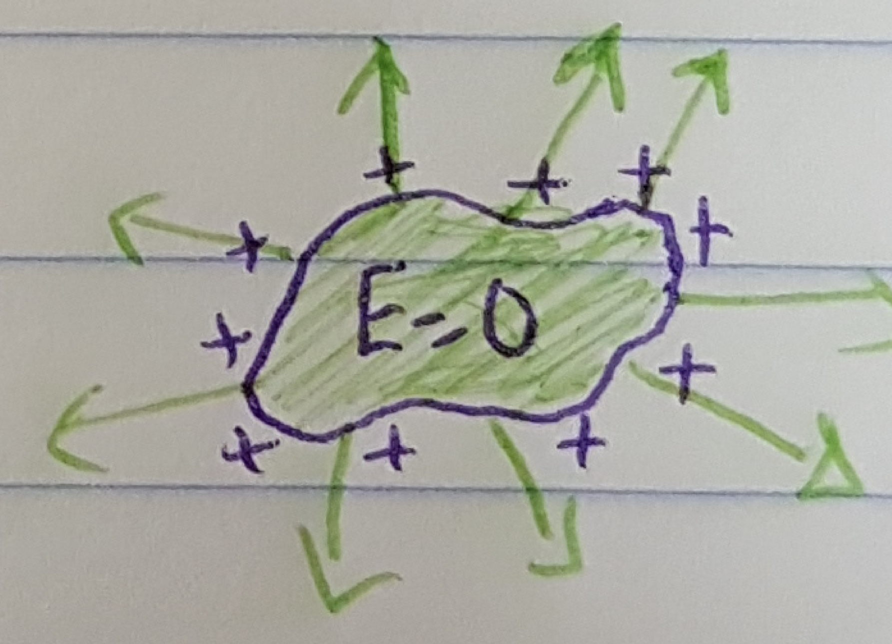
لو كان الوجهان كان 2σ

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$ constant

Charged conductor "solid" معباة من الداخل

خطوط المجال عمودية على السطح.



$E = 0$ inside the conductor

$q = 0$ inside the conductor.

q rest in the outer surface.

E near the outer surface $= \frac{\sigma}{\epsilon_0}$