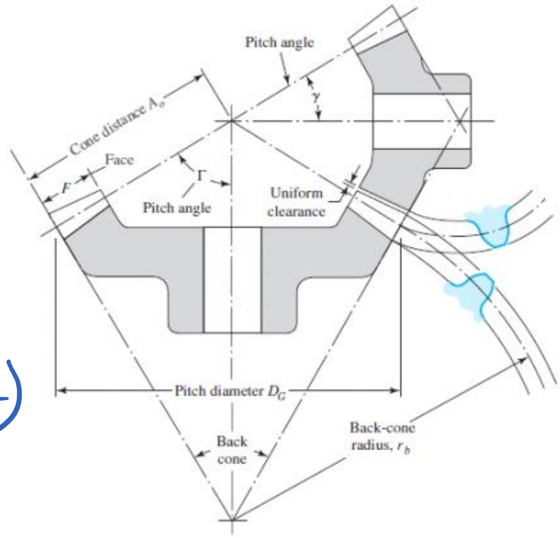
Bevel Gear

Monday, June 21, 2021 8:45 PM

* Geometry:



* The circular pitch : $p = \frac{\pi d}{N}$

* The diameteral pitch : $P = \frac{N}{d}$

* The pitch angle : $\gamma_p = \tan^{-1}\left(\frac{N_p}{N_g}\right)$

$\gamma_g = \tan^{-1}\left(\frac{N_g}{N_p}\right) = \tan^{-1}(VR)$

$\gamma_p + \gamma_g = 90^\circ$

* The face width : $b < \frac{10}{P}$ & $b \leq \frac{L}{3}$

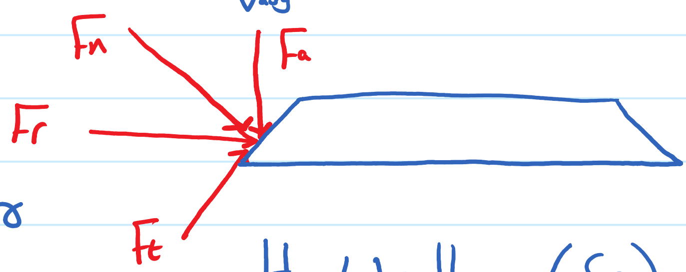
* The Cone pitch length : $L = \frac{d_g}{2\sin\gamma_g} = \frac{d_p}{2\sin\gamma_p}$

* $d_{avg} = d - b\sin\gamma$

* $V_{avg} = \frac{\pi d_{avg} n}{12}$ [US] or $V_{avg} = \frac{\pi d_{avg} n}{60}$

⇒ Force Analysis :

* $F_t = \frac{33000\, H}{V_{avg}}$ [US] or $F_t = \frac{P}{V_{avg}}$ [W]



* $F_n = F_t \tan\phi$

* $F_a = F_t \tan\phi \sin\gamma = F_n \sin\gamma$

* $F_r = F_t \tan\phi \cos\gamma = F_n \cos\gamma$

$H_{rated} = H_{nom}\left(\frac{S_e}{\sigma_b'}\right)$

$H_{rated} = H_{nom}\left(\frac{S_{fe}}{\sigma_c'}\right)$

Bending Stress Analysis (1)

Monday, June 21, 2021 9:01 PM

① Bending Stress

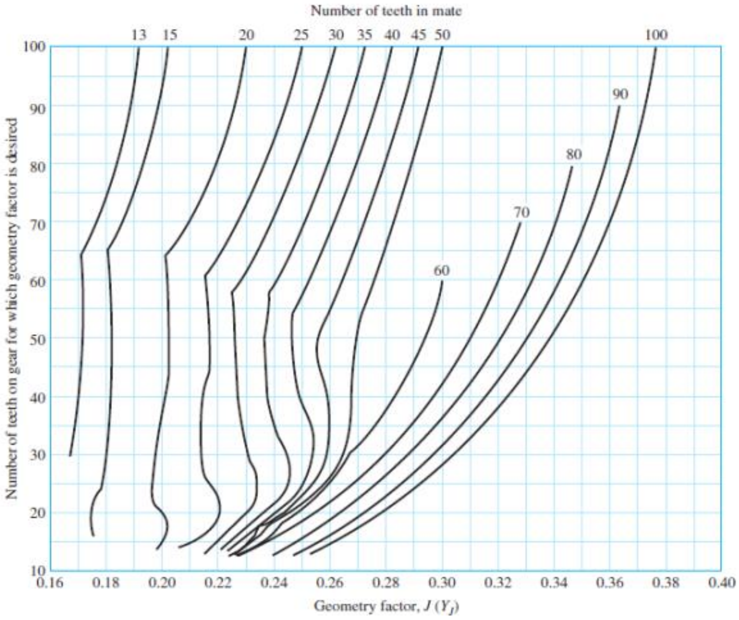
$$\sigma_b = \frac{F_t P}{b J} K_s K_o K_m K_v \quad [US]$$

$$\sigma_b = \frac{F_t}{b J m} K_s K_o K_m K_v \quad [SI]$$

To find (J):

Figure 15-7

Bending factor J (Y_J) for coniflex straight-bevel gears with a 20° normal pressure angle and 90° shaft angle. (*Source: ANSI/AGMA 2003-B97.*)



* Find (J_p) and (J_g) because $\sigma_{bp} \neq \sigma_{bg}$

To find (K_s):

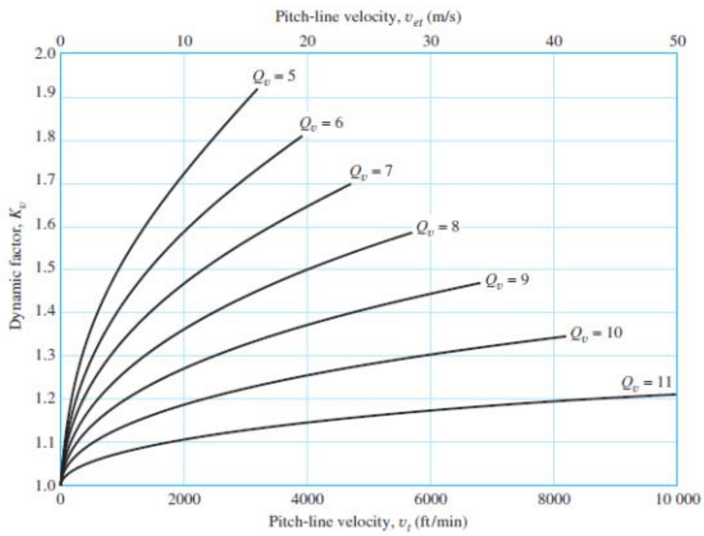
$$K_S = \begin{cases} 0.4867 + 0.2132/P_d & 0.5 \le P_d \le 16 \text{ teeth/in} \\ 0.5 & P_d > 16 \text{ teeth/in} \end{cases} \quad \text{(U.S. customary units)} \quad (15\text{–}10$$

$$= \begin{cases} 0.5 & m_{et} < 1.6 \text{ mm} \\ 0.4867 + 0.008\,339 m_{et} & 1.6 \le m_{et} \le 50 \text{ mm} \end{cases} \quad \text{(SI units)}$$

To find K_v:

Figure 15-5

Dynamic factor K_v. (*Source: ANSI/AGMA 2003-B97.*)



$$K_v = \left(\frac{A + \sqrt{V}}{A}\right)^B \quad [US]$$

$$K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B \quad [SI]$$

$$B = \frac{(12 - Q_v)^{2/3}}{4}$$

$$A = 50 + 56(1 - B)$$

(2)

Monday, June 21, 2021 9:13 PM

To find (K_o):

Table 15–2

Overload Factors K_o (K_A)

Source: ANSI/AGMA 2003-B97.

Character of Prime Mover	Character of Load on Driven Machine: Uniform	Light Shock	Medium Shock	Heavy Shock
Uniform	1.00	1.25	1.50	1.75 or higher
Light shock	1.10	1.35	1.60	1.85 or higher
Medium shock	1.25	1.50	1.75	2.00 or higher
Heavy shock	1.50	1.75	2.00	2.25 or higher

Note: This table is for speed-decreasing drives. For speed-increasing drives, add $0.01(N/n)^2$ or $0.01(z_2/z_1)^2$ to the above factors.

To find (K_m):

Load-Distribution Factor K_m ($K_{H\beta}$)

$$K_m = K_{mb} + 0.0036F^2 \quad \text{(U.S. customary units)}$$
$$K_{H\beta} = K_{mb} + 5.6(10^{-6})b^2 \quad \text{(SI units)} \qquad (15\text{–}11)$$

where

$$K_{mb} = \begin{cases} 1.00 & \text{both members straddle-mounted} \\ 1.10 & \text{one member straddle-mounted} \\ 1.25 & \text{neither member straddle-mounted} \end{cases}$$

+ The endurance limit of bending (S_e):

$$S_e = S_t \frac{K_L K_{ms}}{K_r}$$

To find (S_t):

Table 15–6

Allowable Bending Stress Numbers for Steel Gears, s_{at} ($\sigma_{F\,\text{lim}}$) Source: ANSI/AGMA 2003-B97.

Material Designation	Heat Treatment	Minimum Surface Hardness	Bending Stress Number (Allowable), s_{at} ($\sigma_{F\,\text{lim}}$) lbf/in² (N/mm²): Grade 1*	Grade 2*	Grade 3*
Steel	Through-hardened	Fig. 15–13	Fig. 15–13	Fig. 15–13	
	Flame or induction hardened				
	Unhardened roots	50 HRC	15 000 (85)	13 500 (95)	
	Hardened roots		22 500 (154)		
	Carburized and case hardened†	2003-B97 Table 8	30 000 (205)	35 000 (240)	40 000 (275)
AISI 4140	Nitrided†,‡	84.5 HR15N		22 000 (150)	
Nitralloy 135M	Nitrided†,‡	90.0 HR15N		24 000 (165)	

*See ANSI/AGMA 2003-B97, Tables 8–11, for metallurgical factors for each stress grade of steel gears.

†The allowable stress numbers indicated may be used with the case depths prescribed in 21.1, ANSI/AGMA 2003-B97.

‡The overload capacity of nitrided gears is low. Since the shape of the effective S-N curve is flat, the sensitivity to shock should be

Table 15–7

Allowable Bending Stress Number for Iron Gears, s_{at} ($\sigma_{F\,\text{lim}}$) Source: ANSI/AGMA 2003-B97.

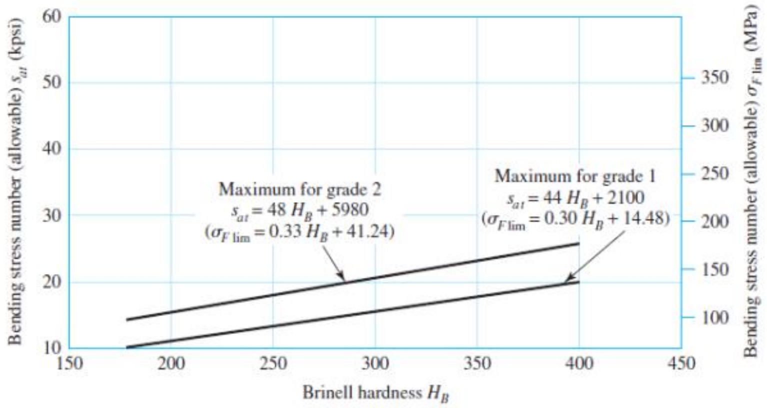
Material	Material Designation: ASTM	ISO	Heat Treatment	Typical Minimum Surface Hardness	Bending Stress Number (Allowable), s_{at} ($\sigma_{F\,\text{lim}}$) lbf/in² (N/mm²)
Cast iron	ASTM A48	ISO/DR 185			
	Class 30	Grade 200	As cast	175 HB	4500 (30)
	Class 40	Grade 300	As cast	200 HB	6500 (45)
Ductile (nodular) iron	ASTM A536	ISO/DIS 1083			
	Grade 80-55-06	Grade 600-370-03	Quenched and tempered	180 HB	10 000 (70)
	Grade 120-90-02	Grade 800-480-02		300 HB	13 500 (95)

(3)

Monday, June 21, 2021 9:21 PM

Figure 15–13

Allowable bending stress number for through-hardened steel gears, $s_{at}(\sigma_{F\,\lim})$. *(Source: ANSI/AGMA 2003-B97.)*



To find (K_{ms}): $K_{ms} = \begin{cases} 1, & \text{for idler gear} \\ 1.4, & \text{for input \& output gears.} \end{cases}$

To find (K_L):

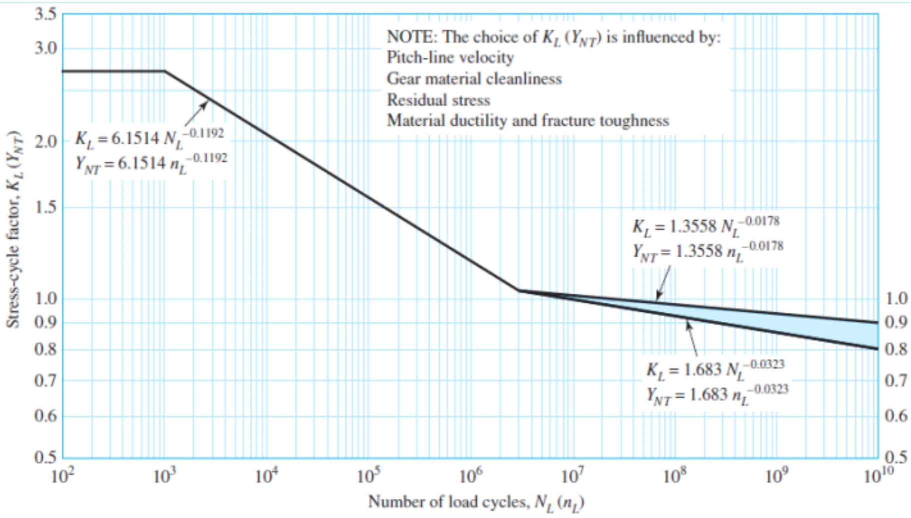


Figure 15–9

Stress-cycle factor for bending strength K_L (Y_{NT}) for carburized case-hardened steel bevel gears. *(Source: ANSI/AGMA 2003-B9*

$$K_L = \begin{cases} 2.7 & 10^2 \le N_L < 10^3 \\ 6.1514 N_L^{-0.1192} & 10^3 \le N_L < 3(10^6) \\ 1.683 N_L^{-0.0323} & 3(10^6) \le N_L \le 10^{10} \quad \text{critical} \\ 1.3558 N_L^{-0.0178} & 3(10^6) \le N_L \le 10^{10} \quad \text{general} \end{cases}$$

To find (K_r):

Table 15–3

Reliability Factors

Source: ANSI/AGMA 2003-B97.

Requirements of Application	Reliability Factors for Steel* C_R (Z_Z)	K_R (Y_Z)†
Fewer than one failure in 10 000	1.22	1.50
Fewer than one failure in 1000	1.12	1.25
Fewer than one failure in 100	1.00	1.00
Fewer than one failure in 10	0.92	0.85‡
Fewer than one failure in 2	0.84	0.70§

$$Y_Z = K_R = \begin{cases} 0.50 - 0.25\log(1-R) & 0.99 \le R \le 0.999 \quad (15\text{–}19) \\ 0.70 - 0.15\log(1-R) & 0.90 \le R < 0.99 \quad (15\text{–}20) \end{cases}$$

Contact Stress (1)

Monday, June 21, 2021 9:28 PM

② Contact Stress

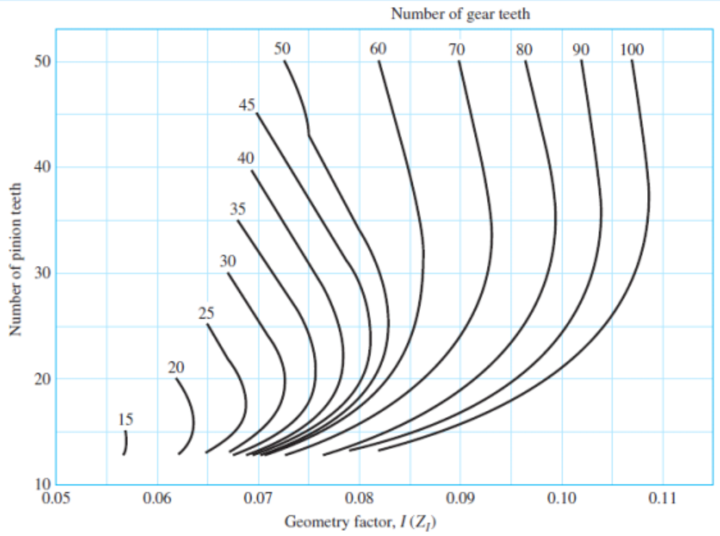
$$\sigma_c = C_p \sqrt{\frac{F_t C_o C_m C_v C_s C_c}{b\, I\, d_p}}$$

$C_p = 190 \sqrt{MPa}$ or $C_p = 2290 \sqrt{MPsi}$ (for Steel)

To find (I):

Figure 15–6

Contact geometry factor I (Z_I) for coniflex straight-bevel gears with a 20° normal pressure angle and a 90° shaft angle. *(Source: ANSI/AGMA 2003-B97.)*



$C_o = K_o$ (overload factor) table (15-2)
$C_m = K_m$ (mounting factor) Equation (15-11)
$C_v = K_v$ (velocity factor) Figure (15-5)

To find (C_s):

Size Factor for Pitting Resistance C_s (Z_x)

F → b = face width

$$C_s = \begin{cases} 0.5 & F < 0.5 \text{ in} \\ 0.125F + 0.4375 & 0.5 \le F \le 4.5 \text{ in} \\ 1 & F > 4.5 \text{ in} \end{cases} \quad \text{(U.S. customary units)}$$

$$\begin{cases} 0.5 & b < 12.7 \text{ mm} \\ 0.00492b + 0.4375 & 12.7 \le b \le 114.3 \text{ mm} \\ 1 & b > 114.3 \text{ mm} \end{cases} \quad \text{(SI units)} \qquad (15\text{–}9)$$

To find (C_c): $C_c = \begin{cases} 1.5, & \text{Crowned teeth.} \\ 2, & \text{uncrowned teeth.} \end{cases}$

(2)

Monday, June 21, 2021 9:37 PM

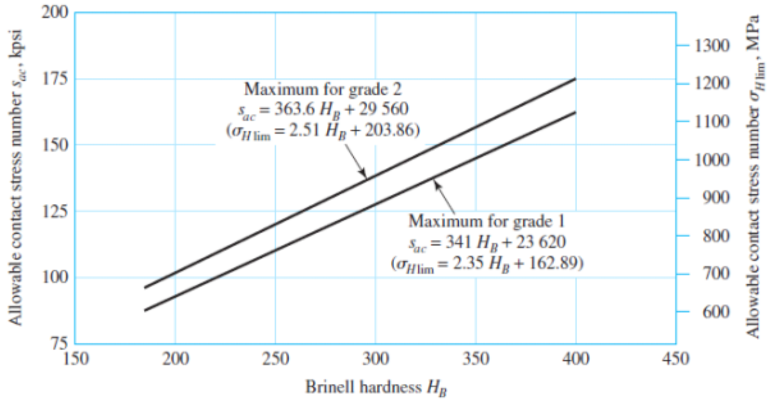
* Contact Fatigue strength:

$$S_{fc} = S_c \frac{C_L C_H}{C_r}$$

To find (S_c):

Figure 15–12

Allowable contact stress number for through-hardened steel gears, s_{ac} ($\sigma_{H\ \text{lim}}$). *(Source: ANSI/AGMA 2003-B97.)*



The equations are

$$s_{ac} = 341H_B + 23\,620 \text{ psi} \qquad \text{grade 1}$$
$$\sigma_{H\,\text{lim}} = 2.35H_B + 162.89 \text{ MPa} \qquad \text{grade 1}$$
$$s_{ac} = 363.6H_B + 29\,560 \text{ psi} \qquad \text{grade 2} \qquad (15\text{–}22)$$
$$\sigma_{H\,\text{lim}} = 2.51H_B + 203.86 \text{ MPa} \qquad \text{grade 2}$$

Table 15–4

Allowable Contact Stress Number for Steel Gears, s_{ac} ($\sigma_{H\ \text{lim}}$) *Source: ANSI/AGMA 2003-B97.*

Material Designation	Heat Treatment	Minimum Surface* Hardness	Allowable Contact Stress Number, s_{ac} ($\sigma_{H\ \text{lim}}$) lbf/in² (N/mm²) Grade 1†	Grade 2†	Grade 3†
Steel	Through-hardened‡	Fig. 15–12	Fig. 15–12	Fig. 15–12	
	Flame or induction hardened§	50 HRC	175 000 (1210)	190 000 (1310)	
	Carburized and case hardened§	2003-B97 Table 8	200 000 (1380)	225 000 (1550)	250 000 (1720)
AISI 4140	Nitrided§	84.5 HR15N		145 000 (1000)	
Nitralloy 135M	Nitrided§	90.0 HR15N		160 000 (1100)	

*Hardness to be equivalent to that at the tooth middepth in the center of the face width.

Table 15–5

Allowable Contact Stress Number for Iron Gears, s_{ac} ($\sigma_{H\ \text{lim}}$) *Source: ANSI/AGMA 2003-B97.*

Material	Material Designation ASTM	ISO	Heat Treatment	Typical Minimum Surface Hardness	Allowable Contact Stress Number, s_{ac} ($\sigma_{H\ \text{lim}}$) lbf/in² (N/mm²)
Cast iron	ASTM A48	ISO/DR 185			
	Class 30	Grade 200	As cast	175 HB	50 000 (345)
	Class 40	Grade 300	As cast	200 HB	65 000 (450)
Ductile (nodular) iron	ASTM A536	ISO/DIS 1083			
	Grade 80-55-06	Grade 600-370-03	Quenched and tempered	180 HB	94 000 (650)
	Grade 120-90-02	Grade 800-480-02		300 HB	135 000 (930)

To find (C_L):

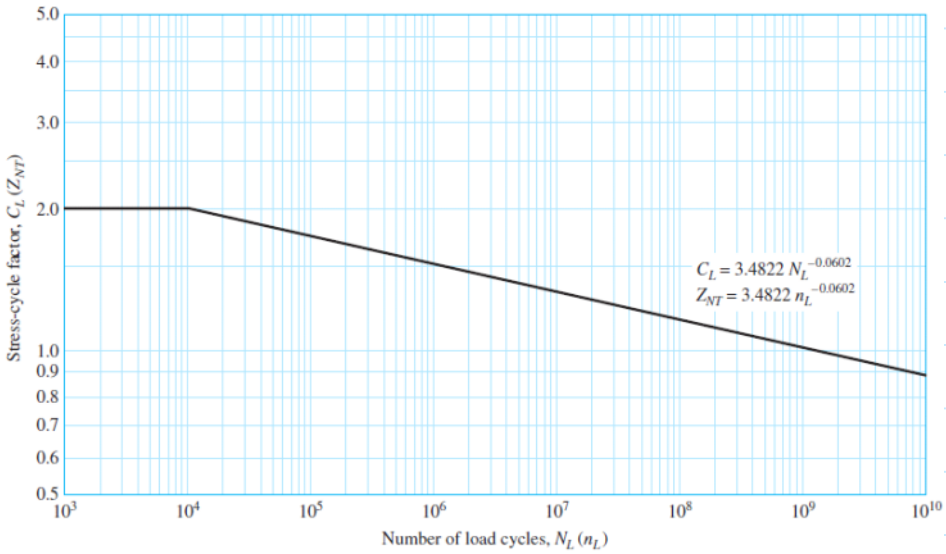


Figure 15–8

Contact stress-cycle factor for pitting resistance C_L (Z_{NT}) for carburized case-hardened steel bevel gears. *(Source: ANSI/AGMA 2003-B97.)*

To find (C_H): (for gear only)

Figure 15–10

Hardness-ratio factor C_H (Z_W) for through-hardened pinion and gear. *(Source: ANSI/AGMA 2003-B97.)*

* If $C_H < 1.2$ let $C_H = 1$

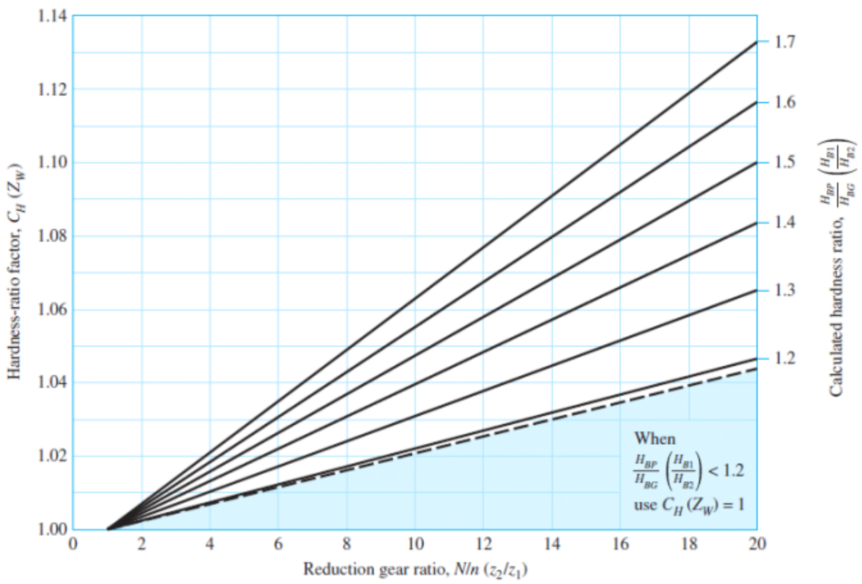
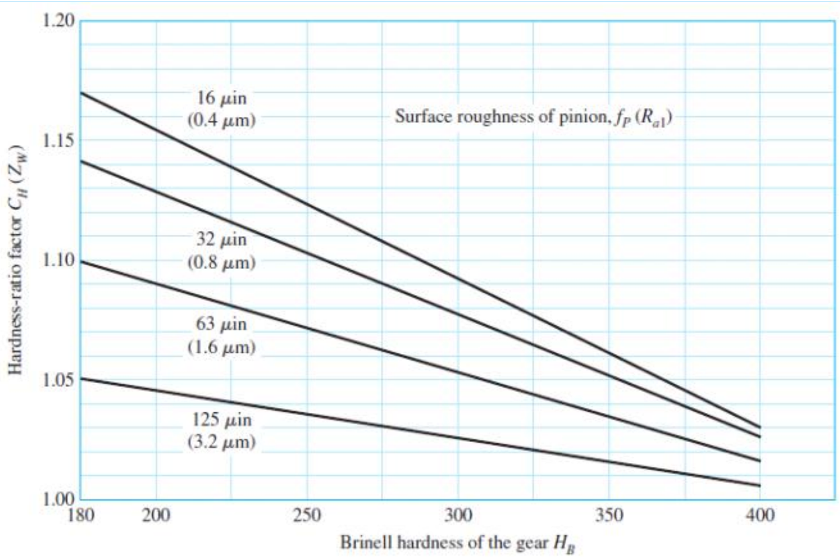


Figure 15–11

Hardness-ratio factor C_H (Z_W) for surface-hardened pinions. *(Source: ANSI/AGMA 2003-B97.)*



(4)

Tuesday, June 22, 2021 12:14 AM

To find (Cr):

Table 15–3

Reliability Factors

Source: ANSI/AGMA 2003-B97.

Requirements of Application	Reliability Factors for Steel* C_R (Z_Z)	K_R (Y_Z)†
Fewer than one failure in 10 000	1.22	1.50
Fewer than one failure in 1000	1.12	1.25
Fewer than one failure in 100	1.00	1.00
Fewer than one failure in 10	0.92	0.85‡
Fewer than one failure in 2	0.84	0.70§

*At the present time there are insufficient data concerning the reliability of bevel gears made from other materials.

†Tooth breakage is sometimes considered a greater hazard than pitting. In such cases a greater value of K_R (Y_Z) is selected for bending.

‡At this value plastic flow might occur rather than pitting.

§From test data extrapolation.

$$Y_Z = K_R = \begin{cases} 0.50 - 0.25\log(1 - R) & 0.99 \le R \le 0.999 \quad (15\text{–}19) \\ 0.70 - 0.15\log(1 - R) & 0.90 \le R < 0.99 \quad (15\text{–}20) \end{cases}$$