

0.3 Integral Exponents

Consider the expression $(1.5)(1.5)(1.5)$. This can be written in exponential form as $(1.5)^3$

In general, if a is a real number and n is positive integer, then:- $a^n = a \cdot a \cdot a \cdot \dots \cdot a$ (n times)
Where a is the base, n is the exponent.

Rules of exponents.

For any real numbers a and b and any integers m and n :-

① $a^m \cdot a^n = a^{m+n}$ عند الضرب تجمع الأسس

Examples: $3^2 \cdot 3^{-4} = 3^{2+(-4)} = 3^{-2}$

$$x \cdot x^2 = x^{1+2} = x^3$$

② $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$ عند القسمة تطرح الأسس

Examples: $\frac{4^7}{4^4} = 4^{7-4} = 4^3$

$$\frac{y^2}{y^{-1}} = y^{2-(-1)} = y^{2+1} = y^3$$

③ $(ab)^m = a^m \cdot b^m$

Example:- $(3x)^2 = (3)^2 x^2 = 9x^2$

$$(x \cdot y)^5 = x^5 \cdot y^5$$

④ $(a^m)^n = a^{m \cdot n}$ عند الرفع نضرب الأس

Example:- $(3^2)^3 = 3^{2 \cdot 3} = 3^6 = 729$
 $(X^2)^2 = X^4$

⑤ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$

Example:- $\left(\frac{-2}{5}\right)^3 = \frac{(-2)^3}{5^3} = \frac{-8}{125}$

$\left(\frac{X^2}{X^{-1}}\right)^3 = \frac{(X^2)^3}{(X^{-1})^3} = \frac{X^6}{X^{-3}} = X^{6-(-3)} = X^9$

⑥ $a^0 = 1$, $a \neq 0$
 0^0 is undefined.

→ Example:- $(-0.1256)^0 = 1$
 $X^0 = 1$

⑦ $a^{-n} = \frac{1}{a^n}$, $a \neq 0$

Example:- $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$(-4)^{-3} = \frac{1}{(-4)^3} = \frac{1}{-64}$

⑧ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$, $a, b \neq 0$

Example:- $\left(\frac{X^2}{3}\right)^{-2} = \left(\frac{3}{X^2}\right)^2 = \frac{(3)^2}{(X^2)^2} = \frac{9}{X^4}$

Example:- Simplify the following with positive exponents

بسط المقادير التالية بحيث الجواب يحتوي على قوى موجبة

$$a) 2(x^2)^{-2} = 2x^{-4} = \frac{2}{x^4}$$

عند الرفع نقرب القوى ↓ القوى سالبة
لنعكس مكانها

$$b) x^{-2} \cdot x^{-5} = x^{-2+(-5)} = x^{-7} = \frac{1}{x^7}$$

عند الضرب نجمع القوى

$$c) \frac{x^{-8}}{x^{-4}} = x^{-8-(-4)} = x^{-8+4} = x^{-4} = \frac{1}{x^4}$$

عند القسمة نطرح القوى

$$d) \left(\frac{2x^3}{3x^{-5}} \right)^{-2} = \left(\frac{2x^{3-(-5)}}{3} \right)^{-2} = \left(\frac{2x^8}{3} \right)^{-2}$$

عند القسمة نطرح القوى

$$= \left(\frac{3}{2x^8} \right)^2 = \frac{(3)^2}{(2)^2 (x^8)^2} = \frac{9}{4x^{16}}$$

Example: Simplify the following so all exponents are positive.

$$\begin{aligned}
 \text{a) } & (2^3 x^{-4} y^5)^{-2} \\
 &= (2^3)^{-2} \cdot (x^{-4})^{-2} \cdot (y^5)^{-2} \\
 &= 2^{-6} \cdot x^8 \cdot y^{-10} \\
 &= \frac{x^8}{2^6 y^{10}} = \frac{x^8}{64 y^{10}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \frac{2 x^4 (x^2 y)^0}{(4 x^{-2} y)^2} \\
 &= \frac{2 x^4}{(4 x^{-2} y)^2} = \frac{2 x^4}{(4)^2 \cdot (x^{-2})^2 \cdot (y)^2} \\
 &= \frac{2 x^4}{8 \cdot 16 x^{-4} y^2} \\
 &= \frac{x^{4-(-4)}}{8 y^2} = \frac{x^8}{8 y^2}
 \end{aligned}$$

Evaluate: $0^4 = 0$

$0^{-4} = \frac{1}{0^4} = \frac{1}{0}$ meaningless (undefined)