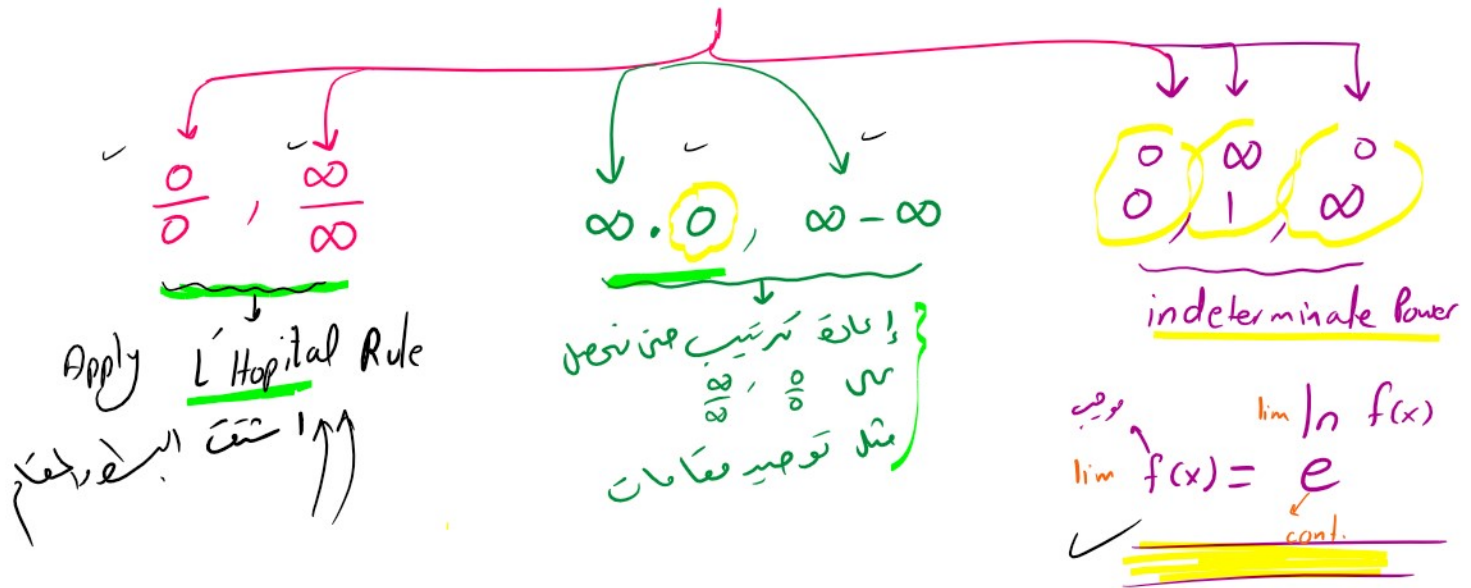


Indeterminate forms



Exp ① $\lim_{x \rightarrow -3} \frac{x+3}{x^2-9} = \lim_{x \rightarrow -3} \frac{1}{2x} = \frac{1}{-6}$ $\frac{0}{0}$

or $\lim_{x \rightarrow -3} \frac{x+3}{(x-3)(x+3)} = \lim_{x \rightarrow -3} \frac{1}{x-3} = \frac{1}{-3-3} = \frac{1}{-6}$

② $\lim_{x \rightarrow -1} \frac{x-2}{x^2-4} = \frac{[-1]-2}{(-1)^2-4} = \frac{-3}{1-4} = \frac{-3}{-3} = 1$

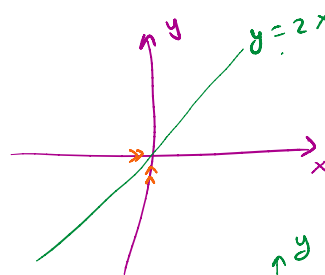
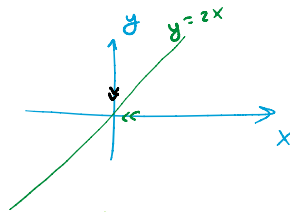
③ $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{3x^2}$ $\frac{0}{0}$

$= \lim_{x \rightarrow 0} \frac{\sin x}{6x} = \lim_{x \rightarrow 0} \frac{\cos x}{6} = \frac{1}{6}$ $\frac{0}{0}$

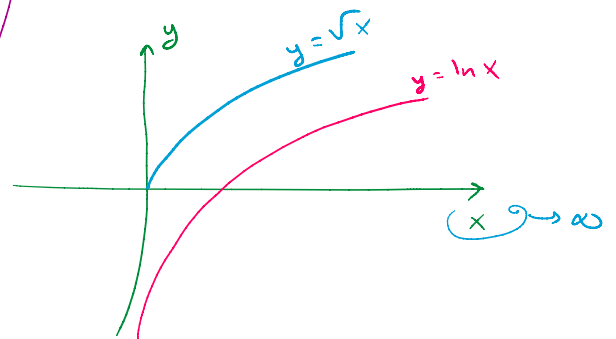
$$\textcircled{4} \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2 + x} = \lim_{x \rightarrow 0} \frac{\sin x}{2x + 1} = \frac{0}{0+1} = \frac{0}{1} = \textcircled{0}$$

$$\textcircled{5} \lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} = \lim_{x \rightarrow 0^+} \frac{\cos x}{2x} = \frac{1}{\text{small}^+} = +\infty \quad \frac{0}{0}$$

$$\textcircled{6} \lim_{x \rightarrow 0^-} \frac{\sin x}{x^2} = \lim_{x \rightarrow 0^-} \frac{\cos x}{2x} = \frac{1}{\text{small}^-} = -\infty \quad \frac{0}{0}$$



$\frac{0}{0}$



$$\textcircled{7} \lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$$

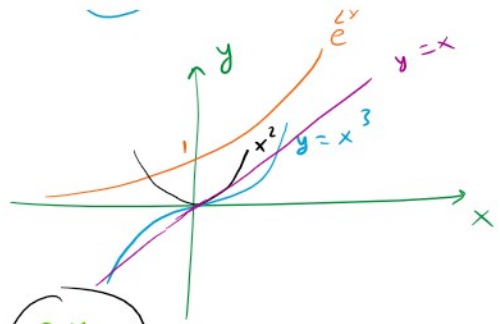
$$\lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \frac{2\sqrt{x}}{1} = \lim_{x \rightarrow \infty} \frac{2\sqrt{x}}{x}$$



$$= \lim_{x \rightarrow \infty} \frac{2}{\sqrt{x}} = 2 \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}} = 2 \cdot (0) = 0$$



8 $\lim_{x \rightarrow \infty} \frac{e^{2x}}{x^3} \quad \frac{\infty}{\infty}$



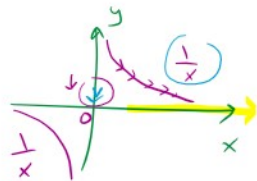
$$\lim_{x \rightarrow \infty} \frac{e^{2x} (2) \ln e}{3x^2} = \lim_{x \rightarrow \infty} \frac{2e^{2x}}{3x^2} \quad \frac{\infty}{\infty}$$

$$= \frac{2}{3} \lim_{x \rightarrow \infty} \frac{e^{2x} (2) \ln e}{2x} = \frac{2}{3} \lim_{x \rightarrow \infty} \frac{e^{2x}}{x}$$

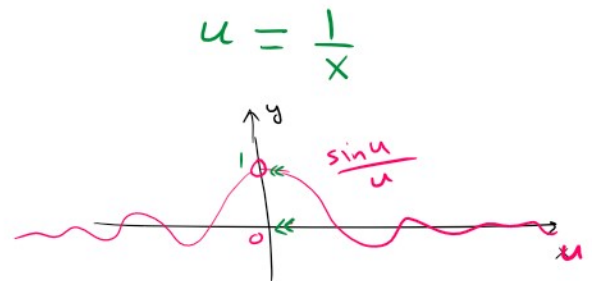
$$= \frac{2}{3} \lim_{x \rightarrow \infty} \frac{e^{2x} (2) \ln e}{1} = \frac{4}{3} \lim_{x \rightarrow \infty} e^{2x} = \frac{4}{3} \cdot \infty = \infty$$

9 $\lim_{x \rightarrow \infty} \left(x \sin \frac{1}{x} \right)$

$\infty \cdot \sin 0$
 $\infty \cdot 0$



$$= \lim_{x \rightarrow \infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}}$$



$$= \lim_{u \rightarrow 0^+} \frac{\sin u}{u}$$

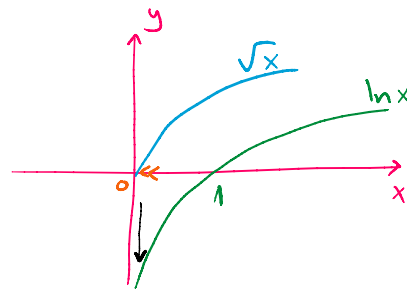
$$\frac{0}{0}$$

$$= \lim_{u \rightarrow 0^+} \frac{\cos u}{1} = \cos 0 = 1$$

10 $\lim_{x \rightarrow 0^+} \sqrt{x} \ln x$

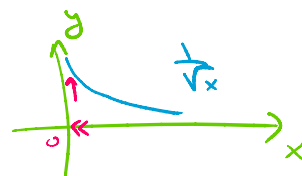
$u \rightarrow 0^+$

$0 \cdot (-\infty) = - (0 \cdot \infty)$



$\lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{\sqrt{x}}}$

$\frac{-\infty}{\infty}$

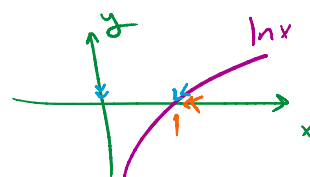


$\lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\left(-\frac{1}{2}\right) x^{-\frac{3}{2}}}$

$\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$= -2 \lim_{x \rightarrow 0^+} x^{\frac{3}{2}} x^{-1} = -2 \lim_{x \rightarrow 0^+} x^{\frac{1}{2}}$

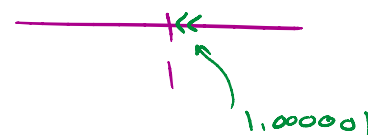
$= -2 \lim_{x \rightarrow 0^+} \sqrt{x} = 2(0) = 0$



11 $\lim_{x \rightarrow 1^+} \left(\frac{1}{x-1} - \frac{1}{\ln x} \right)$

$\frac{1}{\text{small}^+} - \frac{1}{\text{small}^+}$

$\infty - \infty$



$\lim_{x \rightarrow 1^+} \frac{\ln x - (x-1)}{(x-1)(\ln x)}$

$\frac{0 - (1-1)}{(1-1) 0} = \frac{0}{0}$

$$\lim_{x \rightarrow 1^+} \frac{\frac{1}{x} - 1}{\frac{x-1}{x} + \ln x} = \lim_{x \rightarrow 1^+} \frac{\left(\frac{1}{x}\right) - 1}{1 - \left(\frac{1}{x}\right) + \ln x} \quad \frac{1-1}{1-1+0} \quad \left(\frac{0}{0}\right)$$

$$\lim_{x \rightarrow 1^+} \frac{-\frac{1}{x^2}}{\frac{1}{x^2} + \frac{1}{x}} = \frac{-1}{1+1} = -\frac{1}{2}$$

Th (L'Hopital Rule)

Assume

$$f(a) = g(a) = 0 \quad \text{or} \\ f(a) = g(a) = \infty$$

Assume f, g diff such that $g'(x) \neq 0$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)} = \frac{0}{0} \quad \text{or} \quad \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

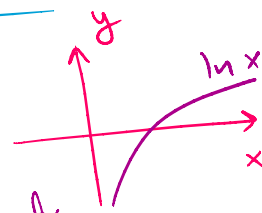
Exp ^{sb} (1)

$$\lim_{x \rightarrow \infty}$$

$$\frac{\frac{1}{\ln x}}{x} \rightarrow f(x)$$

$$\frac{1}{\infty}$$

$\infty = \infty$ indeterminate power



$f(x) = e^{\ln f(x)}$
 $\lim_{x \rightarrow \infty} e^{\ln x} = \lim_{x \rightarrow \infty} e^{\frac{1}{\frac{1}{\ln x}}}$
 $= \lim_{x \rightarrow \infty} e^{\frac{1}{\ln x}}$
 $= \lim_{x \rightarrow \infty} e = e$

$\ln 10$
 $e^{10} = 10$, $\ln \sqrt{5-x}$
 $e^{\sqrt{5-x}} = \sqrt{5-x}$

$\ln x$
 $e^x = x$, $\ln x^{\frac{1}{\ln x}}$
 $e^{\frac{1}{\ln x}} = \frac{1}{x}$

$\frac{\infty}{-\infty}$
 $\frac{\text{رقم بزرگتر}}{\text{منفی}} = \frac{\text{البسته}}{\text{المنام}}$

$\frac{\text{منفی}}{\text{رقم بزرگتر}} =$

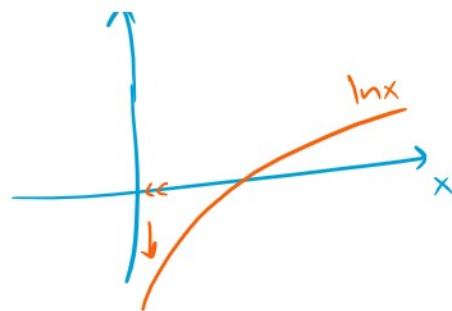
$\lim_{x \rightarrow 0} x$

0

y

lnx

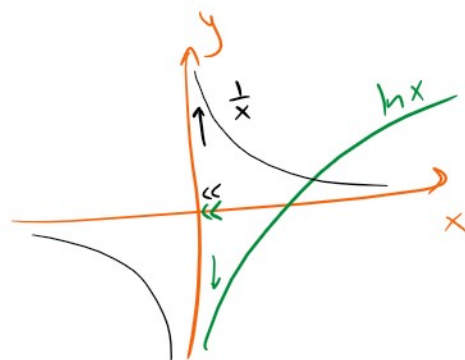
Exp $\lim_{x \rightarrow 0^+} \frac{x}{\ln x}$ $\frac{0}{-\infty}$



$$\left(\lim_{x \rightarrow 0^+} x \right) \left(\lim_{x \rightarrow 0^+} \frac{1}{\ln x} \right) \rightarrow -\frac{1}{\infty} \Rightarrow 0$$

$$0 \cdot 0 = 0$$

Exp $\lim_{x \rightarrow 0^+} \frac{\ln x}{x}$ $\frac{-\infty}{0}$



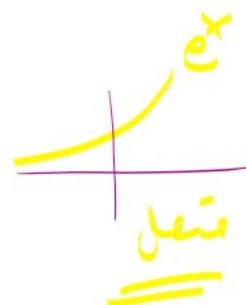
$$\left(\lim_{x \rightarrow 0^+} \frac{1}{x} \right) \left(\lim_{x \rightarrow 0^+} \ln x \right)$$

$$= \boxed{\infty} \cdot \boxed{-\infty} = -\infty$$

3 $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}}$

جواب

$$\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}} = e$$



$$\ln(1+x)^{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{x} \ln(1+x)$$

$$\lim_{x \rightarrow 0^+} e^{\ln(1+x)} = \lim_{x \rightarrow 0^+} e$$

$$\lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x} = e$$

$$\lim_{x \rightarrow 0^+} \frac{\frac{1}{1+x}}{1} = e$$

$\frac{\ln 1}{0} = \frac{0}{0}$

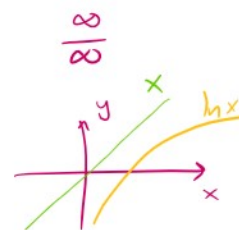
→ 1

$$= e = e$$

$$\lim_{x \rightarrow \infty} x^{\frac{1}{x}} = \lim_{x \rightarrow \infty} e^{\ln x^{\frac{1}{x}}}$$

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0$$

$$= \lim_{x \rightarrow \infty} e^{\frac{1}{x} \ln x} = \lim_{x \rightarrow \infty} e^{\frac{\ln x}{x}}$$



$$= e^{\lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1}} = e^{\lim_{x \rightarrow \infty} \frac{1}{x}} = e^0 = 1$$

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}}$$

(i)

0/0

$$f(x) = \ln x$$

$$= e^x$$

$$\neq \sqrt{x}$$

$$\lim_{x \rightarrow \infty} \frac{x}{x} = \lim_{x \rightarrow \infty} 1 = 1$$

✓ sin x

$$\lim_{x \rightarrow 0^+}$$

$$\frac{\frac{1}{\cancel{2\sqrt{x}}}}{\frac{\cos x}{\cancel{2\sqrt{\sin x}}}}$$

$$\lim_{x \rightarrow 0^+} \frac{1}{\cos x} \sqrt{\frac{\sin x}{x}}$$

$$\lim_{x \rightarrow 0^+} \frac{1}{\cos x} \quad \lim_{x \rightarrow 0^+} \sqrt{\frac{\sin x}{x}}$$

$$\frac{1}{1} \sqrt{\lim_{x \rightarrow 0^+} \frac{\sin x}{x}} = \sqrt{1} = 1$$

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}} = \lim_{x \rightarrow 0^+} \sqrt{\frac{x}{\sin x}}$$

$$= \sqrt{\lim_{x \rightarrow 0^+} \frac{x}{\sin x}} = \sqrt{\lim_{x \rightarrow 0^+} \frac{1}{\cos x}} = \sqrt{1} = 1$$

$$\lim_{x \rightarrow \infty} e^x = e^{\lim_{x \rightarrow \infty} x} = e^\infty = \infty$$

$$\lim_{x \rightarrow \infty} \sqrt{x} = \sqrt{\lim_{x \rightarrow \infty} x} = \sqrt{\infty} = \infty$$

$$\lim_{x \rightarrow \infty} \ln x = \ln \lim_{x \rightarrow \infty} x = \ln \infty = \infty$$