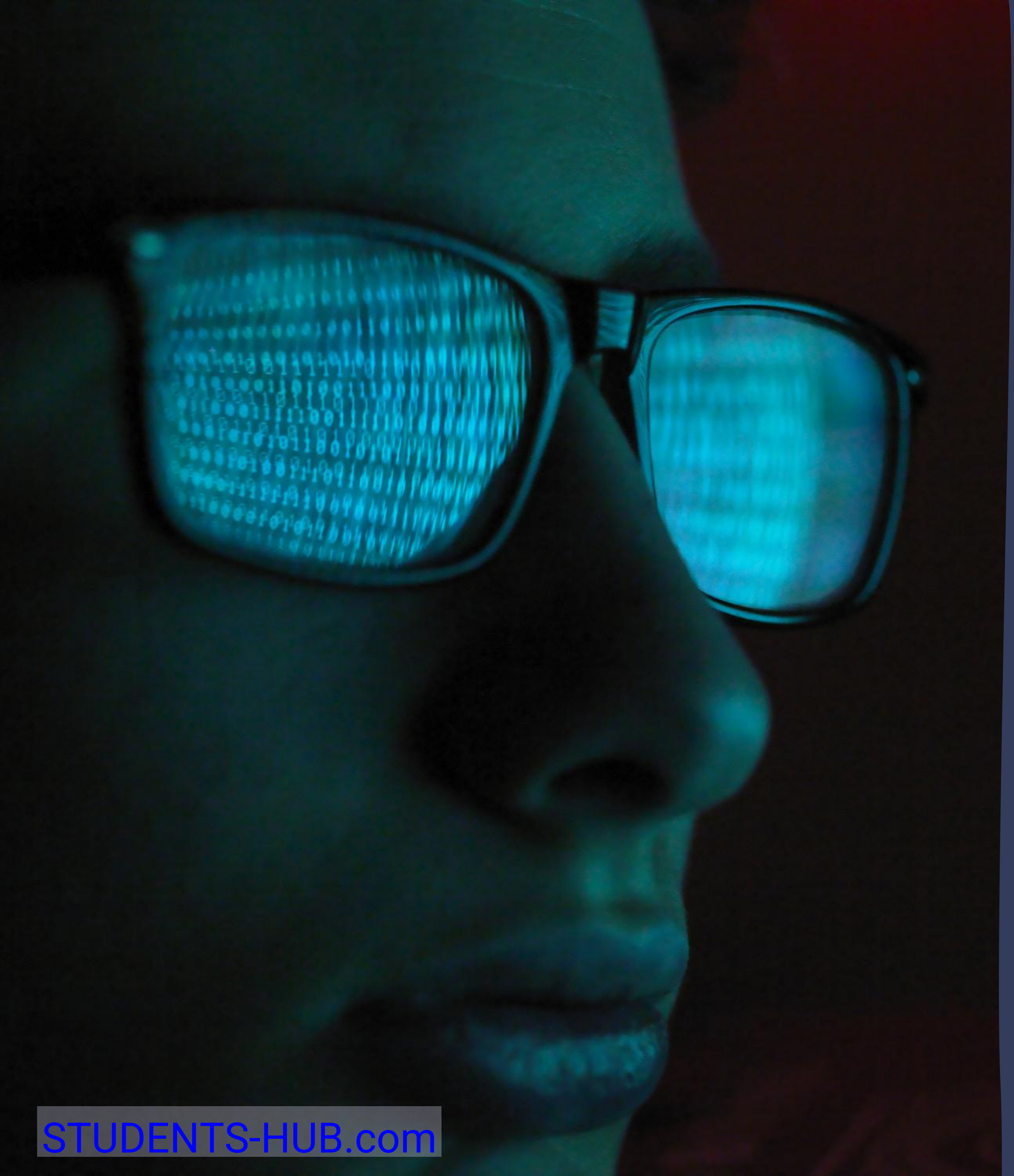

CYBERSECURITY MATHEMATICS

Chapter 4

MATRIX : Describe by number of rows (m) multiply by number of column (n) , A m*n

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

EX :

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}_{2\times 2} , \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}_{2\times 3} , \quad C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}_{3\times 2}$$

SPECIAL MATRIX :

1- Zero matrix (notation O) , all elements are zero $\begin{pmatrix} 0 & 0 & \cdots \\ 0 & 0 & \\ \vdots & & \end{pmatrix}$

2- Identity matrix : it is diagonal are ones

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $I_{m \times n} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

ADDITION & SUBTRACTION :

THE TWO MATRICES MUST HAVE THE SAME NUMBER OF ROWS AND COLUMNS.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 7 & 2 & 3 \\ 1 & 0 & 10 \end{pmatrix}, \quad C \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 0 \end{pmatrix}$$

* $A + B = \begin{pmatrix} 8 & 4 & 6 \\ 5 & 0 & 15 \end{pmatrix}$

* $A - B = \begin{pmatrix} -6 & 0 & 0 \\ 3 & 0 & -5 \end{pmatrix}$

* $A + C \not\rightarrow$ Because A and C don't have the same m, n

MULTIPLICATION :

$$A_{m*r} \times B_{r*n} = (AB)_{m*n}$$

If A , B is squared matrix where m is number of rows and n is number of columns

Then A * B not equal B * A

Rules :

1) $(AB)C = A(BC)$
2) $K(AB) = A(KB) = (KA)B$
3) $A(B \pm C) = AB \pm AC$ and $(B \pm C)A = BA \pm CA$
4) $IA = AI = A \longrightarrow I$: identity matrix
5) $0A = A0 = 0 \longrightarrow 0$: zero matrix

Note:
You can multiply two square matrices, but if the matrices are not square, the number of rows in the first matrix must be equal to the number of columns in the second matrix. Vice versa

EX :

1) $A = \begin{pmatrix} 1 & 3 \\ 2 & 0 \\ 2 & 4 \end{pmatrix}_{3*2}$, $B = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}_{2*3}$

$$A * B = \begin{pmatrix} (1\times1)+(3\times2) & (1*1)+(3*0) & (1*0)+(3*1) \\ (2\times1)+(0*2) & (2*1)+(0*0) & (2*0)+(0*1) \\ (2*1)+(4*2) & (2*1)+(4*0) & (2*0)+(4*1) \end{pmatrix} = \begin{pmatrix} 7 & 1 & 3 \\ 2 & 2 & 0 \\ 10 & 2 & 4 \end{pmatrix}_{3*3}$$

2) $K(A*B)$, $K = -1$

$$K(A*B) = \begin{pmatrix} -7 & -1 & -3 \\ -2 & -2 & 0 \\ -10 & -2 & -4 \end{pmatrix}$$

TRANSPOSE OF MATRIX :

if A_{m*r} then A^T_{m*r}

Example :

$$A = \begin{pmatrix} 2 & 1 & 5 \\ 3 & 4 & 6 \end{pmatrix}_{2\times3} \quad \text{find } A^T ?? \longrightarrow \quad A^T = \begin{pmatrix} 2 & 3 \\ 1 & 4 \\ 5 & 6 \end{pmatrix}_{3\times2}$$

Rules :

1) $(A^T)^T = A$

2) $(KA^T)^T = K A^T$

3) $(A \pm B)^T = A^T \pm B^T$

4) $(AB)^T = B^T * A^T \longrightarrow A^T * B^T \neq B^T * A^T$

MINORS OF MATRICES :

if A is squared matrix , then the minor denoted by Mij of element aij is determinate of submatrix that remains after the i^{th} row and j^{th} column deleted

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} , \quad \det(A) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

then $\boxed{M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}}$, $\boxed{M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}}$

COFACTORS :

The number $C_{ij} = (-1)^{i+j} * M_{ij}$

Adjoint of matrix $adj(A) = M^T$

$$M = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

, the $\boxed{A^{-1} = \frac{1}{\det(A)} * adj(A)}$

EX :

1) $A = \begin{bmatrix} 6 & 1 \\ 5 & 2 \end{bmatrix}$ find A^{-1} ?

$\boxed{\det(A)} = \begin{vmatrix} 6 & 1 \\ 5 & 2 \end{vmatrix} = \boxed{6 \times 2 - 5 \times 1 = 7}$

$\boxed{(-1)^{1+2} * 1}$

$(-1)^{2+1} * 5 = -5$

$M = \begin{bmatrix} 2 & -5 \\ -1 & 6 \end{bmatrix}$

$M^T = \begin{bmatrix} 2 & -1 \\ -5 & 6 \end{bmatrix}$

$A^{-1} = \frac{1}{7} \begin{pmatrix} 2 & -1 \\ -5 & 6 \end{pmatrix} = \begin{pmatrix} \frac{2}{7} & \frac{-1}{7} \\ \frac{-5}{7} & \frac{6}{7} \end{pmatrix}$

Example: $A = \begin{bmatrix} 1 & 5 & 0 \\ 0 & 3 & 2 \\ 1 & 0 & 2 \end{bmatrix}$ find A^{-1} ?

Note: to find the determinate for any matrix we will take one row or on column (But we should aware of the sign of every index)

1. $\det(A) = \begin{vmatrix} \overset{+}{1} & \overset{-}{5} & \overset{+}{0} \\ \overset{-}{0} & \overset{+}{3} & \overset{-}{2} \\ \overset{+}{1} & \overset{-}{0} & \overset{+}{2} \end{vmatrix}$

$$= (1)\underbrace{\begin{vmatrix} 3 & 2 \\ 0 & 2 \end{vmatrix}}_{M_{11}} + (-5)\underbrace{\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix}}_{M_{12}} + (0)\underbrace{\begin{vmatrix} 0 & 3 \\ 1 & 0 \end{vmatrix}}_{M_{13}}$$

$$= 1(6) + -5(-2) + \cancel{0(-3)} = 6 + 10 = 16$$ → in our Example here we took the first row

2. $C_{ij} = (-1)^{i+j} M_{ij}$

$C_{11} = (-1)^2 \cdot 6 = 6$

$C_{12} = (-1)^3 \cdot -2 = 2$

$C_{13} = (-1)^4 \cdot -3 = -3$

$C_{21} = (-1)^3 \cdot 10 = -10$

$C_{22} = (-1)^4 \cdot 2 = 2$

$C_{23} = (-1)^5 \cdot -5 = 5$

$C_{31} = (-1)^4 \cdot 10 = 10$

$C_{32} = (-1)^5 \cdot 2 = -2$

$C_{33} = (-1)^6 \cdot 3 = 3$

$M_{11} = (3\times2)-(0\times2) = 6$ $\quad M_{21} = (5\times2)-(0\times0) = 10$ $\quad M_{31} = (5\times2)-(0\times3) = 10$

$M_{12} = (0\times2)-(2\times1) = -2$ $\quad M_{22} = (1\times2)-(0\times1) = 2$ $\quad M_{32} = (1\times2)-(0\times0) = 2$

$M_{13} = (0\times0)-(3\times1) = -3$ $\quad M_{23} = (1\times0)-(5\times1) = -5$ $\quad M_{33} = (1\times3)-(5\times0) = 3$

$$M = \begin{pmatrix} 6 & 2 & -3 \\ -10 & 2 & 5 \\ 10 & -2 & 3 \end{pmatrix}, \quad M^T = \begin{pmatrix} 6 & -10 & 10 \\ 2 & 2 & -2 \\ -3 & 5 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \boxed{M^T}$$ → transfer: let the row be column and the column be row

$$A^{-1} = \frac{1}{16} \cdot \begin{pmatrix} 6 & -10 & 10 \\ 2 & 2 & -2 \\ -3 & 5 & 3 \end{pmatrix} = \begin{pmatrix} \frac{3}{8} & -\frac{5}{8} & \frac{5}{8} \\ \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \\ -\frac{3}{16} & \frac{5}{16} & \frac{3}{16} \end{pmatrix}$$

RULES :

$\boxed{1}$ $(A^{-1})^T = (A^T)^{-1}$

$\boxed{2}$ $(A.B)^{-1} = A^{-1} * B^{-1}$

$\boxed{3}$ $(A^{-1})^{-1} = A$

$\boxed{4}$ $(A_1, A_2, \ldots, A_n)^{-1} = A_n^{-1}, \ldots, A_2^{-1}, A_1^{-1}$

$\boxed{5}$ $(kA)^{-1} = k^{-1} A^{-1} = \frac{1}{k} A^{-1}$

k non zero