

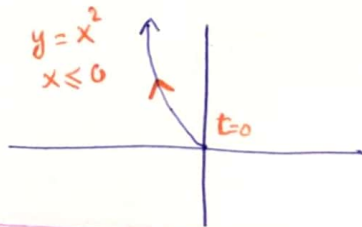
11.1 Parametrization of plane curves.

Identify the particle's path by finding a Cartesian equation. Graph the Cartesian equation. Indicate the portion of the graph traced by the particle & the direction of motion.

2 $x = -\sqrt{t}$, $y = t$, $t \geq 0$

$$x = -\sqrt{y} \quad \text{or} \quad y = x^2 , \quad x \leq 0 , \quad y \geq 0$$

★
(Initial Point) $t=0$
 $\Rightarrow (x,y) = (0,0)$.



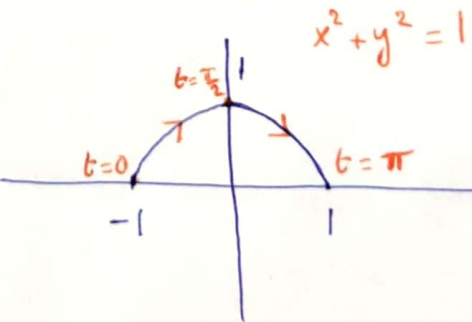
6 $x = \cos(\pi - t)$, $y = \sin(\pi - t)$, $0 \leq t \leq \pi$

$$x^2 + y^2 = \cos^2(\pi - t) + \sin^2(\pi - t) = 1$$

$$\Rightarrow x^2 + y^2 = 1$$

★
IP: when $t=0$

$$\Rightarrow (x,y) = (\cos(\pi), \sin(\pi)) = (-1, 0)$$



★
TP: when $t = \pi$

$$\Rightarrow (x,y) = (\cos 0, \sin 0) = (1, 0)$$

★ To check the direction:

$$\Rightarrow t = \frac{\pi}{2}$$

$$\Rightarrow (x,y) = \left(\cos \frac{\pi}{2}, \sin \frac{\pi}{2}\right) = (0, 1)$$

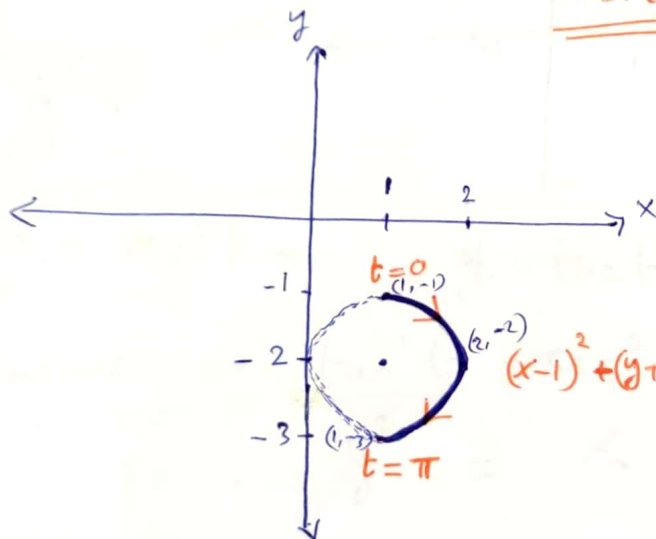
10 $x = 1 + \sin t$, $y = \cos t - 2$, $0 \leq t \leq \pi$

$$x = 1 + \sin t \Rightarrow \sin t = x - 1 \Rightarrow \boxed{\sin^2 t = (x - 1)^2}$$

$$y = \cos t - 2 \Rightarrow \cos t = y + 2 \Rightarrow \boxed{\cos^2 t = (y + 2)^2}$$

$$\sin^2 t + \cos^2 t = (x - 1)^2 + (y + 2)^2 = 1$$

$(x - 1)^2 + (y + 2)^2 = 1$ (it is a circle equation with center $(1, -2)$ & a radius of 1).



Note that $\Rightarrow$ we know

$$-1 \leq \cos t \leq 1$$

$$-3 \leq \cos t - 2 \leq -1$$

$$-3 \leq y \leq -1$$

$$\text{So, } \boxed{y < 0}$$

$$\& -1 \leq \sin t \leq 1$$

$$0 \leq 1 + \sin t \leq 2$$

$$0 \leq x \leq 2$$

$$\text{So, } \boxed{x \geq 0}$$

★ IP: $t = 0$

$$(x, y) = (1 + \sin(0), \cos(0) - 2) = (1, -1) \quad \text{when } y < 0 \& x \geq 0$$

$\Rightarrow$ the curve is in the fourth quadrant.

★ TP: $t = \pi$

$$(x, y) = (1 + \sin(\pi), \cos(\pi) - 2) = (1 + 0, -1 - 2) = (1, -3)$$

★ $t = \frac{\pi}{2} \Rightarrow (x, y) = (2, -2)$

14 $x = \sqrt{t+1}$, $y = \sqrt{t}$, $t \geq 0$

$$y = \sqrt{t} \Rightarrow y^2 = t$$

$$\Rightarrow x = \sqrt{y^2 + 1} , y \geq 0$$

$$\Rightarrow x^2 = y^2 + 1$$

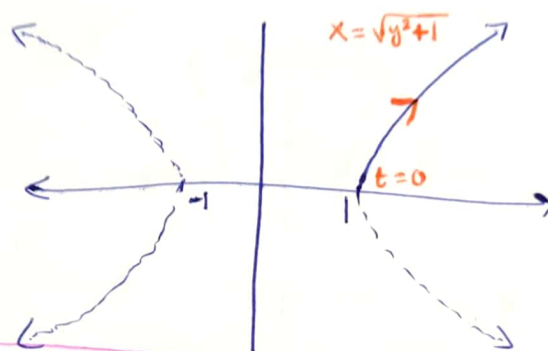
$$\Rightarrow x^2 - y^2 = 1$$

it's a hyperbola equation with $y \geq 0$ & $x \geq 1$

★ when $t=0$

IP $(x, y) = (1, 0)$

★ No TP.



$t=0$ $(1, 0)$

$t=1$ $(\sqrt{2}, 1)$

15 $x = \sec^2 t - 1$, $y = \tan t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$

remember $\Rightarrow \tan^2 t = \sec^2 t - 1$

so , $y^2 = x$

★ No IP & No TP.

⇒ To check the direction, take two points:

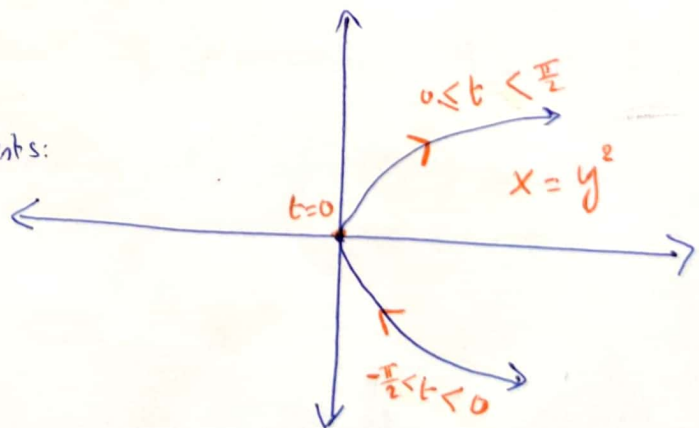
★ when $t=0$

$\Rightarrow (x, y) = (0, 0)$

★ when $t = \frac{\pi}{4}$

$\Rightarrow (x, y) = (1, 1)$

since $x = y^2 \Rightarrow x \geq 0$



18 $x = 2 \sinh t$, $y = 2 \cosh t$, $-\infty < t < \infty$.

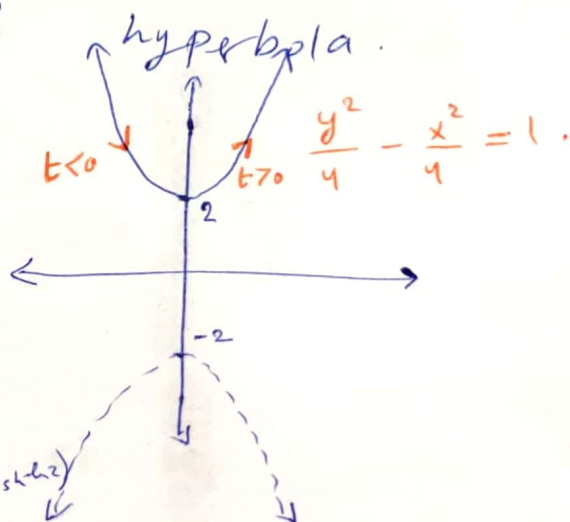
$$\sinh t = \frac{x}{2}$$

$$\cosh t = \frac{y}{2} \quad , \quad y > 0 \quad (y \geq 2).$$

$$\cosh^2 t - \sinh^2 t = 1.$$

$$\left(\frac{y}{2}\right)^2 - \left(\frac{x}{2}\right)^2 = 1.$$

$$\frac{y^2}{4} - \frac{x^2}{4} = 1$$



★ No IP & No TP.

when $t = 0$
 $\Rightarrow (x, y) = (0, 2)$

★ when $t = \ln 2$
 $\Rightarrow (x, y) = (2 \sinh \ln 2, 2 \cosh \ln 2)$
 $= \left(\frac{3}{2}, \frac{5}{2}\right).$

20 Find Parametric equations & Parametric interval for the motion of a particle that starts at $(a, 0)$ & traces the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

g) Once clockwise

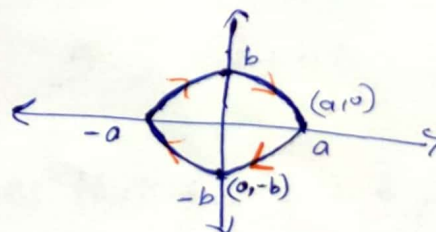
$$\frac{x}{a} = \sin t \Rightarrow x = a \sin t$$

$$\frac{y}{b} = \cos t \Rightarrow y = b \cos t.$$

$$\frac{\pi}{2} \leq t \leq \frac{5\pi}{2}$$

★ IP when $t = \frac{\pi}{2} \Rightarrow (x, y) = (a, 0)$

★ TP when $t = \frac{5\pi}{2} \Rightarrow (x, y) = (a, 0)$



★ To check the direction:

take $t = \pi$.
 $x = a \sin \pi = 0$
 $y = b \cos \pi = -b$

(b) Once counterclockwise.

$$\frac{x}{a} = \cos t \Rightarrow x = a \cos t.$$

$$\frac{y}{b} = \sin t \Rightarrow y = b \sin t.$$

$$0 \leq t \leq 2\pi$$

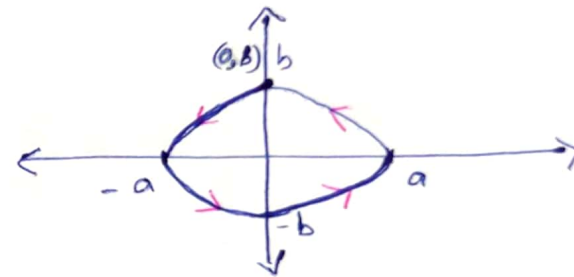
$$\text{IP: } t=0 \Rightarrow (x, y) = (a, 0)$$

$$\text{TP: } t=2\pi \Rightarrow (x, y) = (a, 0)$$

To check the direction, take $t = \frac{\pi}{2}$.

$$x = a \cos \frac{\pi}{2} = 0$$

$$y = b \sin \frac{\pi}{2} = b \quad \uparrow (0, b)$$



(c) Twice clockwise.

$$x = a \sin t$$

$$y = b \cos t$$

$$\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$$

d) Twice counterclockwise.

$$x = a \cos t.$$

$$y = a \sin t \quad 0 \leq t \leq 4\pi.$$

22 Find a parametrization of the curve:

The line segment with end points $(-1, 3)$ & $(3, -2)$.
 (x_1, y_1) (x_2, y_2)

$$m_{\text{slope}} = \frac{-2-3}{3-(-1)} = -\frac{5}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{5}{4}(x + 1) \Rightarrow y = -\frac{5}{4}x + \frac{7}{4}$$

Let $x = t$
so, $y = -\frac{5}{4}t + \frac{7}{4}$, $-1 \leq t \leq 3$.

another possible:

Let $t = \frac{x+1}{4}$, when $x = -1 \Rightarrow t = 0$
 $x = 3 \Rightarrow t = 1$

$$\Rightarrow x = 4t - 1, \quad 0 \leq t \leq 1$$
$$y = -5t + 3$$

or

Let $t = x + 1$

$$x = t - 1, \quad 0 \leq t \leq 4$$

$$y = -\frac{5}{4}t + 3$$

26 Find a parametrization for the ray with initial point $(-1, 2)$ that passes through the point $(0, 0)$.

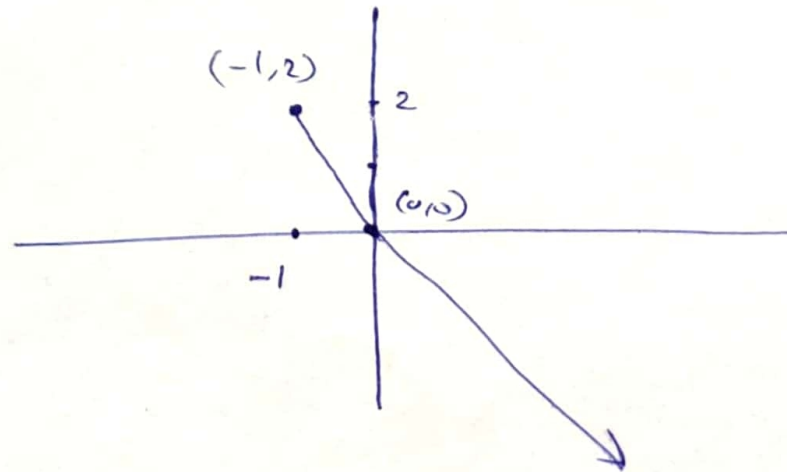
$$m = \frac{2-0}{-1-0} = -2.$$

$$y-0 = -2(x-0) \Rightarrow y = -2x.$$

$$\text{let } x = t$$

$$t \geq -1$$

$$\Rightarrow y = -2t$$



11.2 : Calculus with Parametric Curves.

2 If $x = \sec t$, $y = \tan t$, $t = \frac{\pi}{6}$

Find the tangent line & $\frac{d^2y}{dx^2}$.

When $t = \frac{\pi}{6}$: $x = \sec \frac{\pi}{6} = \frac{2}{\sqrt{3}}$ } The point is $(\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}})$
 $y = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$ }

slope = $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec^2 t}{\sec t \tan t} = \frac{\sec t}{\tan t}$$

$$= \frac{1}{\cos t} \cdot \frac{\cos t}{\sin t} = \csc t$$

So, $\left. \frac{dy}{dx} \right|_{t=\frac{\pi}{6}} = \csc \frac{\pi}{6} = 2$.

The tangent line is:

$$y - \frac{1}{\sqrt{3}} = 2 \left(x - \frac{2}{\sqrt{3}} \right)$$

$$y = 2x - \sqrt{3}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} \bigg|_{t=\frac{\pi}{6}} = \frac{-\csc t \cot t}{\sec t \tan t} \bigg|_{t=\frac{\pi}{6}}$$

$$= \frac{-2 \cdot \sqrt{3}}{\frac{2}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}}$$

$$= \frac{-2\sqrt{3}}{\frac{2}{3}} = -2\sqrt{3} \cdot \frac{3}{2} = \boxed{-3\sqrt{3}}$$

14 Find the equation of the line tangent to the curve $x = t + e^t$, $y = 1 - e^t$, $t = 0$. & find $\frac{d^2y}{dx^2}$ at this point.

$$\text{at } t=0 \Rightarrow \left. \begin{array}{l} x = 0 + e^0 = 1 \\ y = 1 - e^0 = 1 - 1 = 0 \end{array} \right\} \begin{array}{l} (x, y) \\ (1, 0) \end{array}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \bigg|_{t=0} = \frac{-e^t}{1+e^t} \bigg|_{t=0} = \frac{-1}{1+1} = -\frac{1}{2}$$

So, The line is $\Rightarrow \boxed{y = -\frac{1}{2}x + \frac{1}{2}}$

$$\frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy'}{dt} = \frac{(1+e^t)(-e^t) + e^t e^t}{(1+e^t)^2}$$

$$= \frac{-e^t - e^{2t} + e^{2t}}{(1+e^t)^2} = \frac{-e^t}{(1+e^t)^2}$$

$$\text{So, } \left. \frac{dy'}{dx^2} \right|_{t=0} = \frac{\frac{-e^t}{(1+e^t)^2}}{1+e^t} \bigg|_{t=0} = \frac{-e^t}{(1+e^t)^3} \bigg|_{t=0}$$

$$= \frac{-1}{(1+1)^3} = \boxed{-\frac{1}{8}}$$

20 Find the slope of the curve $x = f(t)$, $y = g(t)$ at $t=0$ & the tangent line.

$$t = \ln(x-t) \quad \& \quad y = t e^t.$$

To find $\frac{dx}{dt} \Rightarrow$ implicit derivative

$$\Rightarrow 1 = \frac{1}{x-t} \left(\frac{dx}{dt} - 1 \right).$$

$$x-t = \frac{dx}{dt} - 1$$

$$\frac{dx}{dt} = x-t+1$$

$$\text{but: when } t=0 \Rightarrow 0 = \ln(x-0) \Rightarrow x = e^0 = 1 \quad \left. \begin{array}{l} (x, y) \\ (1, 0) \end{array} \right\}$$

$$y = 0e^0 = 0$$

$$\text{So, } \frac{dx}{dt} = 1-t+1 = \boxed{2-t}$$

$$\frac{dy}{dt} = \boxed{te^t + e^t}$$

$$\left. \frac{dy}{dx} \right|_{t=0} = \left. \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \right|_{t=0} = \left. \frac{te^t + e^t}{2-t} \right|_{t=0}$$

$$= \frac{0+1}{2-0} = \boxed{\frac{1}{2}}$$

So, the tangent line is

$$y = \frac{1}{2}x - \frac{1}{2}$$

22 Find the area enclosed by the y-axis & the curve $x = t - t^2$, $y = 1 + e^{-t}$.

$$\int x \, dy$$

$$x = t - t^2$$

$$dy = -e^{-t} dt$$

$$\int_0^1 |(t - t^2) \cdot -e^{-t} dt|$$

$$= \int_0^1 |(t^2 - t)e^{-t} dt|$$

$$= \left| (t \bar{e}^t (t^2 - t) - \bar{e}^t (2t - 1) - 2\bar{e}^t) \right|_0^1$$

$$= \left| -\bar{e}^t(0) - \bar{e}^t(-1) - 2\bar{e}^t - (0 - \bar{e}^0(-1) - 2(\bar{e}^0)) \right|_0^1$$

$$= \left| -\frac{3}{e} + 1 \right|$$

y-axis $x=0$
& $x = t - t^2$

Find the intersection points.

$$t - t^2 = 0$$

$$t(1 - t) = 0$$

$$t = 0, 1$$

$$\begin{array}{lcl} t^2 - t & \xrightarrow{+} & e^{-t} \\ 2t - 1 & \xrightarrow{-} & -e^{-t} \\ 2 & \xrightarrow{+} & \bar{e}^{-t} \\ & \xrightarrow{+} & -e^{-t} \end{array}$$

25 Find the length of the curve $x = \cos t$, $y = t + \sin t$. $0 \leq t \leq \pi$

$$L = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$

$$\Rightarrow \frac{dx}{dt} = -\sin t \Rightarrow \left(\frac{dx}{dt}\right)^2 = (-\sin t)^2 = \sin^2 t.$$

$$\Rightarrow \frac{dy}{dt} = 1 + \cos t \Rightarrow \left(\frac{dy}{dt}\right)^2 = (1 + \cos t)^2 = 1 + 2\cos t + \cos^2 t.$$

$$\begin{aligned}\Rightarrow \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= \sin^2 t + 1 + 2\cos t + \cos^2 t \\&= 1 + 1 + 2\cos t \\&= 2 + 2\cos t \\&= 2(1 + \cos t).\end{aligned}$$

$$\begin{aligned}\Rightarrow L &= \int_0^{\pi} \sqrt{2(1 + \cos t)} dt = \sqrt{2} \int_0^{\pi} \sqrt{1 + \cos t \cdot \frac{1 - \cos t}{1 - \cos t}} dt \\&= \sqrt{2} \int_0^{\pi} \sqrt{\frac{1 - \cos^2 t}{1 - \cos t}} dt \\&= \sqrt{2} \int_0^{\pi} \frac{|\sin t|}{\sqrt{1 - \cos t}} dt \\&= \sqrt{2} \int_0^{\pi} \frac{\sin t}{\sqrt{1 - \cos t}} dt. \quad \begin{array}{l} \text{Since } 0 \leq \sin t \leq 1 \\ \text{when } 0 \leq t \leq \pi \end{array} \\&= \sqrt{2} \int \frac{du}{\sqrt{u}} \quad \begin{array}{l} \text{Let } u = 1 - \cos t \\ du = \sin t dt \end{array} \\&= \sqrt{2} \cdot 2\sqrt{u} \Big|_0^{\pi} \\&= \sqrt{2} \cdot 2\sqrt{1 - \cos t} \Big|_0^{\pi} \\&= 2\sqrt{2} (\sqrt{2} - 0) = \boxed{4}\end{aligned}$$

27 Find the length of the curve

$$x = \frac{t^2}{2}, \quad y = \frac{(2t+1)^{3/2}}{3}, \quad 0 \leq t \leq 4.$$

$$\Rightarrow \frac{dx}{dt} = \frac{2t}{2} = t \Rightarrow \left(\frac{dx}{dt}\right)^2 = t^2.$$

$$\begin{aligned} \Rightarrow \frac{dy}{dt} &= \frac{\frac{3}{2}(2t+1)^{1/2}}{3} \cdot 2 = \frac{3 \cdot 2 (2t+1)^{1/2}}{3 \cdot 2} \\ &= (2t+1)^{1/2} \end{aligned}$$

$$\left(\frac{dy}{dt}\right)^2 = \left((2t+1)^{1/2}\right)^2 = |2t+1| = 2t+1$$

$$0 \leq t \leq 4.$$

$$\begin{aligned} \Rightarrow \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= t^2 + 2t + 1 \\ &= (t+1)^2 \end{aligned}$$

$$\begin{aligned} L &= \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^4 \sqrt{(t+1)^2} dt \\ &= \int_0^4 (t+1) dt \\ &= \left(\frac{t^2}{2} + t\right) \Big|_0^4 \\ &= \frac{16}{2} + 4 - 0 \\ &= 8 + 4 \\ &= \boxed{12} \end{aligned}$$

11.3 : Polar Coordinates.

1) Which polar coordinate pairs label the same point.

a) $(3, 0)$

b) $(-3, 0)$

c) $(2, \frac{2\pi}{3})$

d) $(2, \frac{7\pi}{3})$

e) $(-3, \pi)$

f) $(2, \frac{\pi}{3})$

g) $(-3, 2\pi)$

h) $(-2, -\frac{\pi}{3})$

Remember :- $(r, \theta) = (r, \theta + 2\pi n)$ 
 $= (-r, \theta + (2n+1)\pi)$   $n = 0, \pm 1, \pm 2, \pm 3, \dots$

a) = e)

$(3, 0) = (-3, \pi)$ (compare with   with $n=0$)

b) = g)

$(-3, 0) = (-3, 2\pi)$ ( with $n=1$).

c) = h)

$(2, \frac{2\pi}{3}) = (-2, -\frac{\pi}{3})$ (  with $n=-1$)

d) = f)

$(2, \frac{\pi}{3}) = (2, \frac{7\pi}{3})$ ( with $n=1$).

6) Find the Cartesian coordinates of the following points :-

$$(r, \theta) \rightarrow (x, y)$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$a) \left(\sqrt{2}, \frac{\pi}{4} \right) \Rightarrow x = \sqrt{2} \cos \frac{\pi}{4} = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1$$

$$y = \sqrt{2} \sin \frac{\pi}{4} = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1$$

$$(x, y) = (1, 1)$$

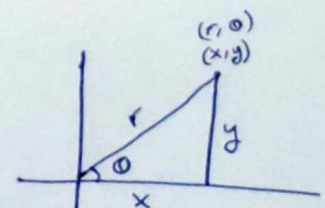
$$b) (1, 0) \Rightarrow \left. \begin{aligned} x &= r \cos 0 = 1 \\ y &= 1 \sin 0 = 0 \end{aligned} \right\} (x, y) = (1, 0)$$

$$c) \left(0, \frac{\pi}{2} \right) \Rightarrow \left. \begin{aligned} x &= 0 \cos \frac{\pi}{2} = 0 \\ y &= 0 \sin \frac{\pi}{2} = 0 \end{aligned} \right\} (x, y) = (0, 0)$$

$$d) \left(-\sqrt{2}, \frac{\pi}{4} \right) \Rightarrow \left. \begin{aligned} x &= -\sqrt{2} \cos \frac{\pi}{4} = -1 \\ y &= -\sqrt{2} \sin \frac{\pi}{4} = -1 \end{aligned} \right\} (x, y) = (-1, -1)$$

$$e) \left(-3, \frac{5\pi}{6} \right) \Rightarrow \left. \begin{aligned} x &= -3 \cos \left(\frac{5\pi}{6} \right) = -3 \left(-\frac{\sqrt{3}}{2} \right) = \frac{3\sqrt{3}}{2} \\ y &= -3 \sin \left(\frac{5\pi}{6} \right) = -3 \cdot \frac{1}{2} = -\frac{3}{2} \end{aligned} \right\} (x, y) = \left(\frac{3\sqrt{3}}{2}, -\frac{3}{2} \right)$$

$$f) \left(5, \tan^{-1} \left(\frac{4}{3} \right) \right) \Rightarrow \left. \begin{aligned} x &= 3 \\ y &= 4 \end{aligned} \right\} (x, y) = (3, 4)$$



$$\tan \theta = \frac{y}{x}$$

$$\tan^{-1} \left(\frac{y}{x} \right) = \theta$$

$$g) (-1, 7\pi) \Rightarrow \left. \begin{aligned} x &= -1 \cos(7\pi) = (-1)(-1) = 1 \\ y &= -1 \sin(7\pi) = -1(0) = 0 \end{aligned} \right\} (x, y) = (1, 0)$$

7) Find the polar coordinates, $0 \leq \theta \leq 2\pi$ & $r \geq 0$.

$$(x, y) \rightarrow (r, \theta) : \left\{ \begin{aligned} r^2 &= x^2 + y^2, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right) \\ \cos \theta &= \frac{x}{r} \quad \& \quad \sin \theta = \frac{y}{r} \end{aligned} \right.$$

$$a) (1, 1) = (x, y) \Rightarrow r^2 = x^2 + y^2 = 1^2 + 1^2 = 2 \Rightarrow r = \sqrt{2} \left\{ \begin{aligned} \cos \theta &= \frac{1}{\sqrt{2}} \\ \sin \theta &= \frac{1}{\sqrt{2}} \end{aligned} \right\} \theta = \frac{\pi}{4} \quad (r, \theta) = \left(\sqrt{2}, \frac{\pi}{4}\right)$$

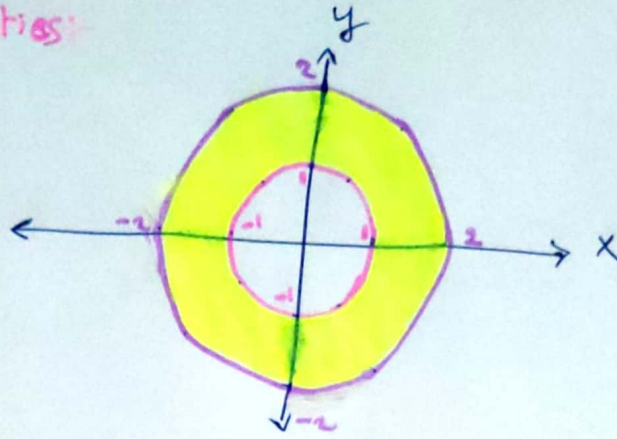
$$b) (-3, 0) \Rightarrow r = \sqrt{(-3)^2 + 0^2} = 3 \left\{ \begin{aligned} \cos \theta &= -\frac{3}{3} = -1 \\ \sin \theta &= \frac{0}{3} = 0 \end{aligned} \right\} \theta = \pi \quad (r, \theta) = (3, \pi)$$

$$c) (\sqrt{3}, -1) \Rightarrow r = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2. \left\{ \begin{aligned} \cos \theta &= \frac{\sqrt{3}}{2} \\ \sin \theta &= -\frac{1}{2} \end{aligned} \right\} \begin{array}{l} \text{in the 4th quadrant} \\ \theta = \frac{11\pi}{6} \end{array} \quad (r, \theta) = \left(2, \frac{11\pi}{6}\right)$$

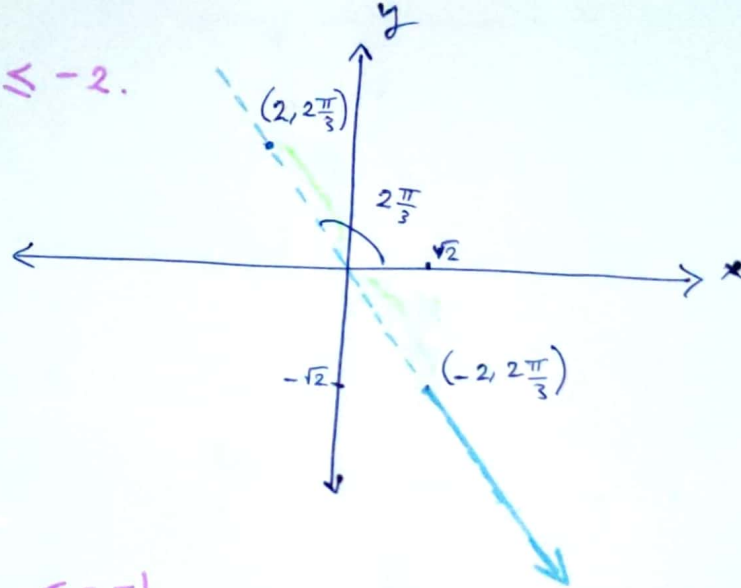
$$d) (-3, 4) \Rightarrow r = \sqrt{(-3)^2 + 4^2} = 5. \left\{ \begin{aligned} \cos \theta &= -\frac{3}{5} \\ \sin \theta &= \frac{4}{5} \end{aligned} \right\} \begin{array}{l} \text{in the 2nd quadrant} \\ \theta = \pi - \tan^{-1}\left(\frac{4}{3}\right) \end{array} \quad (r, \theta) = \left(5, \pi - \tan^{-1}\left(\frac{4}{3}\right)\right)$$

Graph the sets of points whose polar coordinates satisfy the equations & inequalities:

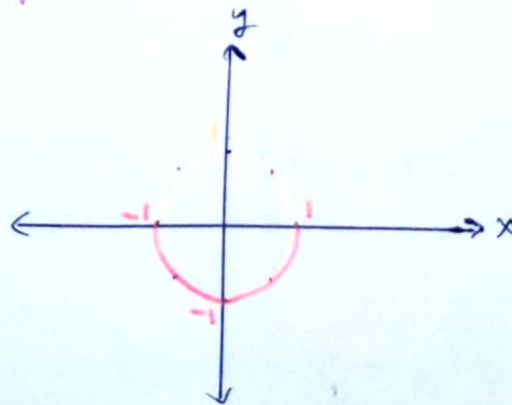
14 $1 \leq r \leq 2$.



16 $\theta = \frac{2\pi}{3}, r \leq -2$.

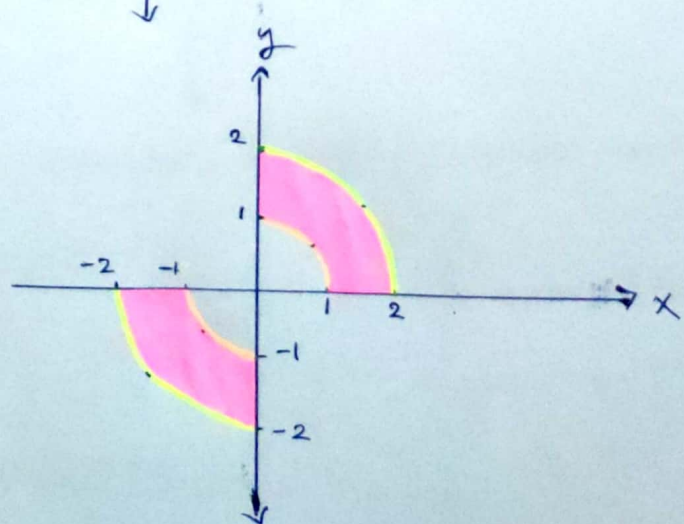


22 $0 \leq \theta \leq \pi, r = -1$



26 $0 \leq \theta \leq \frac{\pi}{2}, 1 \leq |r| \leq 2$.

which mean
 $1 \leq r \leq 2$
 or $-1 \leq -r \leq -2$
 $\Rightarrow -2 \leq r \leq -1$.

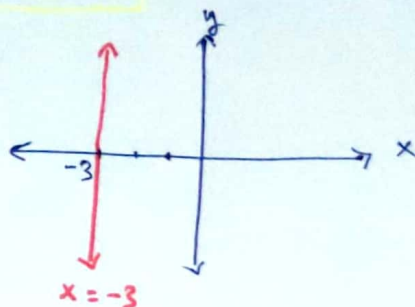


Replace the polar equations with equivalent Cartesian equations.
Then describe the graph.

32 $r = -3 \sec \theta$

$$r = -3 \cdot \frac{1}{\cos \theta} \Rightarrow r \cos \theta = -3$$

$$x = -3 \quad (\text{vertical line through } (-3, 0))$$



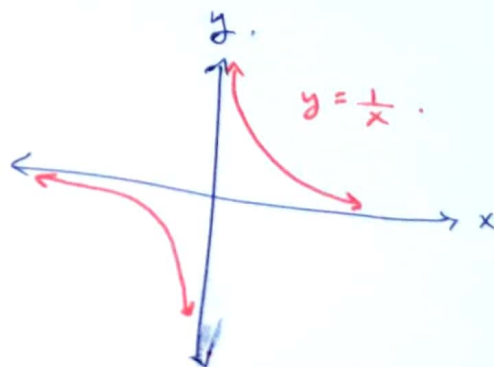
38 $r^2 \sin(2\theta) = 2$

$$r^2 \cdot 2 \sin \theta \cos \theta = 2$$

$$r \sin \theta \cdot r \cos \theta = 1$$

$$y \cdot x = 1$$

$$y = \frac{1}{x}$$

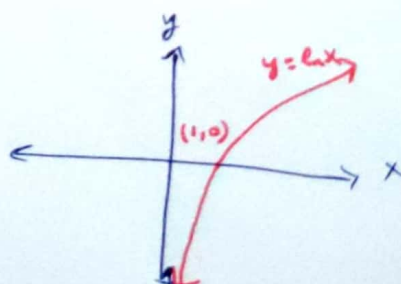


42 $r \sin \theta = \ln r + \ln(\cos \theta)$

$$r \sin \theta = \ln(r \cos \theta)$$

$$y = \ln x$$

$$(\ln a + \ln b = \ln(ab))$$



62 $r \sin\left(\frac{2\pi}{3} - \theta\right) = 5$

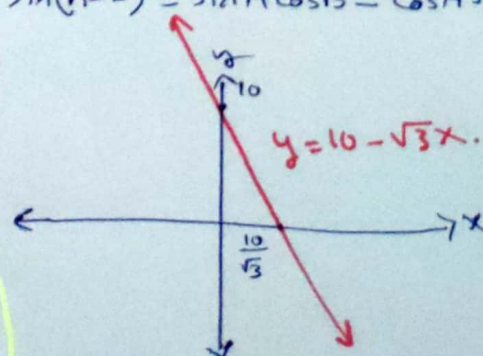
Remember: $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$r \left(\sin\left(\frac{2\pi}{3}\right) \cos \theta - \cos\left(\frac{2\pi}{3}\right) \sin \theta \right) = 5$$

$$r \left(\frac{\sqrt{3}}{2} \cos \theta - -\frac{1}{2} \sin \theta \right) = 5$$

$$\frac{\sqrt{3}}{2} r \cos \theta + \frac{1}{2} r \sin \theta = 5$$

$$\frac{\sqrt{3}}{2} x + \frac{1}{2} y = 5 \Rightarrow y = 10 - \sqrt{3} x$$



62 $x^2 + xy + y^2 = 1$. Find polar equation.

$$x^2 + y^2 + xy = 1.$$

$$r^2 + (r \cos \theta)(r \sin \theta) = 1.$$

$$r^2 + r^2 \cos \theta \sin \theta = 1.$$

$$r^2 \left(1 + \frac{1}{2} \sin 2\theta \right) = 1.$$