Chapter 12: Introduction to the Laplace Transform:

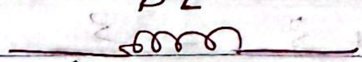
Table 12.1:

	Time-domain		S-domain
	$\delta(t)$	$\longrightarrow$	1
	$u(t)$	$\longrightarrow$	$\frac{1}{s}$
	t	$\longrightarrow$	$\frac{1}{s^2}$
	e^{-at}	$\longrightarrow$	$\frac{1}{s+a}$
مهم	$\sin\omega t$	$\longrightarrow$	$\frac{\omega}{s^2+\omega^2}$
	$\cos\omega t$	$\longrightarrow$	$\frac{s}{s^2+\omega^2}$
مهم	te^{-at}	$\longrightarrow$	$\frac{1}{(s+a)^2}$
	$e^{-at}\sin\omega t$	$\longrightarrow$	$\frac{\omega}{(s+a)^2+\omega^2}$
	$e^{-at}\cos\omega t$	$\longrightarrow$	$\frac{s+a}{(s+a)^2+\omega^2}$

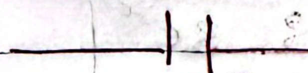
12.2:

	$f(t)$		$F(t)$
مهم	$\frac{df(t)}{dt}$	$\longrightarrow$	$sF(s)-f(0^-)$
مش كثير (بيجيش)	$\frac{d^nf(t)}{dt^n}$	$\longrightarrow$	$s^nF(s)-s^{n-1}f(0^-)-s^{n-2}\frac{df(0^-)}{dt}-s^{n-3}\frac{d^2f(0^-)}{dt}-\frac{d^{n-1}f(0)}{dt^{n-1}}$
مهم	$e^{-at}f(t)$	$\longrightarrow$	$F(s+a)$
مش كثير مهم تعاملهم	$f(at)$, $a>0$	$\longrightarrow$	$\frac{1}{a}F\left(\frac{s}{a}\right)$

sL



$j\omega = s$



Cs

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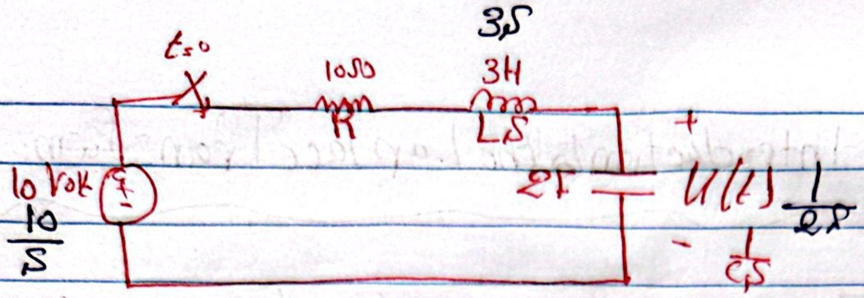


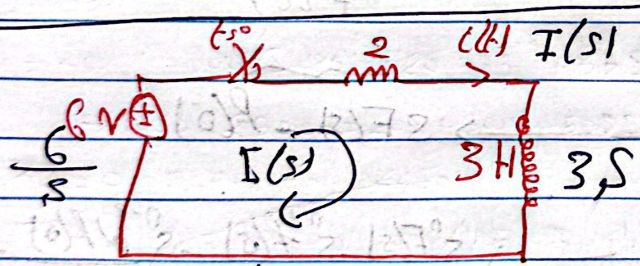
$$V_0(s) = \frac{\frac{1}{2s}}{\frac{1}{2s}+10+3s} * \frac{10}{s}$$

$$V_0(s) = \frac{1}{1+20s+6s^2} \cdot \frac{10}{s} = \frac{10}{s(6s^2+20s+1)}$$

$$= \frac{\frac{10}{6}}{s\left(s^2+\frac{20}{6}s+\frac{1}{6}\right)}$$

$$\rightarrow \frac{\frac{1}{cs}}{\frac{1}{cs}+R+Ls} \rightarrow \frac{1}{LCs^2+RCs+1}$$

$$= \frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$



$$-\frac{6}{s} + (2+3s)\,I(s) = 0$$

$$I(s) = \frac{6}{s(2+3s)} \Rightarrow \frac{2}{s\left(s+\frac{2}{3}\right)} = \frac{A}{s} + \frac{B}{s+\frac{2}{3}}$$

$$\rightarrow I(s) = \frac{3}{s} - \frac{3}{s+\frac{2}{3}}$$

$$\therefore \boxed{i(t) = 3 - 3e^{-\frac{2}{3}t}}$$

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12.7 Inverse Transform

1 Partial Fraction Expansion: Distinct Real Roots of D(s)

$$F(s) = \frac{96(s+5)(s+12)}{s(s+8)(s+6)} = \frac{K_1}{s} + \frac{K_2}{s+8} + \frac{K_3}{s+6}$$

$$K_1 = \frac{96(0+5)(0+12)}{(0+8)(0+6)} = \frac{96(5)(12)}{8\times 6} = \boxed{120}$$

$$K_2 = \frac{96(-8+5)(-8+12)}{-8(-8+6)} = \boxed{-72} \qquad K_3 = \frac{96(-6+5)(-6+12)}{-6(-6+8)} = \boxed{48}$$

$$\rightarrow f(t) = \left(120 - 72e^{-8t} + 48e^{-6t}\right) u(t)$$

2 ~~Partial~~: Distinct Complex Root of D(s)

$$F(s) = \frac{100(s+3)}{(s+6)(s^2+6s+25)} = \frac{\overset{-12}{A}}{s+6} + \frac{Bs+C}{s^2+6s+25}$$

$s_{1,2} = -6 \pm \sqrt{36-100}$

$= -3 \pm j4$ (Real, Imag)

$\rightarrow 100(s+3) = -12(s^2+6s+25) + (Bs+C)(s+6) \rightarrow Bs^2 + (6B+C)s + 6C$

$s^2 \rightarrow 100s + 300 = (B-12)s^2 + (6B+C-72)s + (6C-300)$

$B-12=0$, $\boxed{B=12}$

$6B + C - 72 = 100$, $\boxed{6B + C = 172}$, $72 + C = 172$, $\boxed{C=100}$

~~6B+100=172~~

~~6B=72~~

$6C - 300 = 300$

$6C = 600$

$\boxed{C = 100}$

$\therefore F(s) = \frac{-12}{s+6} + \frac{12s+100}{s^2+6s+25+9-9}$

إكمال المربع: نحولها لشكل (S²)

$$= \frac{-12}{(s+6)} + 12\,\frac{s+\frac{25}{3}+3-3}{(s+3)^2+16} \rightarrow = \frac{-12}{s+6} + 12\,\frac{(s+3)+\frac{16}{3}}{(s+3)^2+16}$$

$$= \frac{-12}{s+6} + 12\left[\frac{(s+3)}{(s+3)^2+(4)^2} + \frac{4}{4}\cdot\frac{\frac{16}{3}}{(s+3)^2+(4)^2}\right]$$

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$$F(s) = \frac{-12}{s+6} + 12\left[\frac{(s+3)}{(s+3)^2+4^2} + \frac{4}{3}\frac{4}{(s+3)^2+4^2}\right]$$

$$f(t) = -12e^{-6t} + 12\left[\cos 4t\, e^{-3t} + \frac{4}{3}e^{-3t}\sin 4t\right]$$

$$= -12e^{-6t} + 12e^{-3t}\left[\cos 4t + \frac{4}{3}\sin 4t\right] \qquad , s_{1,2} = -3 \pm j4$$

$$-12e^{-6t} + 12e^{-3t}\left[\frac{5}{3}\cos(4t - 53.13)\right]$$

$$= -12e^{-6t} + 20e^{-3t}\cos(4t - 53.13)$$

$$\cos 4t + \frac{4}{3}\sin 4t$$

$$\cos 4t + \frac{4}{3}\cos(4t - 90)$$

$$1\angle 0 + \frac{4}{3}\angle -90$$

$$1 - j\frac{4}{3}$$

$$\boxed{\frac{5}{3}\angle -53.13}$$

3 : Repeated Real Roots :

$$\frac{100(s+25)}{s(s+5)^3} \quad \left(\frac{k_1}{s} + \frac{k_2}{(s+5)^3} + \frac{k_3}{(s+5)^2} + \frac{k_4}{s+5}\right) s(s+5)^3$$

$(k_1 = 20)$ $\quad 100(s+25) = 20(s+5)^3 + k_2 s + k_3 s(s+5) + k_4 s(s+5)^2$

$$\rightarrow 100s + 2500 = 20s^3 + 300s^2 + 1500s + 2500 + k_2 s + k_3 s^2 + 5k_3 s + k_4 s^3 + 10k_4 s^2 + 25k_4 s$$

$(s+5)^3 = (s^2+10s+25)(s+5)$

$= s^3 + 10s^2 + 25s + 5s^2 + 50s + 125$

$= s^3 + 15s^2 + 75s + 125$

$20 + k_4 = 0$

$\boxed{k_4 = -20}$

$300 + k_3 + 10k_4 = 0$

$k_3 = -300 - 10(-20)$

$\boxed{k_3 = -100}$

$1500 + k_2 + 5k_3 + 25k_4 = 100$

$k_2 = -1500 + 500 + 500 + 100$

$\boxed{k_2 = -400}$

$$\rightarrow \frac{100(s+25)}{s(s+5)^3} = \frac{20}{s} - \frac{400}{(s+5)^3} - \frac{100}{(s+5)^2} - \frac{20}{s+5}$$

$$\rightarrow \boxed{\left(20 - \frac{400}{2}t^2e^{-5t} - 100te^{-5t} - 20e^{-5t}\right)u(t)}$$

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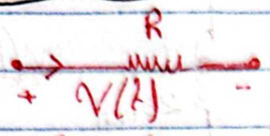
Chapter 13: The Laplace Transform in Circuit Analysis:

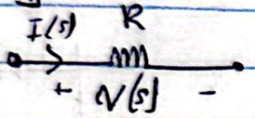


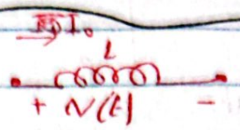
$v(t) = R\,i(t)$

$\downarrow \mathcal{L}$

$V(s) = R\,I(s)$

$R = \frac{V(s)}{I(s)}$

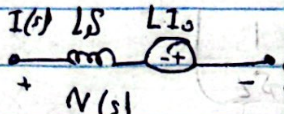


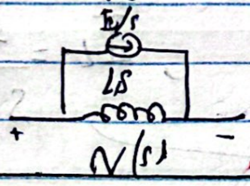


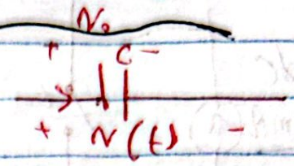
$v(t) = L\frac{di}{dt}$

$V(s) = L\left[S\,I(s) - I_0\right]$

$V(s) = LS\,I(s) - LI_0$



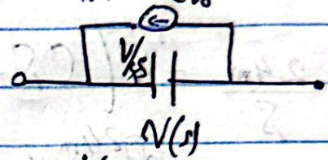


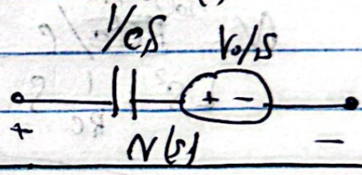


$i(t) = C\frac{d}{dt}v(t)$

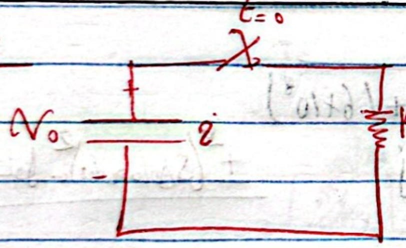
$I(s) = C\left[S\,V(s) - V_0\right]$

$\underbrace{I(s)}_{A} = \underbrace{CS\,V(s)}_{AA} - \underbrace{CV_0}_{A}$

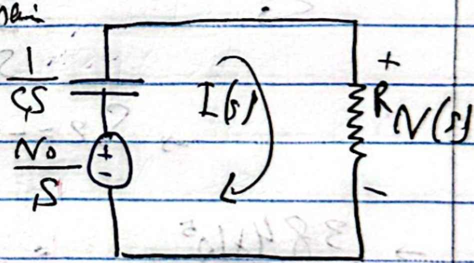




Ex:



S-Domain



$$V(s) = \frac{R}{R + \frac{1}{CS}} \cdot \frac{V_0}{s}$$

$$= \frac{RCS}{RCS+1} \cdot \frac{V_0}{s} = \frac{RCS}{RC\left(S + \frac{1}{RC}\right)} \cdot \frac{V_0}{s}$$

$$= \frac{S}{s\left(S + \frac{1}{RC}\right)} V_0 = \frac{A}{S} + \frac{B}{S + \frac{1}{RC}}$$

$$= \frac{V_0}{S + \frac{1}{RC}} \rightarrow v(t) = V_0 e^{-\frac{1}{RC}t}$$

$$i(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-\frac{1}{RC}t}$$

Note

* Ex $\frac{S+1}{S+3}$

$\frac{S+1}{S+3}$ unstable

$t \to \infty$ $f(s) \to \infty$

$\frac{S-1}{S+3}$

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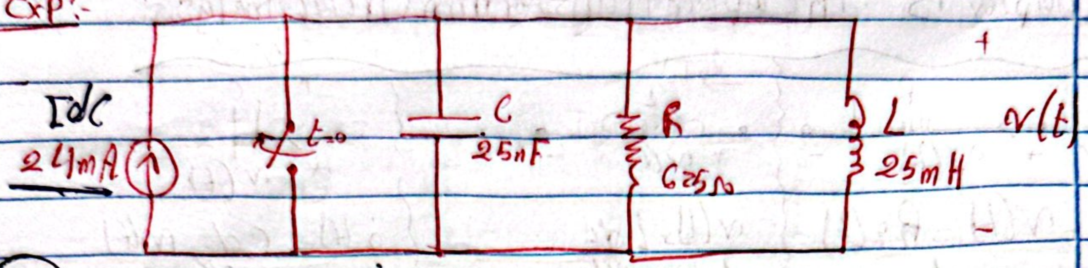
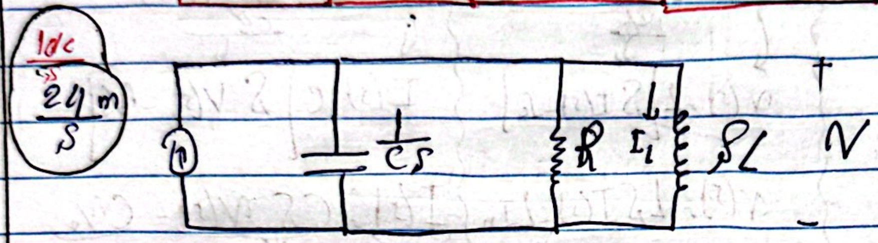
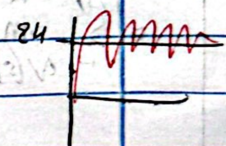
* Exp:-





$$\frac{24m}{S} = V\left(CS + \frac{1}{R} + \frac{1}{SL}\right)$$



$$V = \frac{24m/C}{S^2 + \frac{1}{RC}S + \frac{1}{LC}} \quad , \quad I_L = \frac{24m/LC}{S\left(S^2 + \frac{1}{RC}S + \frac{1}{LC}\right)}$$

$$= \frac{384\times10^5}{S(S^2 + 64000S + 16\times10^8)}$$

$$\rightarrow S_{1,2} = -32000 \pm 24000j$$

الحل بإكمال مربع (نصف معامل S)^2 ±

$$\rightarrow \frac{384\times10^5}{S(S^2+64000S+16\times10^8)} = \frac{384\times10^5}{S\left(\underbrace{S^2+64000S+(32000)^2}+\underbrace{16\times10^8-(32000)^2}_{=576000000}\right)}$$

$$= \frac{384\times10^5}{S\left((S+32000)^2+(24000)^2\right)}$$

بدي ياها بصيغة (رقم)^2

جذر $\rightarrow = \boxed{24000}$

$$= \frac{A}{S} + \frac{BS+C}{(S+32000)^2+(24000)^2}$$

$A = 24m$

$B = -24m$

$C = -1536$

$$I_L = \frac{24m}{s} + \frac{-24m\,s - 1536}{(s+32000)^2 + (24000)^2}$$

$$= \frac{24m}{s} - 24m\left(\frac{s + 64000}{(s+32000)^2 + (24000)^2}\right)$$

$$\cos 24000t + \frac{4}{3}\sin 24000t$$

$$1\angle 0^\circ + \frac{4}{3}\angle -90$$

$$\boxed{\frac{5}{3}\angle 53.13}$$

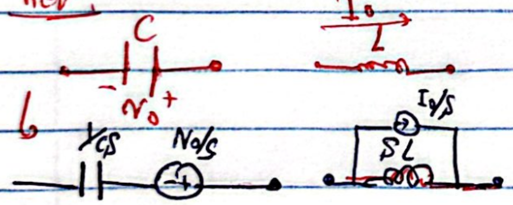
$$= \frac{24m}{s} - 24m\,\frac{(s+32000) + 32000}{(s+32000)^2 + (24000)^2}$$

$$= \frac{24m}{s} - \left(24m\,\frac{(s+32000)}{(s+32000)^2 + (24000)^2}\right) + \left(24m\,\frac{32000}{24000}\cdot\frac{24000}{(s+32000)^2 + (24000)^2}\right)$$

$$\Rightarrow i_L(t) = 24m - 24m\cos(24000t)\,e^{-32000t} + 24m \times \frac{4}{3}\,e^{-32000t}\sin(24000t)$$

$$= 24m - \underset{(24 \times \frac{5}{3})}{40}\,e^{-32000t}\cos(24000t + 126.87)\,mA$$

Rev:



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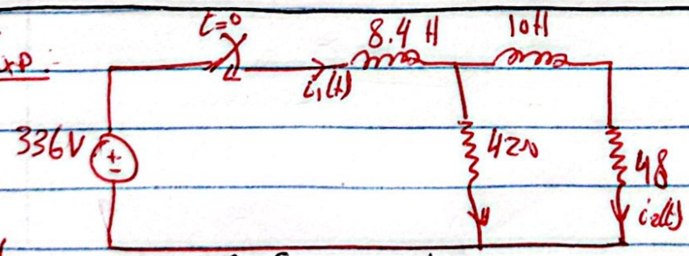

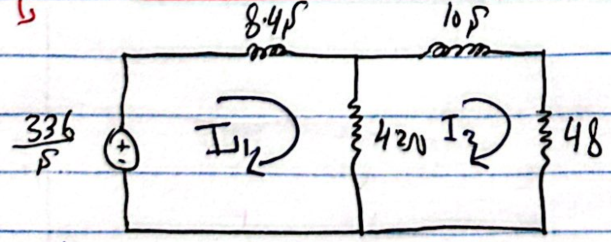

$$(42 - 8.4s)I_1 - 42I_2 = \frac{336}{s} \quad (1)$$

$$-42I_1 + (90 + 10s)I_2 = 0 \quad (2)$$

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$\cancel{\$}$: $(42+8.4S)I_1 - 42I_2 = \frac{336}{S}$ — ①

$-42I_1 + (90+10S)I_2 = 0$ — ②

$\rightarrow I_1 = \dfrac{\begin{vmatrix} \frac{336}{S} & -42 \\ 0 & 90+10S \end{vmatrix}}{\Delta} = \dfrac{40(S+9)}{S(S+2)(S+12)} = \overset{A}{\dfrac{15}{S}} + \overset{B}{\dfrac{-14}{S+2}} + \overset{C}{\dfrac{-1}{S+12}}$

$\Delta = \begin{vmatrix} 42+8.4S & -42 \\ -42 & 90+10S \end{vmatrix}$

$\boxed{i_1(t) = 15 - 14e^{-2t} - e^{-12t}}$, A

$\rightarrow I_2 = \dfrac{\begin{vmatrix} 42+8.4S & \frac{336}{S} \\ -42 & 0 \end{vmatrix}}{\Delta} = \dfrac{168}{S(S+2)(S+12)} = \overset{A}{\dfrac{7}{S}} + \overset{B}{\dfrac{-8.4}{S+2}} + \overset{C}{\dfrac{1.4}{S+12}}$

$\boxed{i_2(t) = 7 - 8.4e^{-2t} + 1.4e^{-12t}}$, A

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