

10.10 : The Binomial series

Def:- for $-1 < x < 1$

$$(1+x)^m = 1 + mx + \frac{m(m-1)x^2}{2!} + \dots + \frac{m(m-1)(m-2)\dots(m-k+1)x^k}{k!} + \dots$$

$$= 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k$$

$$\binom{m}{k} = \frac{m!}{k!(m-k)!}$$

Applications of Taylor series

- You can use it to estimate unknown integrals

s.a:-

$$\int \sin(x^2) dx$$

$$\sin(x) = x - \frac{x^3}{3!} + \dots$$

$$\sin(x^2) = x^2 - \frac{x^6}{3!} + \dots$$

$$\int \sin(x^2) = \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \dots$$

- You can use it to Evaluate limits s.a:-

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$$

$$\lim_{x \rightarrow 0} \frac{(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots) - (1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2x + \frac{x^3}{3} + \dots}{x}$$

$$= \lim_{x \rightarrow 0} \frac{x(2 + \frac{x^2}{3} + \dots)}{x} = 2$$

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- You can use it in finding the sum of the series

$$\frac{\pi}{3} - \frac{\pi^3}{3^3 \cdot 3!} + \frac{\pi^5}{3^5 \cdot 5!} + \dots$$

$$\hookrightarrow \left(\frac{\pi}{3}\right) - \left(\frac{\pi}{3}\right)^3 \frac{1}{3!} + \left(\frac{\pi}{3}\right)^5 \frac{1}{5!} + \dots \quad \text{if we consider } \frac{\pi}{3} = x$$

$$\hookrightarrow x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \quad \text{This is the T.S of } \sin x$$

$$\text{So } \sin x = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

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