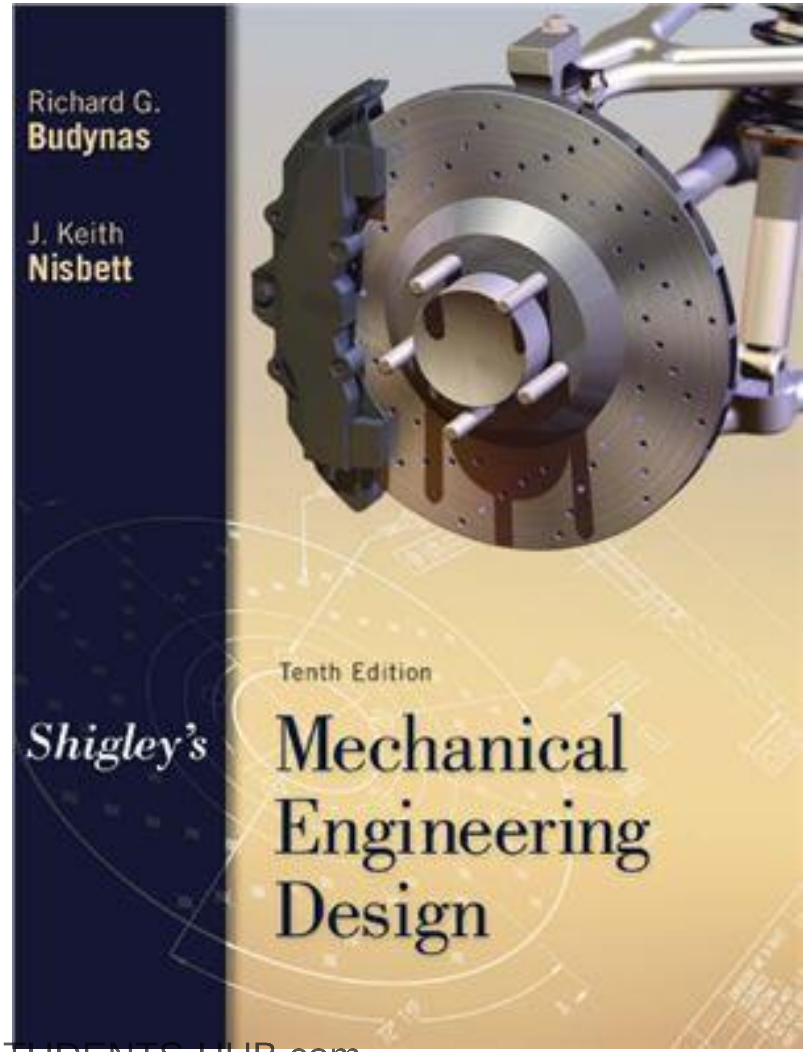
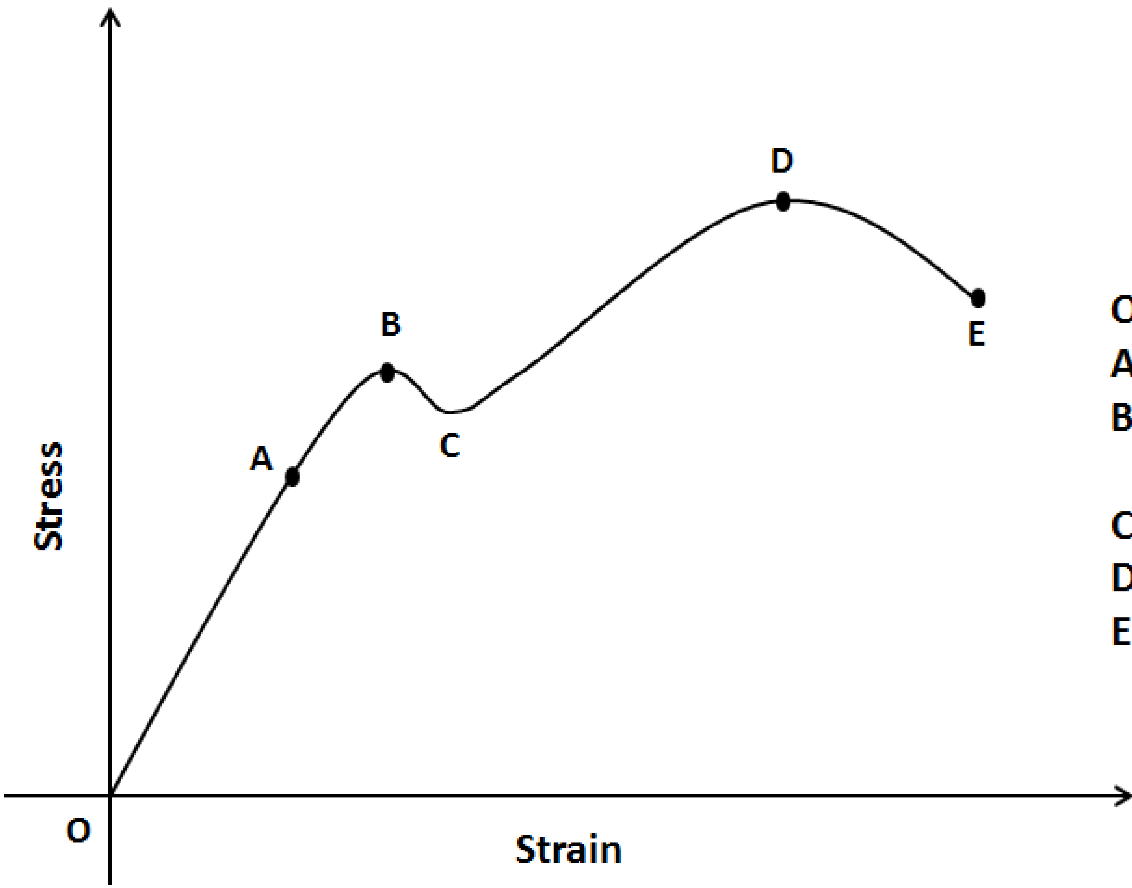
Chapter 6: Fatigue Failure Resulting from Variable Loading (shigley Book)



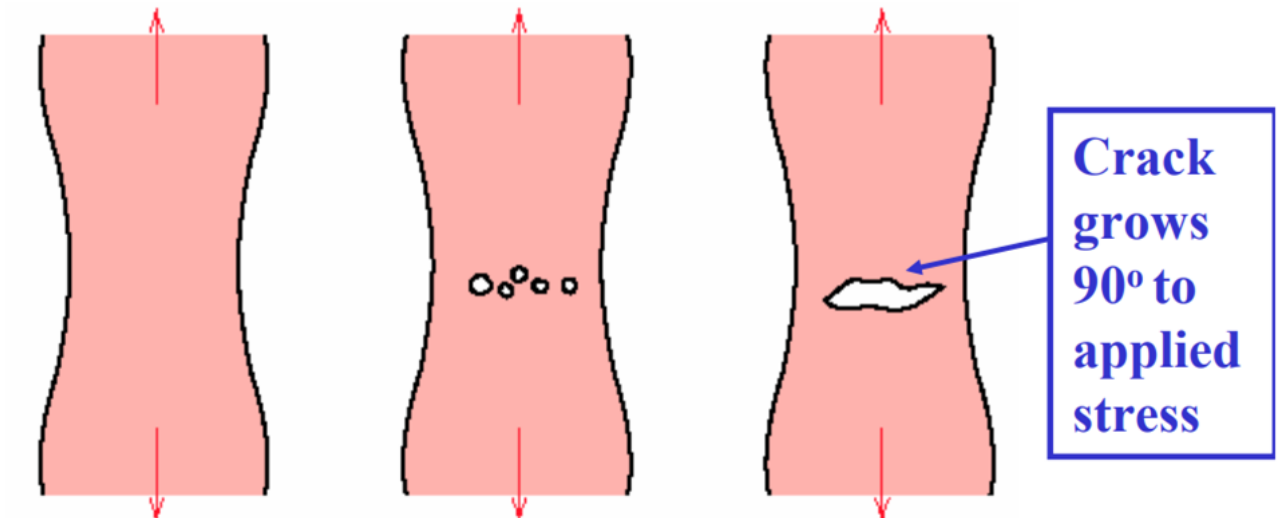
Introduction to Fatigue in Metals – Ductile Fracture

Fatigue: When parts are subjected to fluctuating stress, the parts are likely to fail at stress levels that are much less than yielding or ultimate strengths if subjected to large numbers of cycles.



OA : Proportional limit
A : Elastic limit
B : Yield stress point/upper yield stress point
C : Lower yield stress point
D : Ultimate stress point
E : Breaking or rupture point

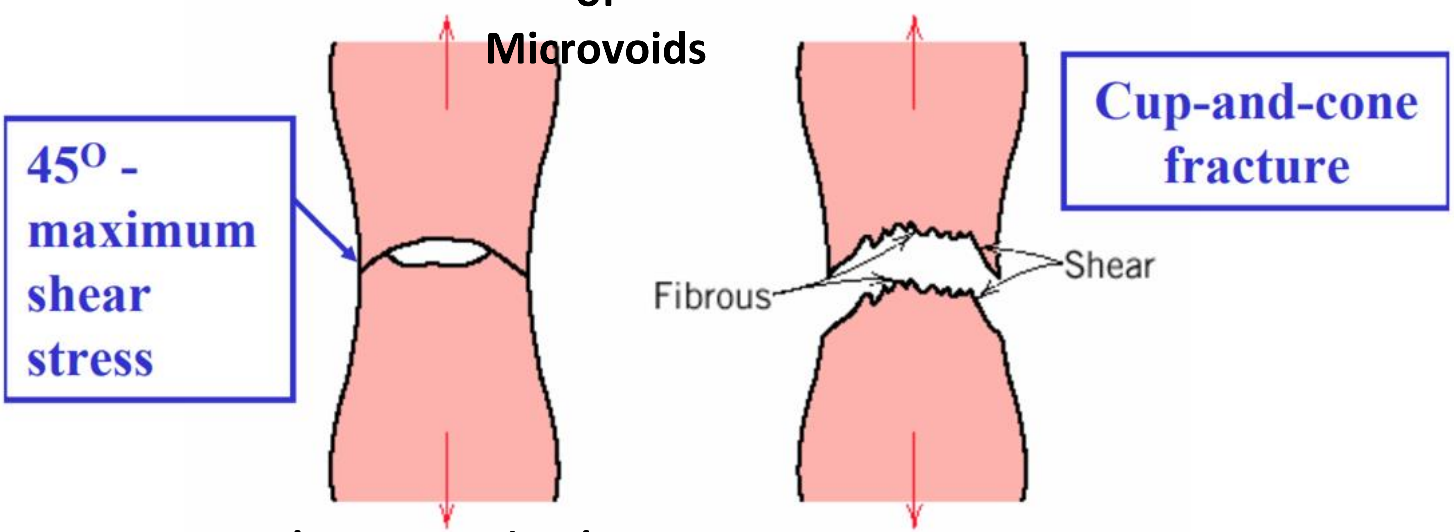
Introduction to Fatigue in Metals – Ductile Fracture



Necking

Formation of Microvoids

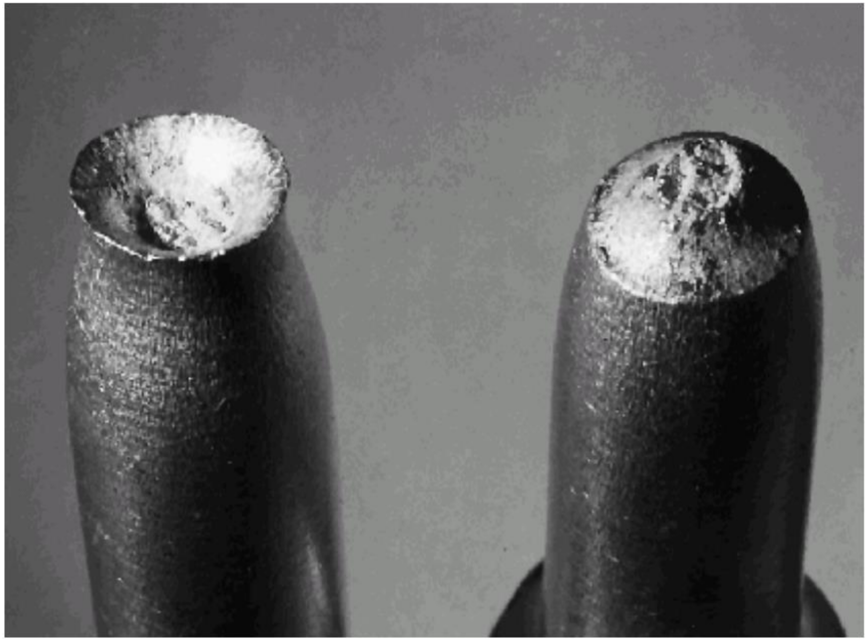
Coalescence of microvoids to form a crack

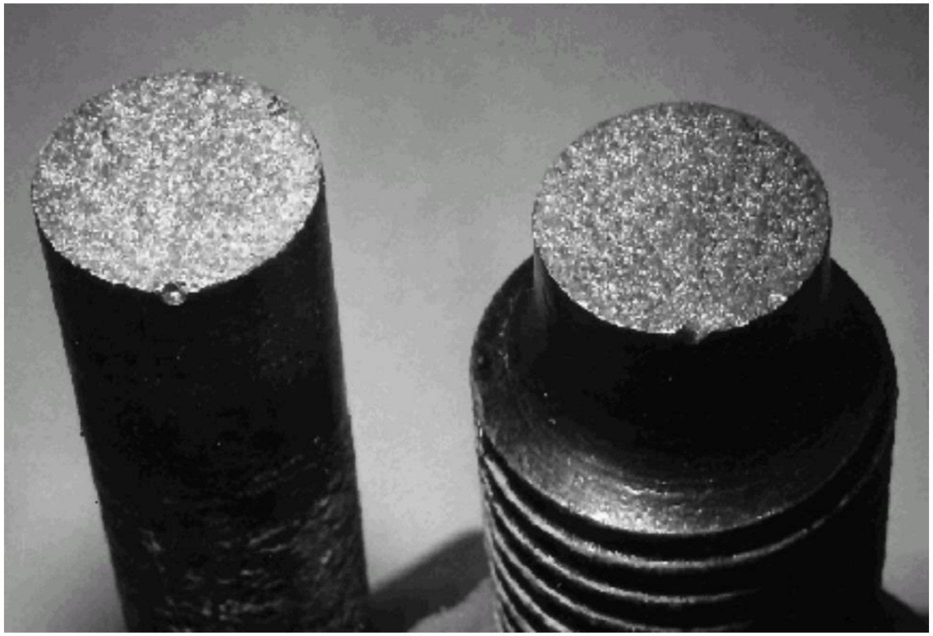


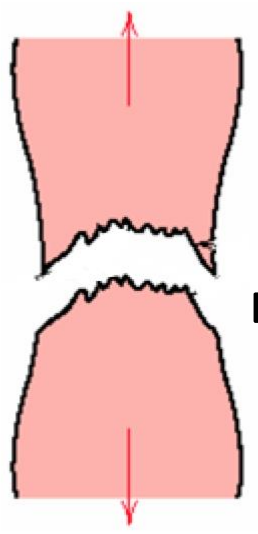
Crack propagation by shear deformation

Fracture

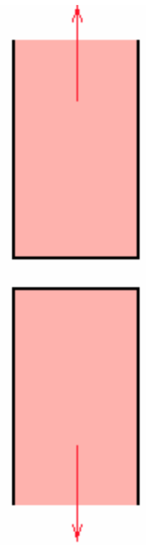
Introduction to Fatigue in Metals – Fracture







Ductile Fracture

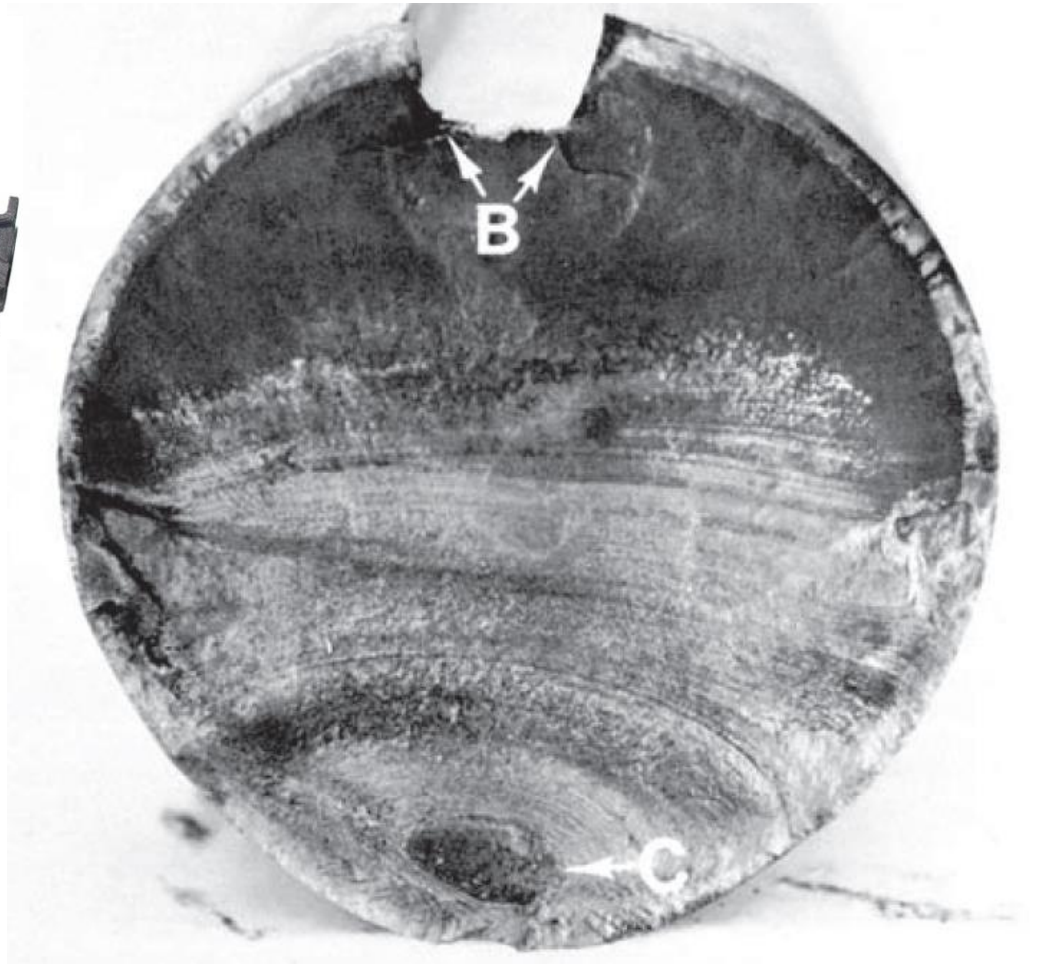


Brittle Fracture

Trees

Trees

Introduction to Fatigue in Metals - Example



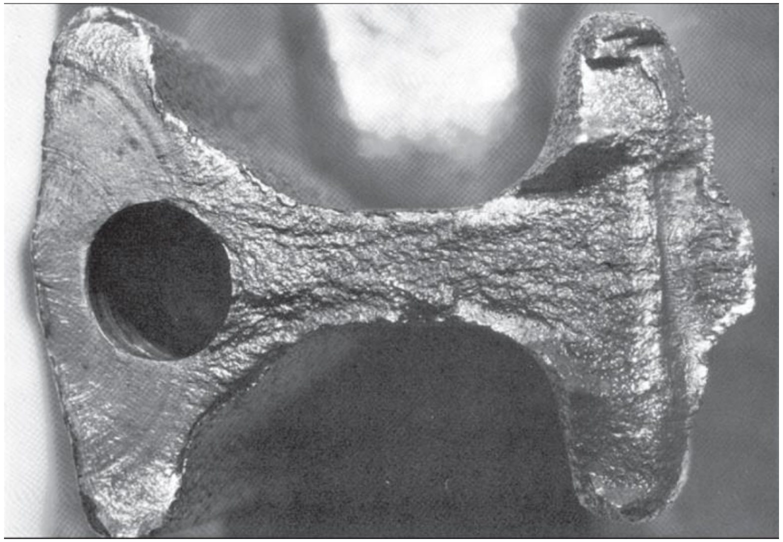
Fatigue fracture of an AISI 4320 drive shaft. The fatigue failure initiated at the end of the keyway at points *B* and progressed to final rupture at *C*. The final rupture zone is small, indicating that loads were low.

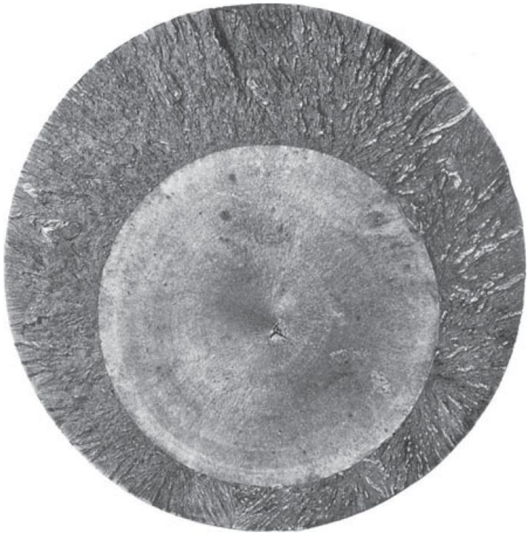
- A fatigue failure has an appearance **similar to a brittle fracture**, as the fracture surface are flat and perpendicular to the stress axis **with absence of necking**

Introduction to Fatigue in Metals - Example

Fatigue Failure Surface:

- Crack Propagation Zone → Smooth Surface
- Final Fracture Zone Rough Surface like Brittle Fracture





6-4 The Stress-Life Method

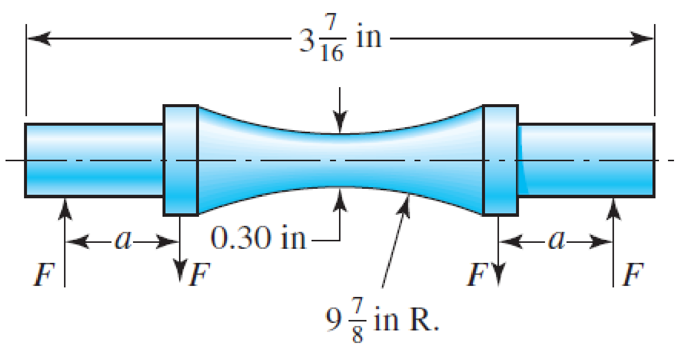
BIRZEIT UNIVERSITY

There are 3 major fatigue life methods

- Stress Life Method ✅
- Strain Life Method ❎
- Linear Elastic fracture mechanics method ❎

➢ The mostly used fatigue testing device is the R. R. Moore high speed rotating beam machine.

➢ This machine subjects the specimen to pure bending (no transverse shear) by means of weights (The specimen should be carefully machined and polished).

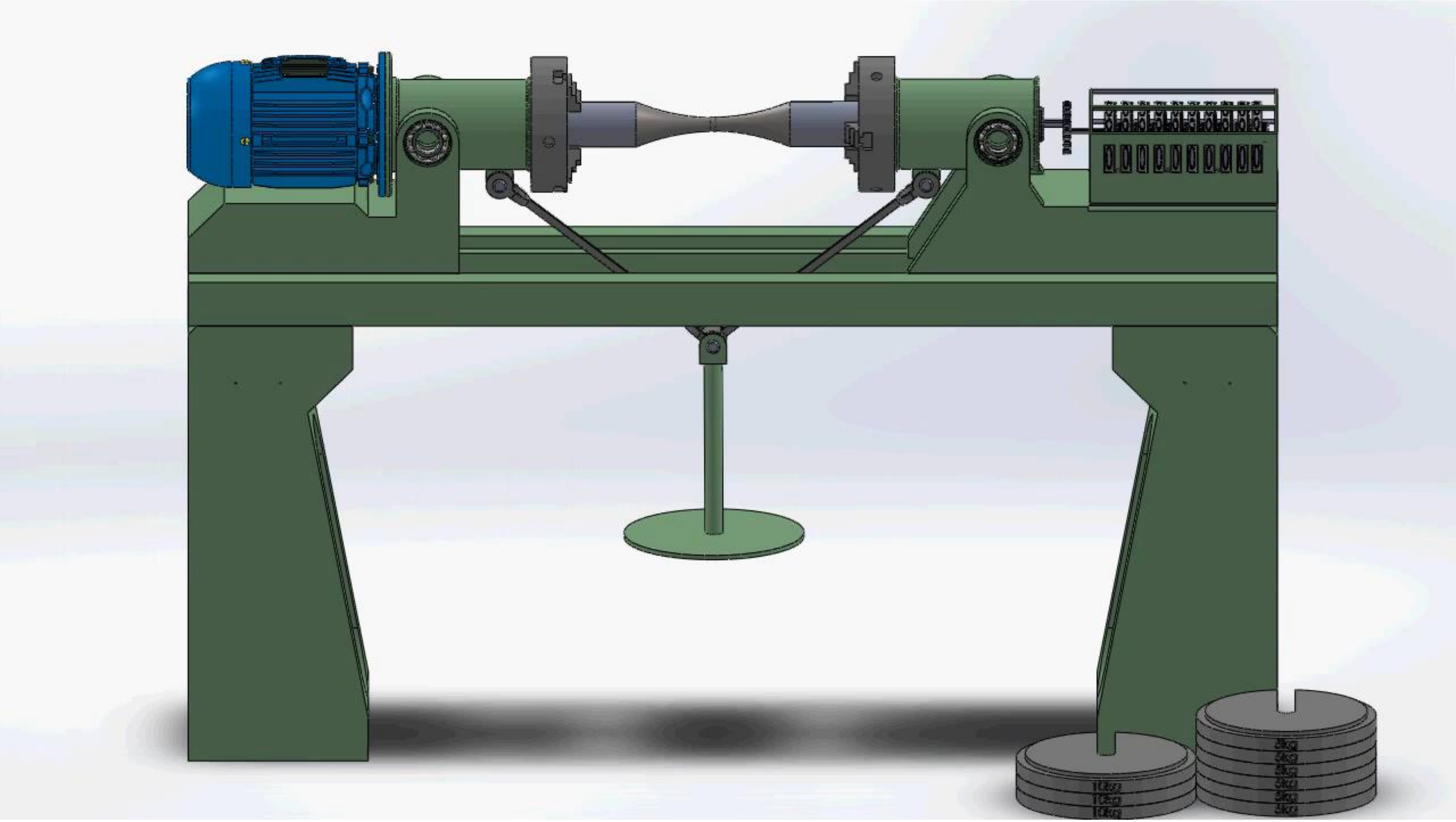


➢ To establish the fatigue strength at the materials, quite a number of tests are necessary because of the statistical nature of fatigue

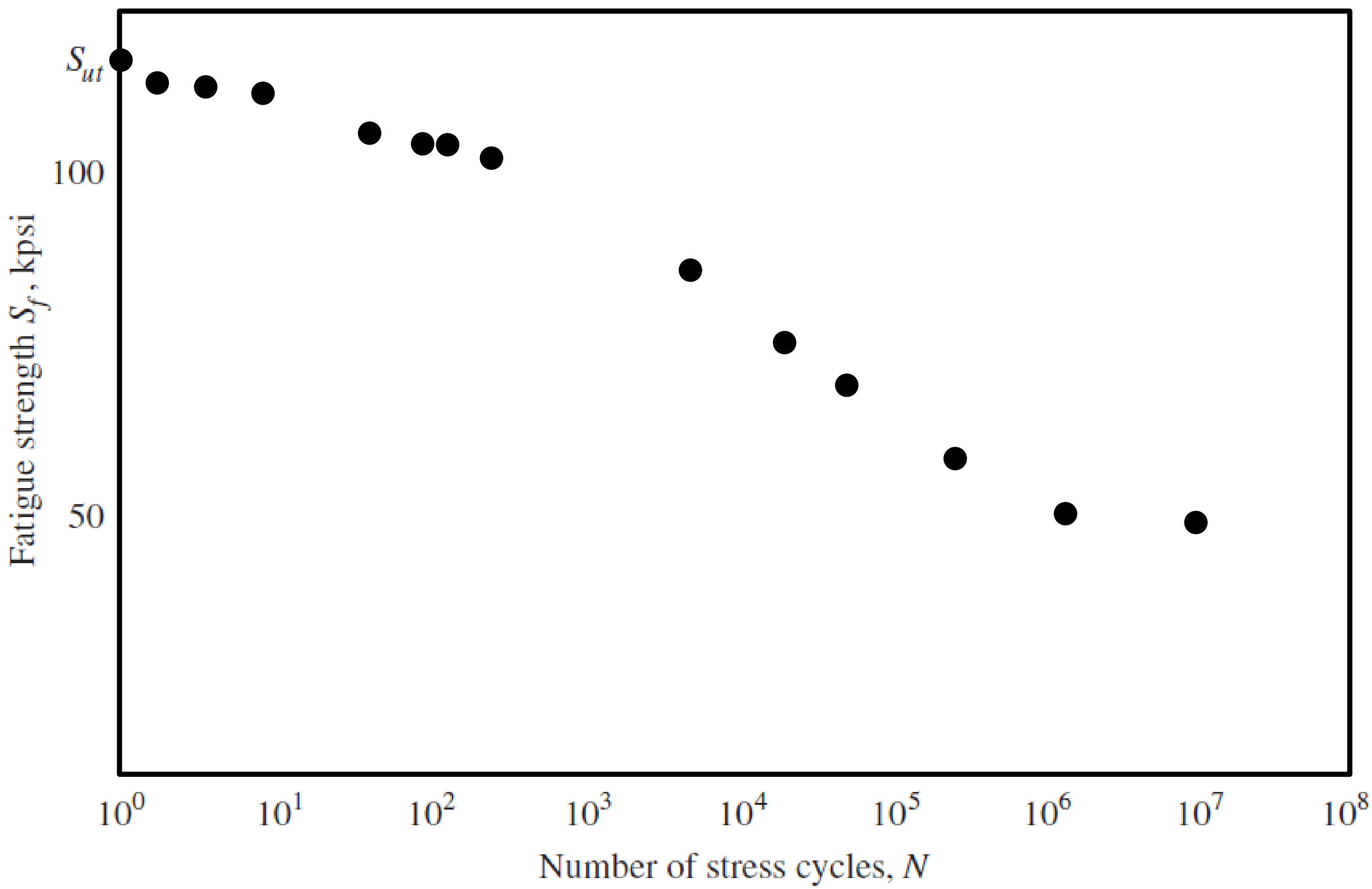
R. R. Moore High-Speed Rotating Beam Machine

https://www.youtube.com/watch?v=52knsY5AWIc

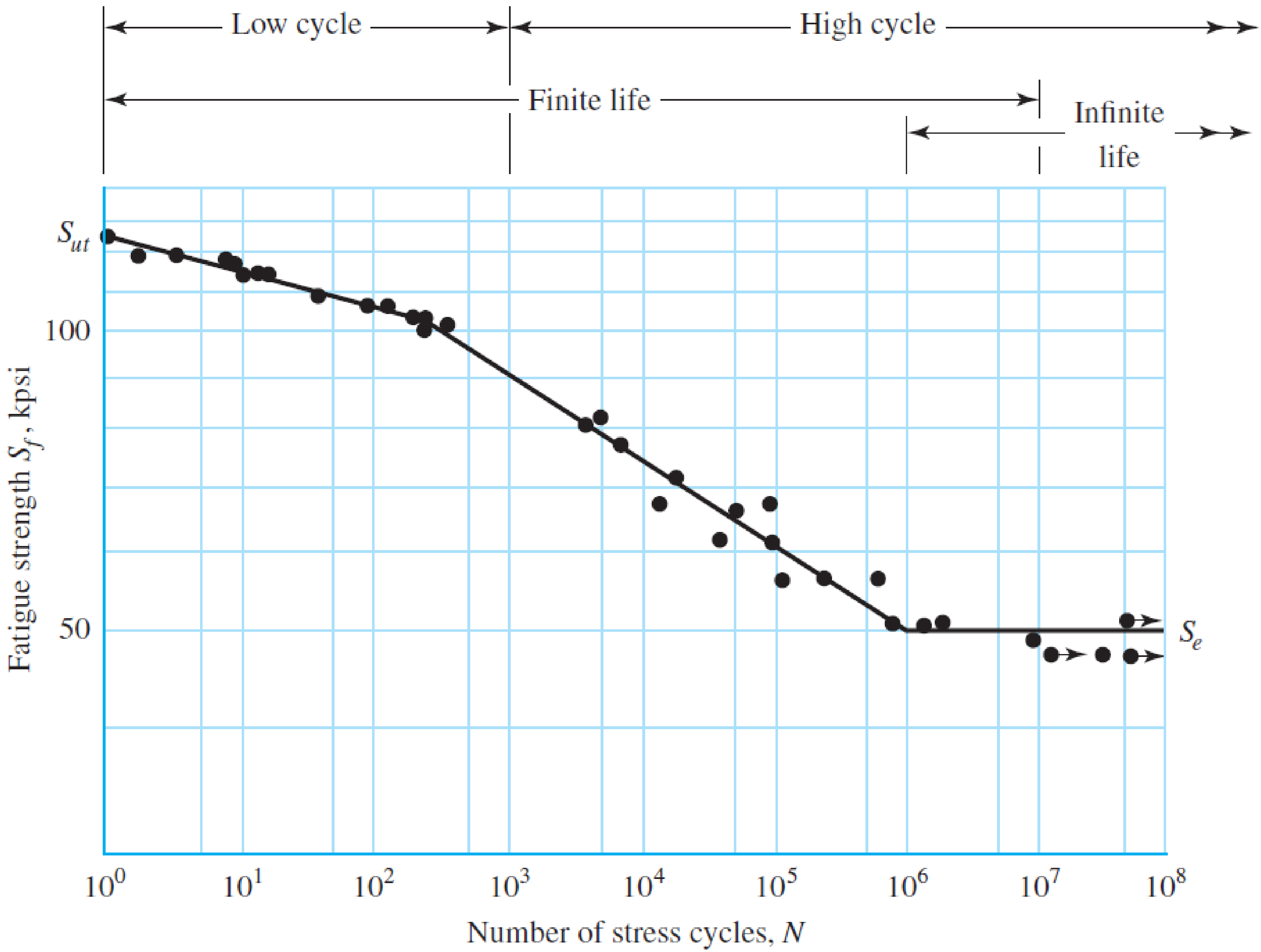
R. R. Moore High-Speed Rotating Beam Machine



6-4 The Stress-Life Method



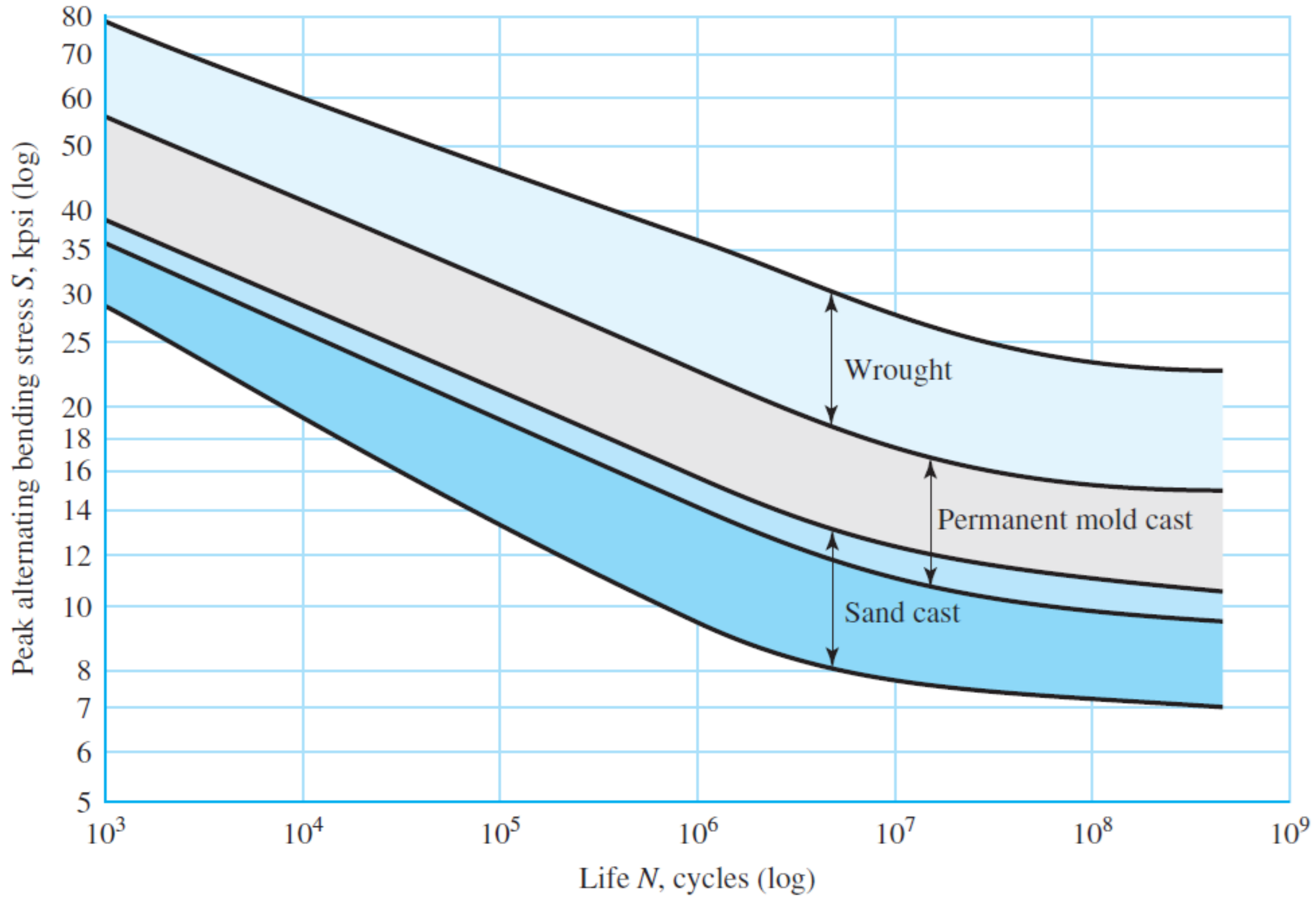
6-4 The Stress-Life Method



6-4 The Stress-Life Method

Figure 6–11

S-N bands for representative aluminum alloys, excluding wrought alloys with $S_{ut} < 38$ kpsi. *(From R. C. Juvinall,* Engineering Considerations of Stress, Strain and Strength. *Copyright © 1967 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*



6-7 The Endurance Limit

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \quad (6\text{–}8)$$

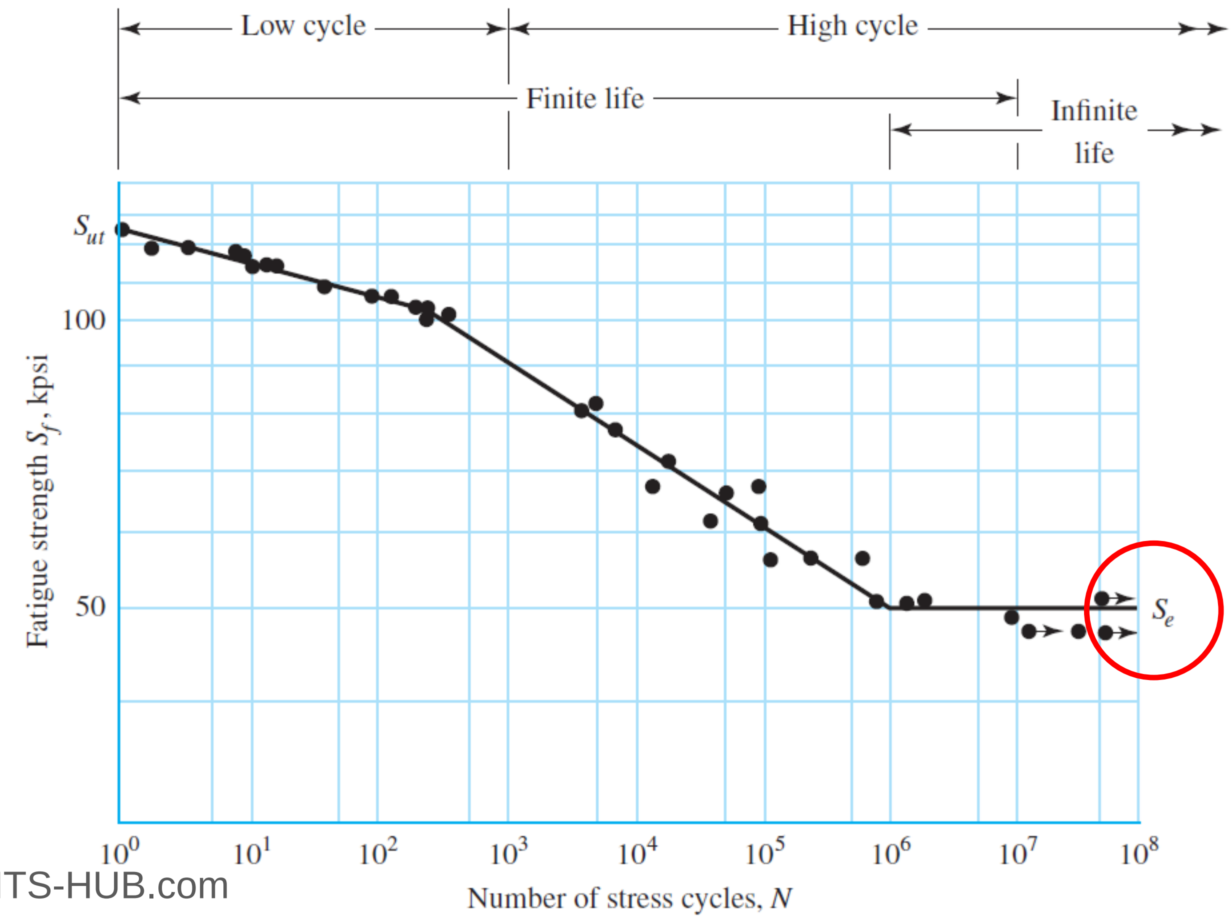


Figure 6 -17 – Endurance Limit (Wrought Irons and Steels)

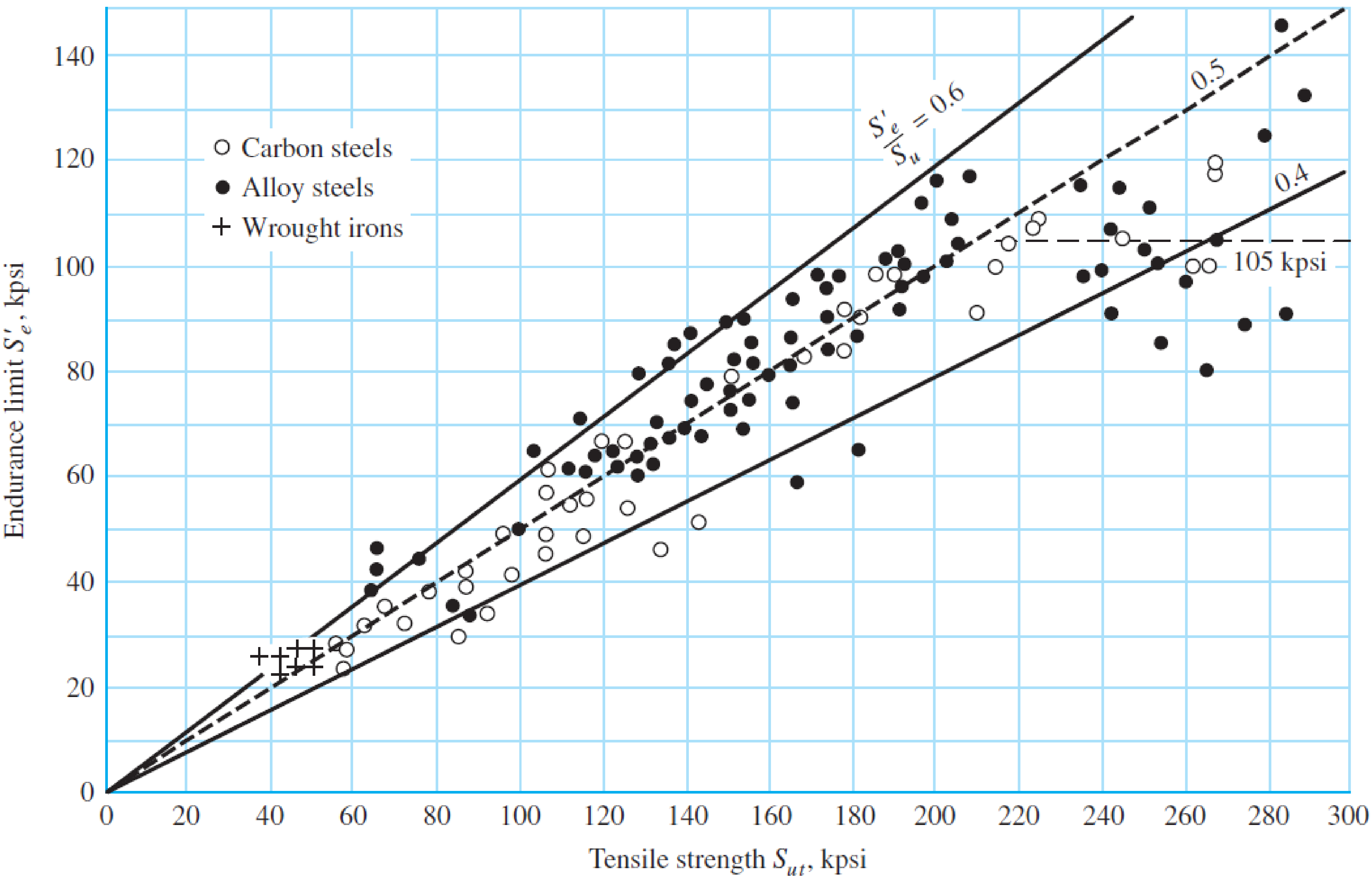


Figure A-24

Mechanical Properties of Three Non-Steel Metals

(a) Typical Properties of Gray Cast Iron

ASTM Number	Tensile Strength S_{ut}, kpsi	Compressive Strength S_{uc}, kpsi	Shear Modulus of Rupture S_{su}, kpsi	Modulus of Elasticity, Mpsi Tension†	Modulus of Elasticity, Mpsi Torsion	Endurance Limit* S_e, kpsi	Brinell Hardness H_B	Fatigue Stress-Concentration Factor K_f
20	22	83	26	9.6–14	3.9–5.6	10	156	1.00
25	26	97	32	11.5–14.8	4.6–6.0	11.5	174	1.05
30	31	109	40	13–16.4	5.2–6.6	14	201	1.10
35	36.5	124	48.5	14.5–17.2	5.8–6.9	16	212	1.15
40	42.5	140	57	16–20	6.4–7.8	18.5	235	1.25
50	52.5	164	73	18.8–22.8	7.2–8.0	21.5	262	1.35
60	62.5	187.5	88.5	20.4–23.5	7.8–8.5	24.5	302	1.50

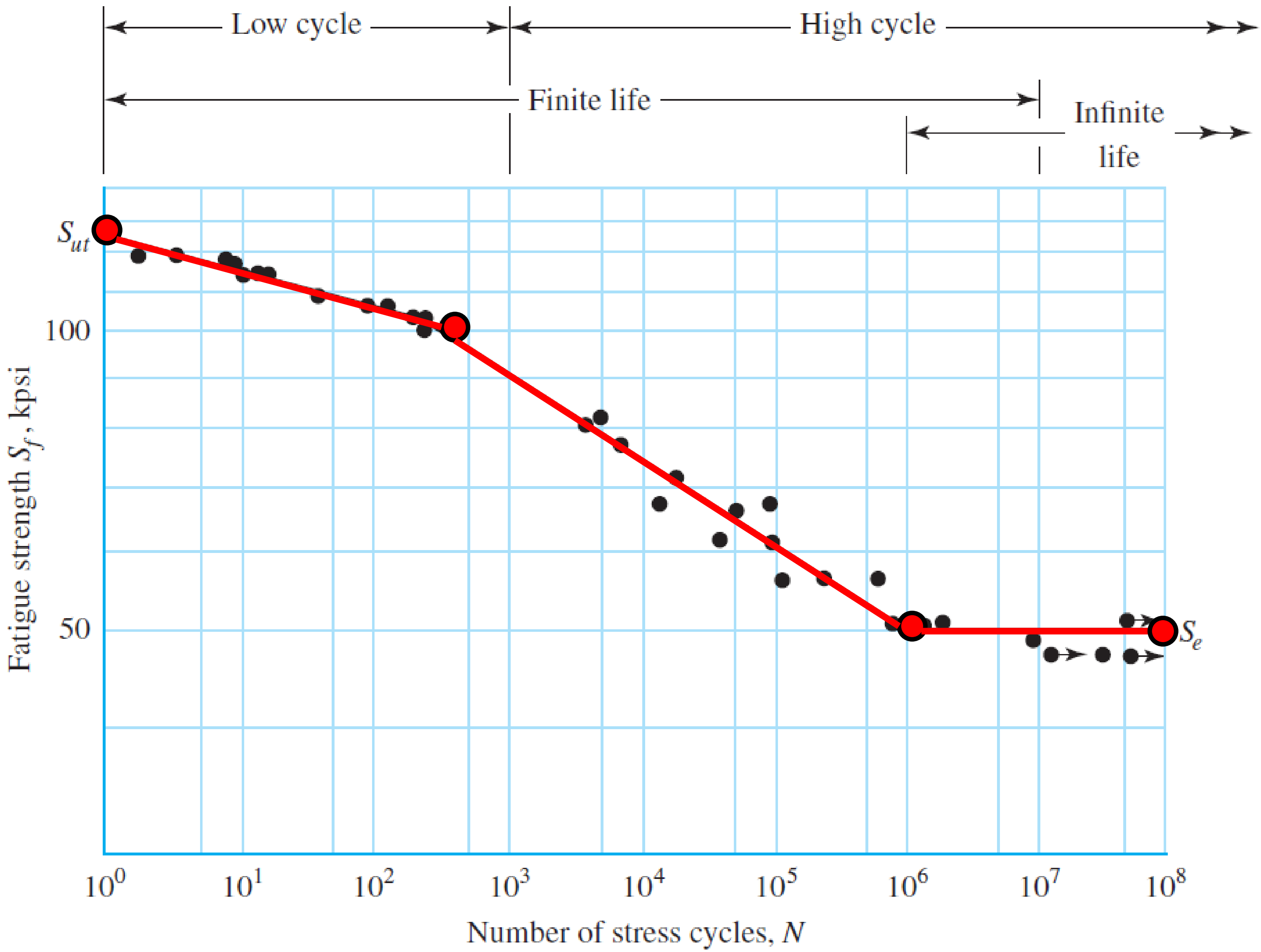
Figure A-24

(b) Mechanical Properties of Some Aluminum Alloys

Aluminum Association Number	Temper	Strength Yield, S_y, MPa (kpsi)	Tensile, S_u, MPa (kpsi)	Fatigue, S_f, MPa (kpsi)	Elongation in 2 in, %	Brinell Hardness H_B
Wrought:						
2017	O	70 (10)	179 (26)	90 (13)	22	45
2024	O	76 (11)	186 (27)	90 (13)	22	47
	T3	345 (50)	482 (70)	138 (20)	16	120
3003	H12	117 (17)	131 (19)	55 (8)	20	35
	H16	165 (24)	179 (26)	65 (9.5)	14	47
3004	H34	186 (27)	234 (34)	103 (15)	12	63
	H38	234 (34)	276 (40)	110 (16)	6	77
5052	H32	186 (27)	234 (34)	117 (17)	18	62
	H36	234 (34)	269 (39)	124 (18)	10	74
Cast:						
319.0*	T6	165 (24)	248 (36)	69 (10)	2.0	80
333.0†	T5	172 (25)	234 (34)	83 (12)	1.0	100
	T6	207 (30)	289 (42)	103 (15)	1.5	105
335.0*	T6	172 (25)	241 (35)	62 (9)	3.0	80
	T7	248 (36)	262 (38)	62 (9)	0.5	85

Aluminum alloys do not have an endurance limit. The fatigue strengths of some aluminum alloys at $5(10^8)$ cycles of reversed stress are given in Table A–24.

6-8 Fatique Strength



6-8 Fatique Strength

$$(S_f)_{10^3} = f\, S_{ut}.$$

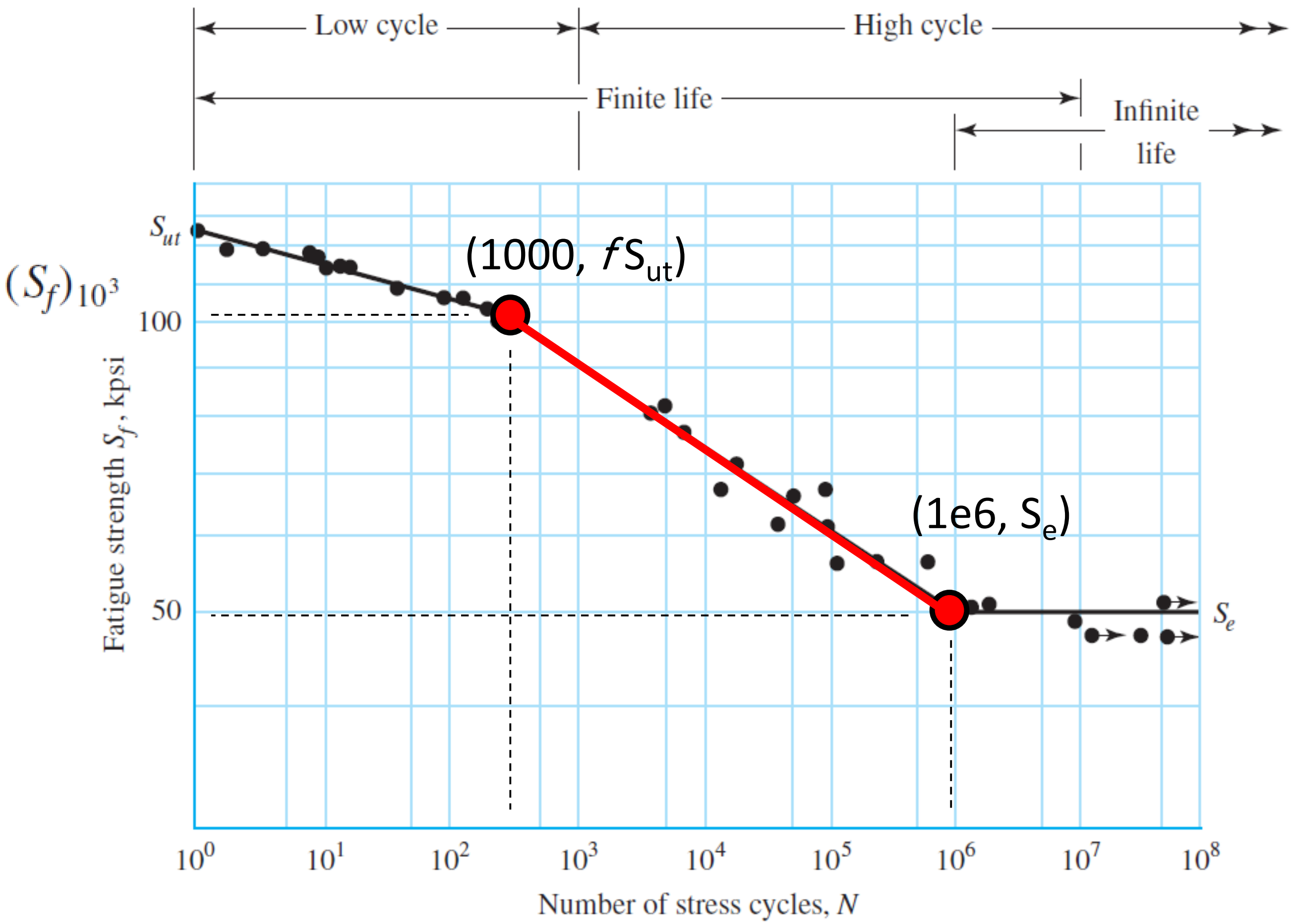
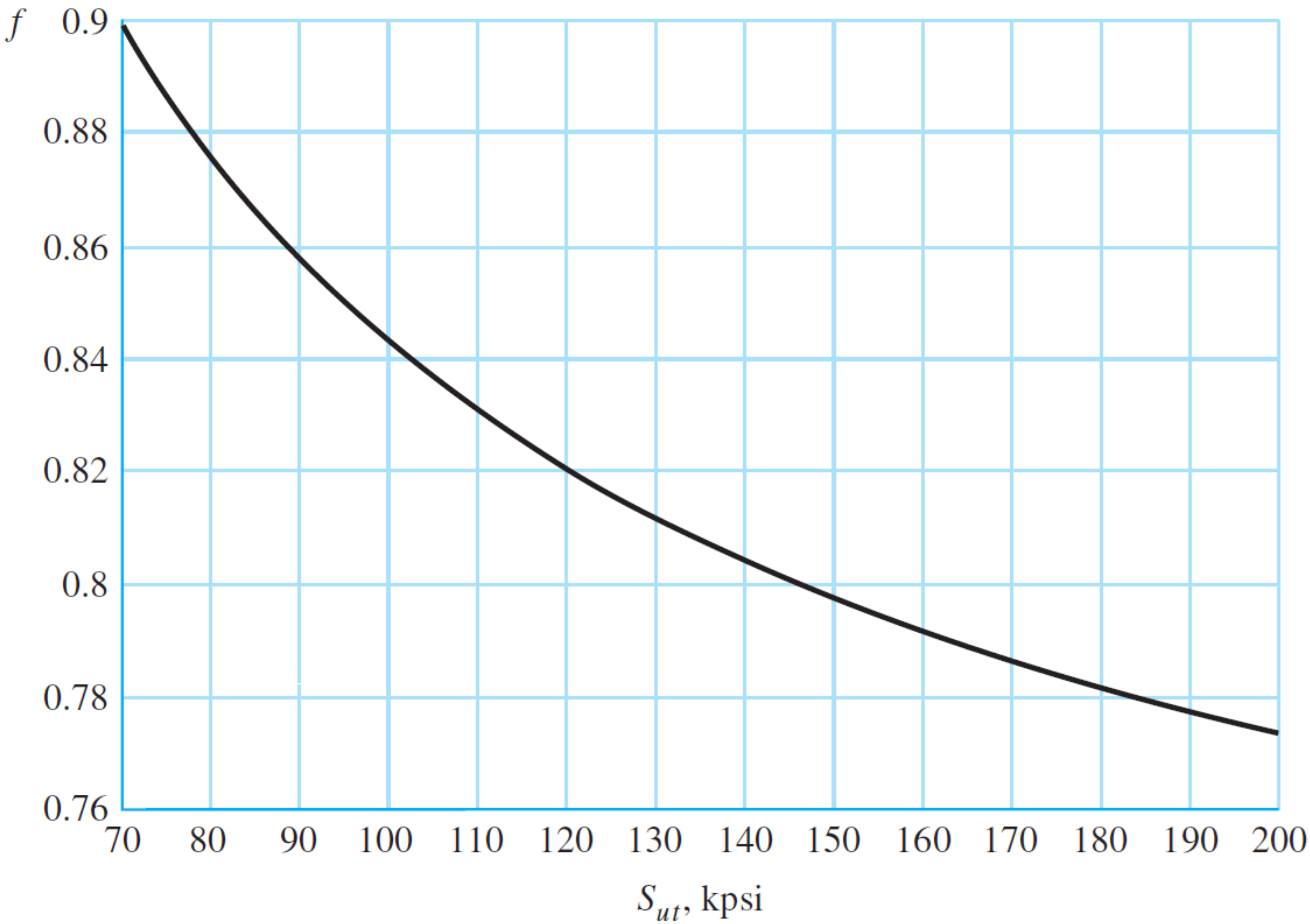


Figure 6-18: Fatigue Strength fraction f of S_{ut} at 10^3 cycles

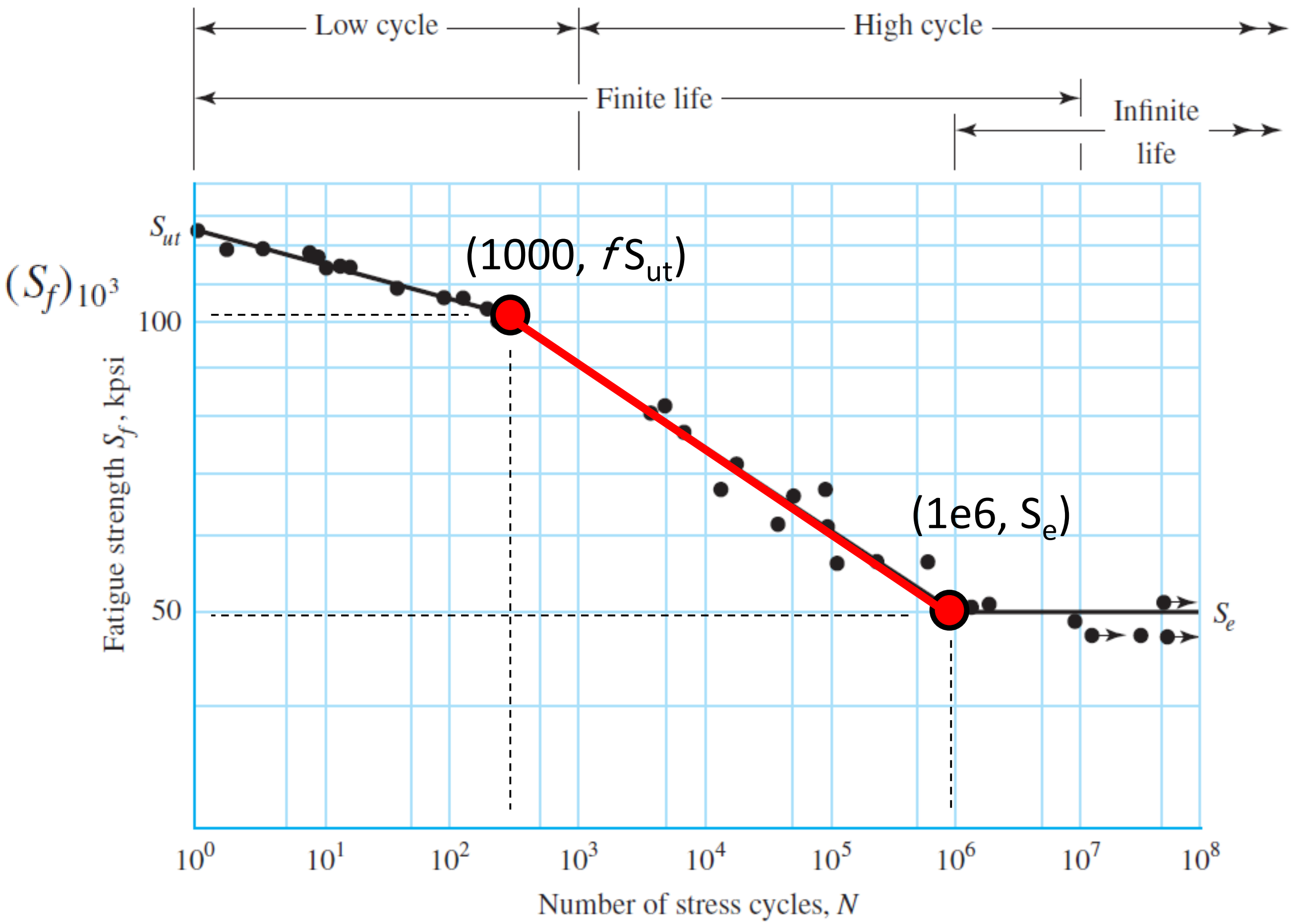


6-8 Fatique Strength

BIRZEIT UNIVERSITY

$$(S_f)_{10^3} = f\, S_{ut}.$$

$$S_f = a\, N^b$$

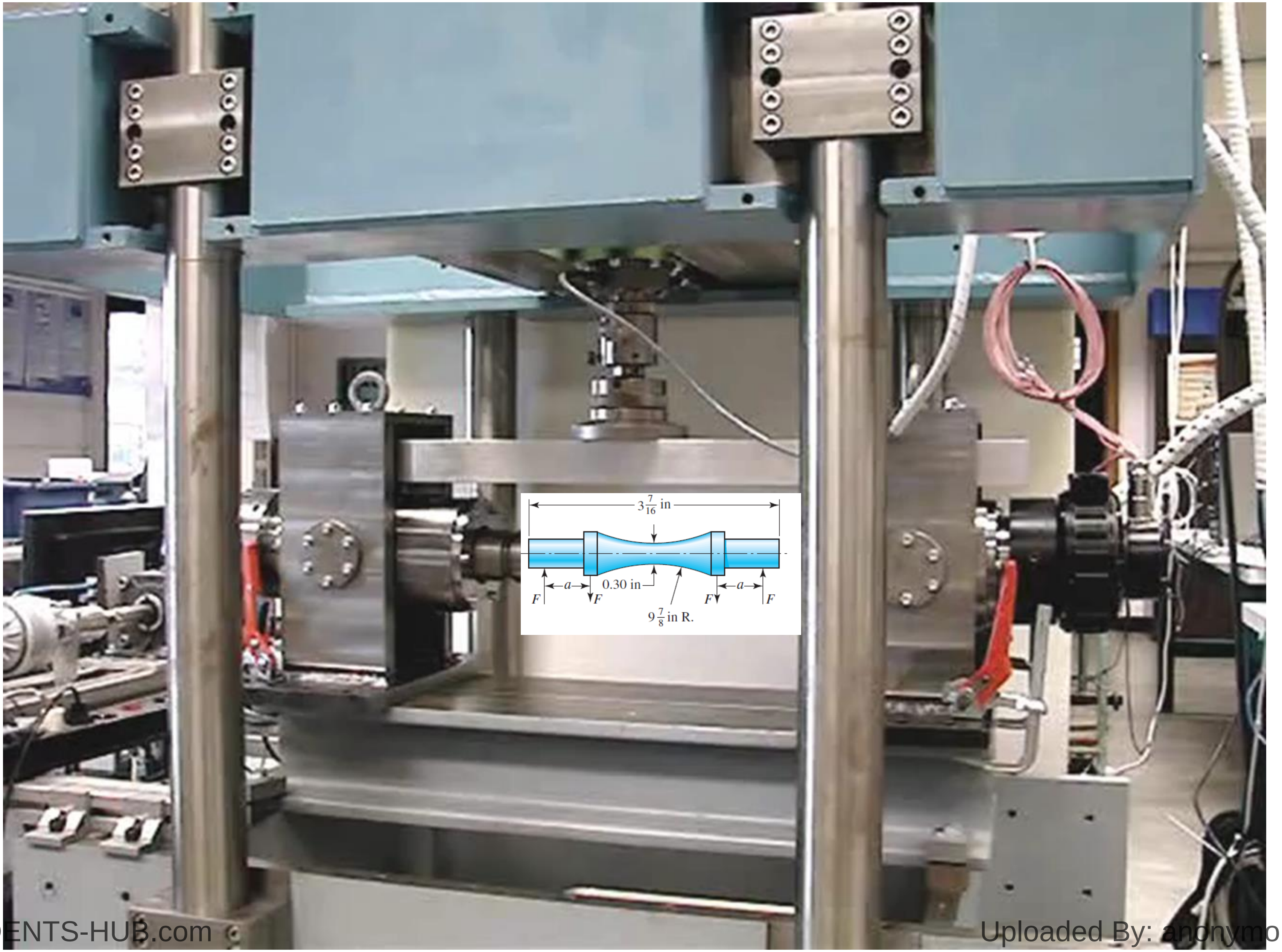


$$N = \left(\frac{\sigma_{\mathrm{rev}}}{a}\right)^{1/b}$$

$$a = \frac{(f\, S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f\, S_{ut}}{S_e}\right)$$

6-9 Endurance Limit Modifying Factors



6-9 Endurance Limit Modifying Factors

BIRZEIT UNIVERSITY

- *Material:* composition, basis of failure, variability
- *Manufacturing:* method, heat treatment, fretting corrosion, surface condition, stress concentration
- *Environment:* corrosion, temperature, stress state, relaxation times
- *Design:* size, shape, life, stress state, speed, fretting, galling

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \qquad (6\text{–}18)$$

where k_a = surface condition modification factor

k_b = size modification factor

k_c = load modification factor

k_d = temperature modification factor

k_e = reliability factor[13]

k_f = miscellaneous-effects modification factor

S'_e = rotary-beam test specimen endurance limit

S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

Table 6-2: Parameters for Marin Surface Modification

BIRZEIT UNIVERSITY

Surface Factor k_a

$$k_a = aS_{ut}^b$$

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

6-9 Endurance Limit Modifying Factors

Size Factor k_b

- The results for bending and torsion may be expressed as:

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \qquad (6\text{–}20)$$

- For axial loading there is no size effect, so

$$k_b = 1 \qquad (6\text{–}21)$$

Rotating Components

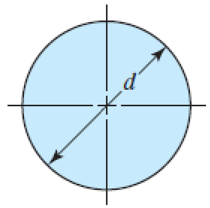
Table 6-3: Areas of Common Nonrotating Structural Shapes

Size Factor k_b

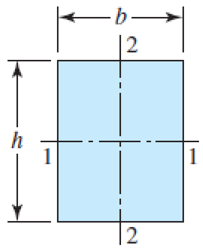
Table 6–3

$A_{0.95\sigma}$ Areas of Common Nonrotating Structural Shapes

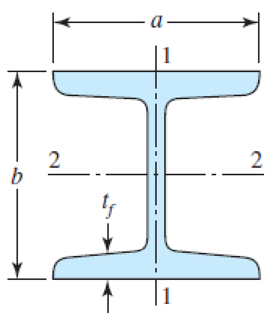
$$d_e = 0.370d$$



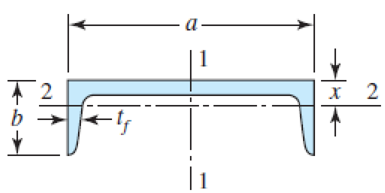
$$A_{0.95\sigma} = 0.01046d^2$$
$$d_e = 0.370d$$



$$A_{0.95\sigma} = 0.05hb$$
$$d_e = 0.808\sqrt{hb}$$



$$A_{0.95\sigma} = \begin{cases} 0.10at_f & \text{axis 1-1} \\ 0.05ba \quad t_f > 0.025a & \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b - x) & \text{axis 2-2} \end{cases}$$

6-9 Endurance Limit Modifying Factors

BIRZEIT UNIVERSITY

Load Factor k_c

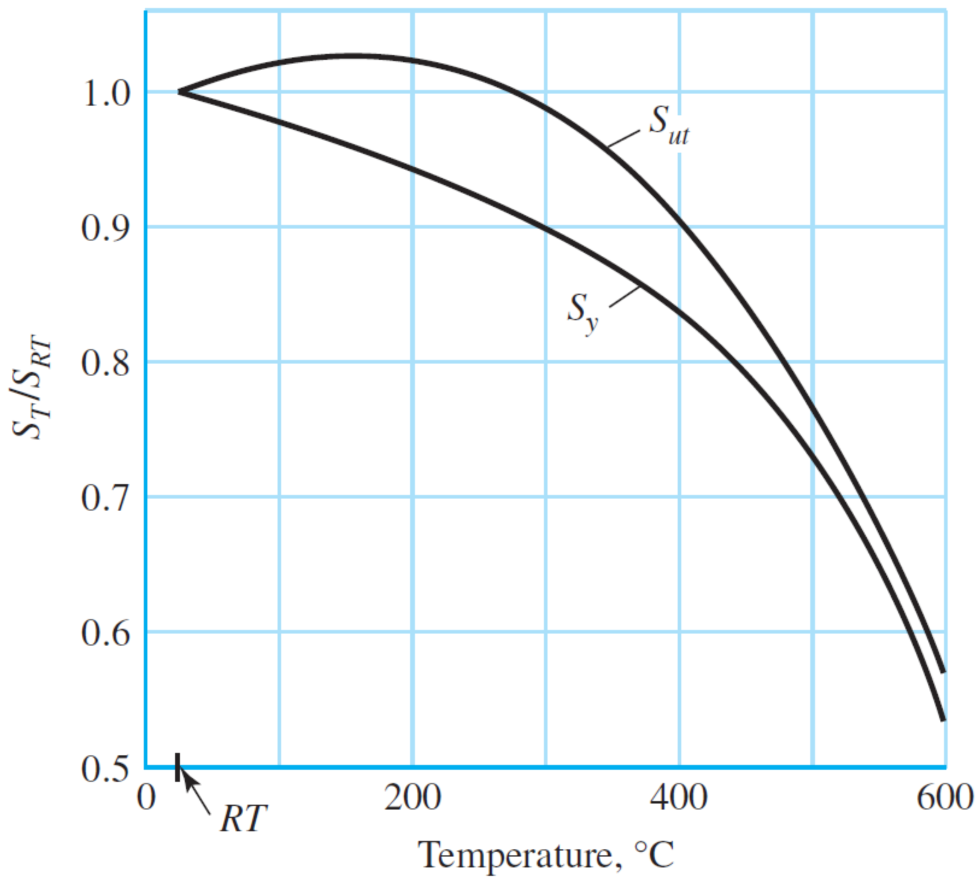
$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{–}26)$$

When torsion is combined with other loading, such as bending, set $k_c = 1$, and the combined loading is managed by using the effective von Mises stress as described in Sec. 6-14

Figure 2-9: Effect of operating temperature on S_y & S_{ut}

Temperature Factor k_d

$$k_d = \frac{S_T}{S_{RT}}$$



A fourth-order polynomial curve fit to the data underlying Fig. 2–9 gives

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \qquad (6\text{–}27)$$

where $70 \leq T_F \leq 1000°F$.

Table 6-4: Effect of operating temperature on S_y & S_{ut}

BIRZEIT UNIVERSITY

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

*Data source: Fig. 2–9.

Figure 6 -17 – Endurance Limit (Wrought Irons and Steels)

BIRZEIT UNIVERSITY

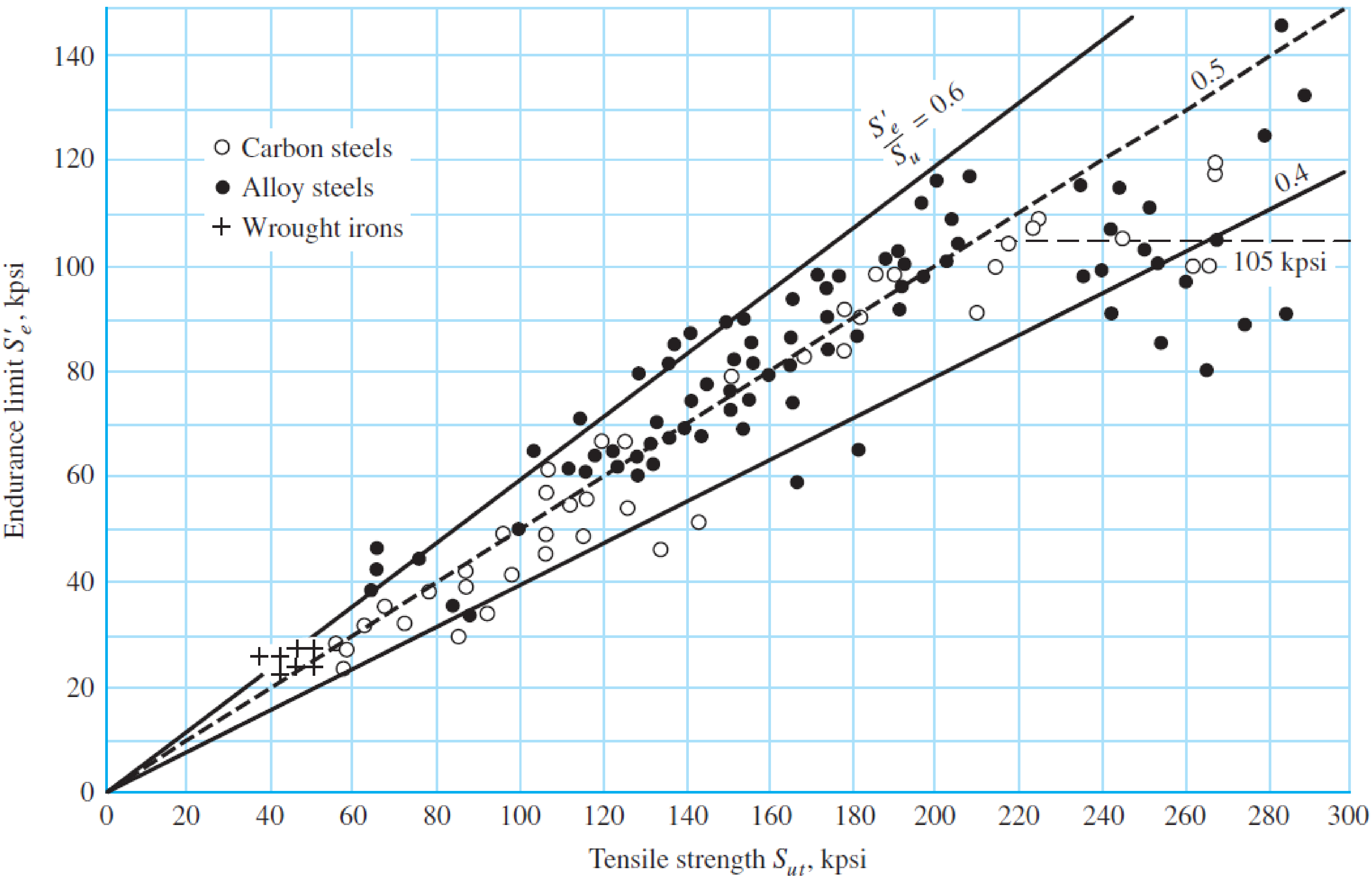


Table 6-5: Reliability Factors K_e

Reliability Factor k_e

$$k_e = 1 - 0.08\, z_a$$

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

6-9 Endurance Limit Modifying Factors

BIRZEIT UNIVERSITY

Miscellaneous Effects Factor k_f

Corrosion

It is to be expected that parts that operate in a corrosive atmosphere will have a lowered fatigue resistance. This is, of course, true, and it is due to the roughening or pitting of the surface by the corrosive material. But the problem is not so simple as the one of finding the endurance limit of a specimen that has been corroded. The reason for this is that the corrosion and the stressing occur at the same time. Basically, this means that in time any part will fail when subjected to repeated stressing in a corrosive atmosphere. There is no fatigue limit. Thus the designer's problem is to attempt to minimize the factors that affect the fatigue life; these are:

- Mean or static stress
- Alternating stress
- Electrolyte concentration
- Dissolved oxygen in electrolyte
- Material properties and composition
- Temperature
- Cyclic frequency
- Fluid flow rate around specimen
- Local crevices

6-9 Endurance Limit Modifying Factors

Miscellaneous Effects Factor k_f

Electrolytic Plating

Metallic coatings, such as chromium plating, nickel plating, or cadmium plating, reduce the endurance limit by as much as 50 percent. In some cases the reduction by coatings has been so severe that it has been necessary to eliminate the plating process. Zinc plating does not affect the fatigue strength. Anodic oxidation of light alloys reduces bending endurance limits by as much as 39 percent but has no effect on the torsional endurance limit.

Metal Spraying

Metal spraying results in surface imperfections that can initiate cracks. Limited tests show reductions of 14 percent in the fatigue strength.

Cyclic Frequency

If, for any reason, the fatigue process becomes time-dependent, then it also becomes frequency-dependent. Under normal conditions, fatigue failure is independent of frequency. But when corrosion or high temperatures, or both, are encountered, the cyclic rate becomes important. The slower the frequency and the higher the temperature, the higher the crack propagation rate and the shorter the life at a given stress level.

Frettage Corrosion

The phenomenon of frettage corrosion is the result of microscopic motions of tightly fitting parts or structures. Bolted joints, bearing-race fits, wheel hubs, and any set of tightly fitted parts are examples. The process involves surface discoloration, pitting, and eventual fatigue. The frettage factor k_f depends upon the material of the mating pairs and ranges from 0.24 to 0.90.

6-9 Endurance Limit Modifying Factors

- *Material:* composition, basis of failure, variability
- *Manufacturing:* method, heat treatment, fretting corrosion, surface condition, stress concentration
- *Environment:* corrosion, temperature, stress state, relaxation times
- *Design:* size, shape, life, stress state, speed, fretting, galling

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \tag{6–18}$$

where k_a = surface condition modification factor

k_b = size modification factor

k_c = load modification factor

k_d = temperature modification factor

k_e = reliability factor[13]

k_f = miscellaneous-effects modification factor

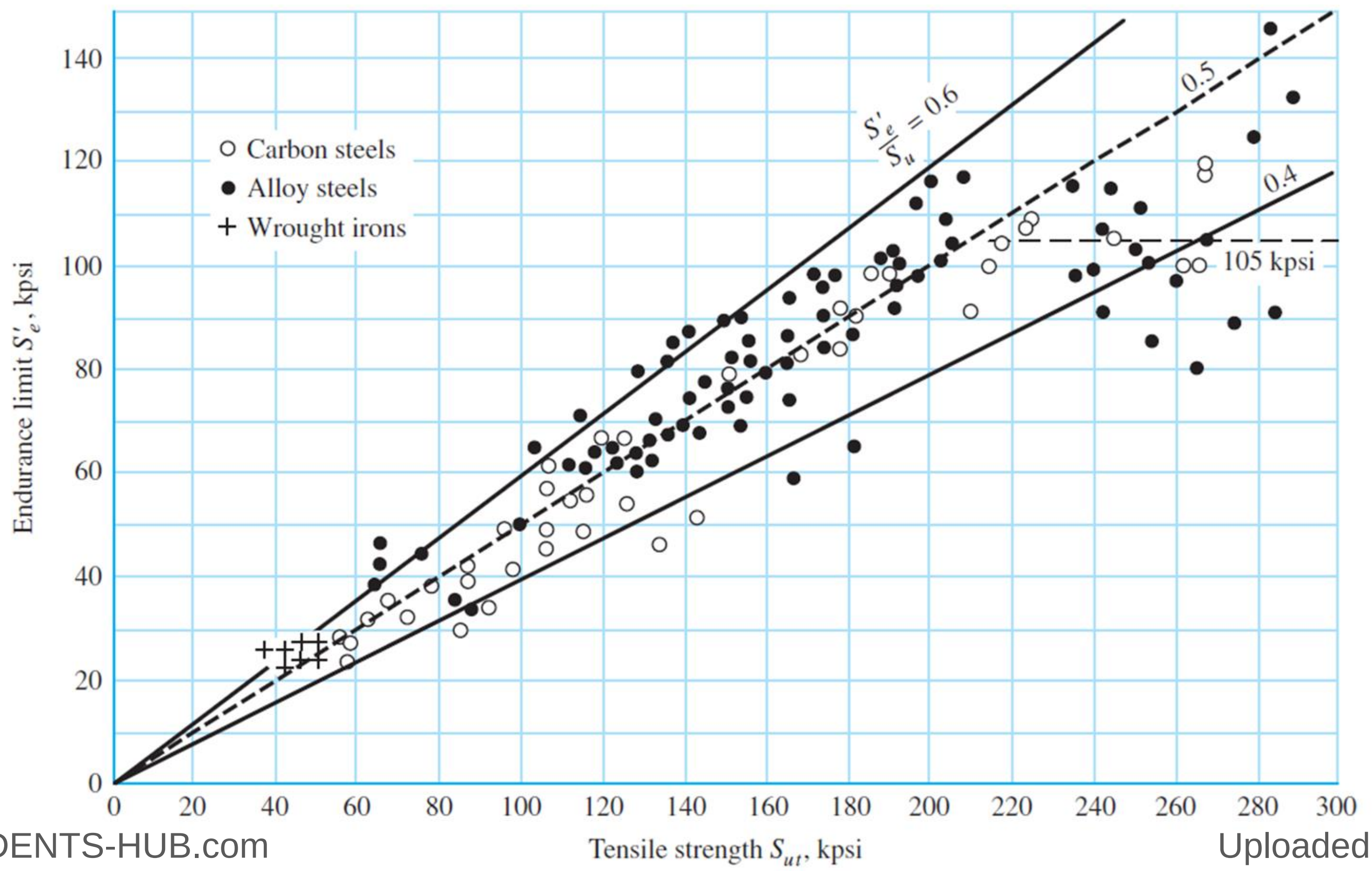
S'_e = rotary-beam test specimen endurance limit

S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

6-9 Endurance Limit Modifying Factors

BIRZEIT UNIVERSITY

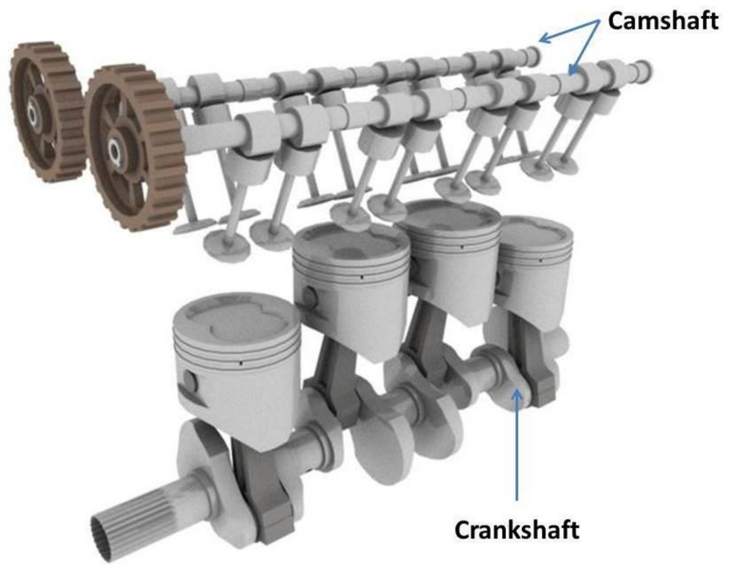
$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \qquad (6\text{–}8)$$



6-10 Stress Concentration and Notch Sensitivity

3–13 Stress Concentration

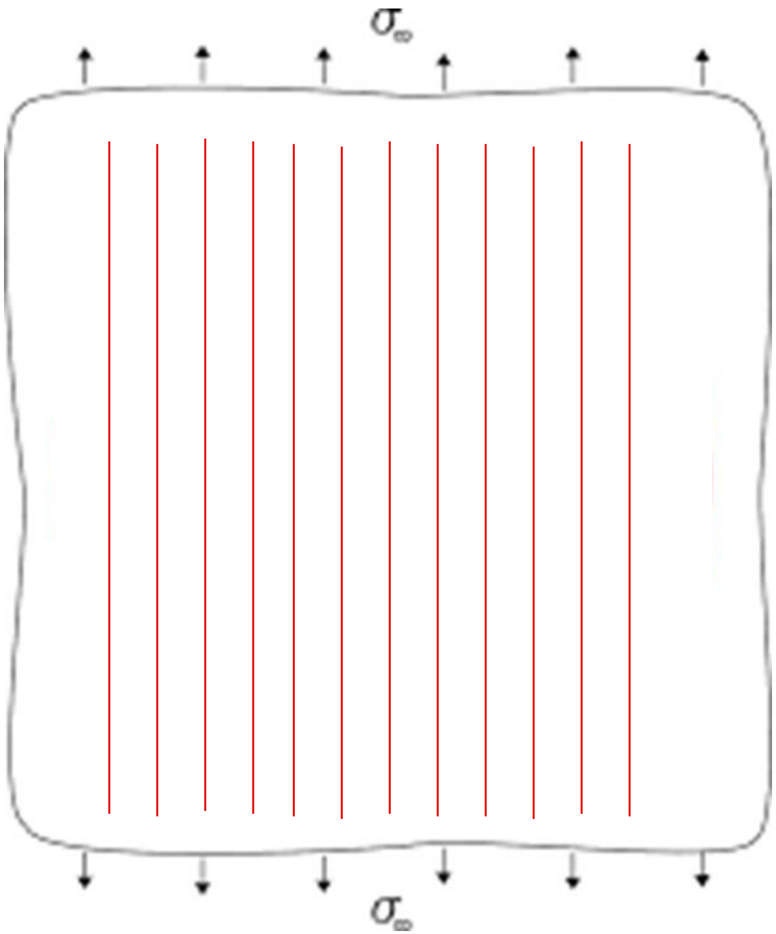
In the development of the basic stress equations for tension, compression, bending, and torsion, it was assumed that no geometric irregularities occurred in the member under consideration. But it is quite difficult to design a machine without permitting some changes in the cross sections of the members.

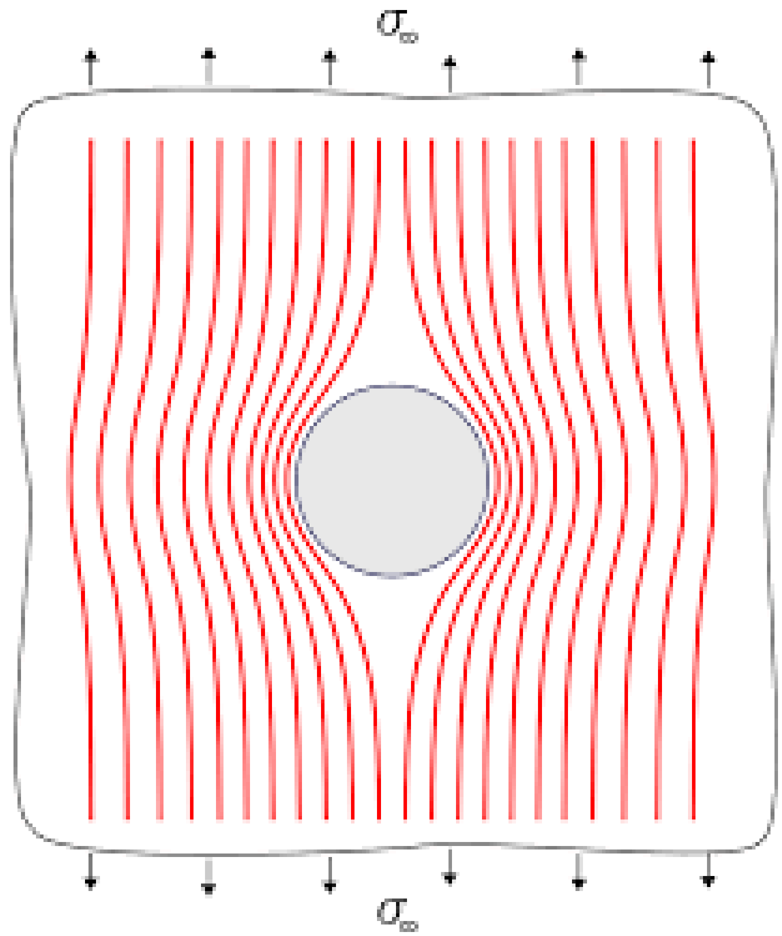


6-10 Stress Concentration and Notch Sensitivity

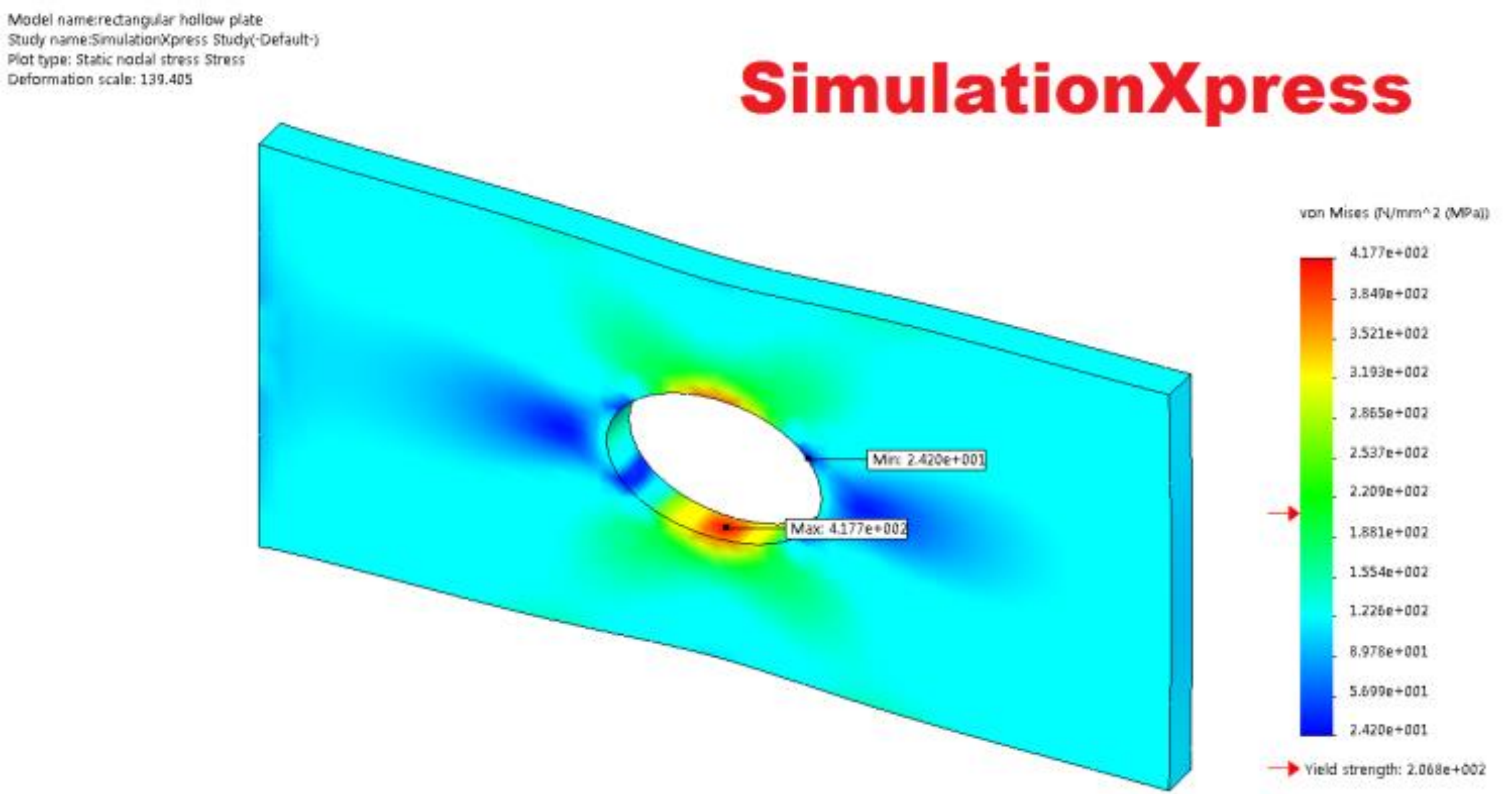
Rotating shafts must have shoulders designed on them so that the bearings can be properly seated and so that they will take thrust loads; and the shafts must have key slots machined into them for securing pulleys and gears. A bolt has a head on one end and screw threads on the other end, both of which account for abrupt changes in the cross section. Other parts require holes, oil grooves, and notches of various kinds.

6-10 Stress Concentration and Notch Sensitivity

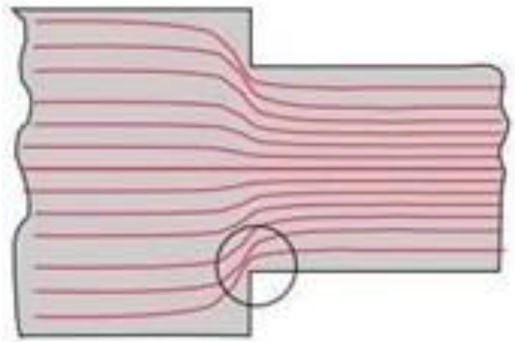




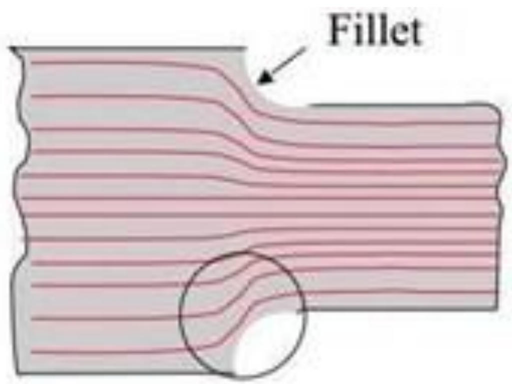
6-10 Stress Concentration and Notch Sensitivity



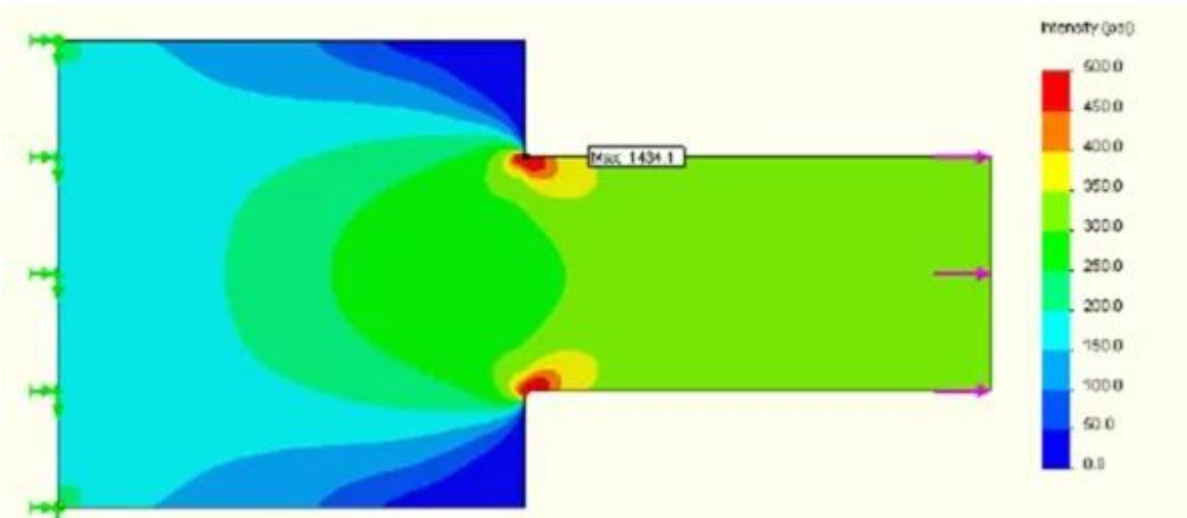
6-10 Stress Concentration and Notch Sensitivity



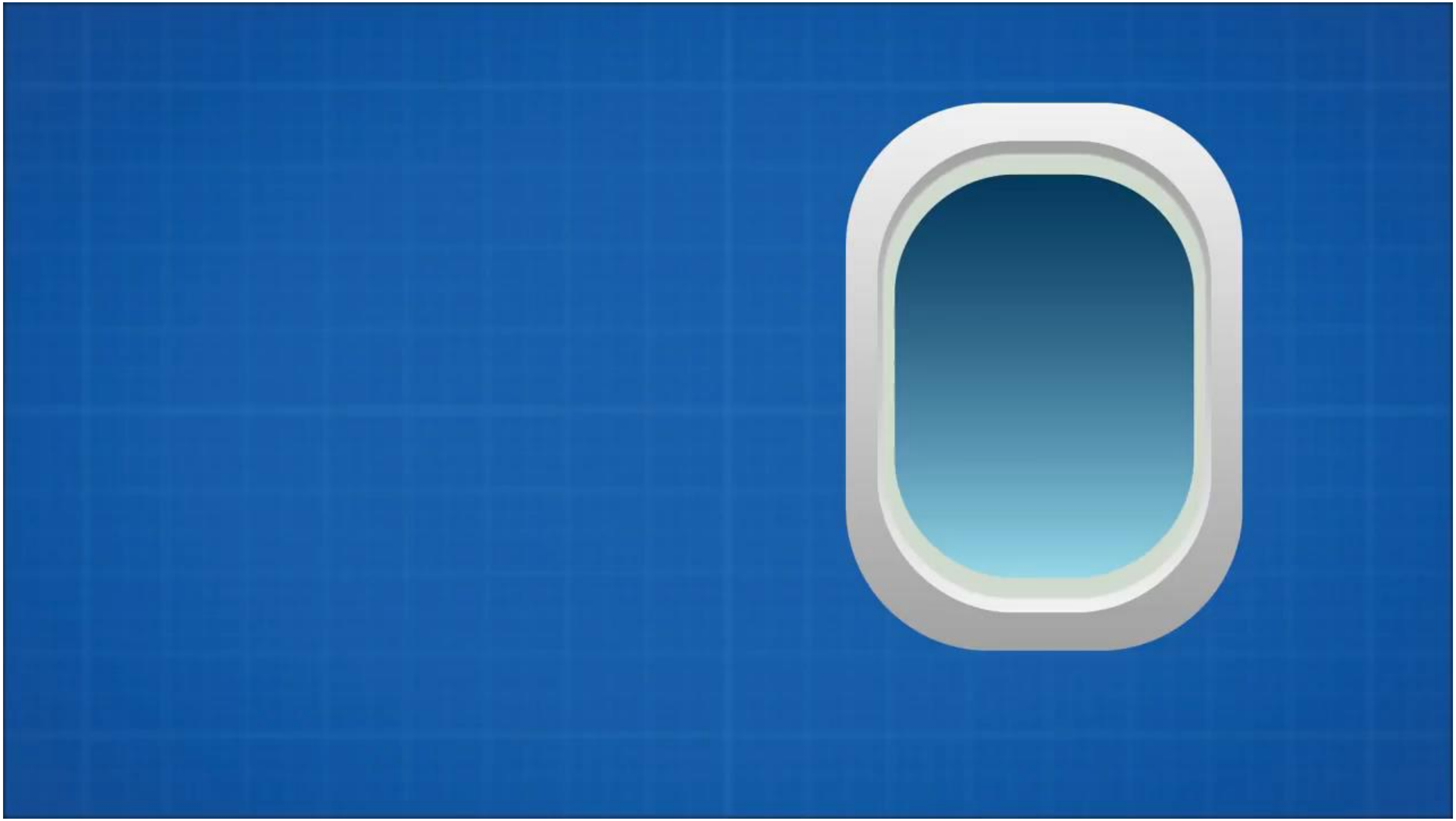
Abrupt change
Stress "flow lines" crowd
High stress concentration



Smoother change
"Flow lines" less crowded
Lower stress concentration



6-10 Stress Concentration



https://www.youtube.com/watch?v=7rXGRPMD-GQ

6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

Stress Concentration factors:

- This index at stress concentration is referred as the stress concentration factor $\mathbf{K_t}$ or $\mathbf{K_{ts}}$

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_0}$$

- The stress concentration factors $\mathbf{K_t}$ or $\mathbf{K_{ts}}$ are taken from experiments

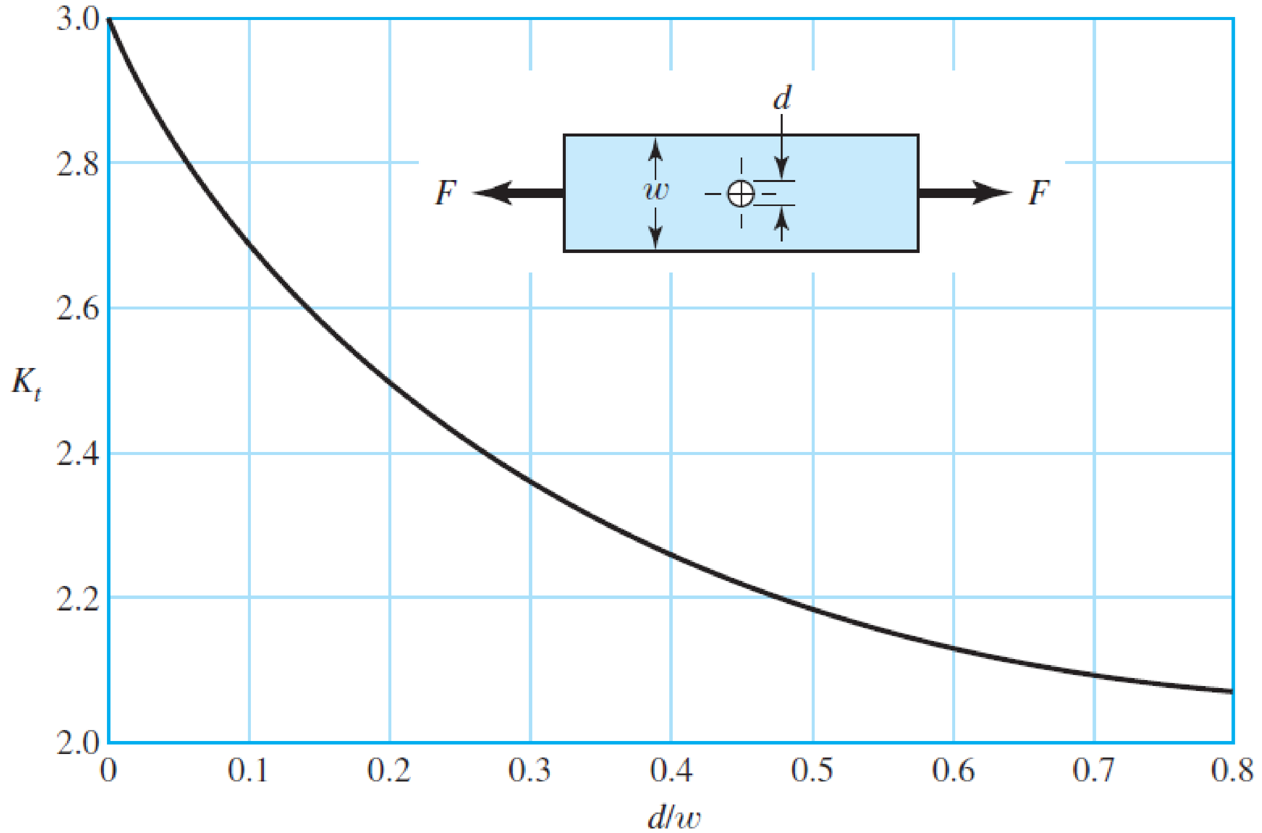
6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

Stress Concentration Factor k_t

Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.



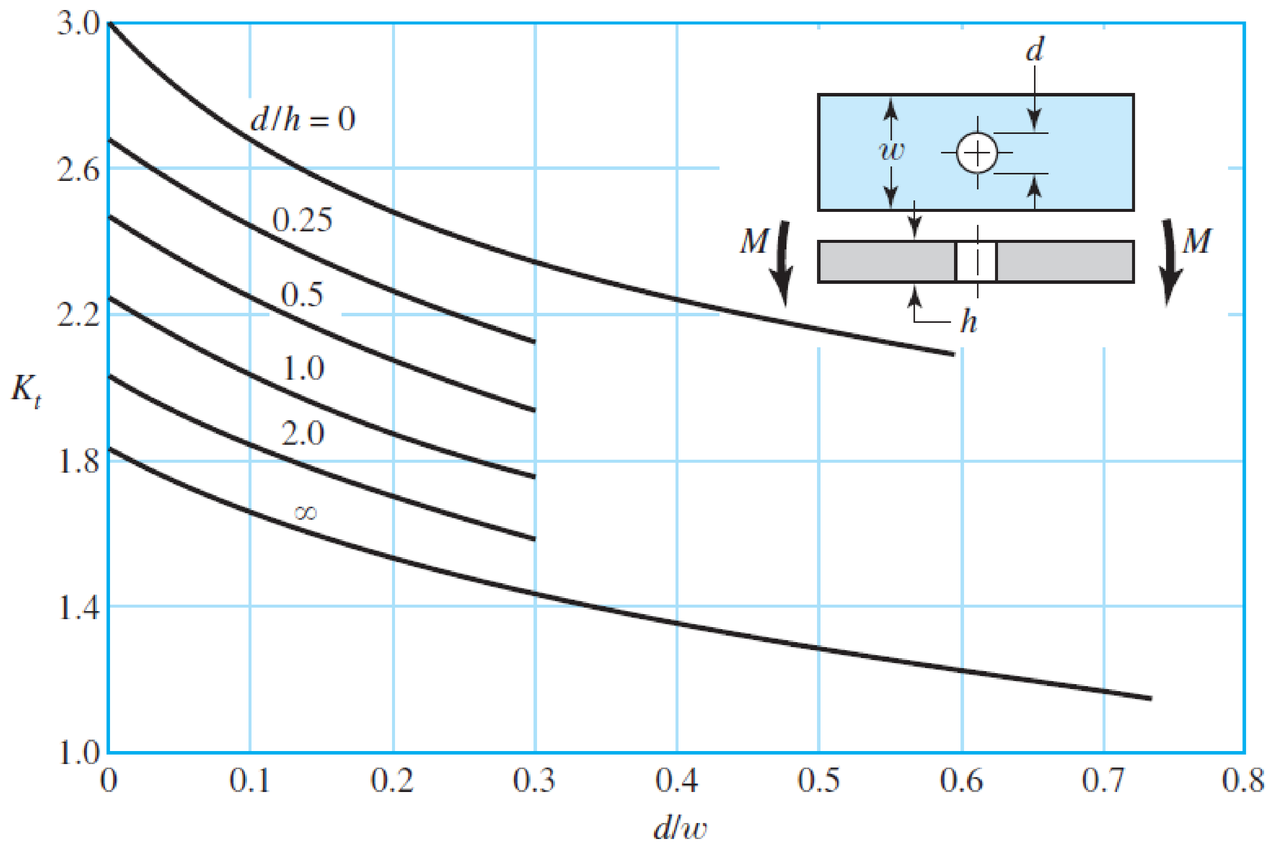
6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

Stress Concentration Factor k_t

Figure A–15–2

Rectangular bar with a transverse hole in bending. $\sigma_0 = Mc/I$, where $I = (w - d)h^3/12$.

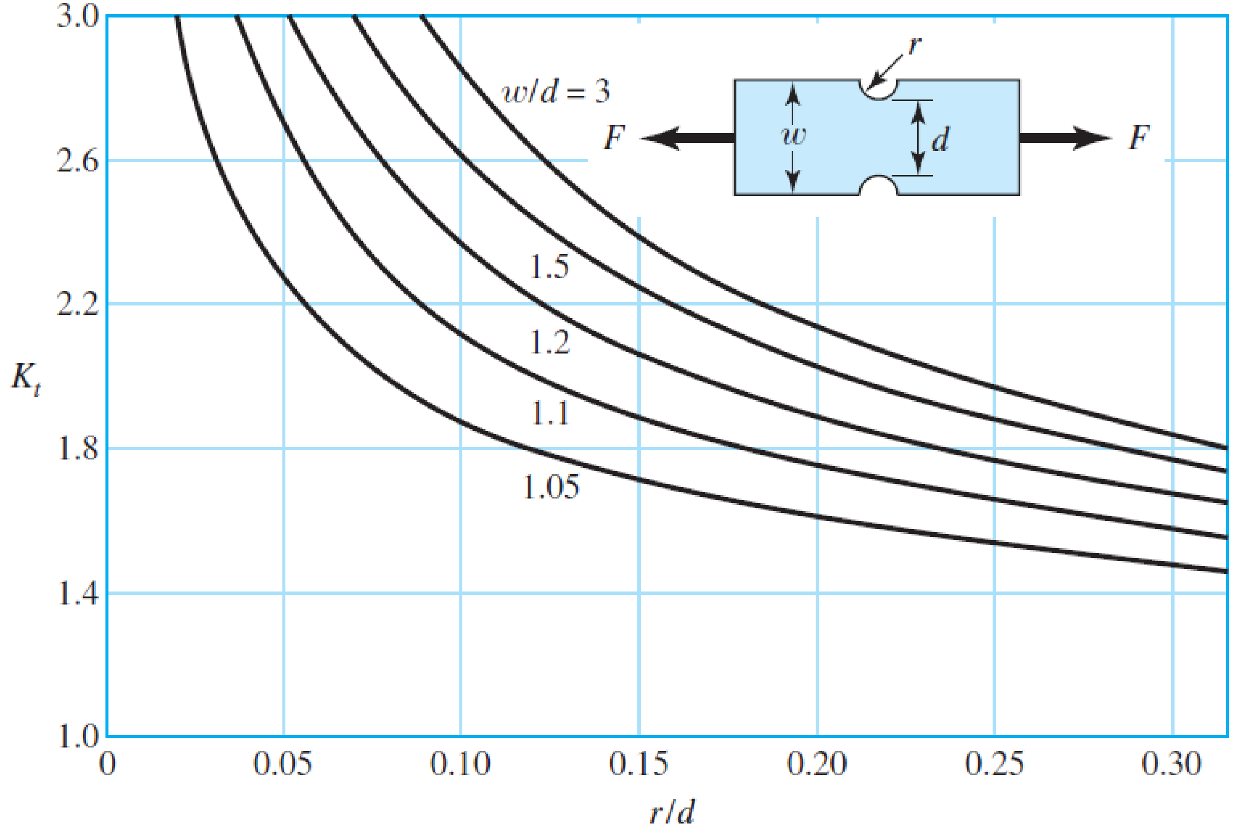


6-10 Stress Concentration and Notch Sensitivity

Stress Concentration Factor k_t

Figure A–15–3

Notched rectangular bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

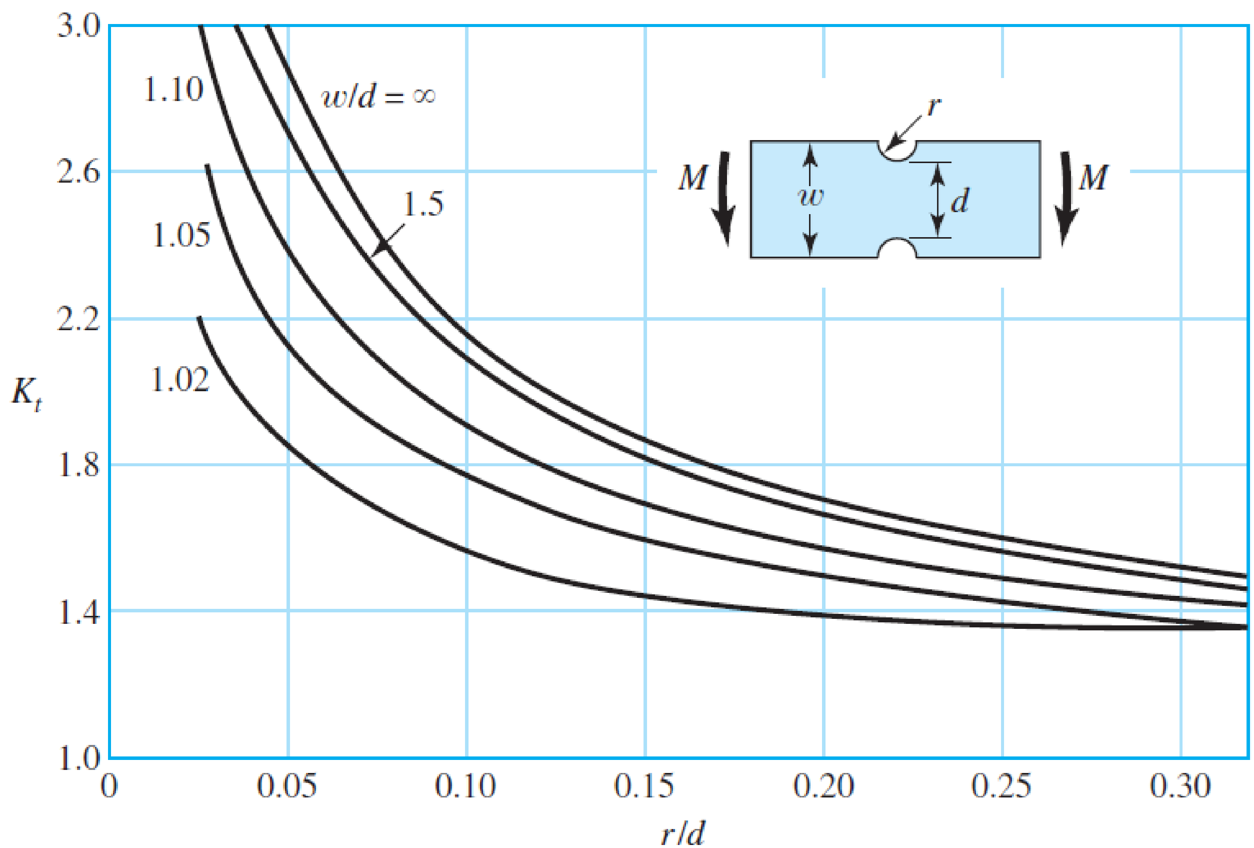


6-10 Stress Concentration and Notch Sensitivity

Stress Concentration Factor k_t

Figure A–15–4

Notched rectangular bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, and t is the thickness.

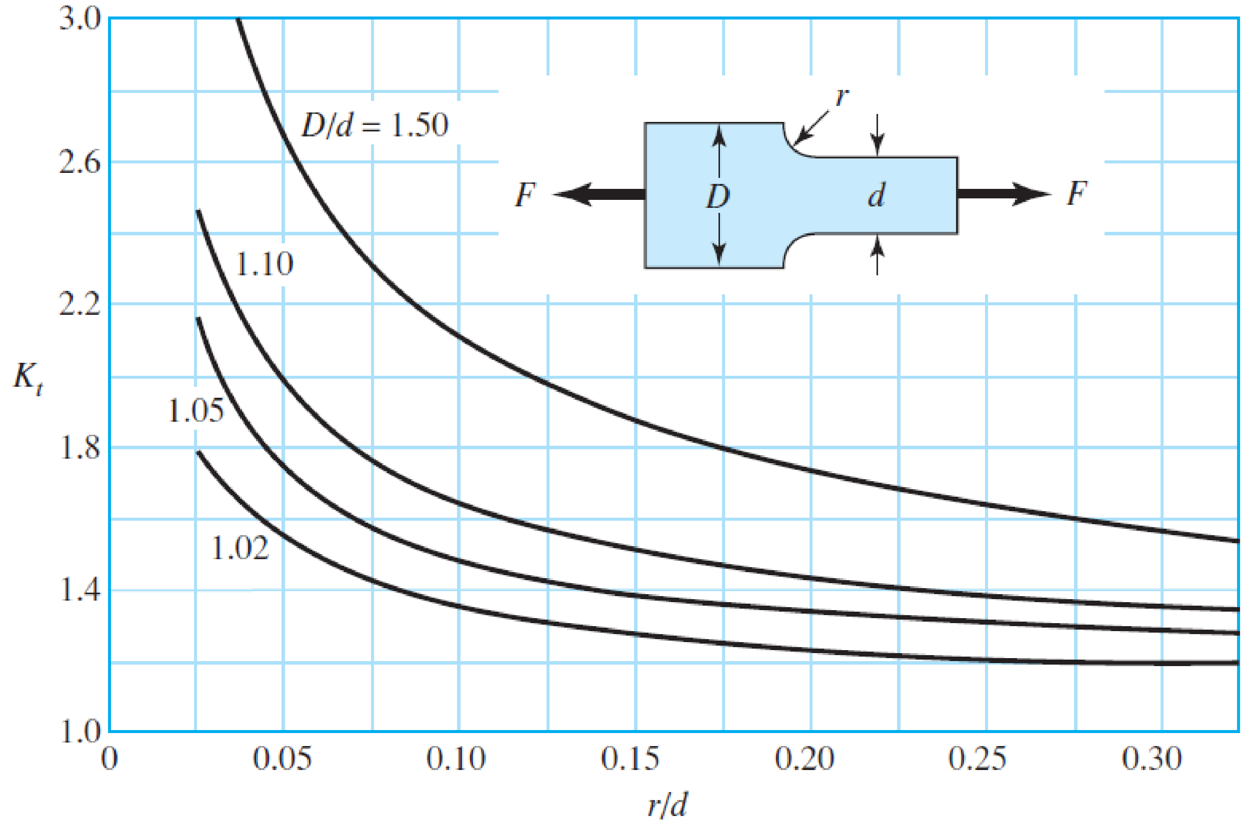


6-10 Stress Concentration and Notch Sensitivity

Stress Concentration Factor k_t

Figure A–15–5

Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

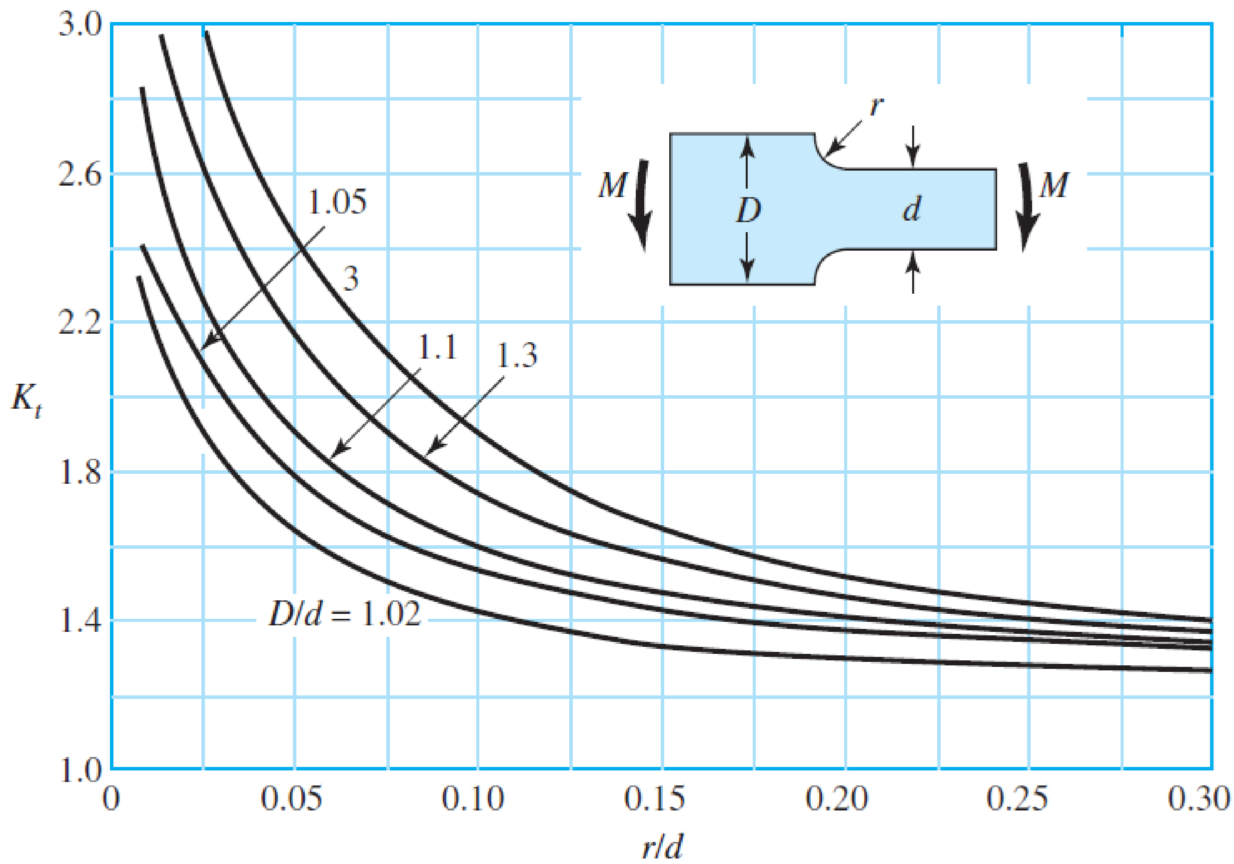


6-10 Stress Concentration and Notch Sensitivity

Stress Concentration Factor k_t

Figure A–15–6

Rectangular filleted bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, t is the thickness.

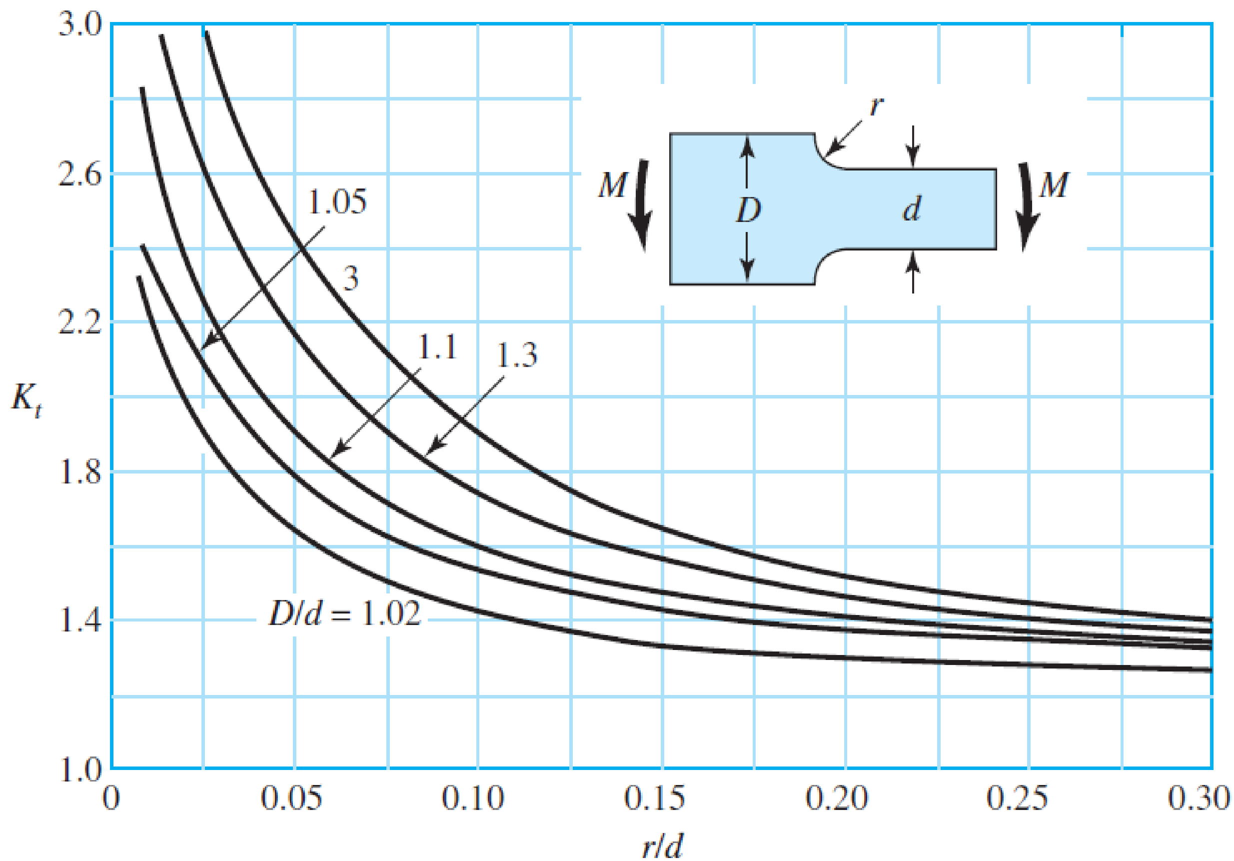


6-10 Stress Concentration and Notch Sensitivity

Stress Concentration Factor k_t

Figure A–15–6

Rectangular filleted bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, t is the thickness.



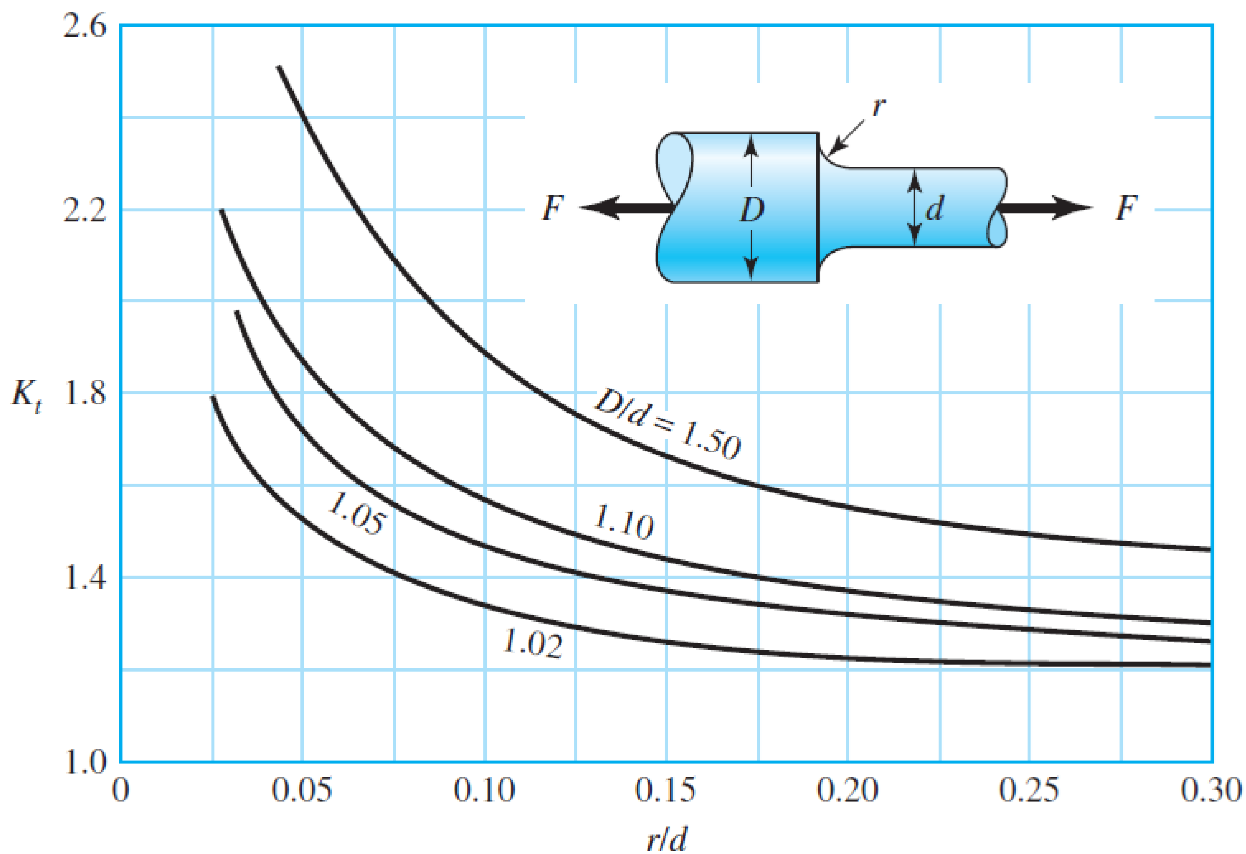
6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

Stress Concentration Factor k_t

Figure A–15–7

Round shaft with shoulder fillet in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.



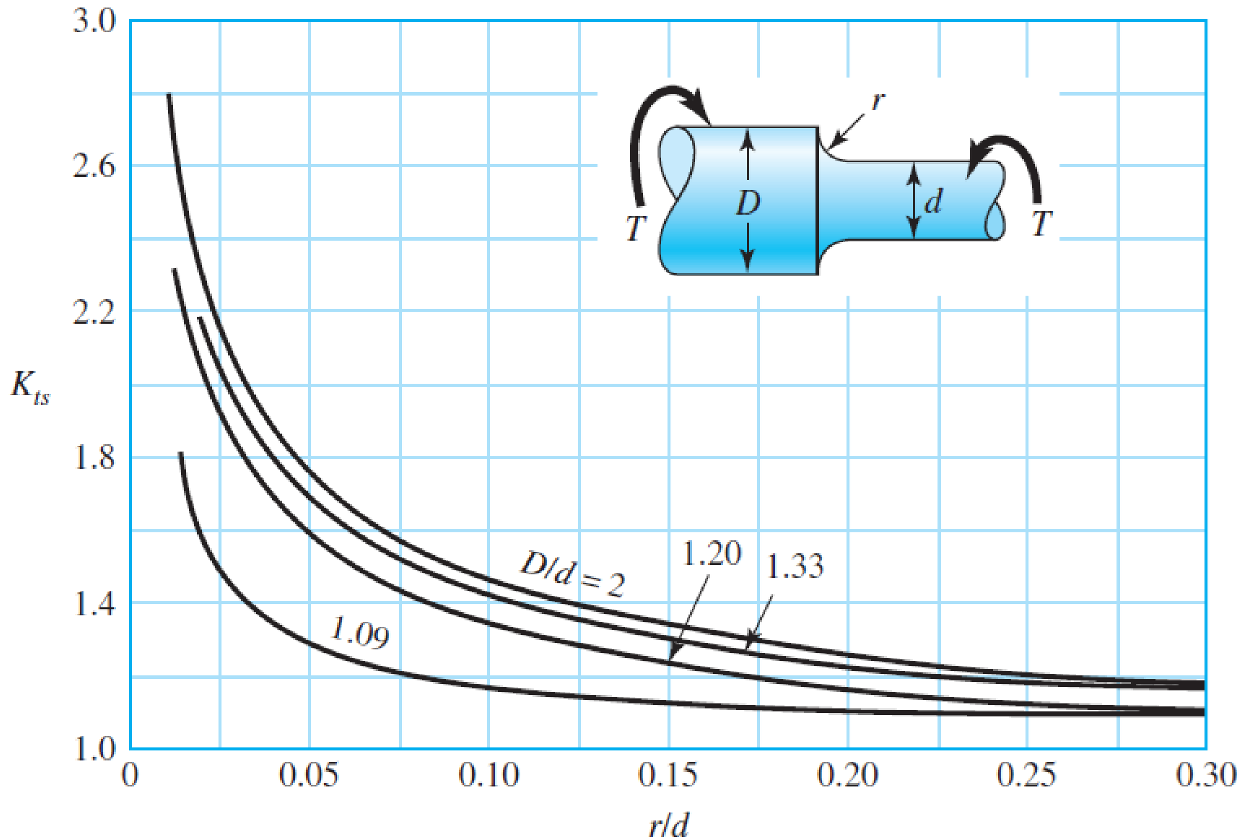
6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

Stress Concentration Factor k_{ts}

Figure A–15–8

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

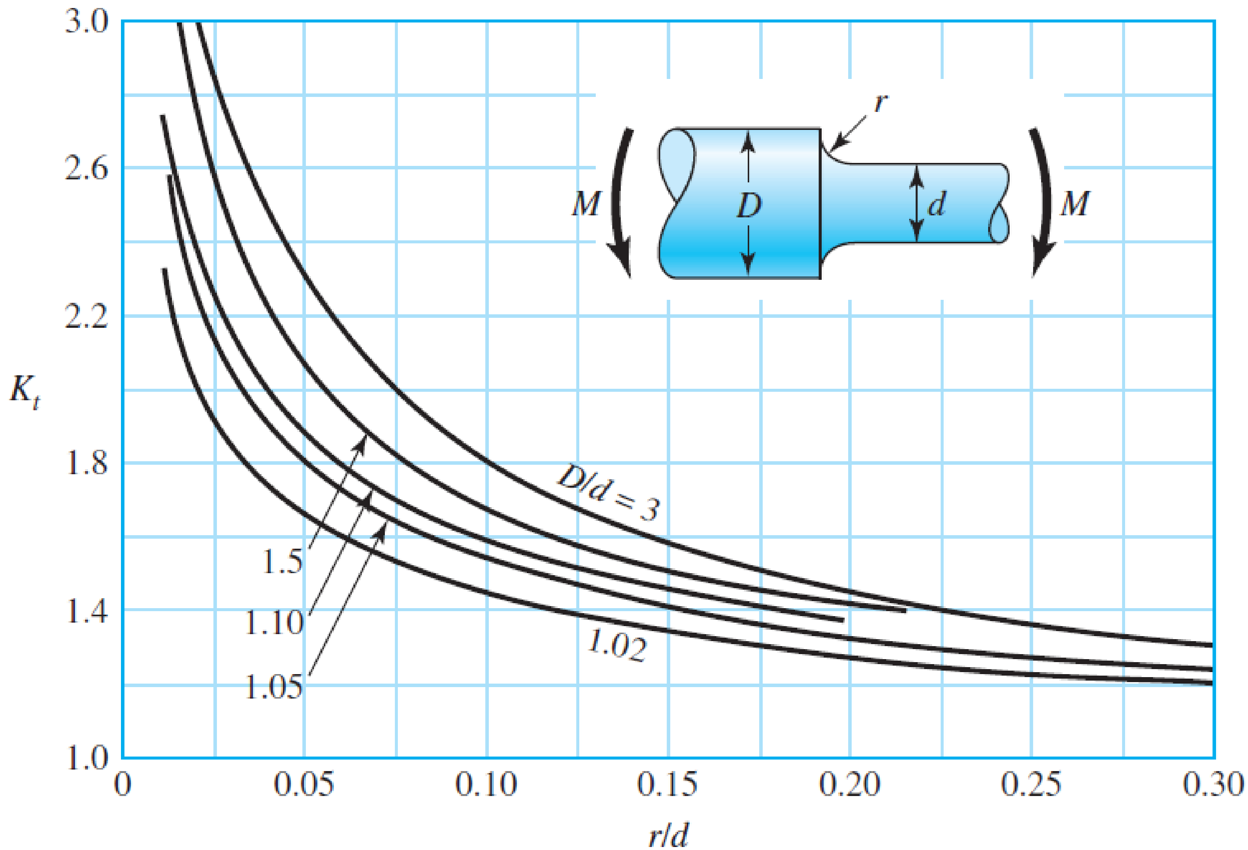


6-10 Stress Concentration and Notch Sensitivity

Stress Concentration Factor k_t

Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



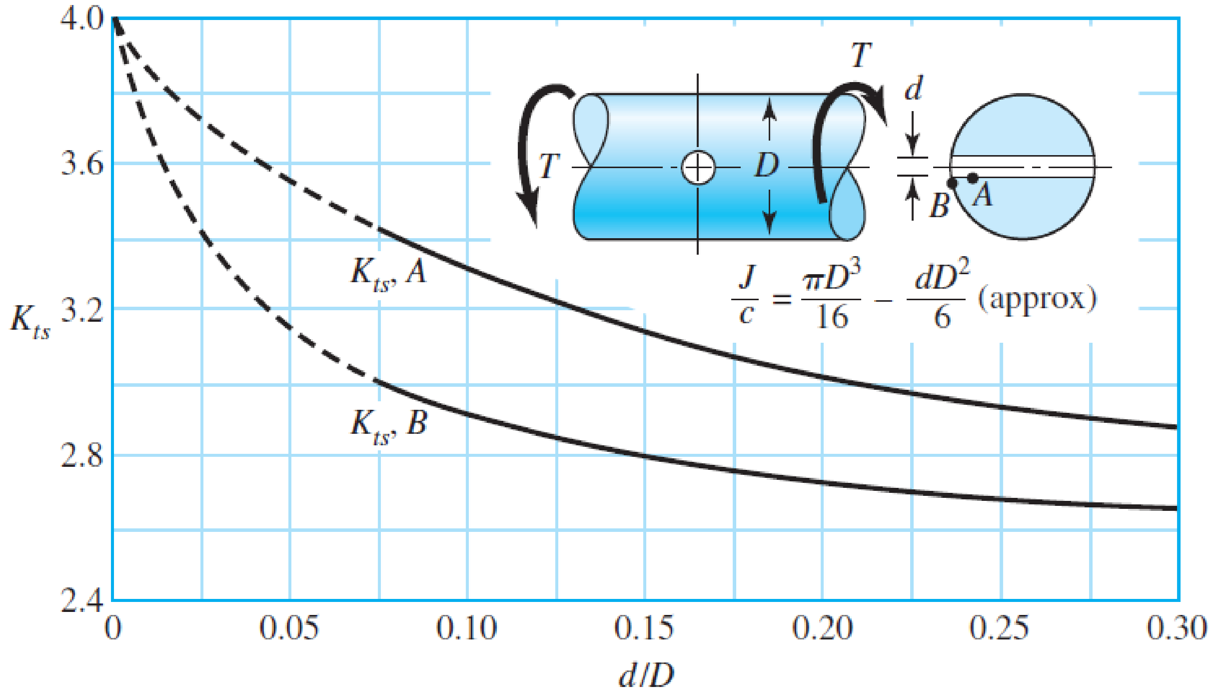
6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

Stress Concentration Factor k_{ts}

Figure A–15–10

Round shaft in torsion with transverse hole.

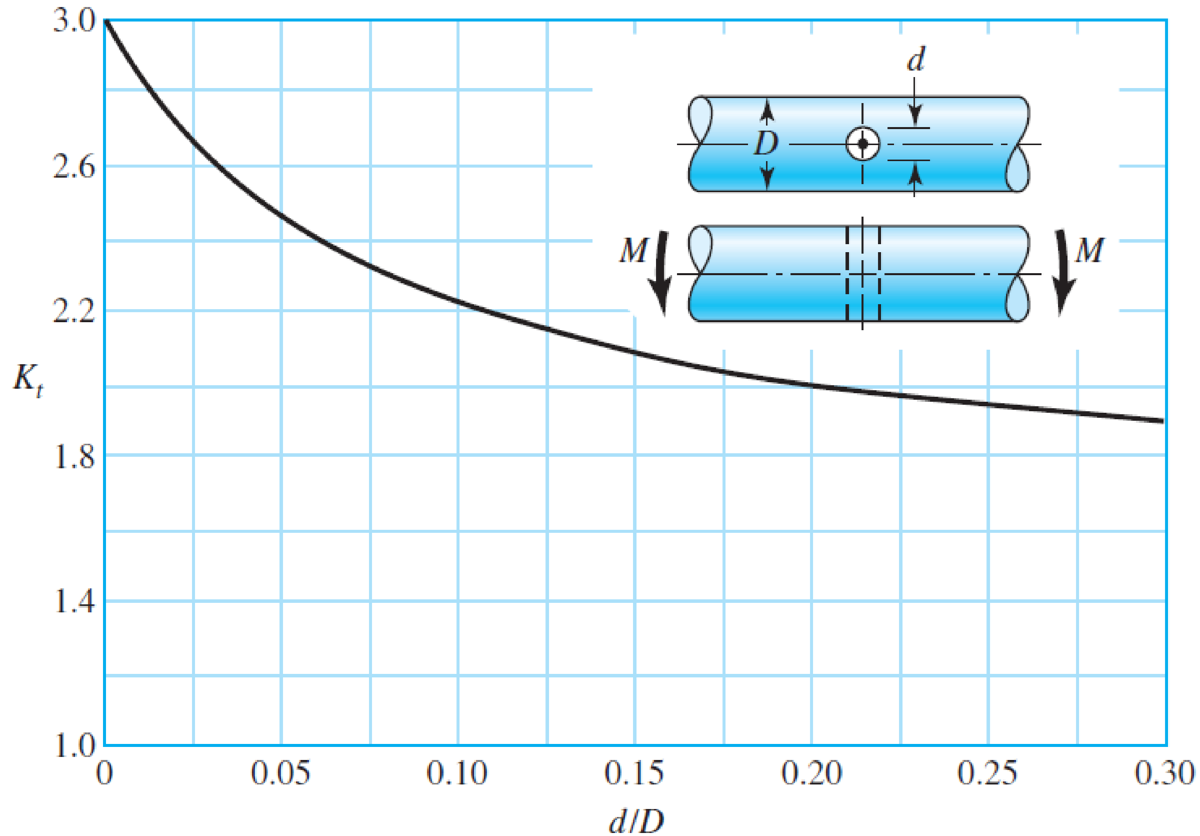


6-10 Stress Concentration and Notch Sensitivity

Stress Concentration Factor k_t

Figure A–15–11

Round shaft in bending with a transverse hole. $\sigma_0 = M/[(\pi D^3/32) - (dD^2/6)]$, approximately.

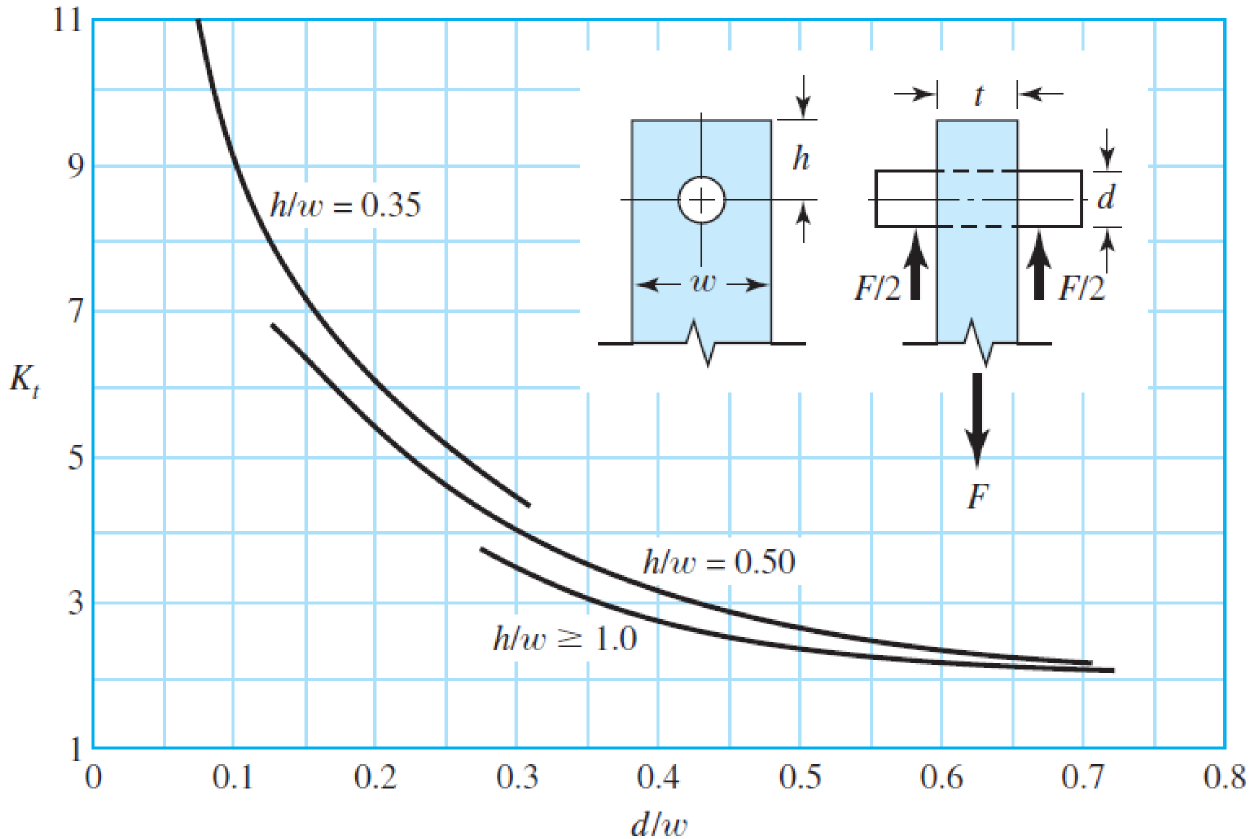


6-10 Stress Concentration and Notch Sensitivity

Stress Concentration Factor k_t

Figure A–15–12

Plate loaded in tension by a pin through a hole. $\sigma_0 = F/A$, where $A = (w - d)t$. When clearance exists, increase K_t 35 to 50 percent. *(M. M. Frocht and H. N. Hill, "Stress-Concentration Factors around a Central Circular Hole in a Plate Loaded through a Pin in Hole," J. Appl. Mechanics, vol. 7, no. 1, March 1940, p. A-5.)*



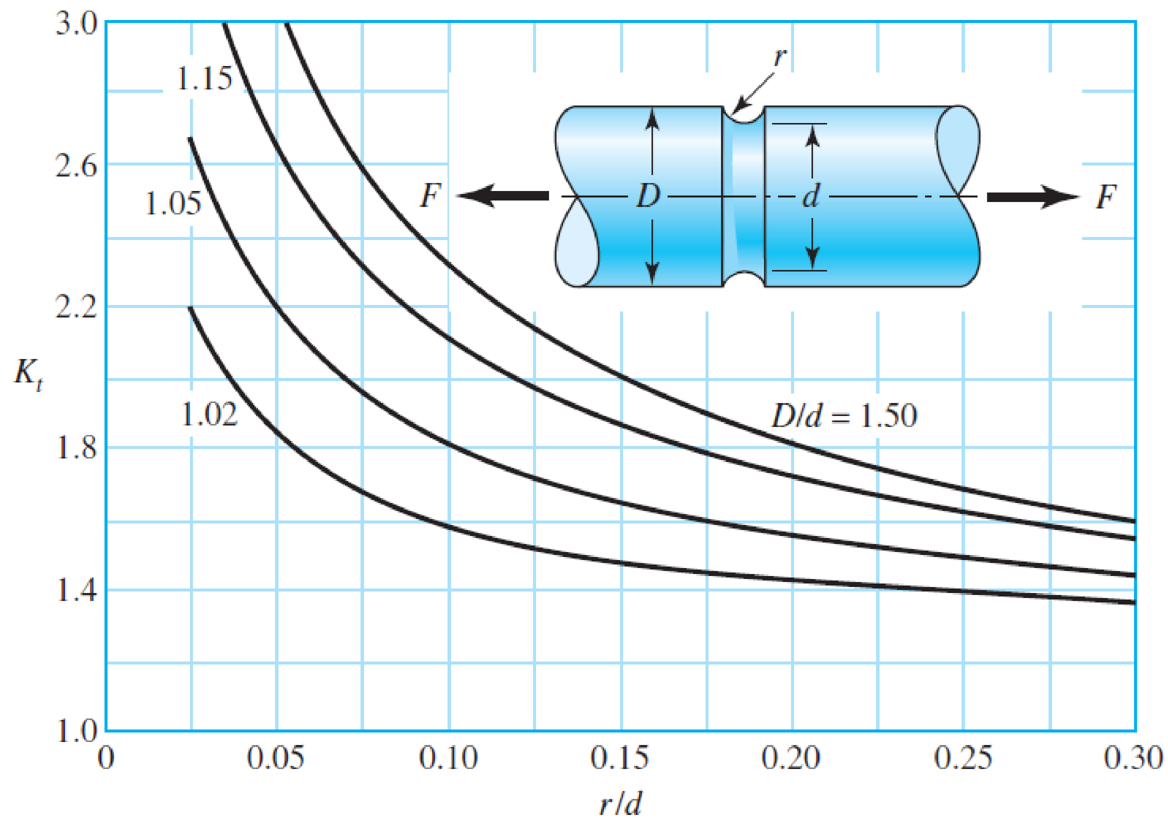
6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

Stress Concentration Factor k_t

Figure A–15–13

Grooved round bar in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

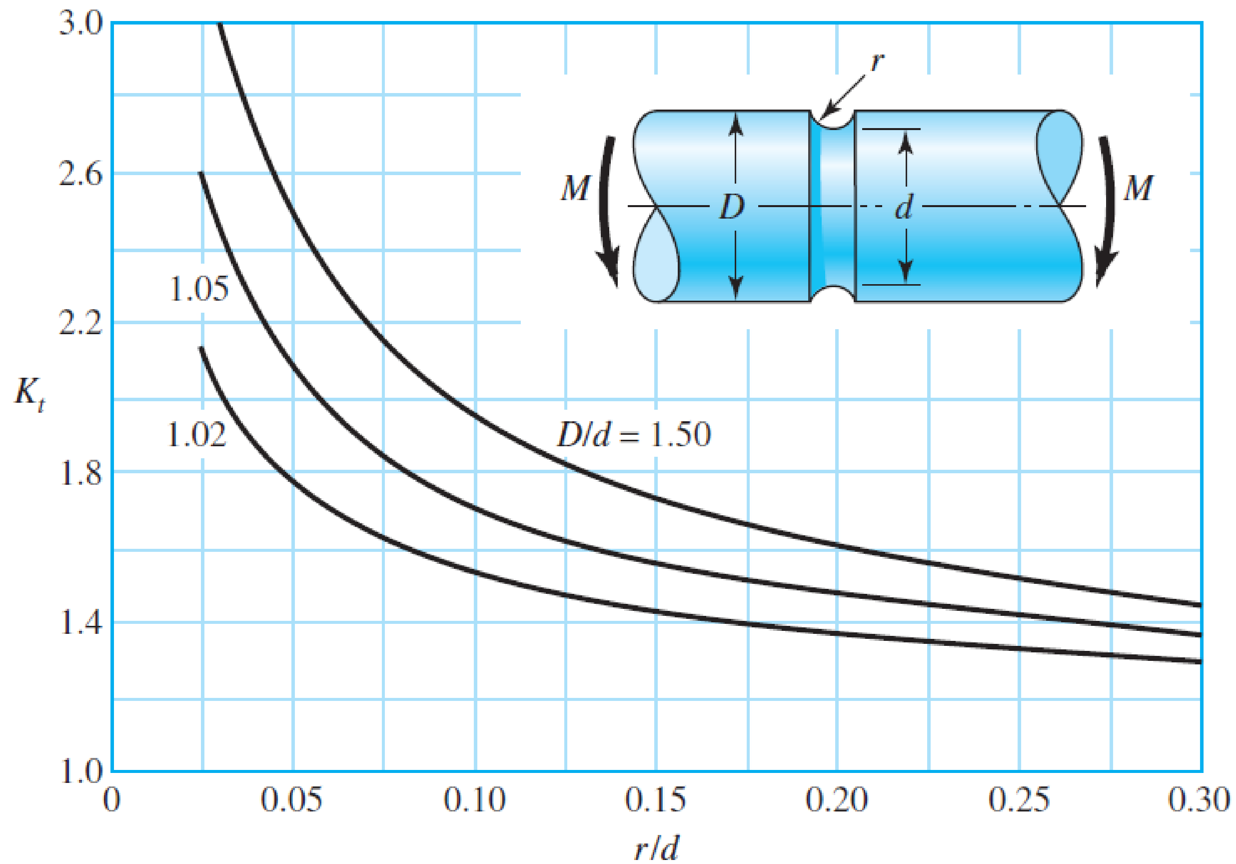


6-10 Stress Concentration and Notch Sensitivity

Stress Concentration Factor k_t

Figure A–15–14

Grooved round bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

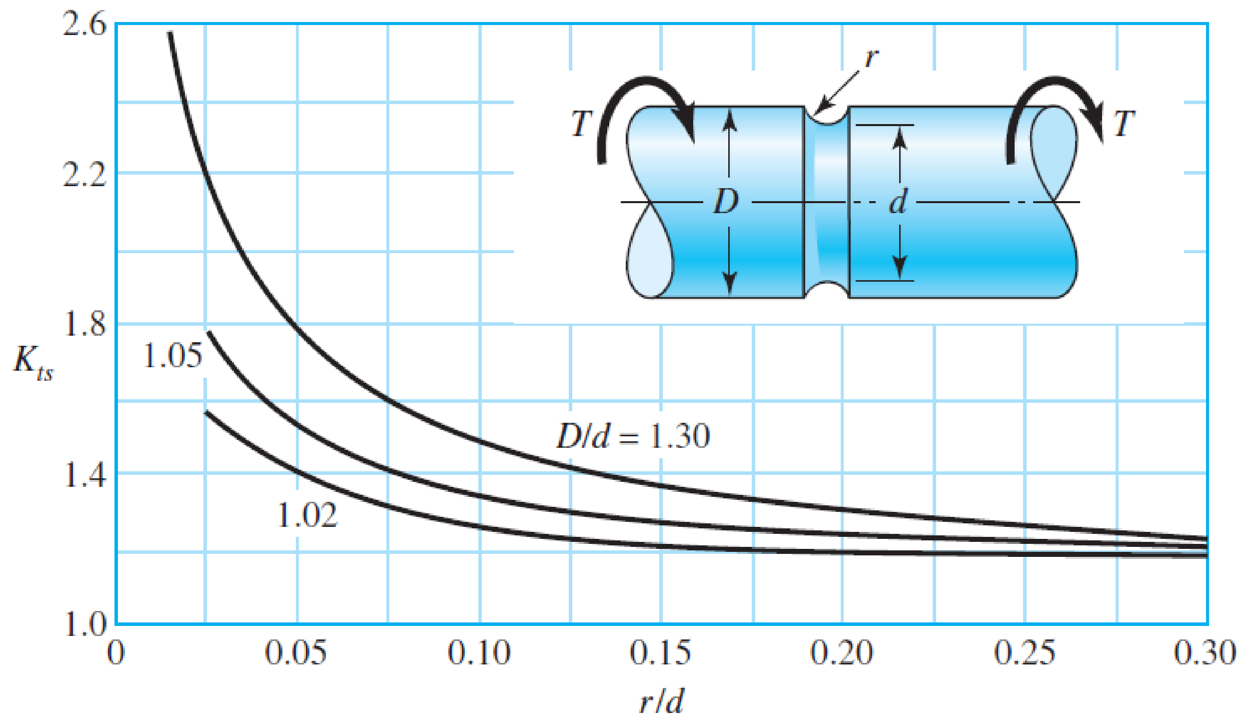


6-10 Stress Concentration and Notch Sensitivity

Stress Concentration Factor k_{ts}

Figure A–15–15

Grooved round bar in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



6-10 Stress Concentration and Notch Sensitivity

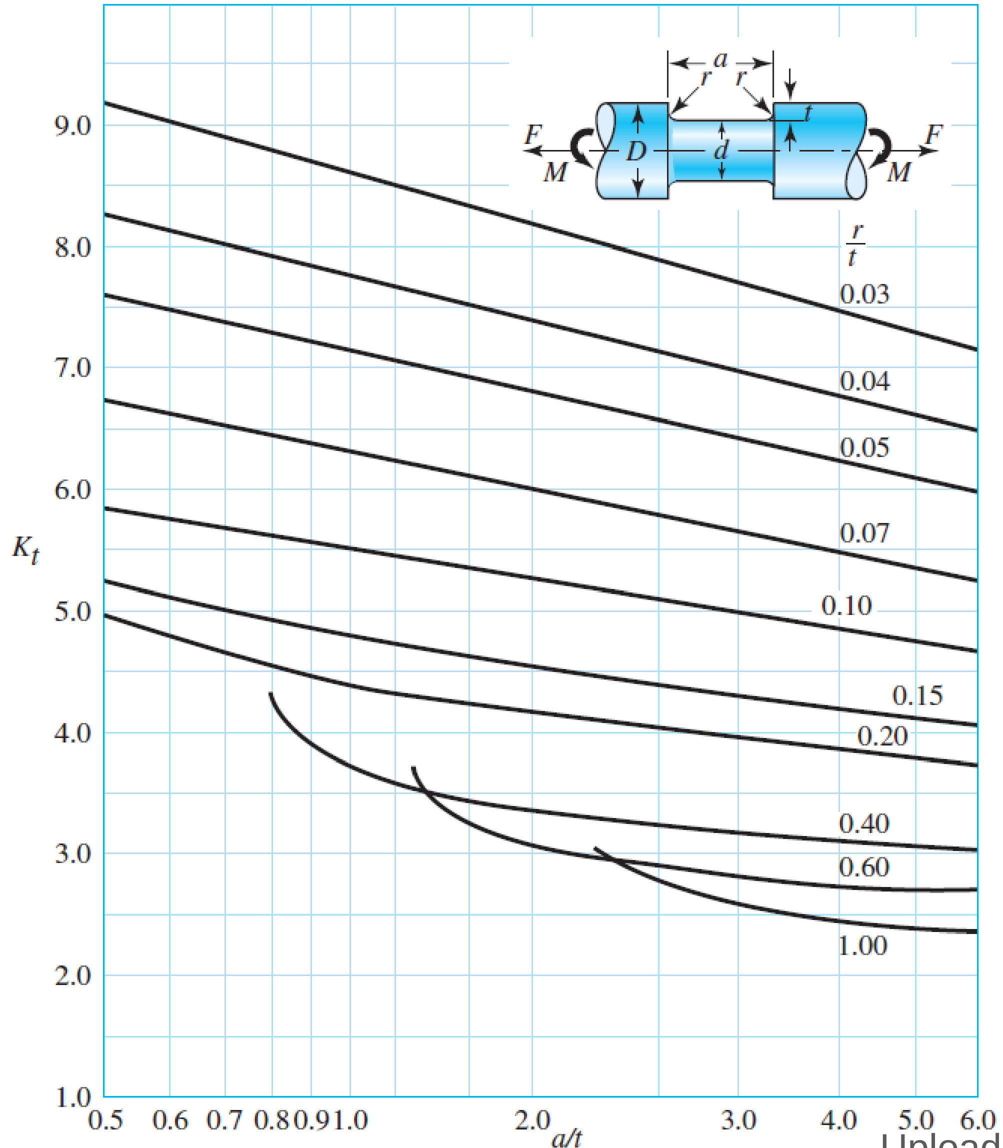
Stress Concentration Factor k_t

Figure A–15–16

Round shaft with flat-bottom groove in bending and/or tension.

$$\sigma_0 = \frac{4F}{\pi d^2} + \frac{32M}{\pi d^3}$$

Source: W. D. Pilkey and D. F. Pilkey, *Peterson's Stress-Concentration Factors,* 3rd ed. John Wiley & Sons, Hoboken, NJ, 2008, p. 115.



6-10 Stress Concentration and Notch Sensitivity

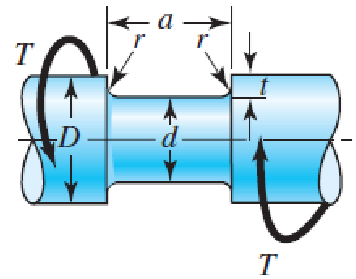
Stress Concentration Factor k_{ts}

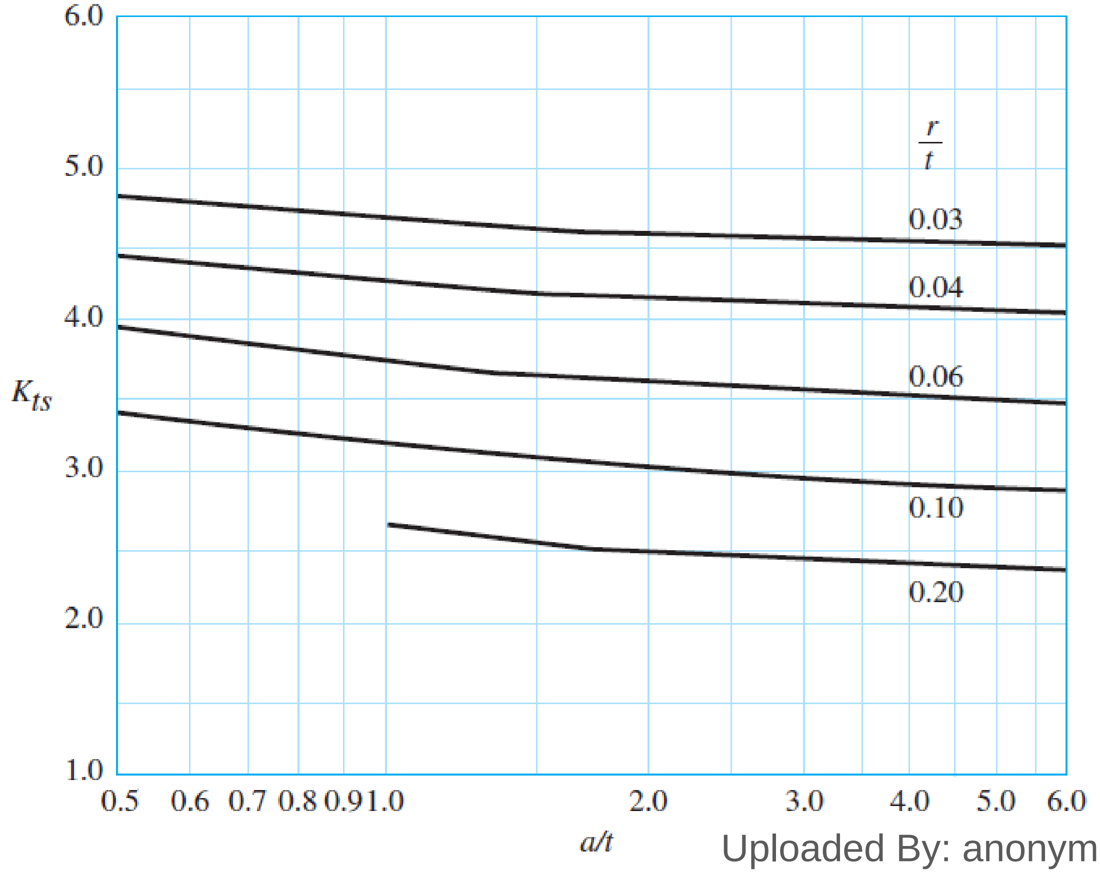
Figure A–15–17

Round shaft with flat-bottom groove in torsion.

$$\tau_0 = \frac{16T}{\pi d^3}$$

Source: W. D. Pilkey and D. F. Pilkey, *Peterson's Stress-Concentration Factors,* 3rd ed. John Wiley & Sons, Hoboken, NJ, 2008, p. 133





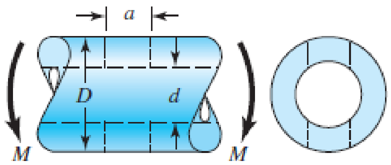
6-10 Stress Concentration and Notch Sensitivity

Stress Concentration Factor k_t

Table A-16

Approximate Stress-Concentration Factor K_t of a Round Bar or Tube with a Transverse Round Hole and Loaded in Bending

Source: R. E. Peterson, *Stress-Concentration Factors,* Wiley, New York, 1974, pp. 146, 235.



The nominal bending stress is $\sigma_0 = M/Z_{\text{net}}$ where Z_{net} is a reduced value of the section modulus and is defined by

$$Z_{\text{net}} = \frac{\pi A}{32D}(D^4 - d^4)$$

Values of A are listed in the table. Use $d = 0$ for a solid bar

	d/D					
	0.9		0.6		0	
a/D	A	K_t	A	K_t	A	K_t
0.050	0.92	2.63	0.91	2.55	0.88	2.42
0.075	0.89	2.55	0.88	2.43	0.86	2.35
0.10	0.86	2.49	0.85	2.36	0.83	2.27
0.125	0.82	2.41	0.82	2.32	0.80	2.20
0.15	0.79	2.39	0.79	2.29	0.76	2.15
0.175	0.76	2.38	0.75	2.26	0.72	2.10
0.20	0.73	2.39	0.72	2.23	0.68	2.07
0.225	0.69	2.40	0.68	2.21	0.65	2.04
0.25	0.67	2.42	0.64	2.18	0.61	2.00
0.275	0.66	2.48	0.61	2.16	0.58	1.97
0.30	0.64	2.52	0.58	2.14	0.54	1.94

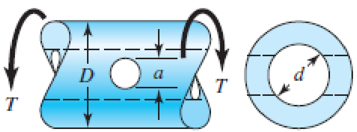
6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

Stress Concentration Factor k_{ts}

Table A–16 *(Continued)*

Approximate Stress-Concentration Factors K_{ts} for a Round Bar or Tube Having a Transverse Round Hole and Loaded in Torsion *Source:* R. E. Peterson, *Stress-Concentration Factors,* Wiley, New York, 1974, pp. 148, 244.



The maximum stress occurs on the inside of the hole, slightly below the shaft surface. The nominal shear stress is $\tau_0 = TD/2J_{net}$, where J_{net} is a reduced value of the second polar moment of area and is defined by

$$J_{net} = \frac{\pi A(D^4 - d^4)}{32}$$

Values of A are listed in the table. Use $d = 0$ for a solid bar.

	d/D									
	0.9		0.8		0.6		0.4		0	
a/D	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}
0.05	0.96	1.78							0.95	1.77
0.075	0.95	1.82							0.93	1.71
0.10	0.94	1.76	0.93	1.74	0.92	1.72	0.92	1.70	0.92	1.68
0.125	0.91	1.76	0.91	1.74	0.90	1.70	0.90	1.67	0.89	1.64
0.15	0.90	1.77	0.89	1.75	0.87	1.69	0.87	1.65	0.87	1.62
0.175	0.89	1.81	0.88	1.76	0.87	1.69	0.86	1.64	0.85	1.60
0.20	0.88	1.96	0.86	1.79	0.85	1.70	0.84	1.63	0.83	1.58
0.25	0.87	2.00	0.82	1.86	0.81	1.72	0.80	1.63	0.79	1.54
0.30	0.80	2.18	0.78	1.97	0.77	1.76	0.75	1.63	0.74	1.51
0.35	0.77	2.41	0.75	2.09	0.72	1.81	0.69	1.63	0.68	1.47
0.40	0.72	2.67	0.71	2.25	0.68	1.89	0.64	1.63	0.63	1.44

6-10 Stress Concentration and Notch Sensitivity

Stress Concentration factors:

- This index at stress concentration is referred as the stress concentration factor **K_t** or **K_{ts}**

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_0}$$

- The stress concentration factors ***K_t*** or ***K_{ts}*** are taken from experiments
- The Fatigue stress concentration factors ***K_f*** or ***K_{fs}***

$$K_f = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch-free specimen}}$$

Notch sensitivity q is defined by the equation

$$q = \frac{K_f - 1}{K_t - 1} \qquad \text{or} \qquad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1}$$

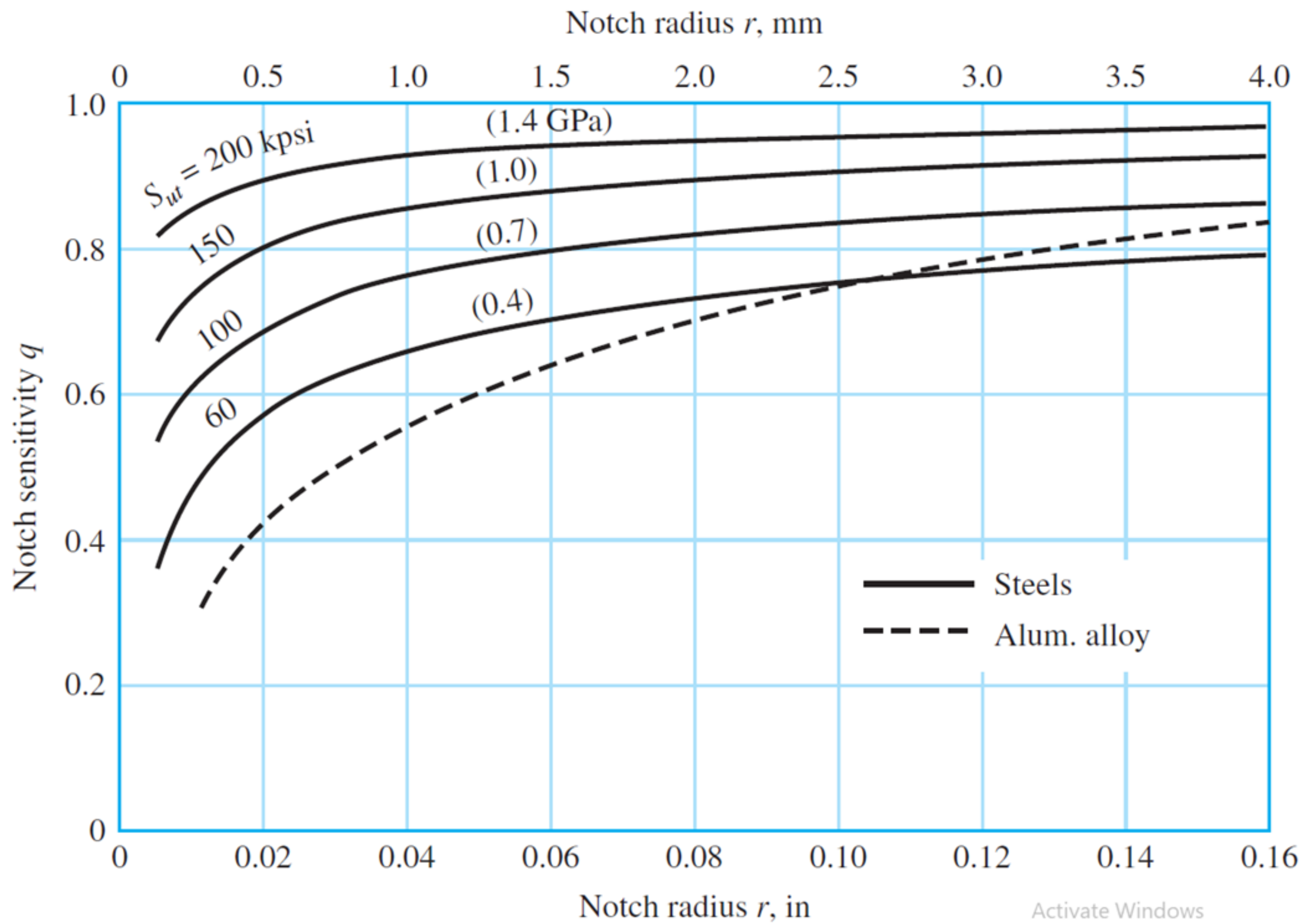
6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

Notch Sensitivity q

Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the r = 0.16-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*

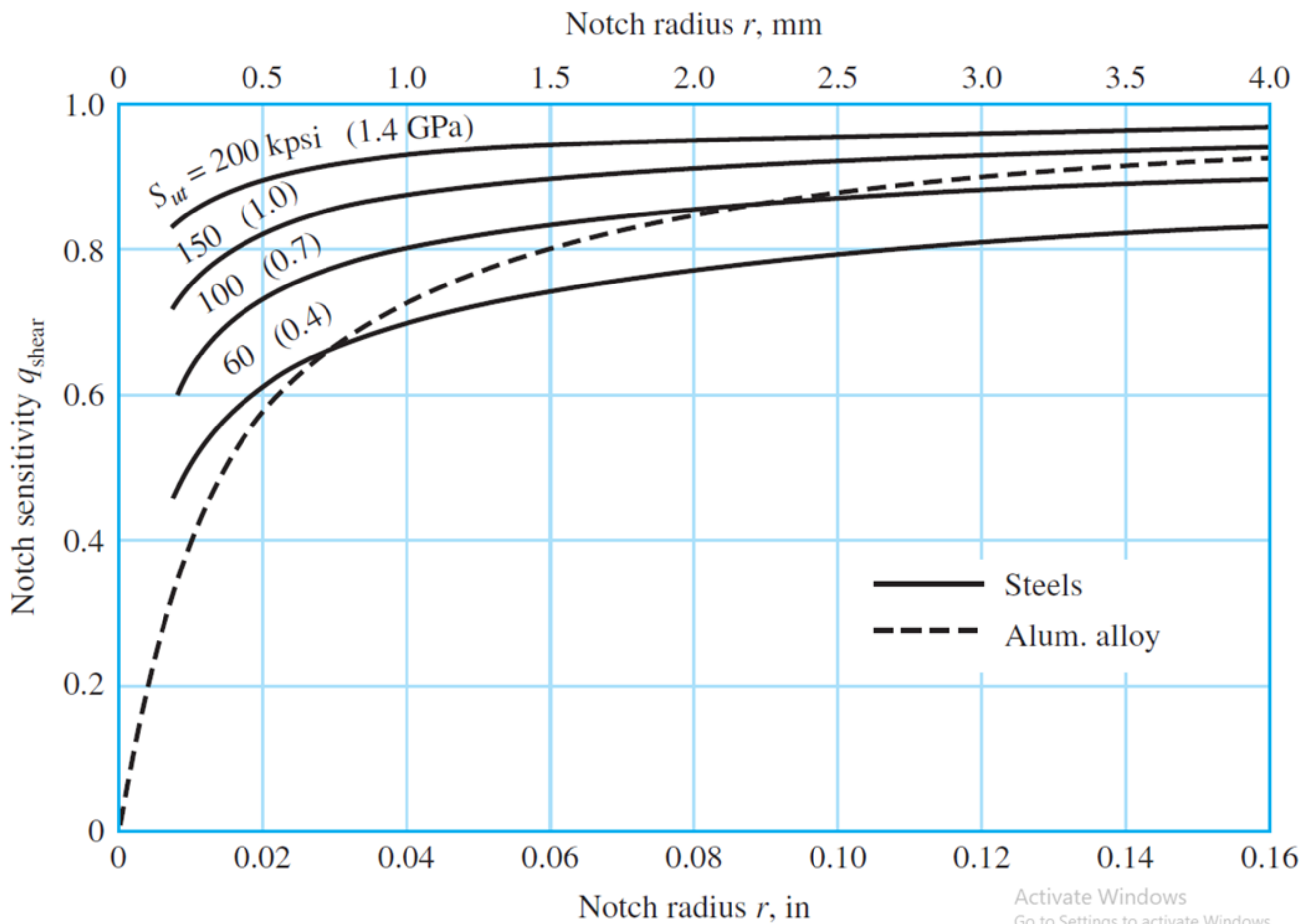


6-10 Stress Concentration and Notch Sensitivity

Notch Sensitivity q_s

Figure 6–21

Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of q_{shear} corresponding to $r = 0.16$ in (4 mm).



6-10 Stress Concentration and Notch Sensitivity

Notch Sensitivity

Figure 6–20 has as its basis the *Neuber equation*, which is given by

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \tag{6–33}$$

where $\sqrt{a}$ is defined as the *Neuber constant* and is a material constant. Equating Eqs. (6–31) and (6–33) yields the notch sensitivity equation

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} \tag{6–34}$$

correlating with Figs. 6–20 and 6–21 as

Bending or axial: $$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \tag{6–35a}$$

Torsion: $$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \tag{6–35b}$$

6-10 Stress Concentration and Notch Sensitivity

EXAMPLE 6–6 A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:
(*a*) Figure 6–20.
(*b*) Equations (6–33) and (6–35).

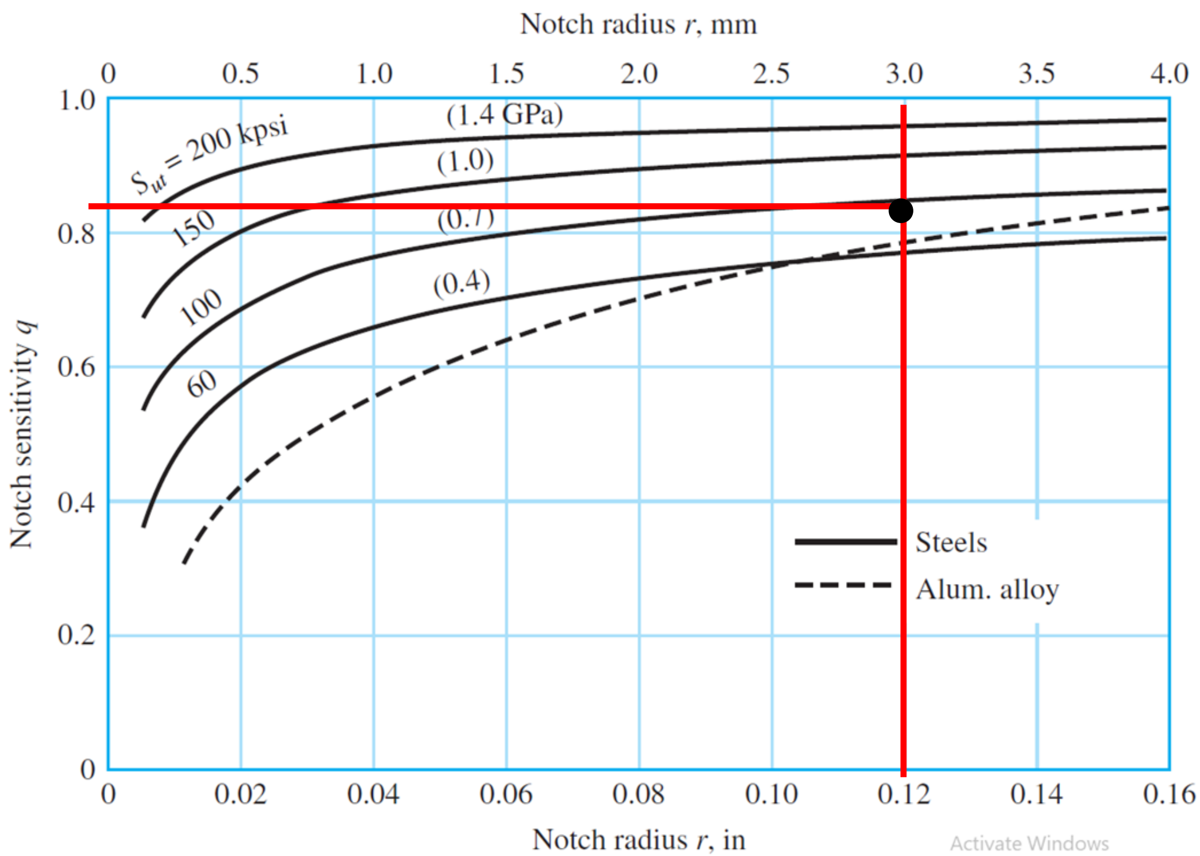
$$K_f = 1 + q(K_t - 1) \qquad \text{or} \qquad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1)$$

6-10 Stress Concentration and Notch Sensitivity

EXAMPLE 6–6 A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:
(*a*) Figure 6–20.
(*b*) Equations (6–33) and (6–35).

Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the r = 0.16-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*



6-10 Stress Concentration and Notch Sensitivity

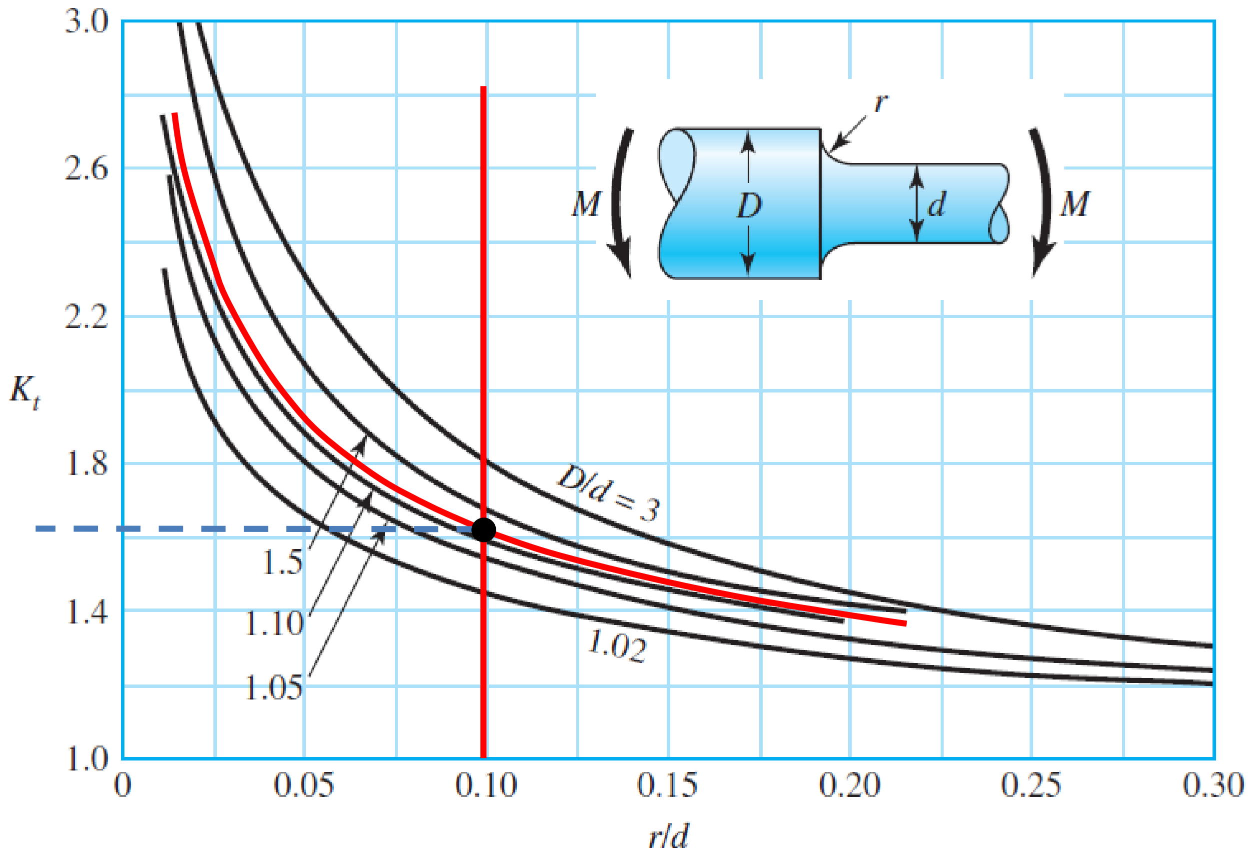
EXAMPLE 6–6 A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:

(*a*) Figure 6–20.

(*b*) Equations (6–33) and (6–35).

Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



$$K_f = 1 + q(K_t - 1)$$

6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

EXAMPLE 6–6 A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:
(*a*) Figure 6–20.
(*b*) Equations (6–33) and (6–35).

Bending or axial: $$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \qquad (6\text{–}35a)$$

S_{ut} is in kpsi.

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \qquad (6\text{–}33)$$

6-10 Stress Concentration and Notch Sensitivity

EXAMPLE 6–6 A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:
(a) Figure 6–20.
(b) Equations (6–33) and (6–35).

EXAMPLE 6–7 For the step-shaft of Ex. 6–6, it is determined that the fully corrected endurance limit is $S_e = 280$ MPa. Consider the shaft undergoes a fully reversing nominal stress in the fillet of $(\sigma_{\text{rev}})_{\text{nom}} = 260$ MPa. Estimate the number of cycles to failure.

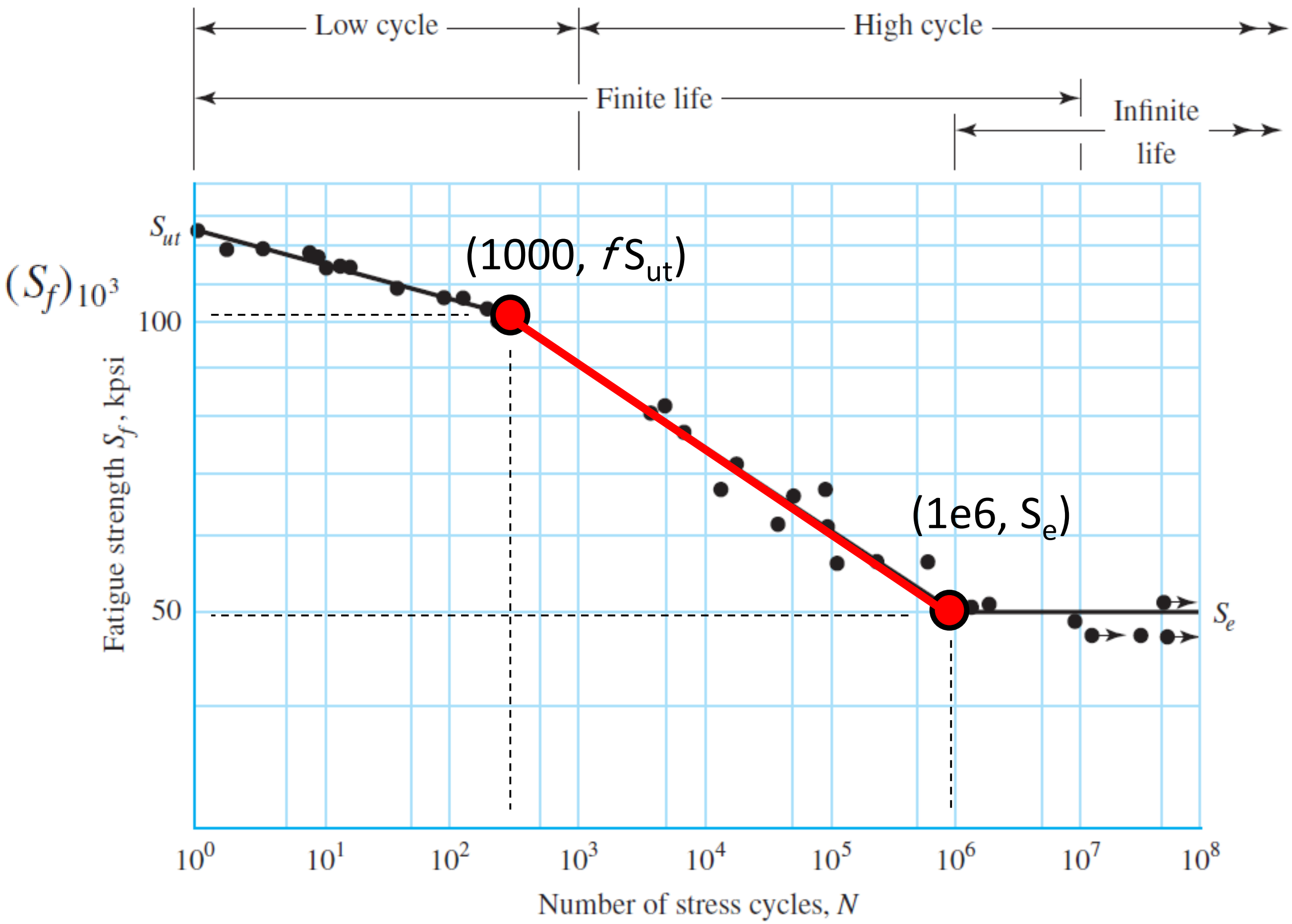
$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b}$$

$$(\sigma_{\text{rev}})_{\text{max}} = K_f(\sigma_{\text{rev}})_{\text{nom}} = 1.55(260) = 403 \text{ MPa}$$

6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

EXAMPLE 6–7 For the step-shaft of Ex. 6–6, it is determined that the fully corrected endurance limit is $S_e = 280$ MPa. Consider the shaft undergoes a fully reversing nominal stress in the fillet of $(\sigma_{rev})_{nom} = 260$ MPa. Estimate the number of cycles to failure.



$$(S_f)_{10^3} = f\,S_{ut}.$$

$$S_f = a\,N^b$$

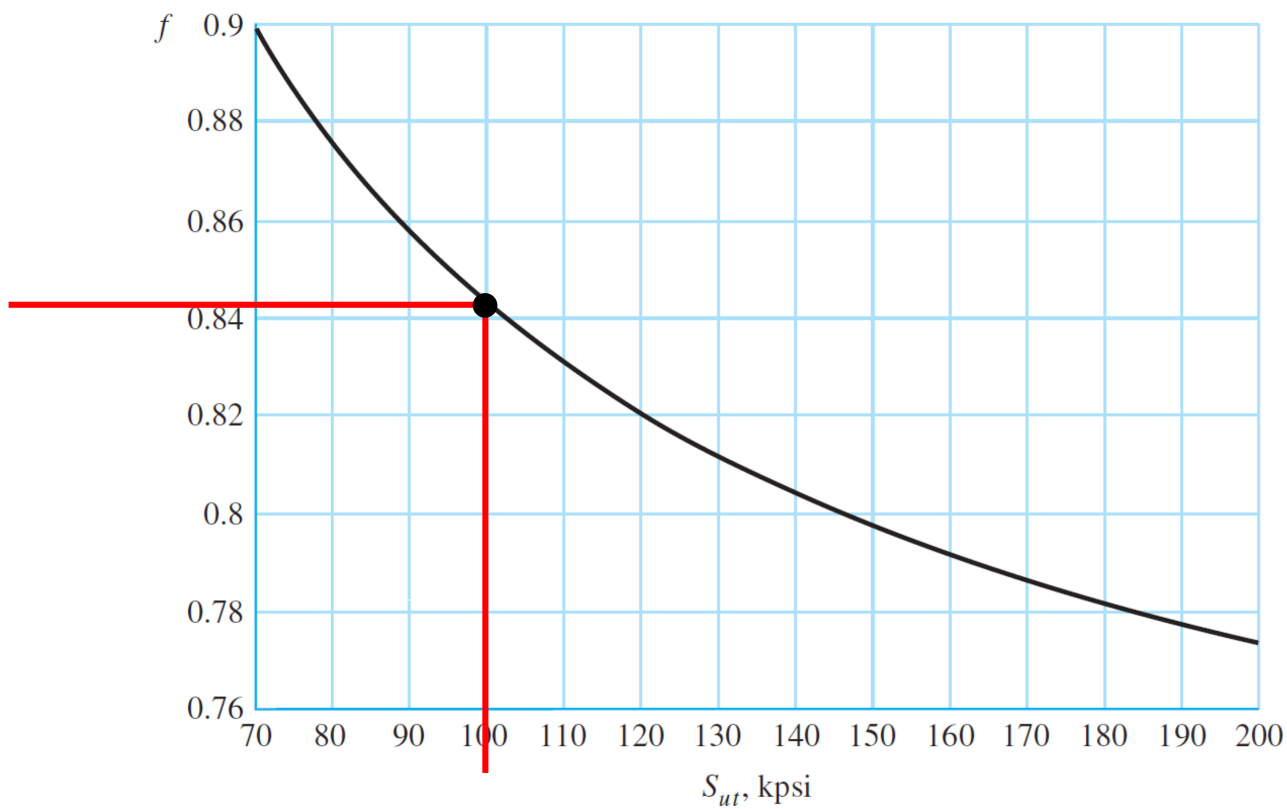
$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b}$$

$$a = \frac{(f\,S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right)$$

6-10 Stress Concentration and Notch Sensitivity

EXAMPLE 6–7 For the step-shaft of Ex. 6–6, it is determined that the fully corrected endurance limit is $S_e = 280$ MPa. Consider the shaft undergoes a fully reversing nominal stress in the fillet of $(\sigma_{rev})_{nom} = 260$ MPa. Estimate the number of cycles to failure.



6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY

EXAMPLE 6–7 For the step-shaft of Ex. 6–6, it is determined that the fully corrected endurance limit is $S_e = 280$ MPa. Consider the shaft undergoes a fully reversing nominal stress in the fillet of $(\sigma_{rev})_{nom} = 260$ MPa. Estimate the number of cycles to failure.

$$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.845(690)]^2}{280} = 1214 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{f\,S_{ut}}{S_e} = -\frac{1}{3}\log\left[\frac{0.845(690)}{280}\right] = -0.1062$$

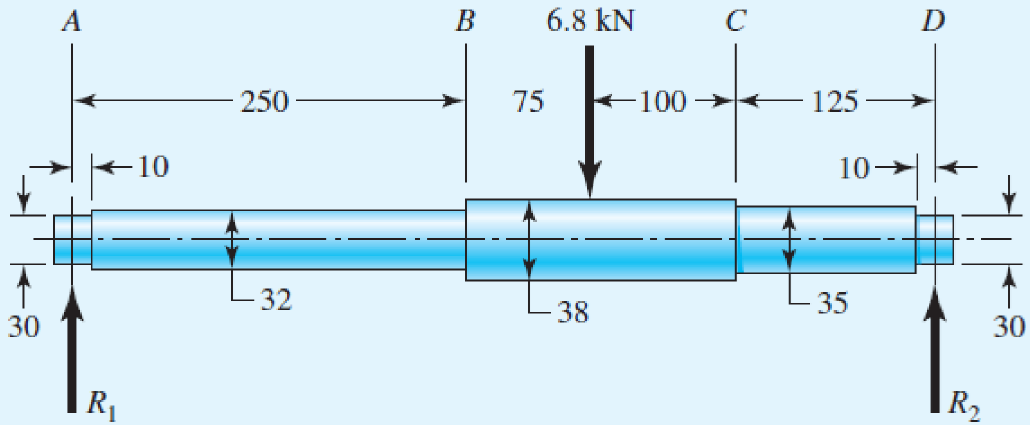
$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b} = \left(\frac{403}{1214}\right)^{1/-0.1062} = 32.3(10^3) \text{ cycles}$$

6-10 Stress Concentration and Notch Sensitivity

EXAMPLE 6–9 Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

Figure 6–22

(*a*) Shaft drawing showing all dimensions in millimeters; all fillets 3-mm radius. The shaft rotates and the load is stationary; material is machined from AISI 1050 cold-drawn steel. (*b*) Bending-moment diagram.



(*a*)

$$\boxed{N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b}}$$

$$(S_f)_{10^3} = f\,S_{ut}.$$

$$a = \frac{(f\,S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right)$$

6-10 Stress Concentration - Example 6-9

EXAMPLE 6–9 Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

From Table A–20 we find $S_{ut} = 690$ MPa and $S_y = 580$ MPa. The endurance limit S'_e is estimated as

1 UNS No.	2 SAE and/or AISI No.	3 Process-ing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197

6-10 Stress Concentration - Example 6-9

EXAMPLE 6–9 Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

From Table A–20 we find $S_{ut} = 690$ MPa and $S_y = 580$ MPa. The endurance limit S'_e is estimated as

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \qquad (6\text{–}8)$$

$$S'_e = 0.5(690) = 345 \text{ MPa}$$

6-10 Stress Concentration - Example 6-9

EXAMPLE 6–9 Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

From Eq. (6–19) and Table 6–2,

$$k_a = 4.51(690)^{-0.265} = 0.798$$

Surface Finish	Factor *a* S_{ut}, kpsi	Factor *a* S_{ut}, MPa	Exponent *b*
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

6-10 Stress Concentration - Example 6-9

EXAMPLE 6–9 Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

From Eq. (6–19) and Table 6–2,

$$k_a = 4.51(690)^{-0.265} = 0.798$$

From Eq. (6–20),

$$k_b = (32/7.62)^{-0.107} = 0.858$$

6-9 Endurance Limit Modifying Factors

Size Factor k_b

- The results for bending and torsion may be expressed as:

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \qquad (6\text{–}20)$$

- For axial loading there is no size effect, so

$$k_b = 1 \qquad (6\text{–}21)$$

Rotating Components

6-10 Stress Concentration - Example 6-9

EXAMPLE 6–9 Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

From Eq. (6–19) and Table 6–2,

$$k_a = 4.51(690)^{-0.265} = 0.798$$

From Eq. (6–20),

$$k_b = (32/7.62)^{-0.107} = 0.858$$

Since $k_c = k_d = k_e = k_f = 1$,

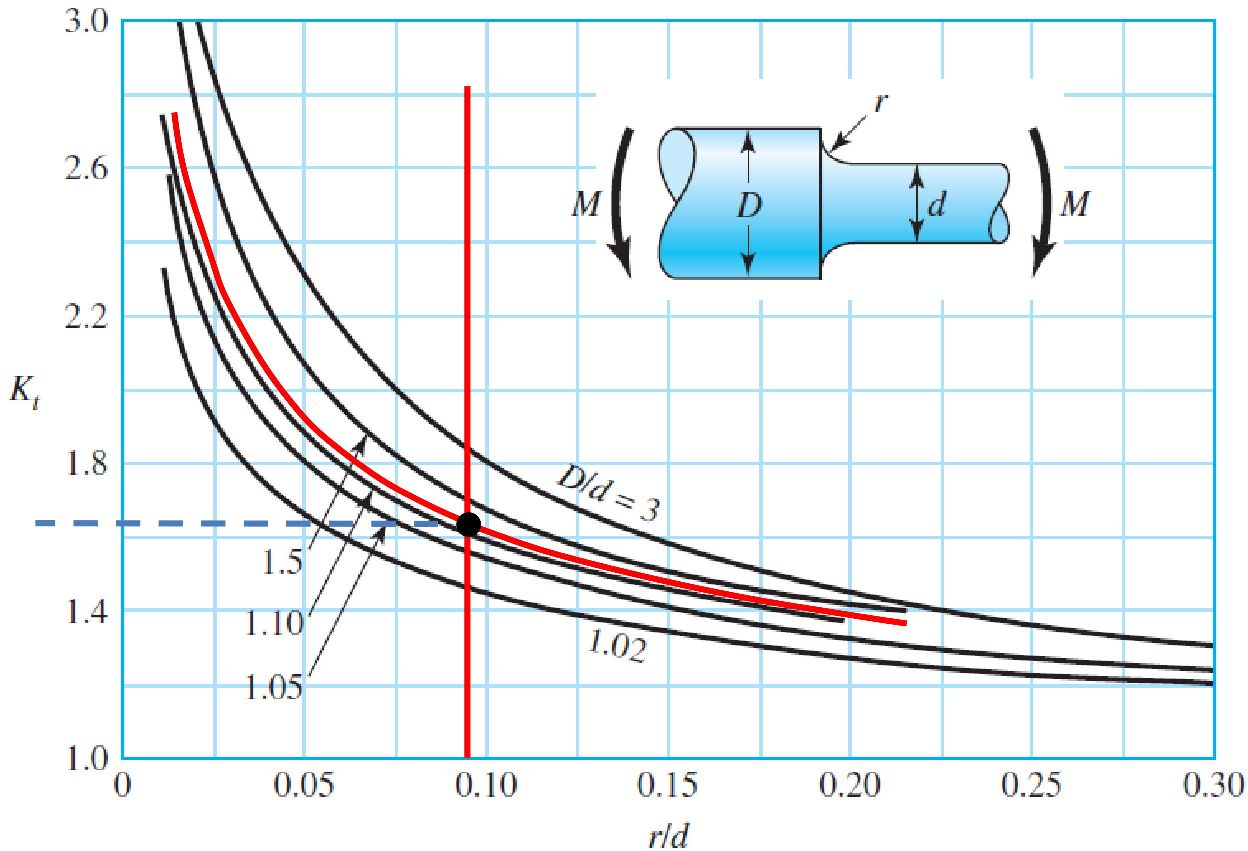
$$S_e = 0.798(0.858)345 = 236 \text{ MPa}$$

6-10 Stress Concentration - Example 6-9

EXAMPLE 6–9 Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

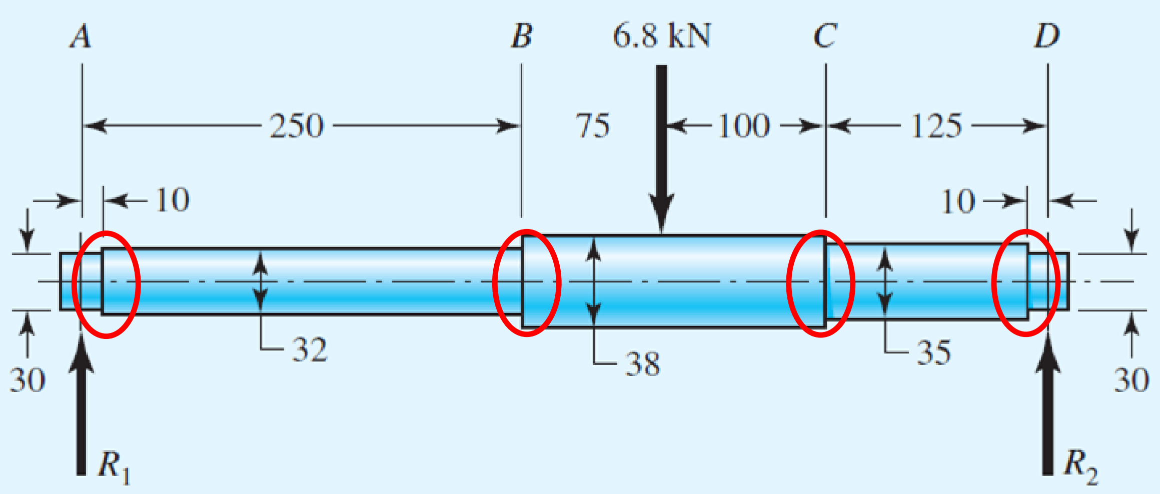


6-10 Stress Concentration - Example 6-9

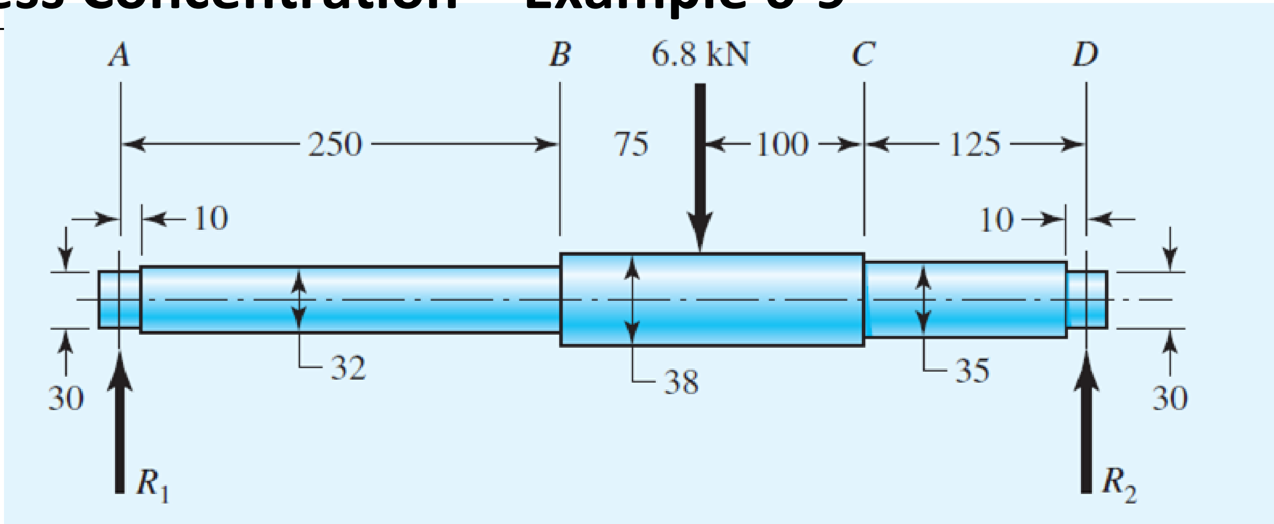
EXAMPLE 6–9 Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

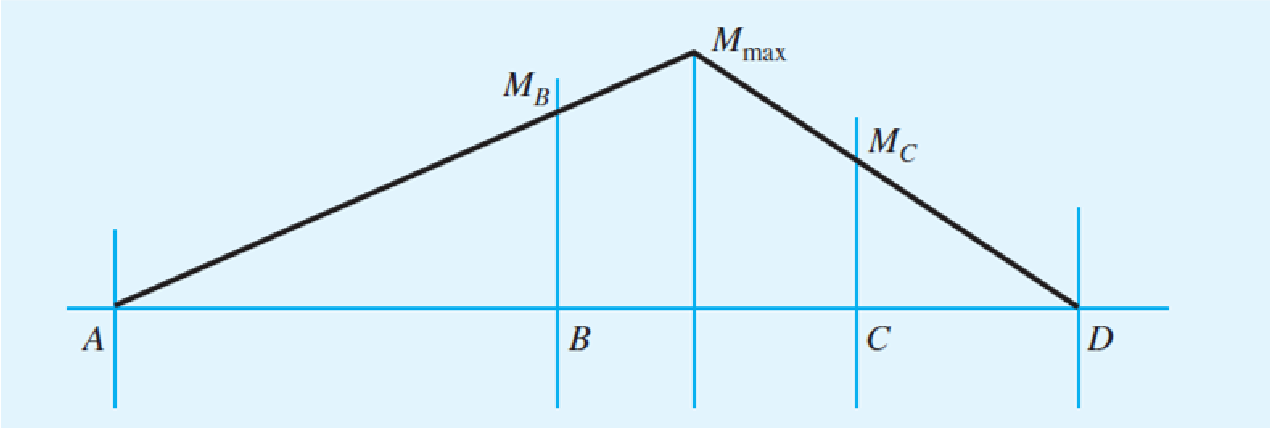
To find the geometric stress-concentration factor K_t we enter Fig. A–15–9 with $D/d = 38/32 = 1.1875$ and $r/d = 3/32 = 0.093\,75$ and read $K_t = 1.65$. Substituting $S_{ut} = 690/6.89 = 100$ kpsi into Eq. (6–35*a*) yields $\sqrt{a} = 0.0622\sqrt{\text{in}} = 0.313\sqrt{\text{mm}}$. Substituting this into Eq. (6–33) gives

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} = 1 + \frac{1.65 - 1}{1 + 0.313/\sqrt{3}} = 1.55$$



6-10 Stress Concentration - Example 6-9





(b)

$$\sigma_{rev} = K_f \frac{M_B}{I/c} = 1.55 \frac{695.5}{3.217}(10)^{-6} = 335.1(10^6)\ \text{Pa} = 335.1\ \text{MPa}$$

6-10 Stress Concentration - Example 6-9

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b}$$

For finite life, we will need to use Eq. (6–16). The ultimate strength, $S_{ut} = 690$ MPa = 100 kpsi. From Fig. 6–18, $f = 0.844$. From Eq. (6–14)

$$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.844(690)]^2}{236} = 1437 \text{ MPa}$$

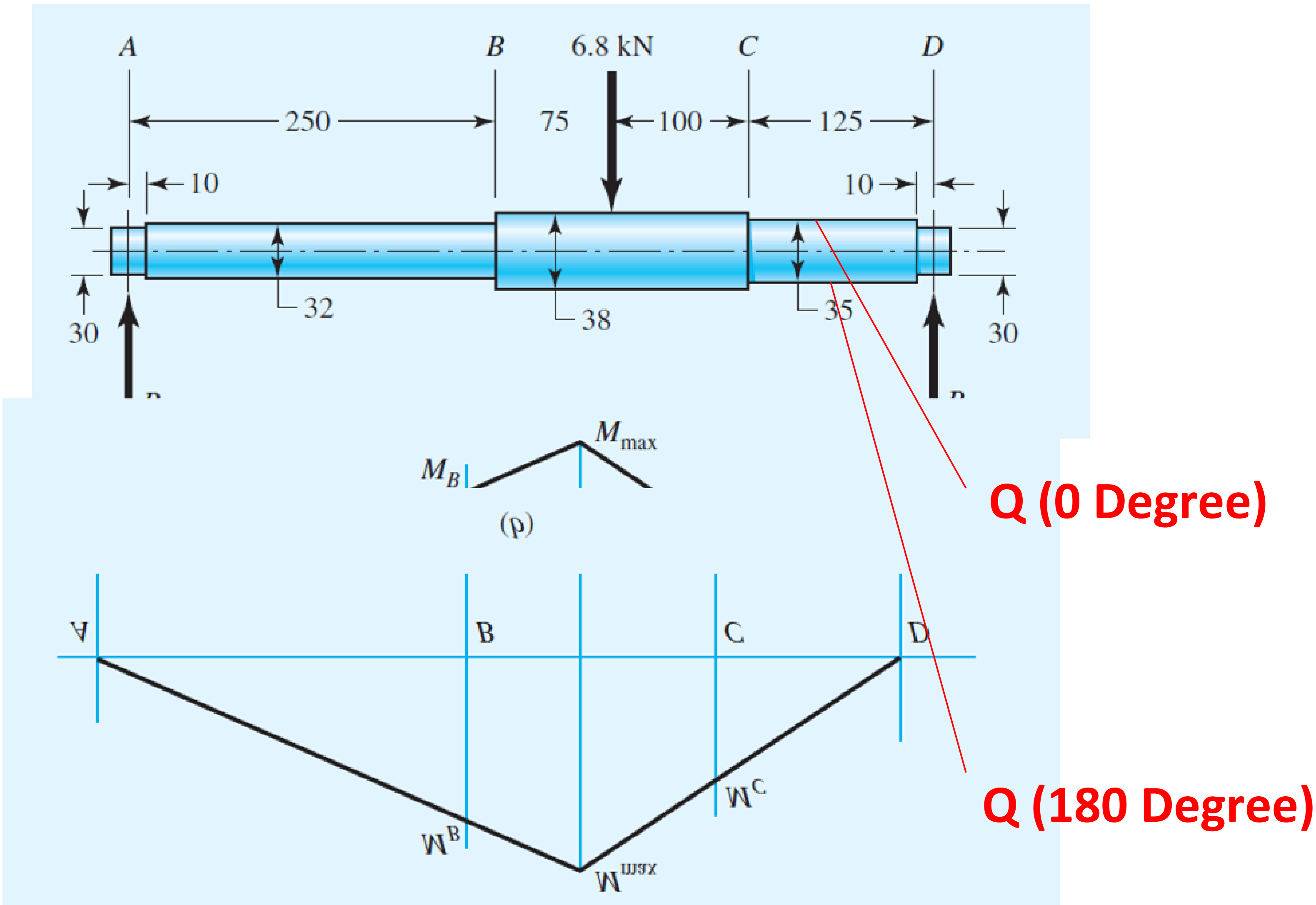
and from Eq. (6–15)

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.844(690)}{236}\right] = -0.1308$$

From Eq. (6–16),

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{335.1}{1437}\right)^{-1/0.1308} = 68(10^3) \text{ cycles}$$

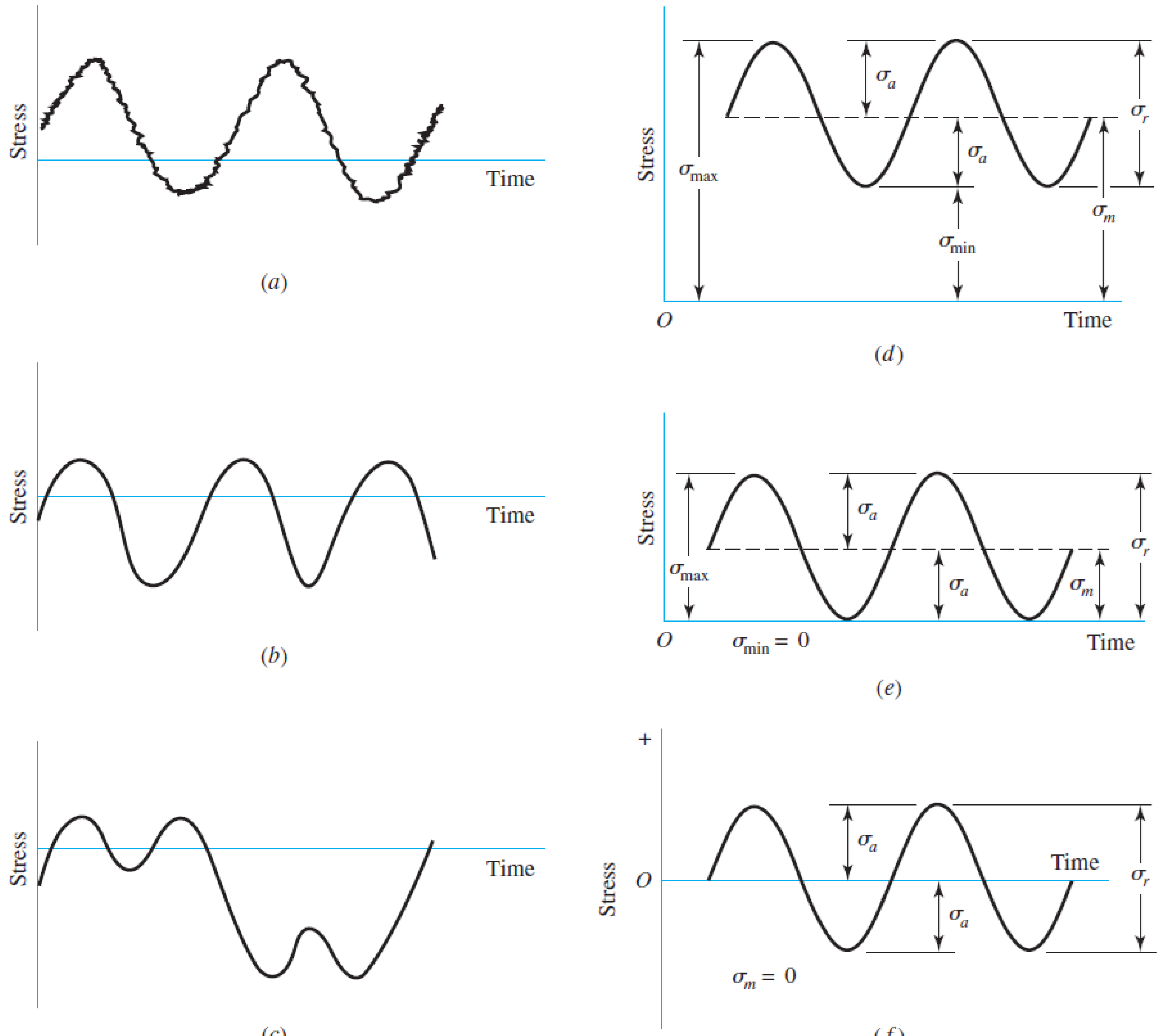
6 – 11 Characterizing Fluctuating Stresses



6 – 11 Characterizing Fluctuating Stresses

Figure 6–23

Some stress-time relations: (*a*) fluctuating stress with high-frequency ripple; (*b* and *c*) nonsinusoidal fluctuating stress; (*d*) sinusoidal fluctuating stress; (*e*) repeated stress; (*f*) completely reversed sinusoidal stress.



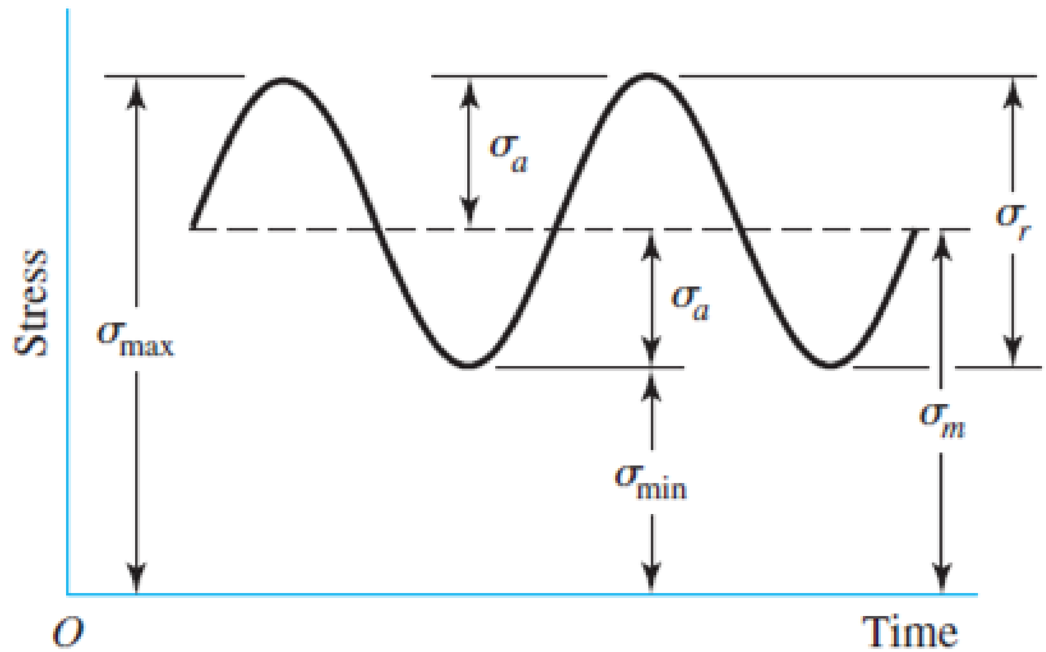
6 – 11 Characterizing Fluctuating Stresses

$$F_m = \frac{F_{\max} + F_{\min}}{2} \qquad F_a = \left| \frac{F_{\max} - F_{\min}}{2} \right|$$

$\sigma_{\min}$ = minimum stress

$\sigma_{\max}$ = maximum stress

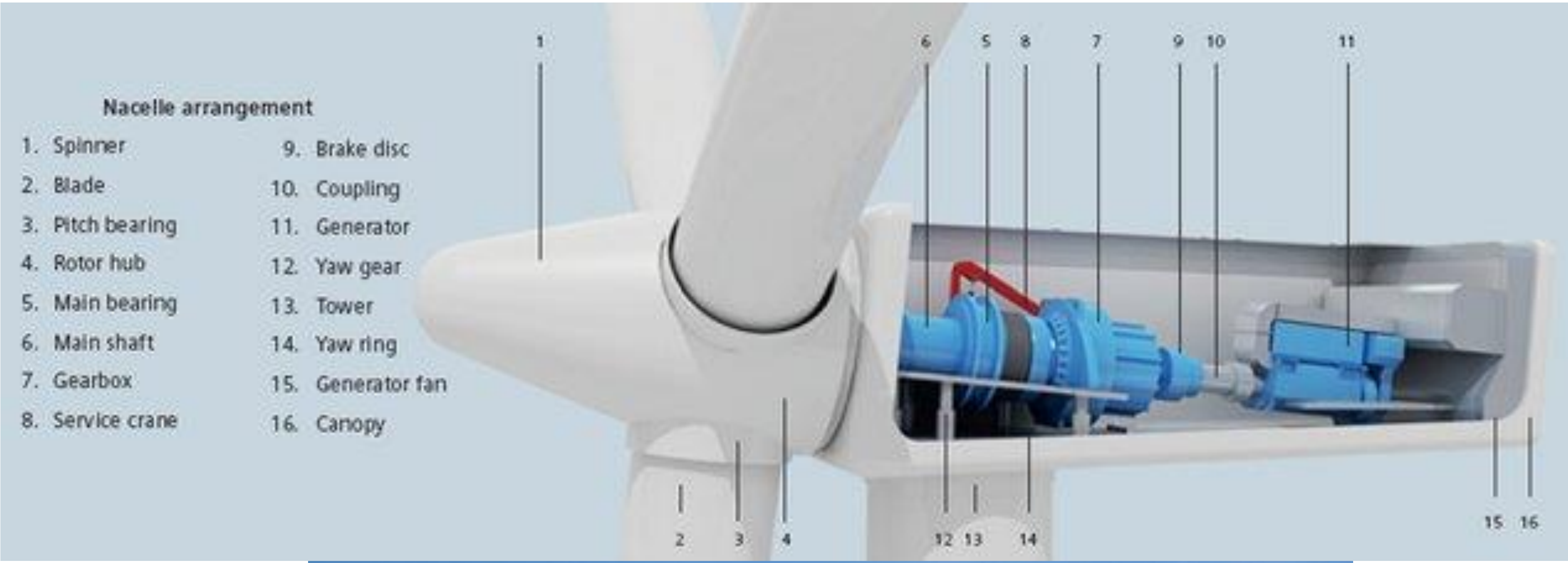
σ_a = amplitude component



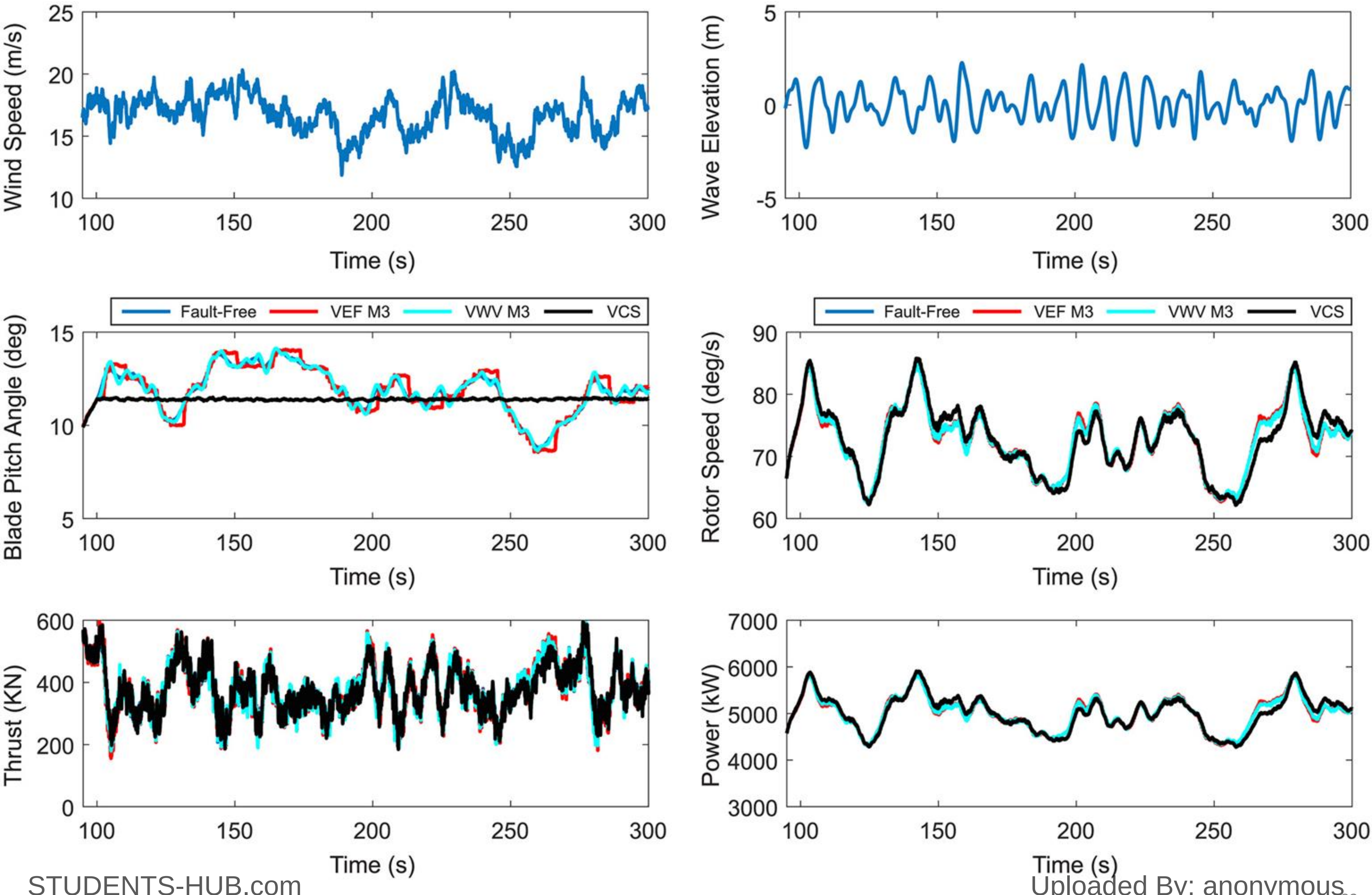
$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

$$\sigma_a = \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right|$$

6 – 11 Characterizing Fluctuating Stresses


Nacelle arrangement
1. Spinner
2. Blade
3. Pitch bearing
4. Rotor hub
5. Main bearing
6. Main shaft
7. Gearbox
8. Service crane
9. Brake disc
10. Coupling
11. Generator
12. Yaw gear
13. Tower
14. Yaw ring
15. Generator fan
16. Canopy
1
6
5
8
7
9
10
11
2
3
4
12
13
14
15
16

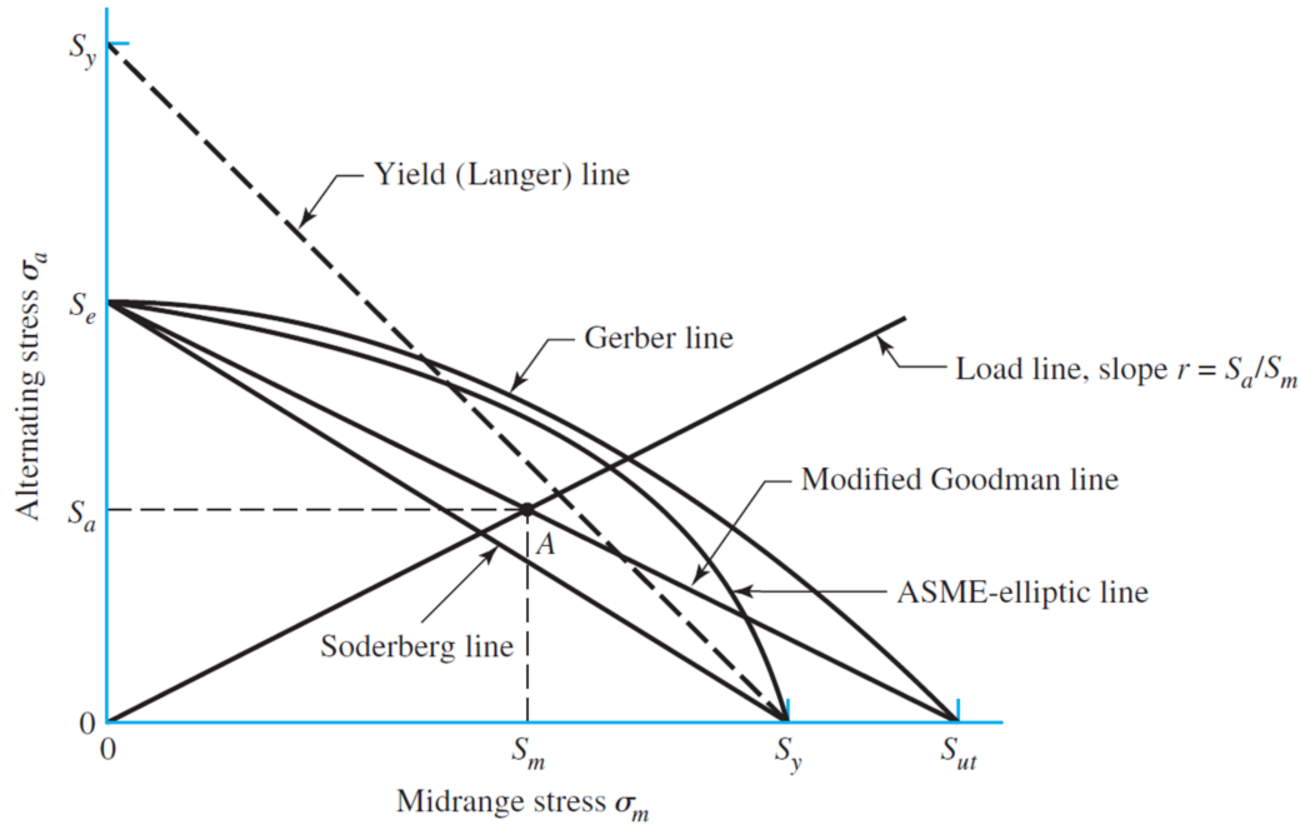
6 – 11 Characterizing Fluctuating Stresses



6- 12 Fatigue Failure Criteria for Fluctuating Stress

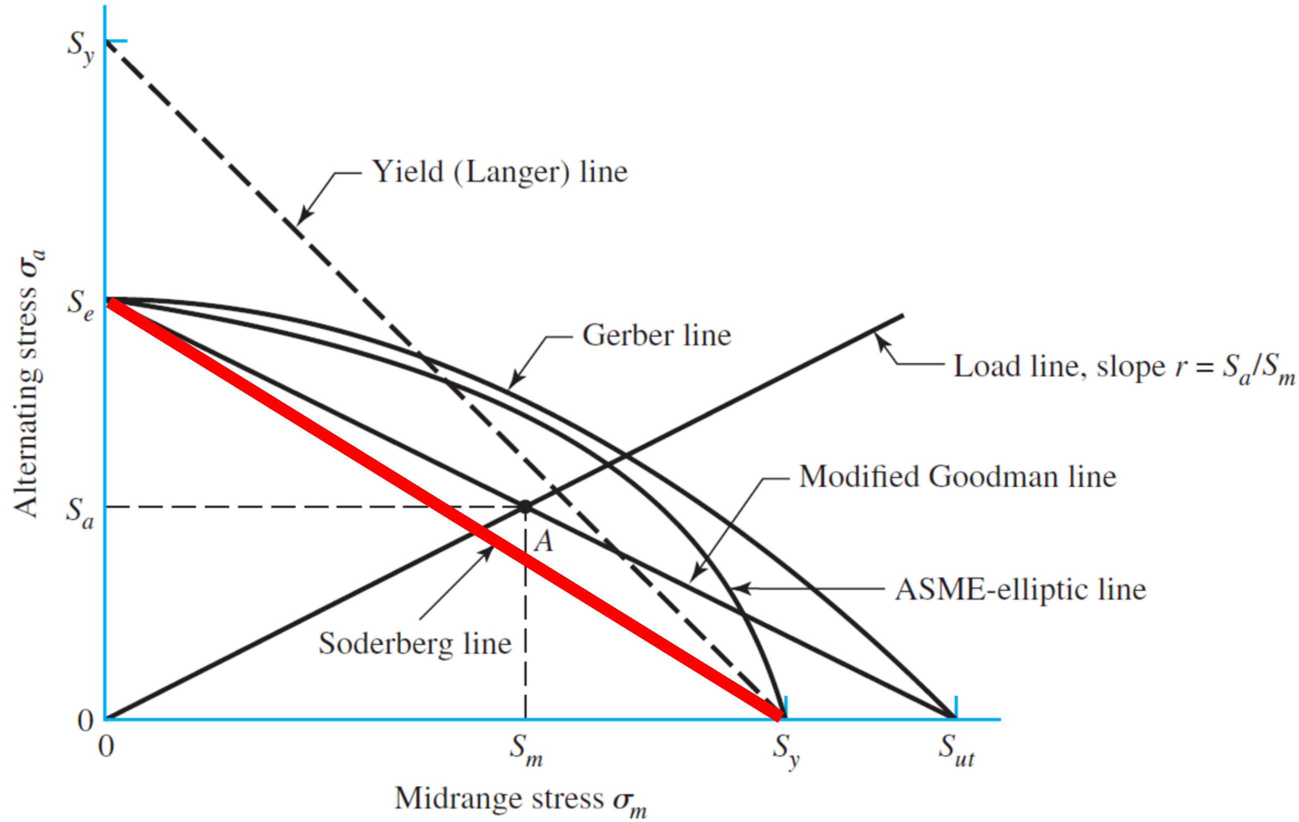
Figure 6–27

Fatigue diagram showing various criteria of failure. For each criterion, points on or "above" the respective line indicate failure. Some point *A* on the Goodman line, for example, gives the strength S_m as the limiting value of σ_m corresponding to the strength S_a, which, paired with σ_m, is the limiting value of σ_a.



6- 12 Fatigue Failure Criteria for Fluctuating Stress

Soderberg Line

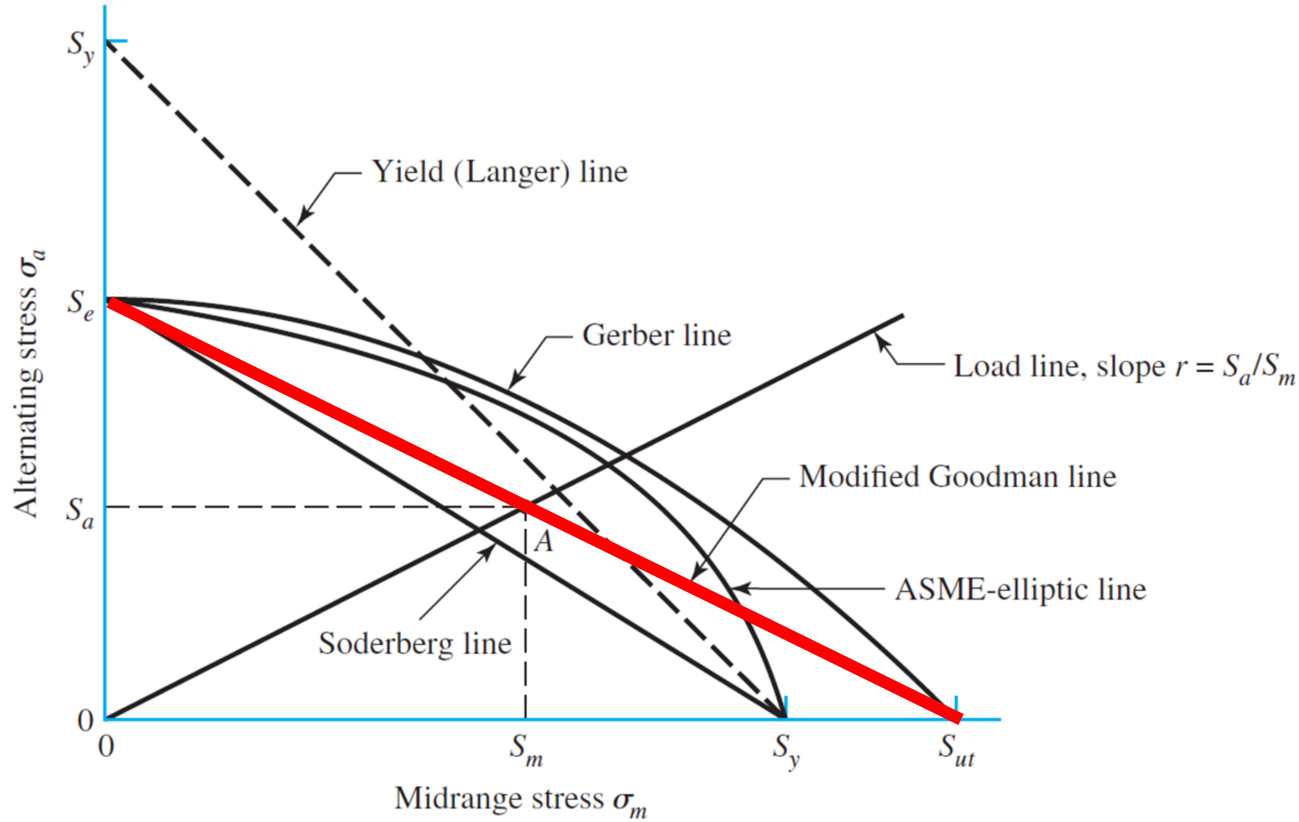


$$\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

Modified Goodman line



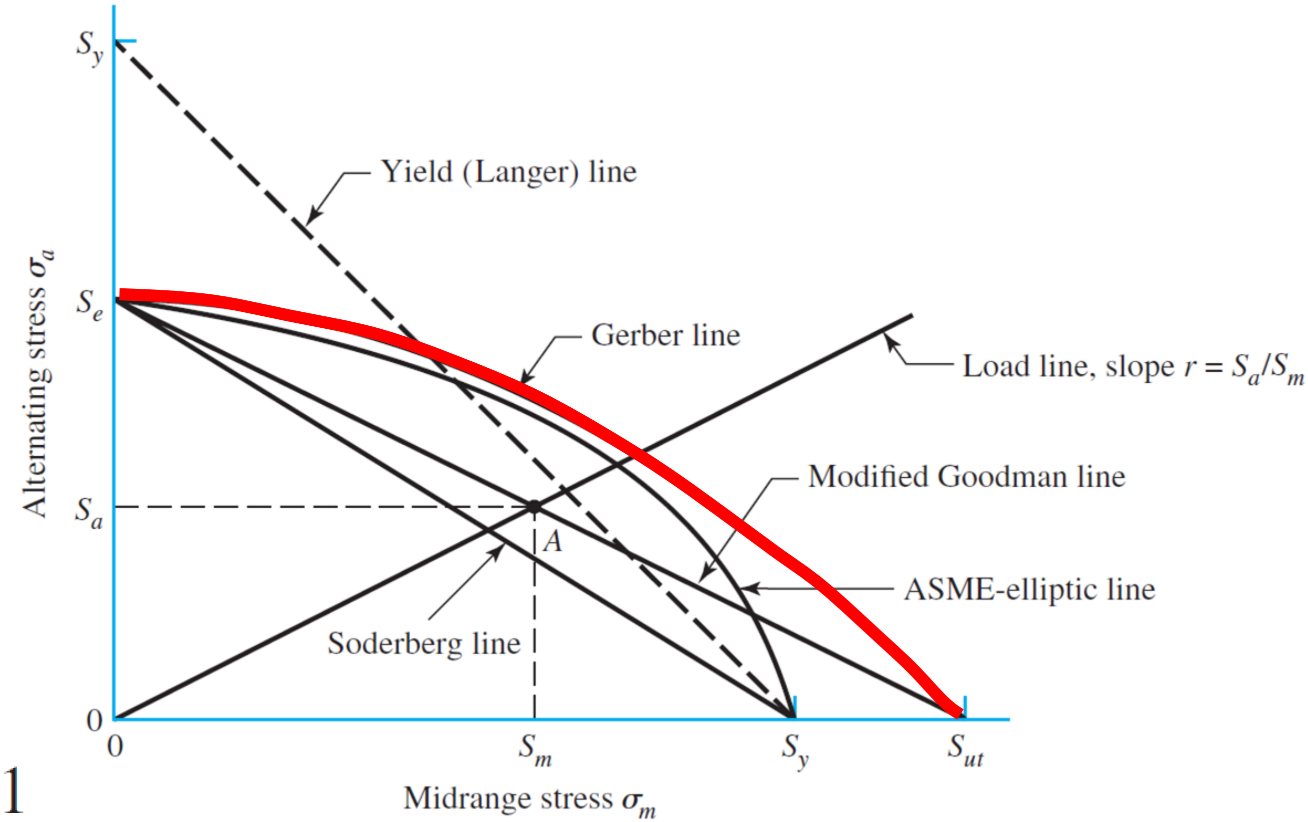
$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

BIRZEIT UNIVERSITY

Gerber line



$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

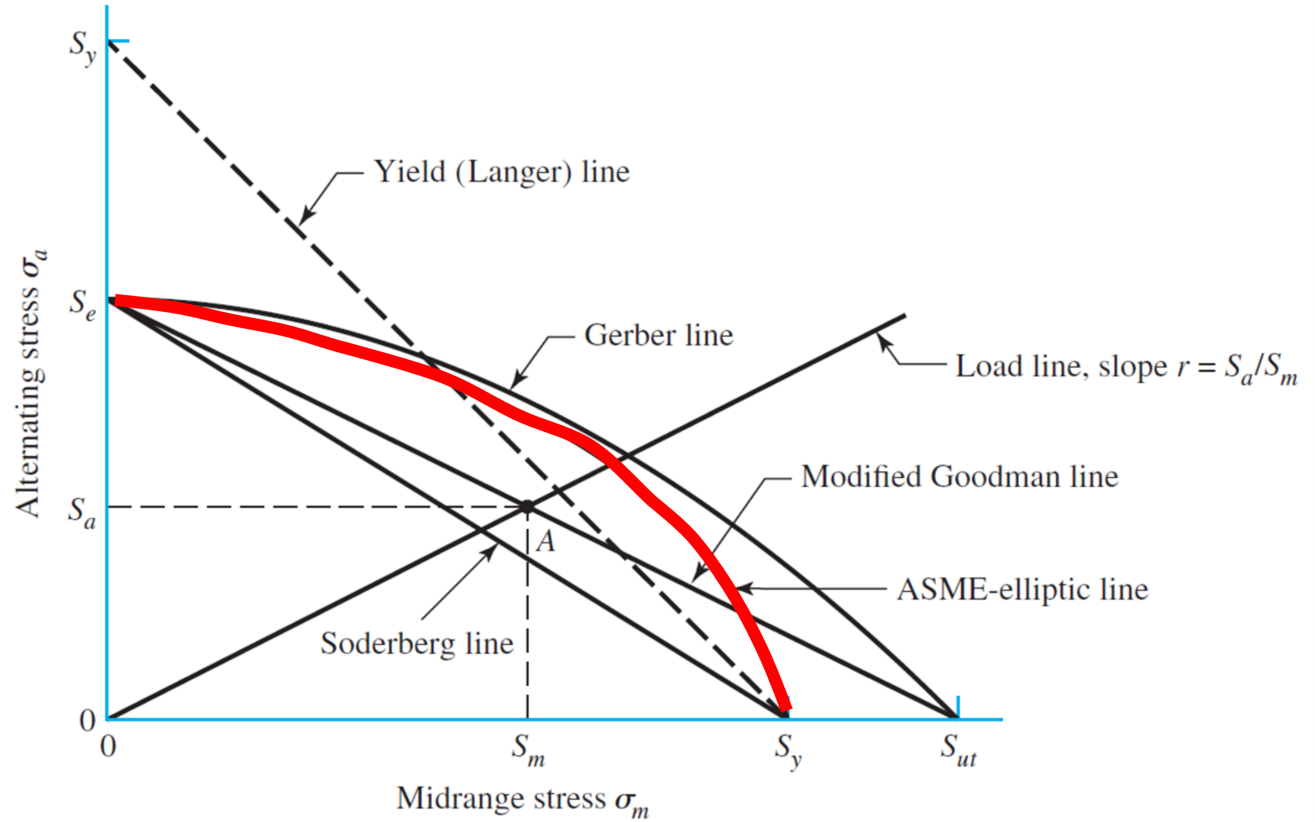
$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

BIRZEIT UNIVERSITY

ASME-Elliptic line



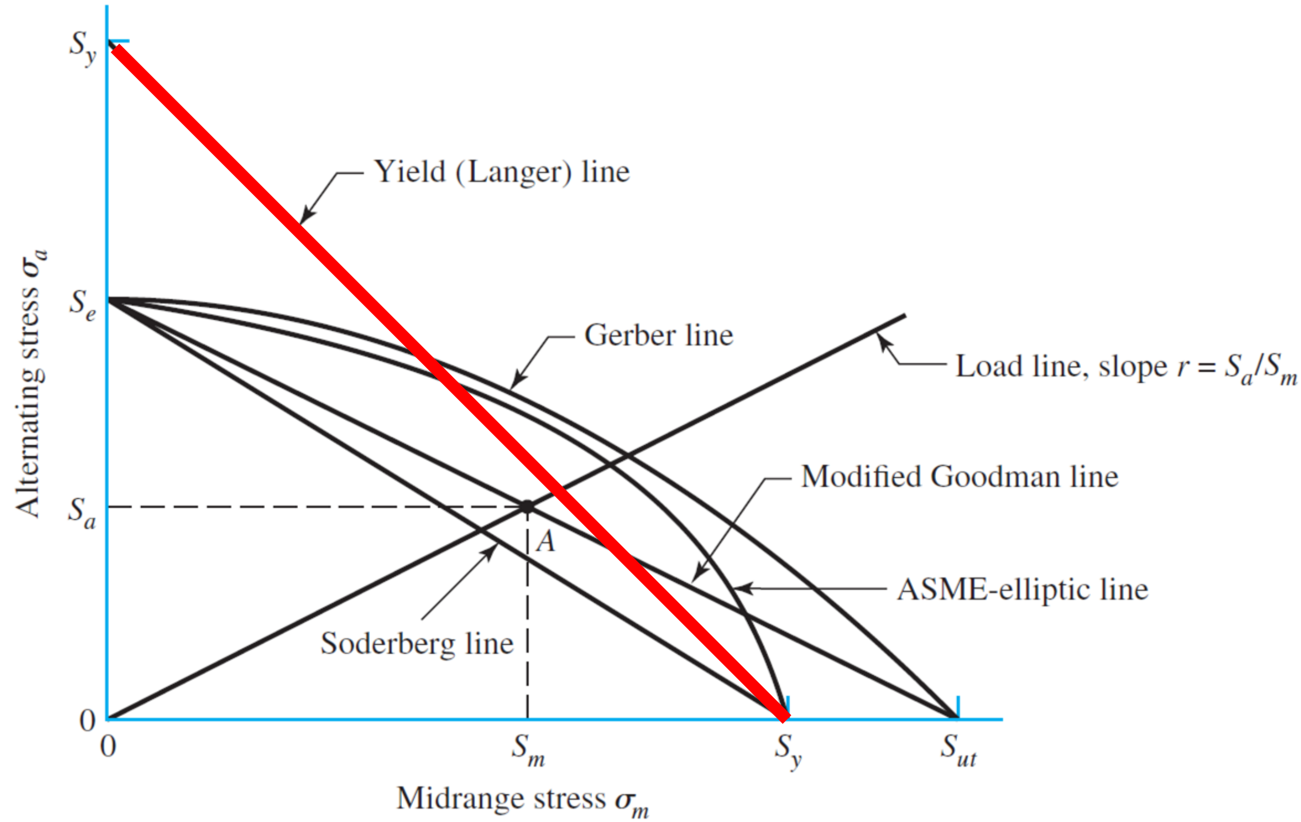
$$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$$

$$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$$

$$n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

Langer First Cycle Yielding



$$\sigma_a + \sigma_m = \frac{S_y}{n} \qquad S_a + S_m = S_y$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

EXAMPLE 6–10 A 1.5-in-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 16 kip. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line and (*b*) the ASME-elliptic fatigue line.

We begin with some preliminaries. From Table A–20, $S_{ut} = 100$ kpsi and $S_y = 84$ kpsi. Note that $F_a = F_m = 8$ kip. The Marin factors are, deterministically,

$k_a = 2.70(100)^{-0.265} = 0.797$: Eq. (6–19), Table 6–2, p. 296

Surface Factor k_a

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

6- 12 Fatigue Failure Criteria for Fluctuating Stress

EXAMPLE 6–10 A 1.5-in-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 16 kip. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line and (*b*) the ASME-elliptic fatigue line.

We begin with some preliminaries. From Table A–20, $S_{ut} = 100$ kpsi and $S_y = 84$ kpsi. Note that $F_a = F_m = 8$ kip. The Marin factors are, deterministically,

$k_a = 2.70(100)^{-0.265} = 0.797$: Eq. (6–19), Table 6–2, p. 296

$k_b = 1$ (axial loading, see k_c)

6-9 Endurance Limit Modifying Factors

Size Factor k_b

➢ The results for bending and torsion may be expressed as:

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \qquad (6\text{–}20)$$

➢ For axial loading there is no size effect, so

$$k_b = 1 \qquad (6\text{–}21)$$

Rotating Components

6-9 Endurance Limit Modifying Factors

Load Factor k_c

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{–}26)$$

When torsion is combined with other loading, such as bending, set $k_c = 1$, and the combined loading is managed by using the effective von Mises stress as described in Sec. 6-14

6- 12 Fatigue Failure Criteria for Fluctuating Stress

EXAMPLE 6–10 A 1.5-in-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 16 kip. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line and (*b*) the ASME-elliptic fatigue line.

We begin with some preliminaries. From Table A–20, $S_{ut} = 100$ kpsi and $S_y = 84$ kpsi. Note that $F_a = F_m = 8$ kip. The Marin factors are, deterministically,

$k_a = 2.70(100)^{-0.265} = 0.797$: Eq. (6–19), Table 6–2, p. 296

$k_b = 1$ (axial loading, see k_c)

$k_c = 0.85$: Eq. (6–26), p. 298

$k_d = k_e = k_f = 1$

$S_e = 0.797(1)0.850(1)(1)(1)0.5(100) = 33.9$ kpsi: Eqs. (6–8), (6–18), p. 290, p. 295

6- 12 Fatigue Failure Criteria for Fluctuating Stress

BIRZEIT UNIVERSITY

EXAMPLE 6–10 A 1.5-in-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 16 kip. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line and (*b*) the ASME-elliptic fatigue line.

The nominal axial stress components σ_{ao} and σ_{mo} are

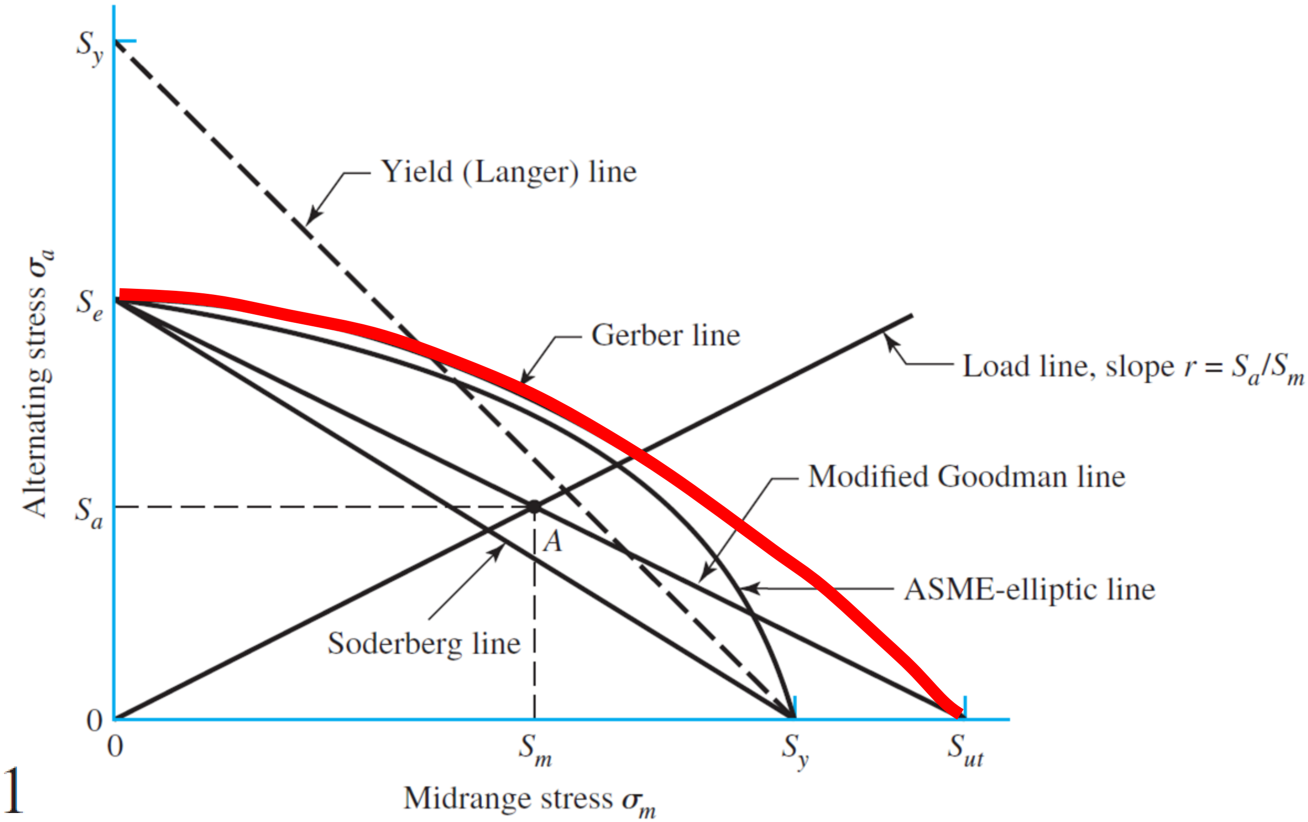
$$\sigma_{ao} = \frac{4F_a}{\pi d^2} = \frac{4(8)}{\pi 1.5^2} = 4.53 \text{ kpsi} \qquad \sigma_{mo} = \frac{4F_m}{\pi d^2} = \frac{4(8)}{\pi 1.5^2} = 4.53 \text{ kpsi}$$

Applying K_f to both components σ_{ao} and σ_{mo} constitutes a prescription of no notch yielding:

$$\sigma_a = K_f \sigma_{ao} = 1.85(4.53) = 8.38 \text{ kpsi} = \sigma_m$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

Gerber line



$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

BIRZEIT UNIVERSITY

EXAMPLE 6–12

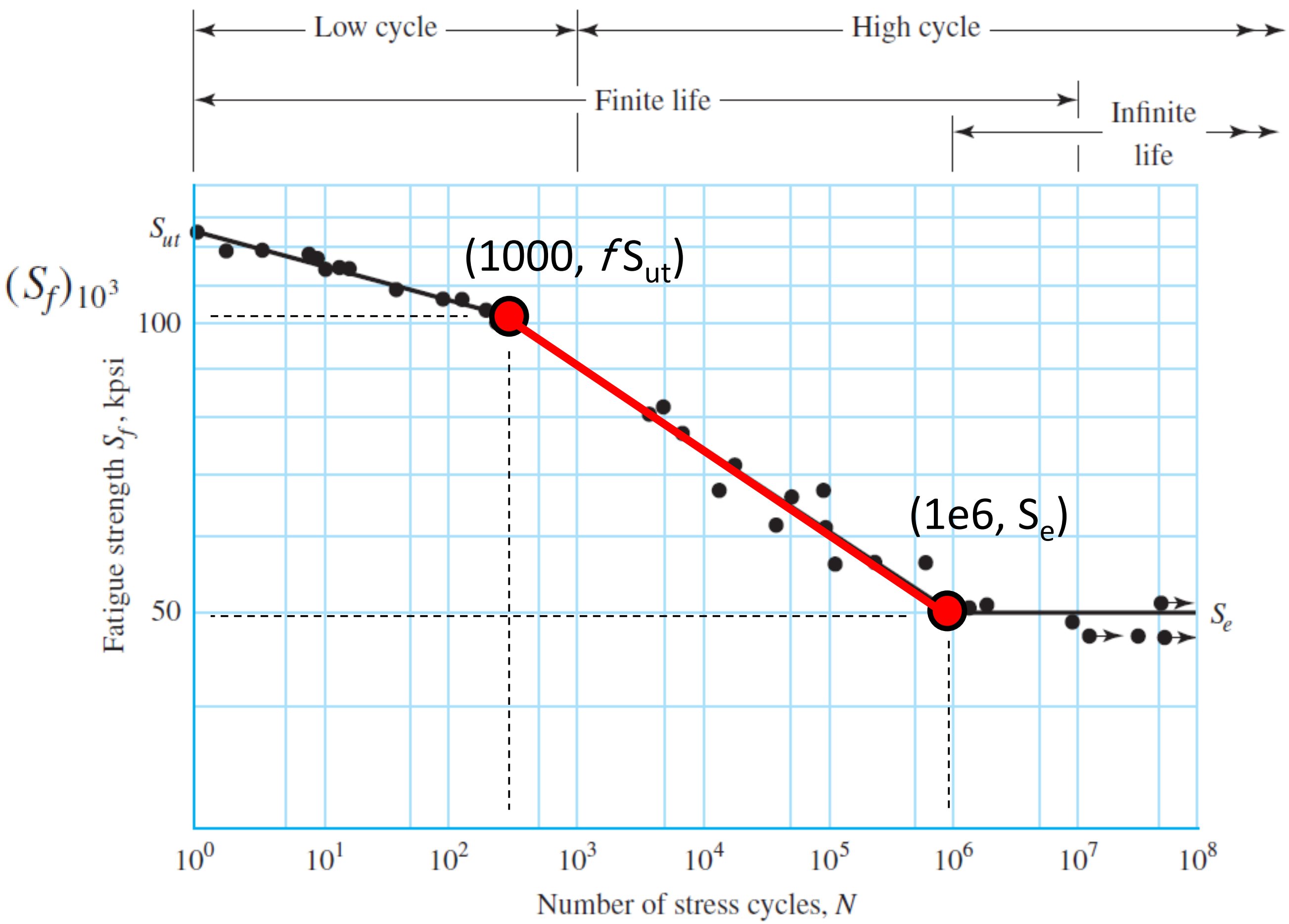
A steel bar undergoes cyclic loading such that $\sigma_{\max} = 60$ kpsi and $\sigma_{\min} = -20$ kpsi. For the material, $S_{ut} = 80$ kpsi, $S_y = 65$ kpsi, a fully corrected endurance limit of $S_e = 40$ kpsi, and $f = 0.9$. Estimate the number of cycles to a fatigue failure using:
(*a*) Modified Goodman criterion.
(*b*) Gerber criterion.

From the given stresses,

$$\sigma_a = \frac{60 - (-20)}{2} = 40 \text{ kpsi} \qquad \sigma_m = \frac{60 + (-20)}{2} = 20 \text{ kpsi}$$

6-10 Stress Concentration and Notch Sensitivity

BIRZEIT UNIVERSITY



$$(S_f)_{10^3} = f\,S_{ut}.$$

$$S_f = a\,N^b$$

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b}$$

$$a = \frac{(f\,S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right)$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

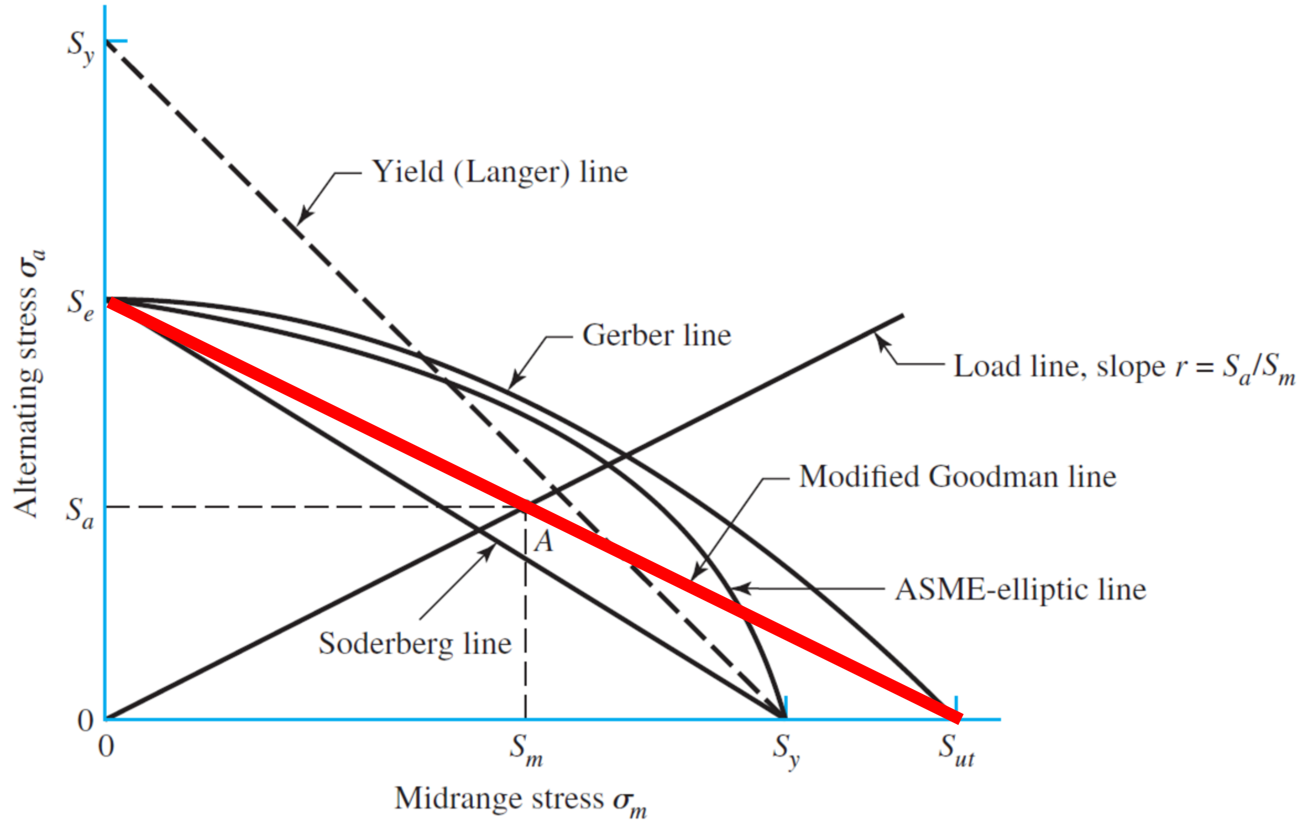
EXAMPLE 6–12 A steel bar undergoes cyclic loading such that $\sigma_{max} = 60$ kpsi and $\sigma_{min} = -20$ kpsi. For the material, $S_{ut} = 80$ kpsi, $S_y = 65$ kpsi, a fully corrected endurance limit of $S_e = 40$ kpsi, and $f = 0.9$. Estimate the number of cycles to a fatigue failure using:

(*a*) Modified Goodman criterion.

(*b*) Gerber criterion.

6- 12 Fatigue Failure Criteria for Fluctuating Stress

Modified Goodman line



$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \qquad \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

BIRZEIT UNIVERSITY

EXAMPLE 6–12 A steel bar undergoes cyclic loading such that $\sigma_{max} = 60$ kpsi and $\sigma_{min} = -20$ kpsi. For the material, $S_{ut} = 80$ kpsi, $S_y = 65$ kpsi, a fully corrected endurance limit of $S_e = 40$ kpsi, and $f = 0.9$. Estimate the number of cycles to a fatigue failure using:
(*a*) Modified Goodman criterion.
(*b*) Gerber criterion.

This completely reversed stress can be obtained by replacing S_e with σ_{rev} in Eq. (6–46) for the modified Goodman line resulting in

$$\sigma_{rev} = \frac{\sigma_a}{1 - \dfrac{\sigma_m}{S_{ut}}} = \frac{40}{1 - \dfrac{20}{80}} = 53.3 \text{ kpsi}$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

EXAMPLE 6–12 A steel bar undergoes cyclic loading such that $\sigma_{max} = 60$ kpsi and $\sigma_{min} = -20$ kpsi. For the material, $S_{ut} = 80$ kpsi, $S_y = 65$ kpsi, a fully corrected endurance limit of $S_e = 40$ kpsi, and $f = 0.9$. Estimate the number of cycles to a fatigue failure using:

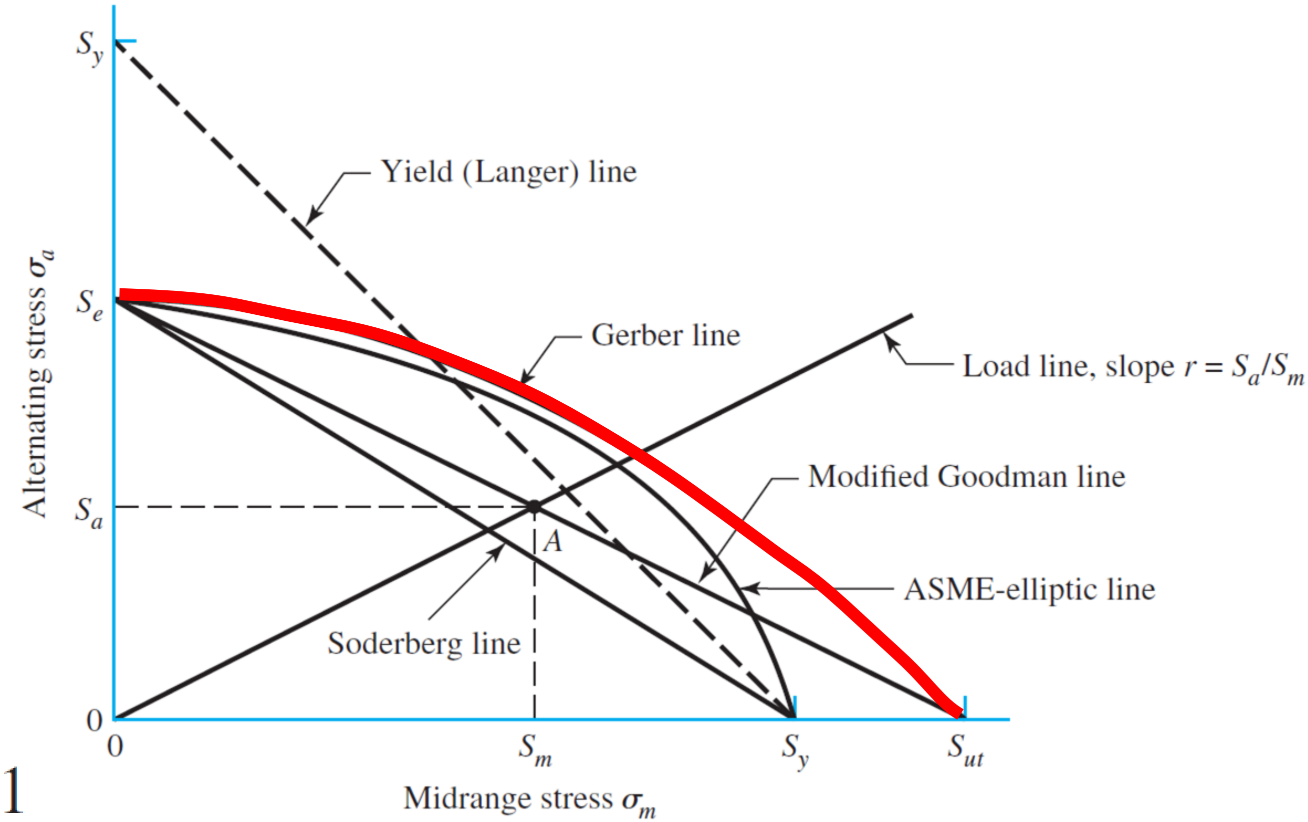
(*a*) Modified Goodman criterion.

(*b*) Gerber criterion.

6- 12 Fatigue Failure Criteria for Fluctuating Stress

BIRZEIT UNIVERSITY

Gerber line



$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

BIRZEIT UNIVERSITY

EXAMPLE 6–12 A steel bar undergoes cyclic loading such that $\sigma_{max} = 60$ kpsi and $\sigma_{min} = -20$ kpsi. For the material, $S_{ut} = 80$ kpsi, $S_y = 65$ kpsi, a fully corrected endurance limit of $S_e = 40$ kpsi, and $f = 0.9$. Estimate the number of cycles to a fatigue failure using:

(*a*) Modified Goodman criterion.

$\boxed{(b)\ \text{Gerber criterion.}}$

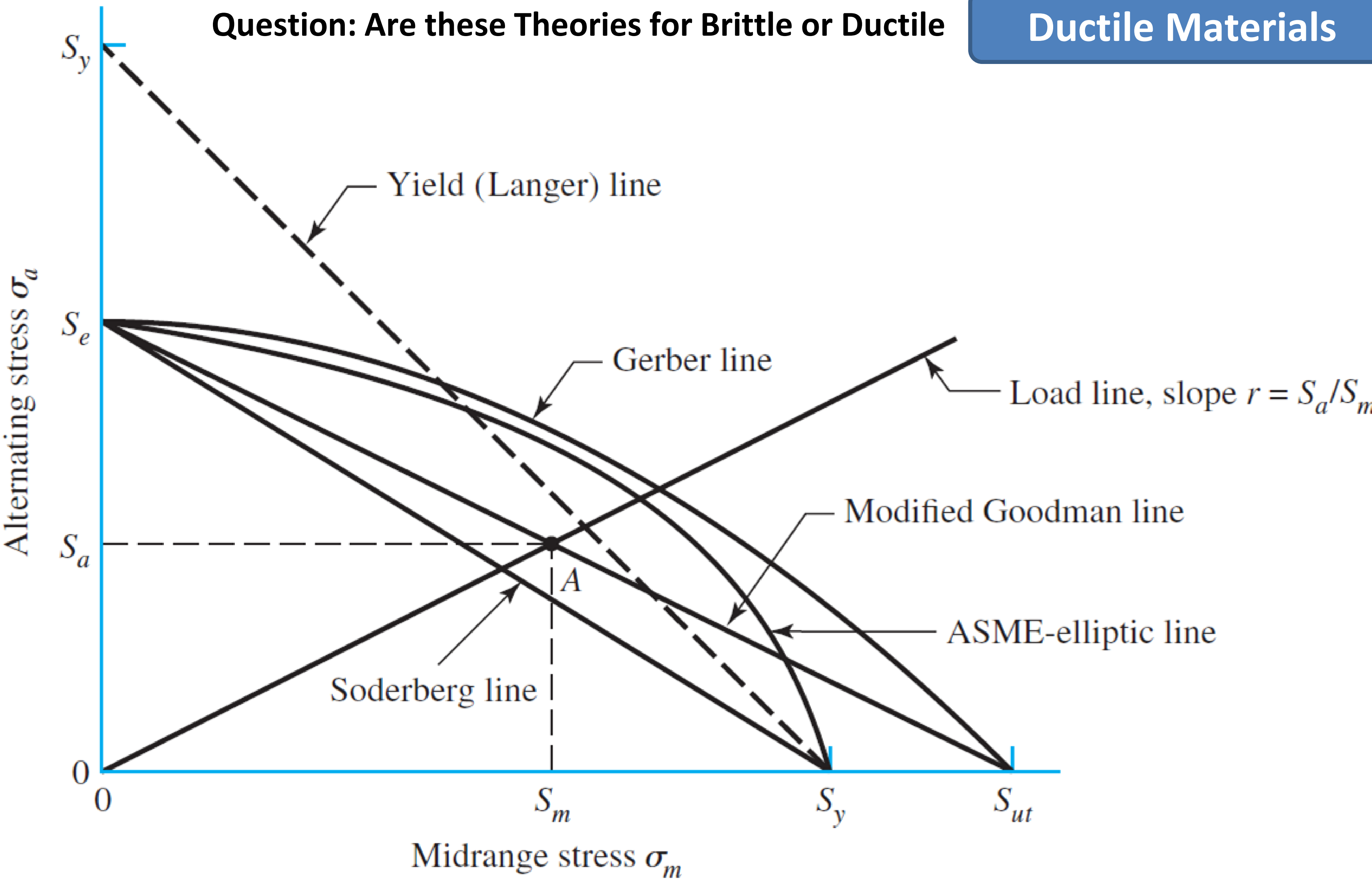
(*b*) For Gerber, similar to part (*a*), from Eq. (6–47),

$$\sigma_{rev} = \frac{\sigma_a}{1 - \left(\dfrac{\sigma_m}{S_{ut}}\right)^2} = \frac{40}{1 - \left(\dfrac{20}{80}\right)^2} = 42.7 \text{ kpsi}$$

6- 12 Fatigue Failure Criteria for Fluctuating Stress

Question: Are these Theories for Brittle or Ductile

Ductile Materials



6-12 Fatigue Failure Criteria for Fluctuating Stress

BIRZEIT UNIVERSITY

Brittle Materials Smith Dolan

$$\frac{S_a}{S_e} = \frac{1 - S_m/S_{ut}}{1 + S_m/S_{ut}} \tag{6–50}$$

$$\frac{n\sigma_a}{S_e} = \frac{1 - n\sigma_m/S_{ut}}{1 + n\sigma_m/S_{ut}} \tag{6–51}$$

$$S_a = \frac{rS_{ut} + S_e}{2}\left[-1 + \sqrt{1 + \frac{4rS_{ut}S_e}{(rS_{ut} + S_e)^2}}\right] \tag{6–52}$$

6-13 Torsional Fatigue Strength under Fluctuating Stresses

Joerres uses

$$S_{su} = 0.67 S_{ut} \qquad (6\text{–}54)$$

Also, from Chap. 5, $S_{sy} = 0.577 S_{yt}$ from distortion-energy theory, and the mean load factor k_c is given by Eq. (6–26), or 0.577.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{–}26)$$

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

6-9 Endurance Limit Modifying Factors

Load Factor k_c

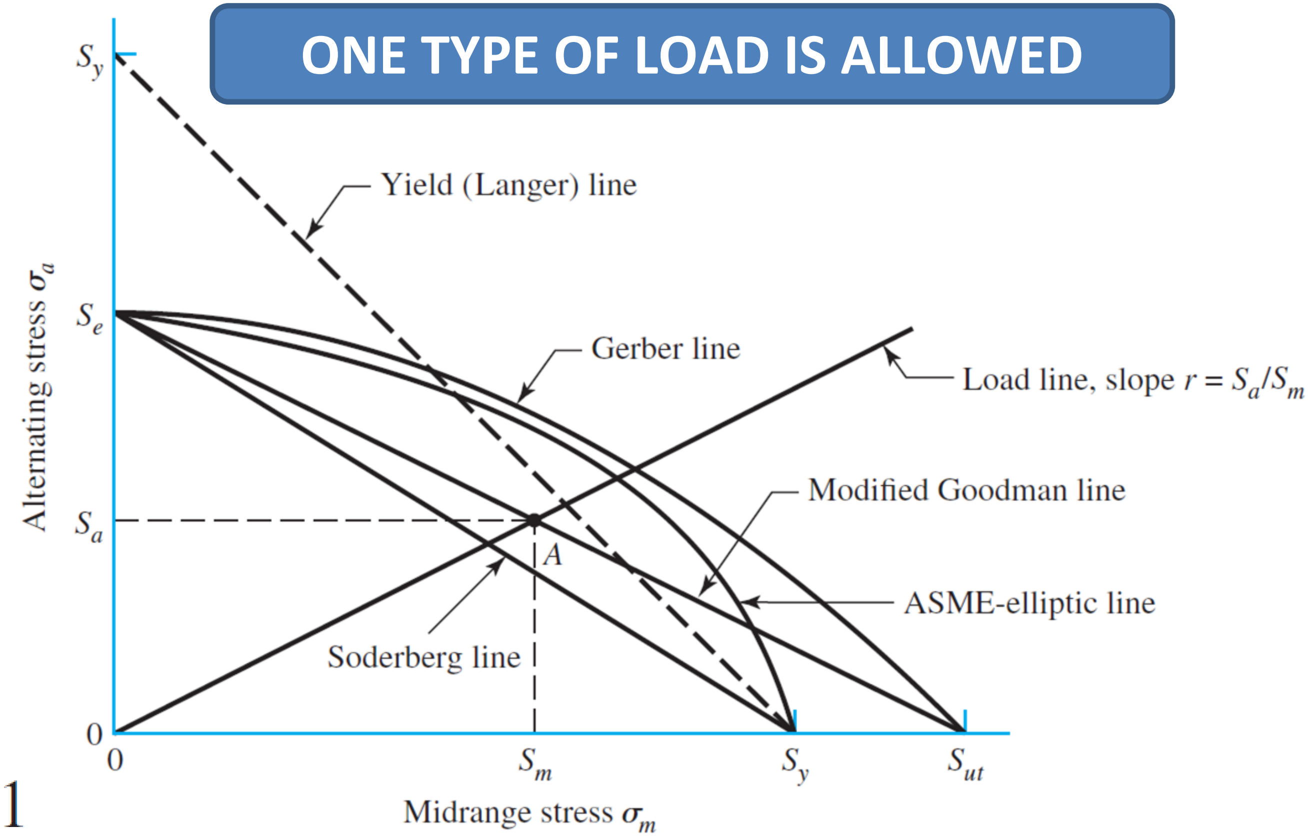
$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{–}26)$$

When torsion is combined with other loading, such as bending, set $k_c = 1$, and the combined loading is managed by using the effective von Mises stress as described in Sec. 6-14

6-12 Fatigue Failure Criteria for Fluctuating Stress

BIRZEIT UNIVERSITY

ONE TYPE OF LOAD IS ALLOWED



Gerber line

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

6-14 Combinations of Loading Modes

$$\sigma' = (\sigma_x^2 + 3\tau_{xy}^2)^{1/2}.$$

$$\sigma'_a = \left\{ \left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}} \frac{(\sigma_a)_{\text{axial}}}{0.85} \right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}} \right]^2 \right\}^{1/2} \tag{6–55}$$

$$\sigma'_m = \{ [(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}}]^2 \}^{1/2} \tag{6–56}$$

6-14 Combinations of Loading Modes

EXAMPLE 6–14 A shaft is made of 42- × 4-mm AISI 1018 cold-drawn steel tubing and has a 6-mm-diameter hole drilled transversely through it. Estimate the factor of safety guarding against fatigue and static failures using the Gerber and Langer failure criteria for the following loading conditions:

(*a*) The shaft is rotating and is subjected to a completely reversed torque of 120 N · m in phase with a completely reversed bending moment of 150 N · m.

(*b*) The shaft is subjected to a pulsating torque fluctuating from 20 to 160 N · m and a steady bending moment of 150 N · m.

$$\sigma'_a = \left\{\left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}}\frac{(\sigma_a)_{\text{axial}}}{0.85}\right]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}}]^2\right\}^{1/2} \tag{6–55}$$

$$\sigma'_m = \{[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}}]^2\}^{1/2} \tag{6–56}$$

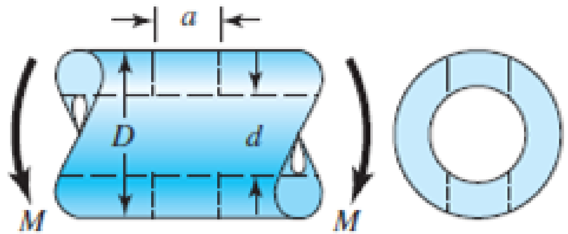
6-14 Combinations of Loading Modes

EXAMPLE 6–14

A shaft is made of 42- × 4-mm AISI 1018 cold-drawn steel tubing and has a 6-mm-diameter hole drilled transversely through it. Estimate the factor of safety guarding against fatigue and static failures using the Gerber and Langer failure criteria for the following loading conditions:

(*a*) The shaft is rotating and is subjected to a completely reversed torque of 120 N · m in phase with a completely reversed bending moment of 150 N · m.

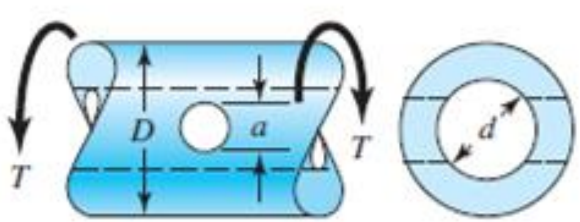
(*b*) The shaft is subjected to a pulsating torque fluctuating from 20 to 160 N · m and a steady bending moment of 150 N · m.



a = 6 mm

d = 34 mm

D = 42 mm



6-14 Combinations of Loading Modes

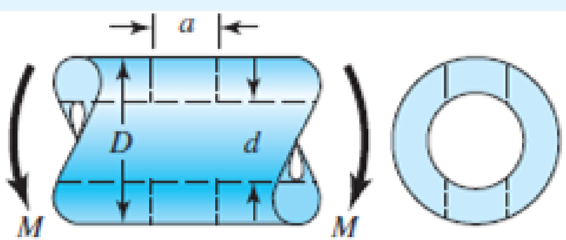
From Table A–20 we find the minimum strengths to be $S_{ut} = 440$ MPa and $S_y = 370$ MPa. The endurance limit of the rotating-beam specimen is $0.5(440) = 220$ MPa.

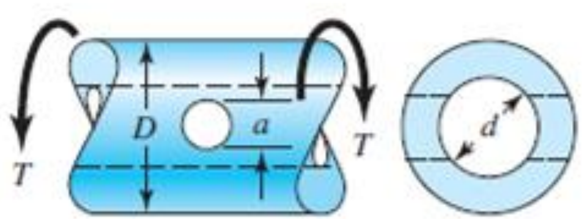
The surface factor, obtained from Eq. (6–19) and Table 6–2, p. 296, is

$$k_a = 4.51 S_{ut}^{-0.265} = 4.51(440)^{-0.265} = 0.899$$

From Eq. (6–20) the size factor is

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{\quad}{7.62}\right)^{-0.107} = 0.833$$





a = 6 mm
d = 34 mm
D = 42 mm

6-14 Combinations of Loading Modes

From Table A–20 we find the minimum strengths to be S_{ut} = 440 MPa and S_y = 370 MPa. The endurance limit of the rotating-beam specimen is 0.5(440) = 220 MPa.

The surface factor, obtained from Eq. (6–19) and Table 6–2, p. 296, is

$$k_a = 4.51 S_{ut}^{-0.265} = 4.51(440)^{-0.265} = 0.899$$

From Eq. (6–20) the size factor is

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{42}{7.62}\right)^{-0.107} = 0.833$$

The remaining Marin factors are all unity, so the modified endurance strength S_e is

$$S_e = 0.899(0.833)220 = 165 \text{ MPa}$$

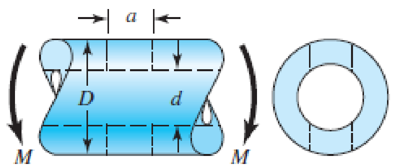
Stress Concentration

Stress Concentration Factor k_t

Table A–16

Approximate Stress-Concentration Factor K_t of a Round Bar or Tube with a Transverse Round Hole and Loaded in Bending

Source: R. E. Peterson, *Stress-Concentration Factors,* Wiley, New York, 1974, pp. 146, 235.



The nominal bending stress is $\sigma_0 = M/Z_{net}$ where Z_{net} is a reduced value of the section modulus and is defined by

$$Z_{net} = \frac{\pi A}{32D}(D^4 - d^4)$$

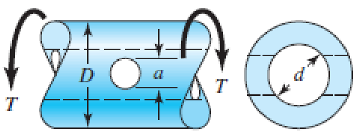
Values of A are listed in the table. Use $d = 0$ for a solid bar

	d/D					
	0.9		0.6		0	
a/D	A	K_t	A	K_t	A	K_t
0.050	0.92	2.63	0.91	2.55	0.88	2.42
0.075	0.89	2.55	0.88	2.43	0.86	2.35
0.10	0.86	2.49	0.85	2.36	0.83	2.27
0.125	0.82	2.41	0.82	2.32	0.80	2.20
0.15	0.79	2.39	0.79	2.29	0.76	2.15
0.175	0.76	2.38	0.75	2.26	0.72	2.10
0.20	0.73	2.39	0.72	2.23	0.68	2.07
0.225	0.69	2.40	0.68	2.21	0.65	2.04
0.25	0.67	2.42	0.64	2.18	0.61	2.00
0.275	0.66	2.48	0.61	2.16	0.58	1.97
0.30	0.64	2.52	0.58	2.14	0.54	1.94

Stress Concentration Factor k_{ts}

Table A–16 *(Continued)*

Approximate Stress-Concentration Factors K_{ts} for a Round Bar or Tube Having a Transverse Round Hole and Loaded in Torsion *Source:* R. E. Peterson, *Stress-Concentration Factors,* Wiley, New York, 1974, pp. 148, 244.



The maximum stress occurs on the inside of the hole, slightly below the shaft surface. The nominal shear stress is $\tau_0 = TD/2J_{net}$, where J_{net} is a reduced value of the second polar moment of area and is defined by

$$J_{net} = \frac{\pi A(D^4 - d^4)}{32}$$

Values of A are listed in the table. Use $d = 0$ for a solid bar.

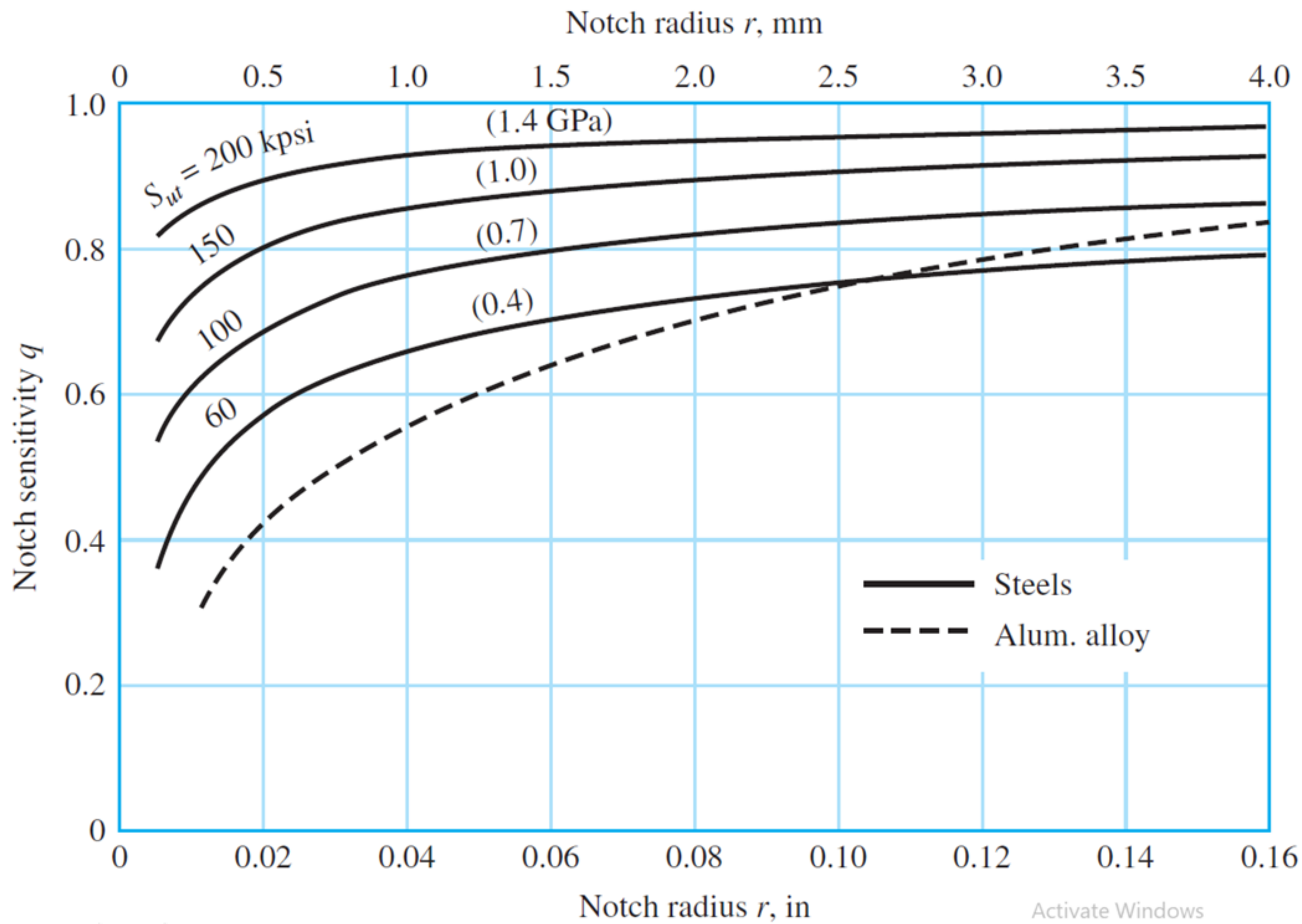
	d/D									
	0.9		0.8		0.6		0.4		0	
a/D	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}
0.05	0.96	1.78							0.95	1.77
0.075	0.95	1.82							0.93	1.71
0.10	0.94	1.76	0.93	1.74	0.92	1.72	0.92	1.70	0.92	1.68
0.125	0.91	1.76	0.91	1.74	0.90	1.70	0.90	1.67	0.89	1.64
0.15	0.90	1.77	0.89	1.75	0.87	1.69	0.87	1.65	0.87	1.62
0.175	0.89	1.81	0.88	1.76	0.87	1.69	0.86	1.64	0.85	1.60
0.20	0.88	1.96	0.86	1.79	0.85	1.70	0.84	1.63	0.83	1.58
0.25	0.87	2.00	0.82	1.86	0.81	1.72	0.80	1.63	0.79	1.54
0.30	0.80	2.18	0.78	1.97	0.77	1.76	0.75	1.63	0.74	1.51
0.35	0.77	2.41	0.75	2.09	0.72	1.81	0.69	1.63	0.68	1.47
0.40	0.72	2.67	0.71	2.25	0.68	1.89	0.64	1.63	0.63	1.44

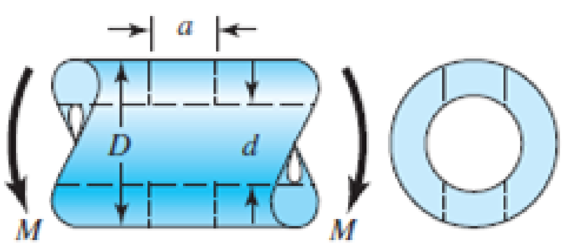
Stress Concentration

Notch Sensitivity q

Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the r = 0.16-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*





Notch Radius
3 mm

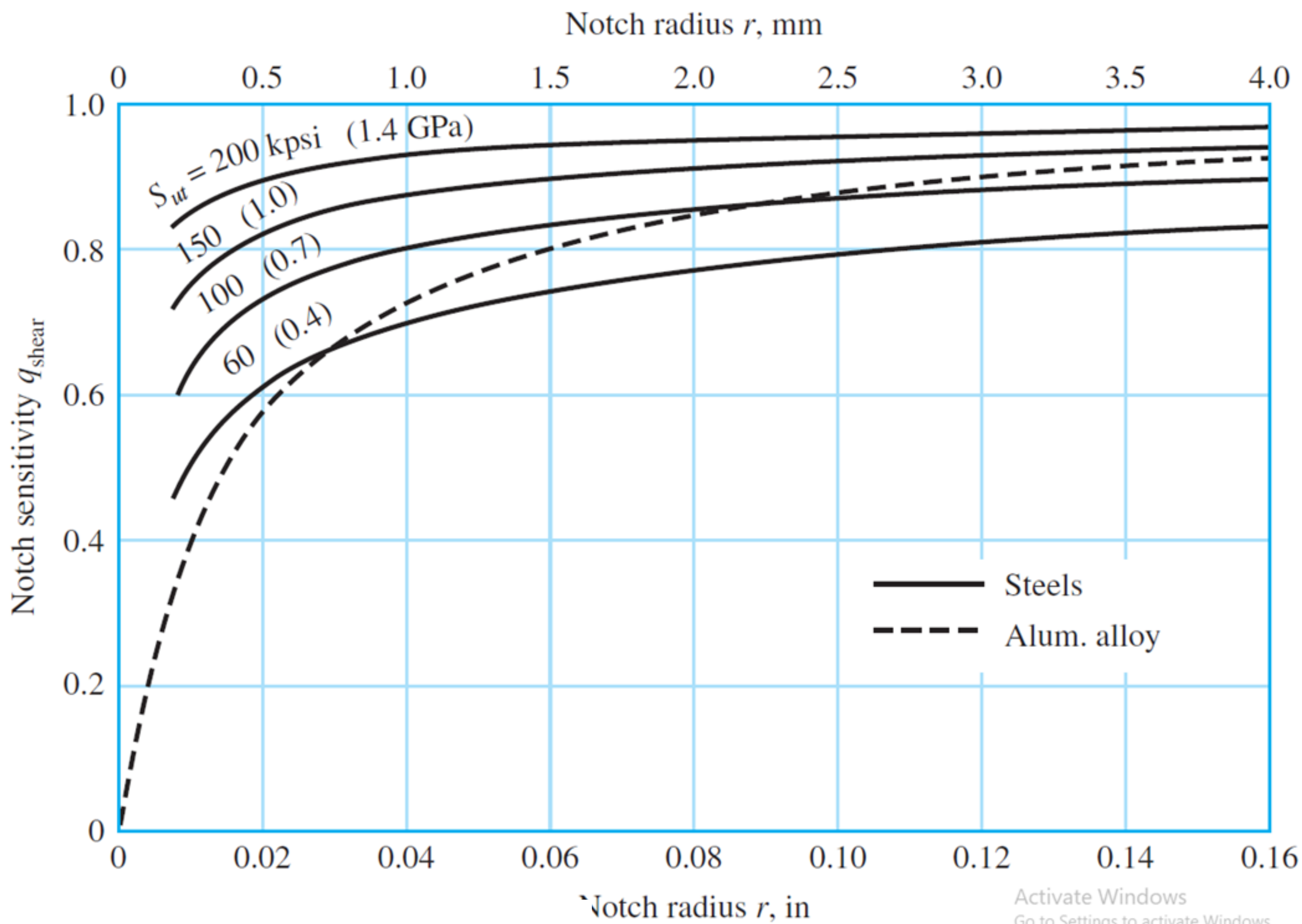
a = 6 mm
d = 34 mm
D = 42 mm

Stress Concentration

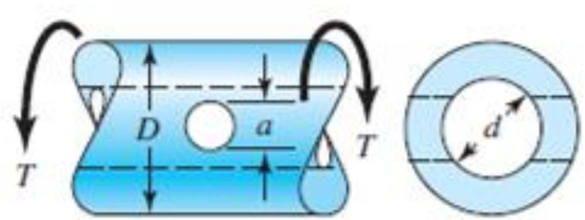
Notch Sensitivity q_s

Figure 6–21

Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of q_{shear} corresponding to $r = 0.16$ in (4 mm).



Notch Radius
3 mm



a = 6 mm
d = 34 mm
D = 42 mm

6-14 Combinations of Loading Modes

Fatigue Stress Concentration factors

$$K_f = 1 + q(K_t - 1) = 1 + 0.78(2.366 - 1) = 2.07$$

$$K_{fs} = 1 + 0.81(1.75 - 1) = 1.61$$

Thus, for bending,

$$Z_{\text{net}} = \frac{\pi A}{32D}(D^4 - d^4) = \frac{\pi(0.798)}{32(42)}[(42)^4 - (34)^4] = 3.31(10^3)\,\text{mm}^3$$

and for torsion

$$J_{\text{net}} = \frac{\pi A}{32}(D^4 - d^4) = \frac{\pi(0.89)}{32}[(42)^4 - (34)^4] = 155(10^3)\,\text{mm}^4$$

6-14 Combinations of Loading Modes

The alternating bending stress is now found to be

$$\sigma_{xa} = K_f \frac{M}{Z_{\text{net}}} = 2.07 \frac{150}{3.31(10^{-6})} = 93.8(10^6)\,\text{Pa} = 93.8\ \text{MPa}$$

and the alternating torsional stress is

$$\tau_{xya} = K_{fs} \frac{TD}{2J_{\text{net}}} = 1.61 \frac{120(42)(10^{-3})}{2(155)(10^{-9})} = 26.2(10^6)\,\text{Pa} = 26.2\ \text{MPa}$$

The midrange von Mises component σ'_m is zero. The alternating component σ'_a is given by

$$\sigma'_a = (\sigma_{xa}^2 + 3\tau_{xya}^2)^{1/2} = [93.8^2 + 3(26.2^2)]^{1/2} = 104.2\ \text{MPa}$$

6-14 Combinations of Loading Modes

$$\sigma' = (\sigma_x^2 + 3\tau_{xy}^2)^{1/2}.$$

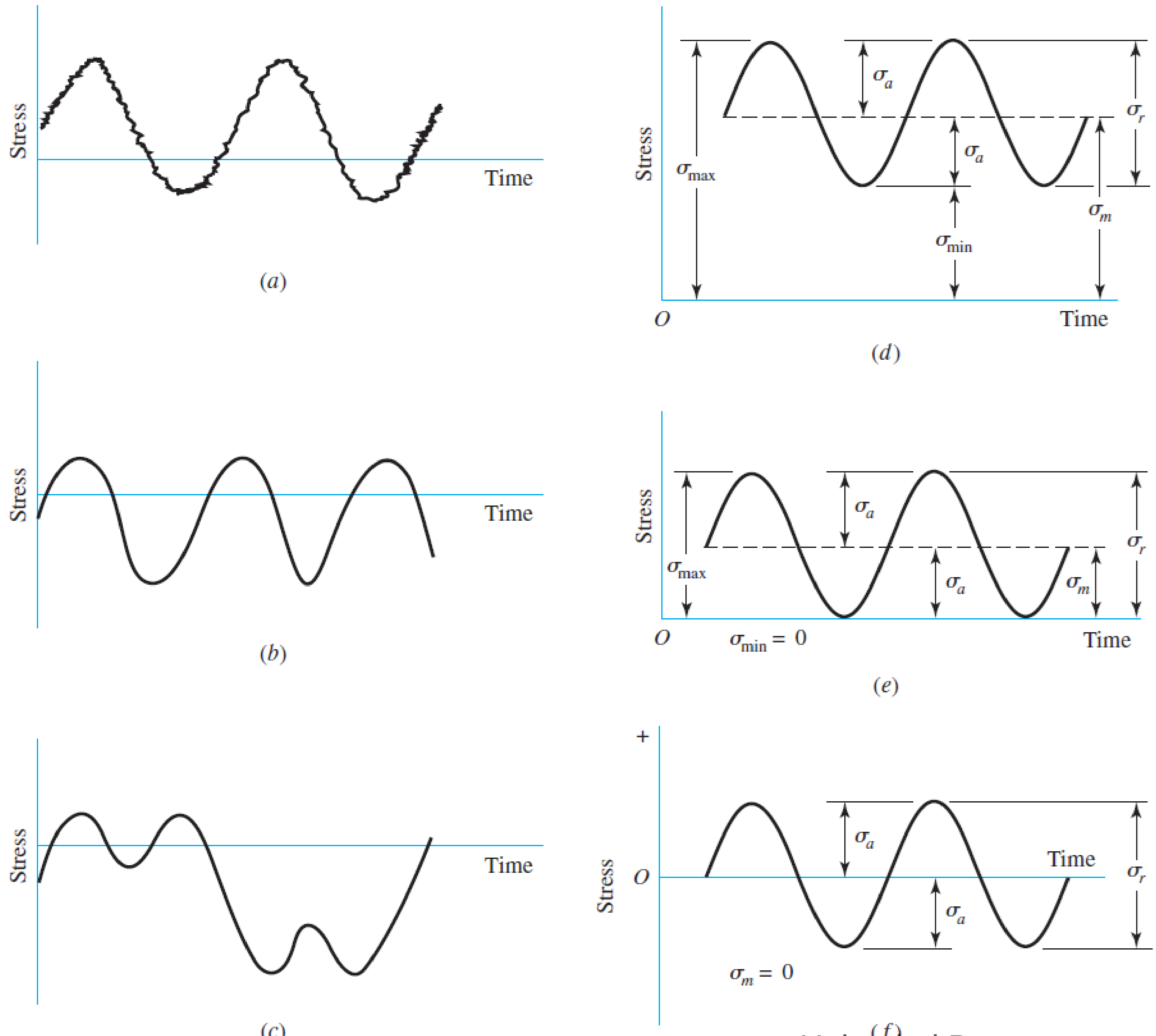
$$\sigma'_a = \left\{\left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}}\frac{(\sigma_a)_{\text{axial}}}{0.85}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}}\right]^2\right\}^{1/2} \tag{6–55}$$

$$\sigma'_m = \left\{\left[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}}\right]^2\right\}^{1/2} \tag{6–56}$$

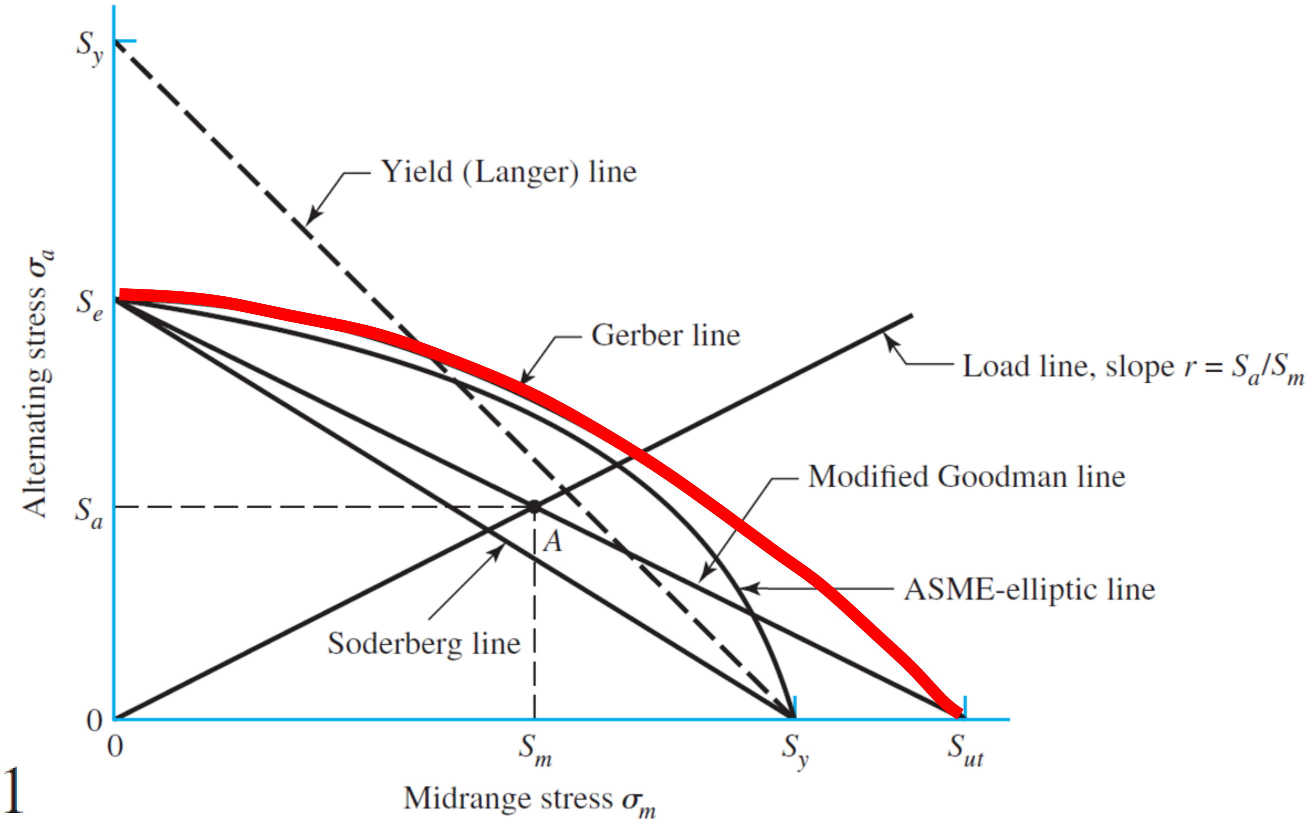
6 – 11 Characterizing Fluctuating Stresses

Figure 6–23

Some stress-time relations: (*a*) fluctuating stress with high-frequency ripple; (*b* and *c*) nonsinusoidal fluctuating stress; (*d*) sinusoidal fluctuating stress; (*e*) repeated stress; (*f*) completely reversed sinusoidal stress.



Fatigue Failure Criteria for Fluctuating Stress



Gerber line

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

6-14 Combinations of Loading Modes

The midrange von Mises component σ'_m is zero. The alternating component σ'_a is given by

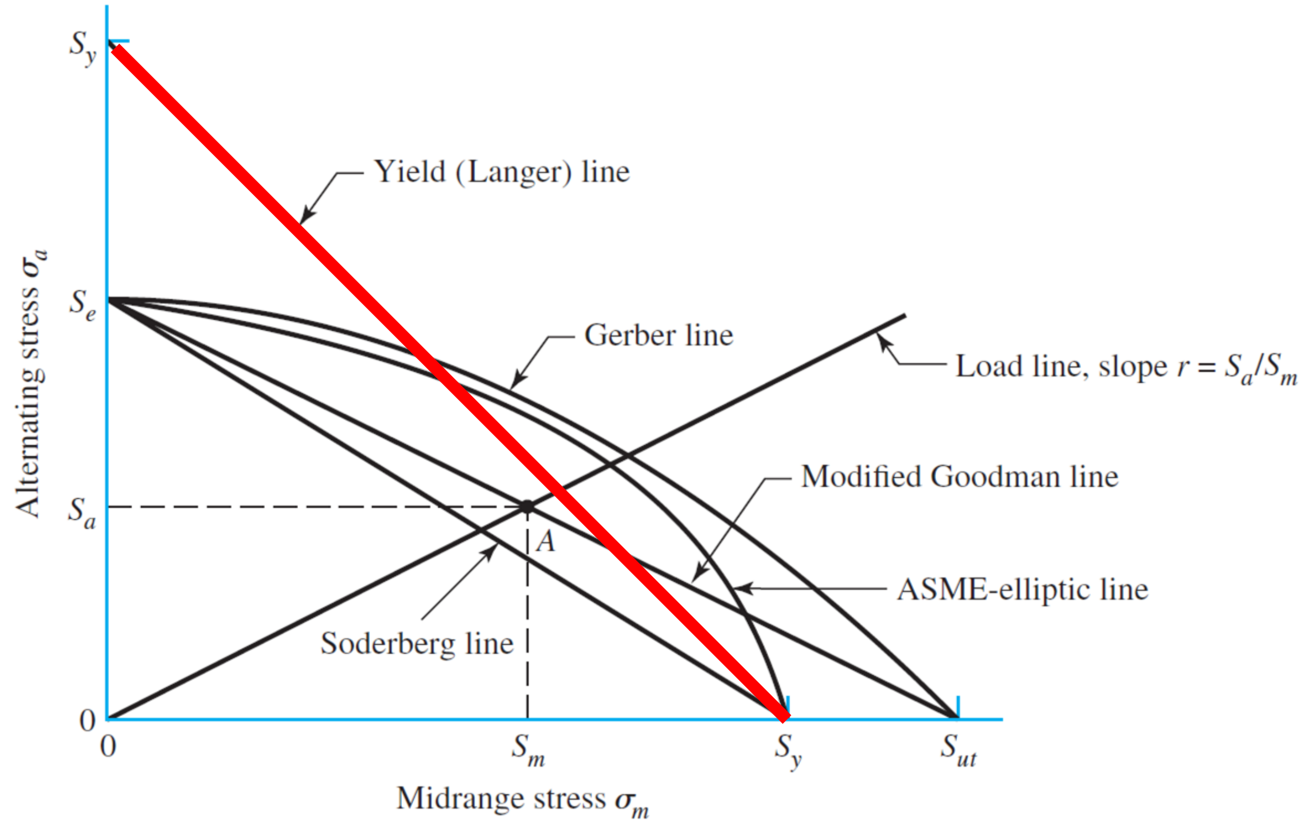
$$\sigma'_a = (\sigma_{xa}^2 + 3\tau_{xya}^2)^{1/2} = [93.8^2 + 3(26.2^2)]^{1/2} = 104.2 \text{ MPa}$$

Since $S_a = S_e$, the fatigue factor of safety n_f is

$$n_f = \frac{S_a}{\sigma'_a} = \frac{165}{104.2} = 1.58$$

Fatigue Failure Criteria for Fluctuating Stress

Langer First Cycle Yielding



$$\sigma_a + \sigma_m = \frac{S_y}{n} \qquad S_a + S_m = S_y$$

6-14 Combinations of Loading Modes

BIRZEIT UNIVERSITY

The midrange von Mises component σ'_m is zero. The alternating component σ'_a is given by

$$\sigma'_a = (\sigma^2_{xa} + 3\tau^2_{xya})^{1/2} = [93.8^2 + 3(26.2^2)]^{1/2} = 104.2 \text{ MPa}$$

Since $S_a = S_e$, the fatigue factor of safety n_f is

$$n_f = \frac{S_a}{\sigma'_a} = \frac{165}{104.2} = 1.58$$

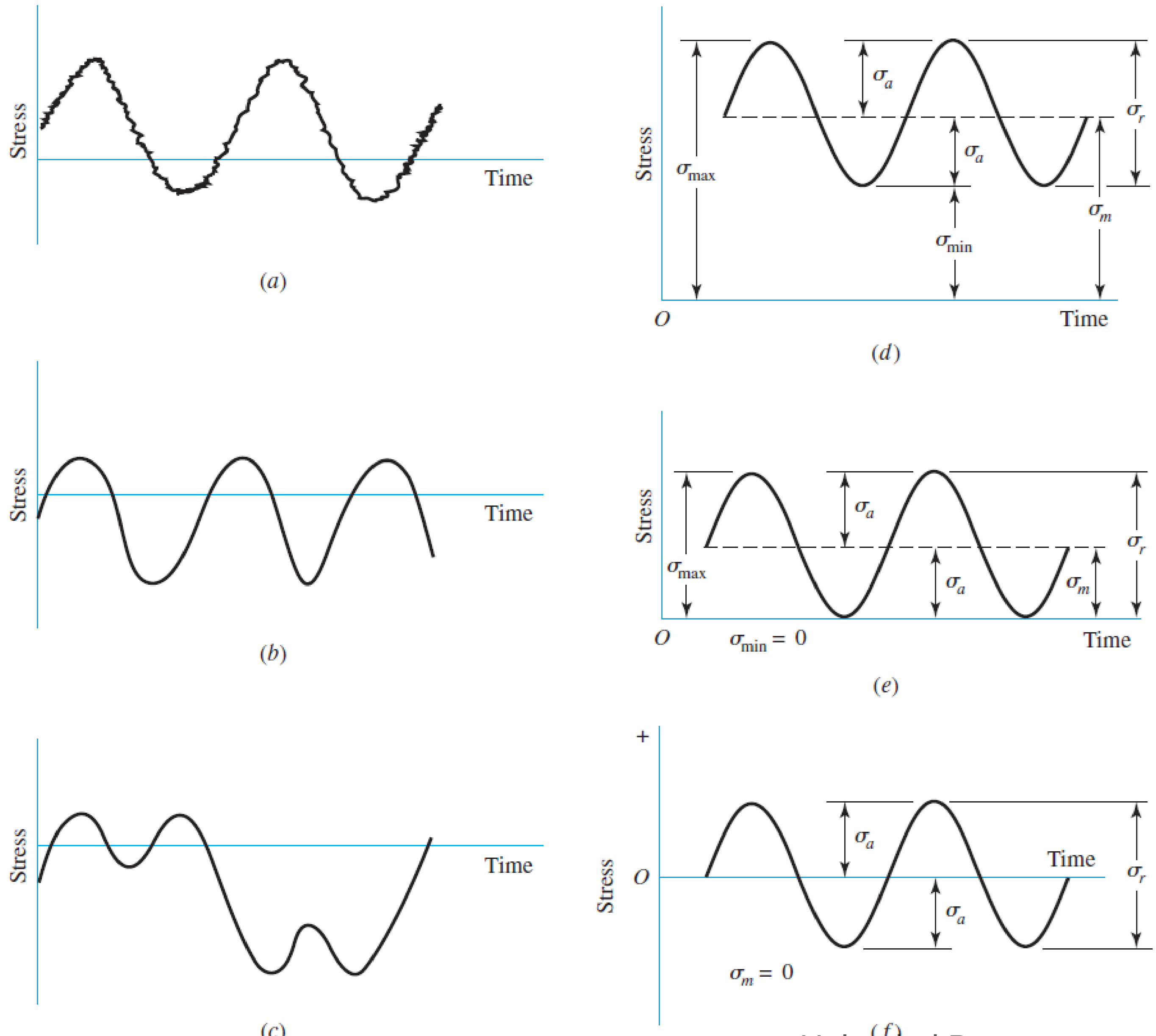
The first-cycle yield factor of safety is

$$n_y = \frac{S_y}{\sigma'_a} = \frac{370}{105.6} = 3.50$$

6 – 11 Characterizing Fluctuating Stresses

Figure 6–23

Some stress-time relations: (*a*) fluctuating stress with high-frequency ripple; (*b* and *c*) nonsinusoidal fluctuating stress; (*d*) sinusoidal fluctuating stress; (*e*) repeated stress; (*f*) completely reversed sinusoidal stress.



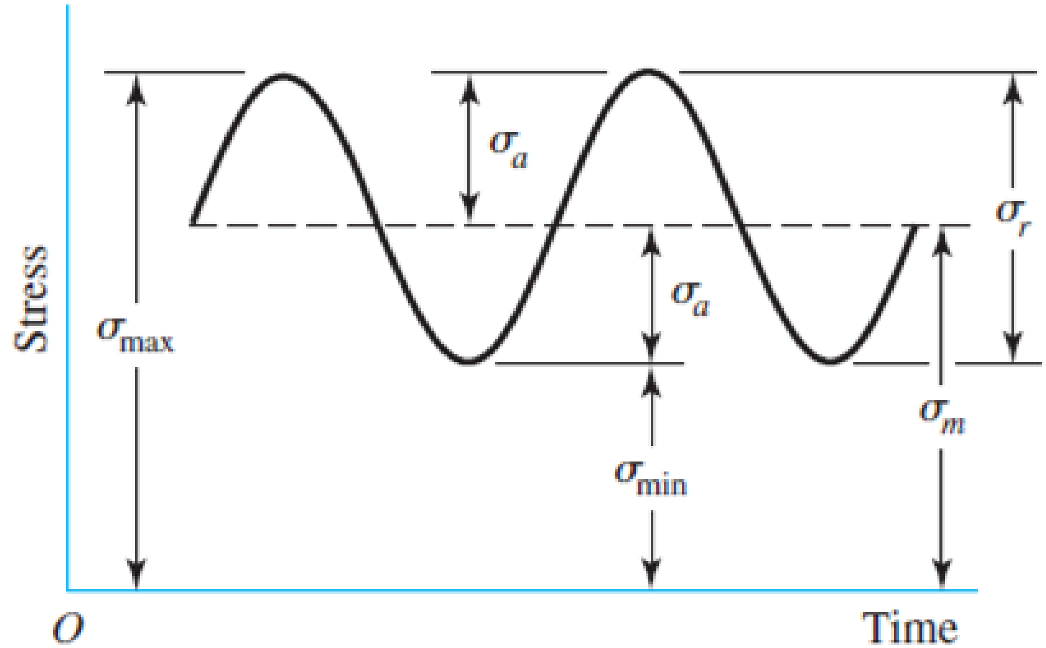
6 – 11 Characterizing Fluctuating Stresses

$$F_m = \frac{F_{\max} + F_{\min}}{2} \qquad F_a = \left| \frac{F_{\max} - F_{\min}}{2} \right|$$

$\sigma_{\min}$ = minimum stress

$\sigma_{\max}$ = maximum stress

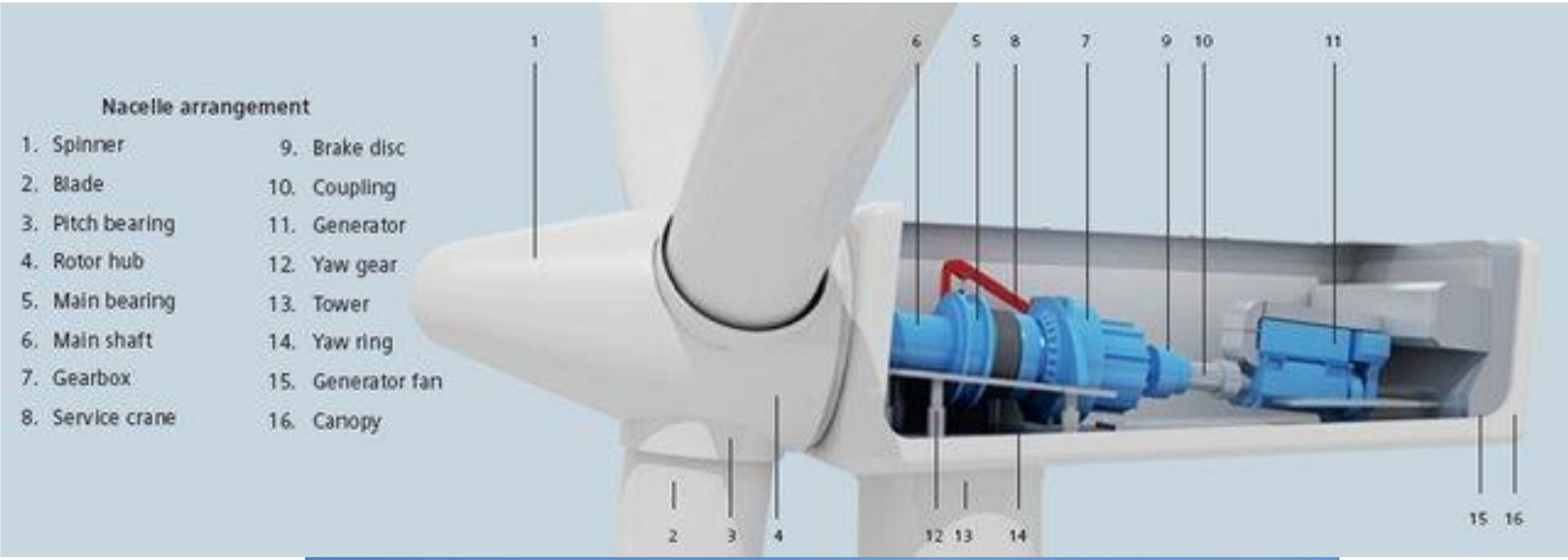
σ_a = amplitude component



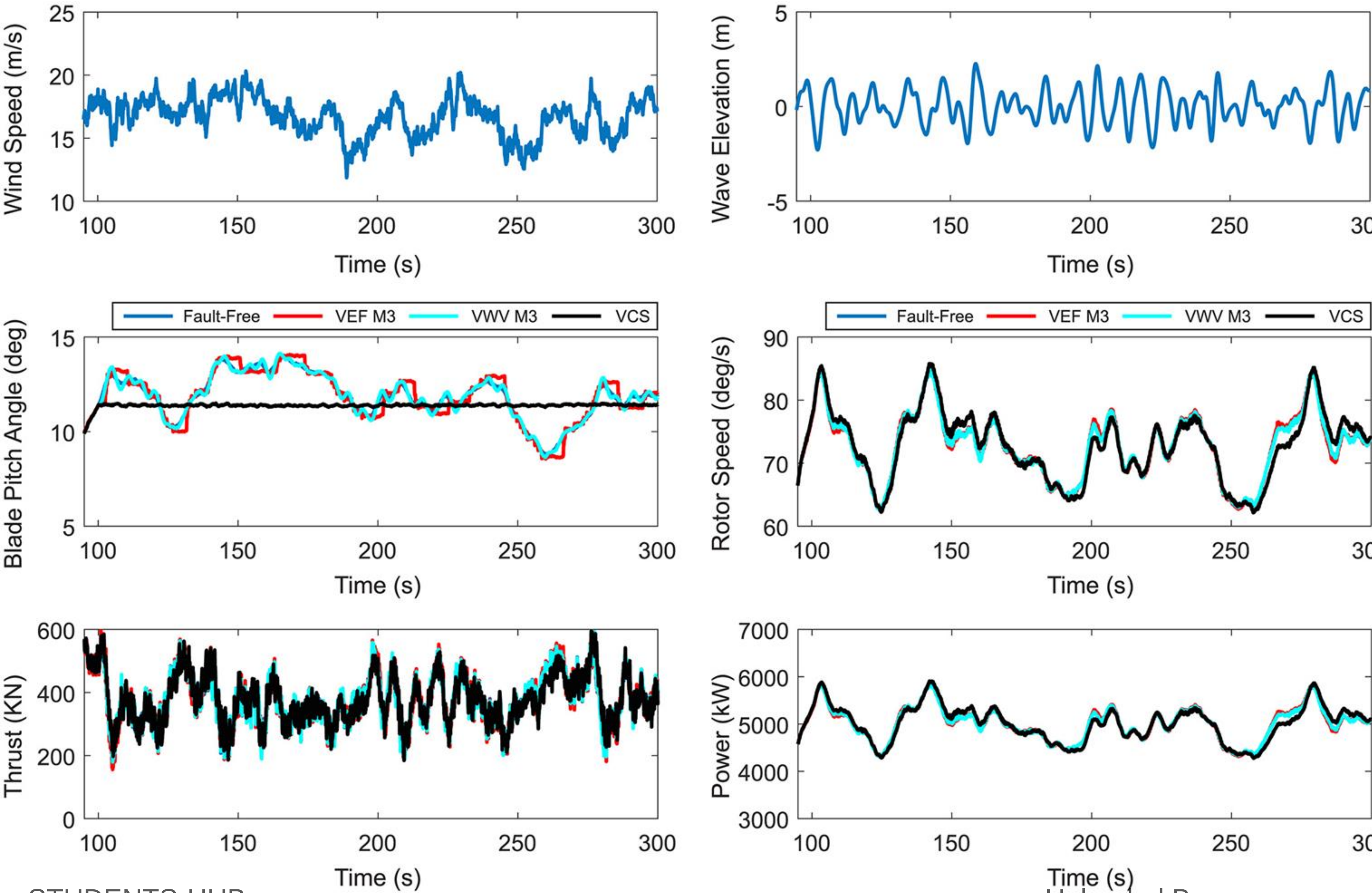
$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

$$\sigma_a = \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right|$$

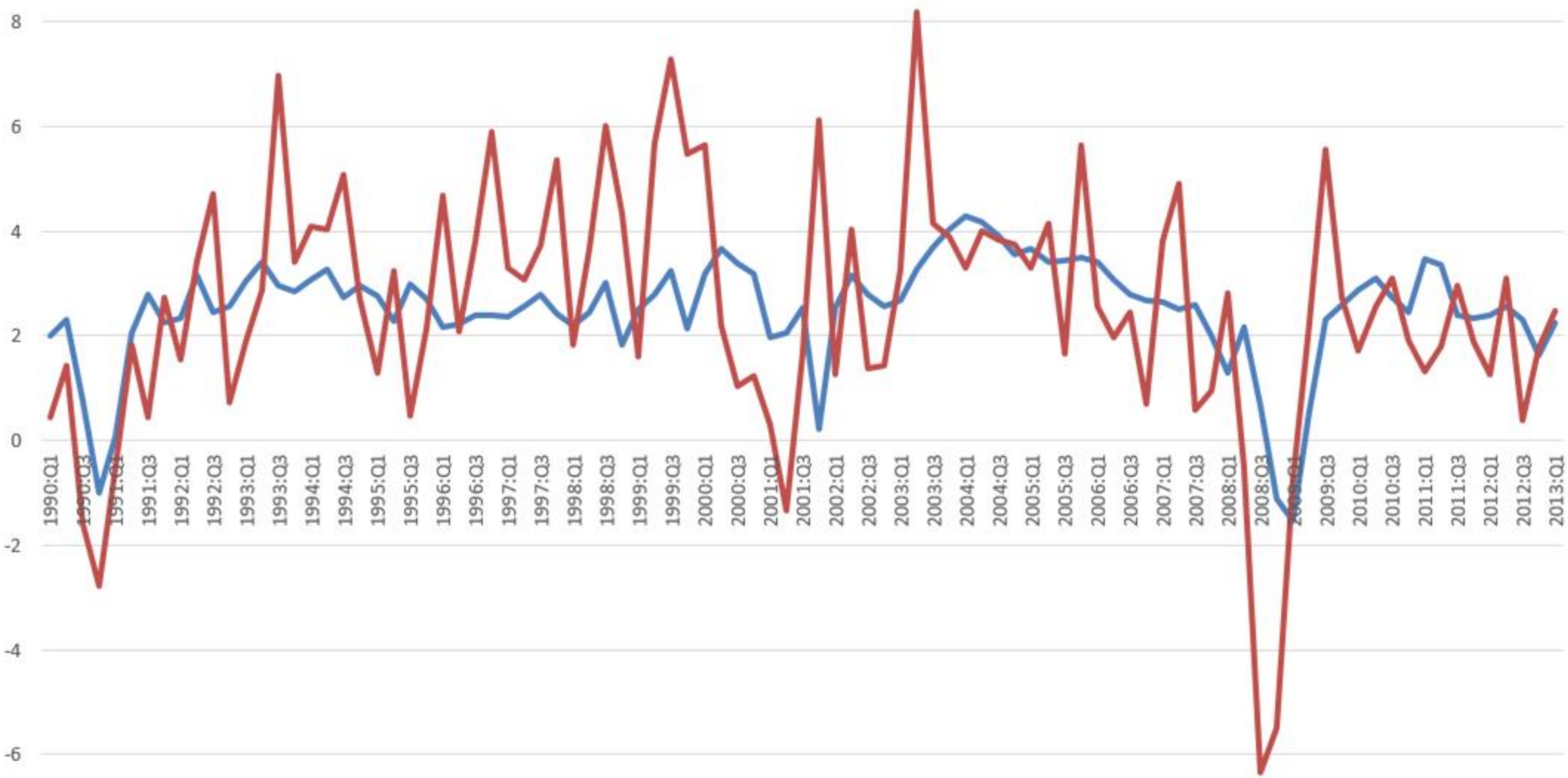
6 – 11 Characterizing Fluctuating Stresses


Nacelle arrangement
1. Spinner
2. Blade
3. Pitch bearing
4. Rotor hub
5. Main bearing
6. Main shaft
7. Gearbox
8. Service crane
9. Brake disc
10. Coupling
11. Generator
12. Yaw gear
13. Tower
14. Yaw ring
15. Generator fan
16. Canopy

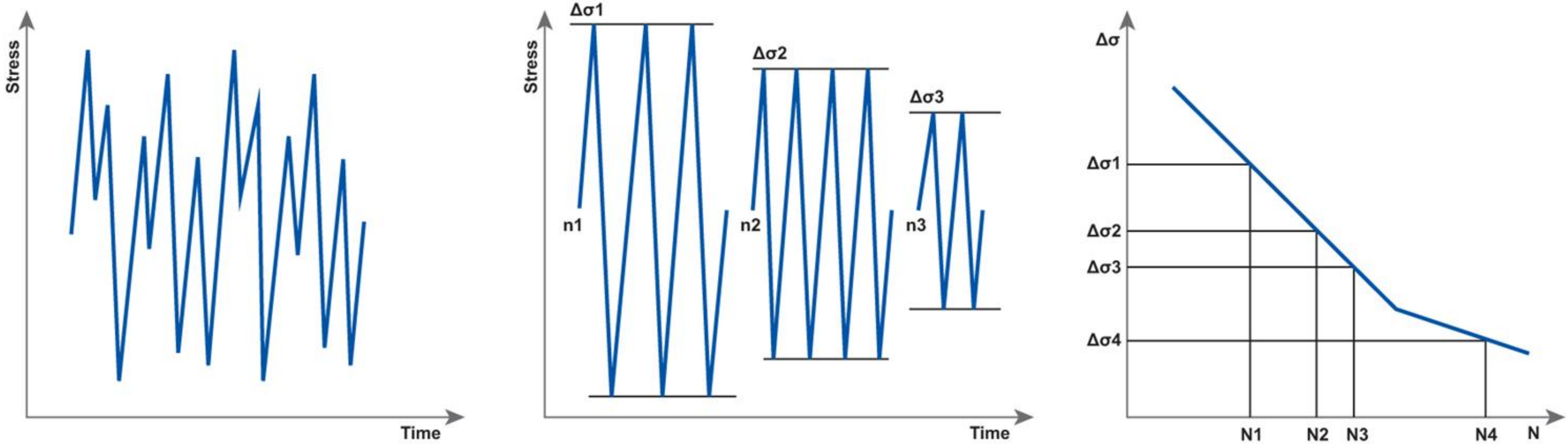
6 – 11 Characterizing Fluctuating Stresses



6-15 Varying, Fluctuating Stresses; Cumulative Fat. Damage

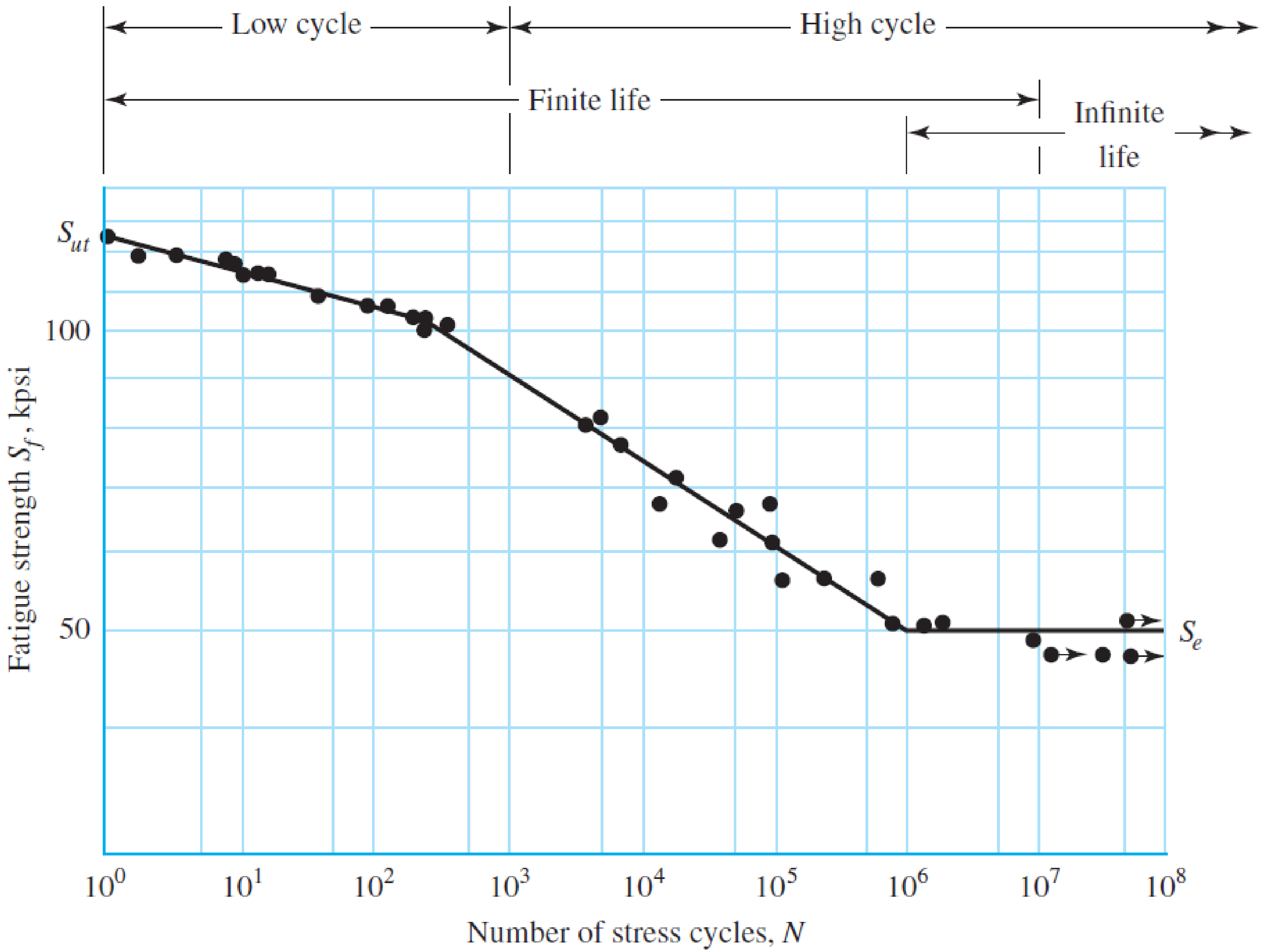


6-15 Varying, Fluctuating Stresses; Cumulative Fat. Damage



$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots \frac{n_i}{N_i} = \sum_{i=1}^{i=\infty} \frac{n_i}{N_i} = 1$$

6-4 The Stress-Life Method



Static Loading Solving Problems

Chapter 6: 12, 13, 14, 15, 16, 17, 18