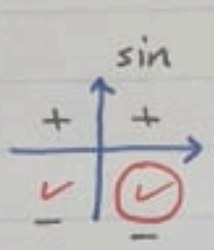
Discussion 7.6

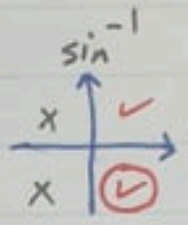
12 $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \cot\left(-\frac{\pi}{3}\right) = \frac{\cos\left(-\frac{\pi}{3}\right)}{\sin\left(-\frac{\pi}{3}\right)} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$= \frac{-1}{\sqrt{3}}$

$\theta = \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

$\sin\theta = -\frac{\sqrt{3}}{2}$





$\theta = -\frac{\pi}{3}$

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33 Find y' if $y = \ln(\tan^{-1}x)$

$y' = \frac{dy}{dx} = \frac{\frac{1}{1+x^2}}{\tan^{-1}x} = \frac{1}{(1+x^2)\tan^{-1}x}$

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63 $\int_0^{\ln\sqrt{3}} \frac{e^x\,dx}{1+e^{2x}}$

$u = e^x$

$du = e^x\,dx$

$u^2 = e^{2x}$

when $x = 0 \Rightarrow u = 1$

$x = \ln\sqrt{3} \Rightarrow u = e^{\ln\sqrt{3}} = \sqrt{3}$

$\int_1^{\sqrt{3}} \frac{du}{1+u^2}$

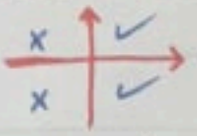
$\tan^{-1}u \Big|_1^{\sqrt{3}} = \tan^{-1}\sqrt{3} - \tan^{-1}1$

$= \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{\pi}{12}$

$\theta = \tan^{-1}\sqrt{3}$

$\tan\theta = \sqrt{3}$



$\theta = \tan^{-1}1$

$\tan\theta = 1$

75 $\int \frac{x+4}{x^2+4}\,dx$

$= \int \frac{x}{x^2+4}\,dx + \int \frac{4}{x^2+4}\,dx$

$u = x^2+4$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

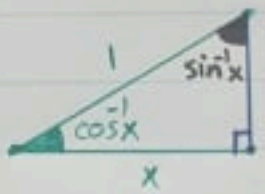
$= \frac{1}{2}\int \frac{2x}{x^2+4}\,dx + 4\int \frac{dx}{x^2+4}$ $\quad a=2$

$= \frac{1}{2}\ln|x^2+4| + 4\left(\frac{1}{2}\right)\tan^{-1}\left(\frac{x}{2}\right) + C$

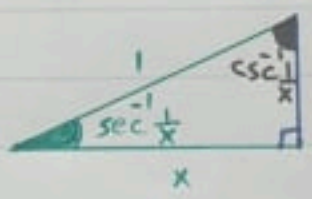
$= \frac{1}{2}\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + C$

Remarks

$\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$



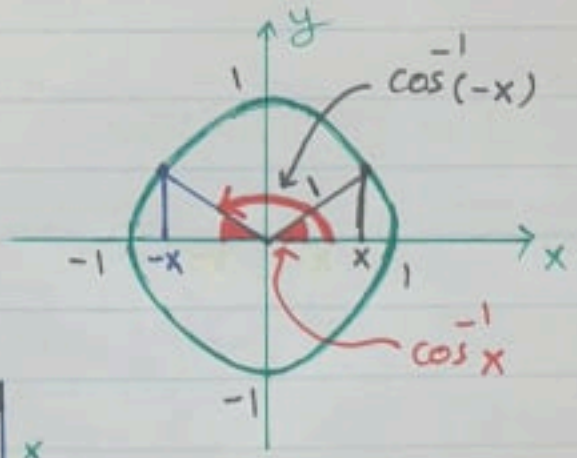
$\sec^{-1}\frac{1}{x} + \csc^{-1}\frac{1}{x} = \frac{\pi}{2}$



Note that $\sec^{-1}x = \cos^{-1}\frac{1}{x} = \frac{\pi}{2} - \sin^{-1}\frac{1}{x}$

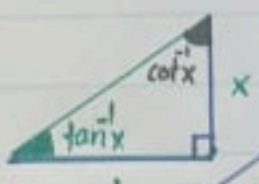
$\csc^{-1}x = \sin^{-1}\frac{1}{x} = \frac{\pi}{2} - \cos^{-1}\frac{1}{x}$

$\cos^{-1}x + \cos^{-1}(-x) = \pi$

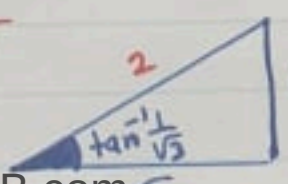


$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

$\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}$



$\tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$



$\sin^{-1}\left(\sin\frac{3\pi}{4}\right) \neq \frac{3\pi}{4}$

$\sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

Exp Find the domain of $f(x) = \sin^{-1}(\ln x)$

$-1 \le \ln x \le 1 \Rightarrow e^{-1} \le e^{\ln x} \le e^{1} \Rightarrow \frac{1}{e} \le x \le e$

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59 $\int \frac{dx}{(2x-1)\sqrt{(2x-1)^2-4}}$

$u = 2x-1$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\frac{1}{2}\int \frac{du}{u\sqrt{u^2-4}} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\sec^{-1}\left|\frac{u}{2}\right| + C$

$= \frac{1}{4}\sec^{-1}\left|\frac{2x-1}{2}\right| + C$

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70 $\int_{\frac{1}{2}}^{1} \frac{6\,dt}{\sqrt{3+4t-4t^2}}$

$3+4t-4t^2$

$= -(4t^2-4t-3)$

$= -[(2t-1)^2-4]$

$= 4-(2t-1)^2$

$\int_{\frac{1}{2}}^{1} \frac{6\,dt}{\sqrt{4-(2t-1)^2}}$

$3\int_{\frac{1}{2}}^{1} \frac{2\,dt}{\sqrt{2^2-(2t-1)^2}}$

$u = 2t-1$

$du = 2\,dt$

$t = \frac{1}{2} \Rightarrow u = 0$

$t = 1 \Rightarrow u = 1$

$3\int_{0}^{1} \frac{du}{\sqrt{2^2-u^2}}$

$3\sin^{-1}\left(\frac{u}{2}\right)\Big|_0^1 = 3\left[\sin^{-1}\frac{1}{2} - \sin^{-1}0\right]$

$= 3\left(\frac{\pi}{6} - 0\right)$

$= \frac{\pi}{2}$