3.2 Measures of Variability

[1] Range [2] Interquartile Range [3] Variance [4] Standard deviation [5] Coefficient of Variation.

[1] **Range:** The simplest measure of variability is the range.

Range = largest value − Smallest value.

[2] **Interquartile Range:** is a measure of variability that overcomes the dependency on extreme values.

↓ used to measure the variability when extrem values are present in the data.

Interquartile Range (IQR) = $Q_3 - Q_1$

Q_3: third quartile (75^{th} percentile)

Q_1: 1^{st} quartile (25^{th} percentile)

Note that IQR is the range for the middle 50% of the data.

[3] **Variance:** is a measure of variability that utilizes all the data.

$$\text{Population variance } \sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$$

μ: population mean

N: population size.

estimator

$$\text{Sample variance } s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

$\bar{x}$: sample mean

n: sample size.

we divid by n-1 to make estimate that is unbaised for the populat

- Note that $\sum_{i=1}^{N} (x_i - \mu)$ or $\sum_{i=1}^{n} (x_i - \bar{x})$ is called the deviation about the mean and it is always zero.
- Thus, the variance is the average of the square deviation.
- The variance is useful in comparing the variability of two or more variables.

[4] **standard deviation** : is positive square root of the variance. (measures the variability)

Sample standard deviation = $s = \sqrt{s^2}$ ⟩ estimator

Population standard deviation = $\sigma = \sqrt{\sigma^2}$

[5] **Coefficient of Variation** : To indicate how large the standard deviation is relative to the mean.

$$\text{Coefficient of variation} = \left(\frac{\text{Standard deviation}}{\text{Mean}} \times 100\right)\%$$

Example: (Q13, Q14 page 95) Consider a sample with data values of 10, 20, 12, 17, 16 Find

[a] Range = largest value − smallest value

= 20 − 10 = 10

[b] Interquartile Range (IQR) = $Q_3 - Q_1$

First we order the data ascending

10 12 16 17 20

Q_3 (75^{th} percentile) $i = \left(\frac{75}{100}\right) \times 5 = 3.75$

i is rounded up to 4^{th} position

$Q_3 = 17$

Q_1 (25^{th} percentile) $i = \left(\frac{25}{100}\right) \times 5 = 1.25$

i is rounded up to 2^{nd} position

$Q_1 = 12$

$IQR = Q_3 - Q_1 = 17 - 12 = 5$

[c] Variance : The sample variance is given by

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})}{n-1}$$

. Note that $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{5} x_i}{5}$

$$= \frac{10+20+12+17+16}{5} = \frac{75}{5} = 15$$

x_i	$x_i - 15$	$(x_i - 15)^2$
10	$10-15 = -5$	$(-5)^2 = 25$
20	$20-15 = 5$	$(5)^2 = 25$
12	$12-15 = -3$	$(-3)^2 = 9$
17	$17-15 = 2$	$(2)^2 = 4$
16	$16-15 = 1$	$(1)^2 = 1$
	Total = 0	$\sum (x_i-15)^2 = 64$

$\sum (x_i - \bar{x}) = 0$

$$s^2 = \frac{\sum_{i=1}^{5}(x_i-15)^2}{5-1} = \frac{64}{4}$$

$s^2 = 16$ sample variance

[d] standard deviation: The sample standard deviation is $s = \sqrt{s^2} = \sqrt{16} = 4$

[e] Coefficient of variations :

Coefficient of variation $= \left(\frac{\text{sample standard deviation}}{\text{sample Mean}} \times 100\right)\%$

$$= \left(\frac{s}{\bar{x}} \times 100\right)\%$$

$$= \left(\frac{4}{15} \times 100\right)\%$$

$= 26.67\%$ This means that the sample standard deviation is 26.67% of the value of the sample mean.