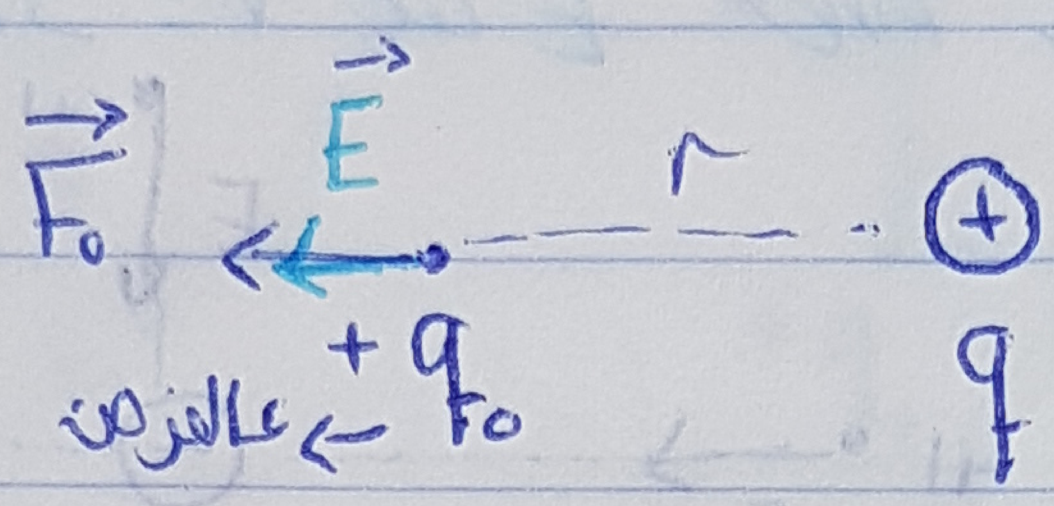
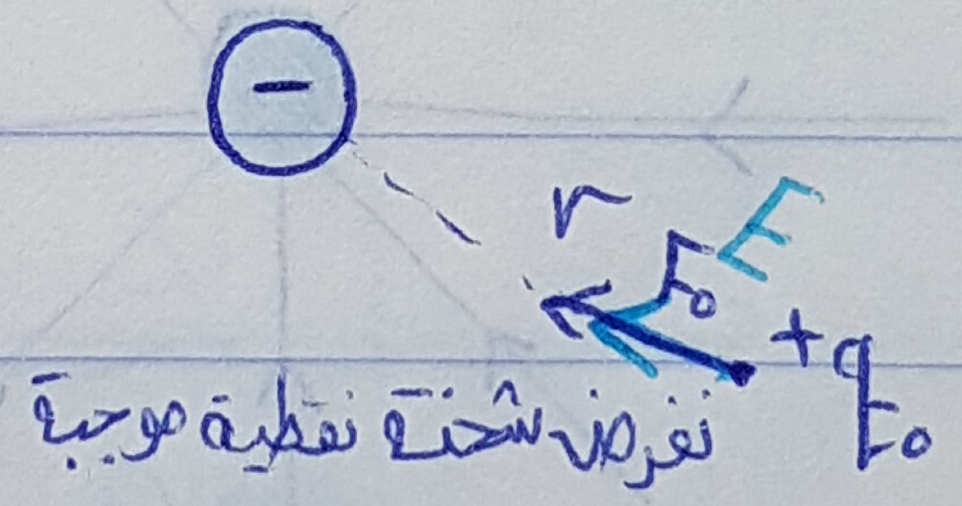
Chapter 22: Electric Fields.

electric fields:



خطوط المجال تبتعد عن الشحنة الموجبة

$$F_o = \frac{K q q_o}{r^2} \; N \Rightarrow \frac{F_o}{q_o} = \boxed{\frac{Kq}{r^2} = E} \; (N/C)$$



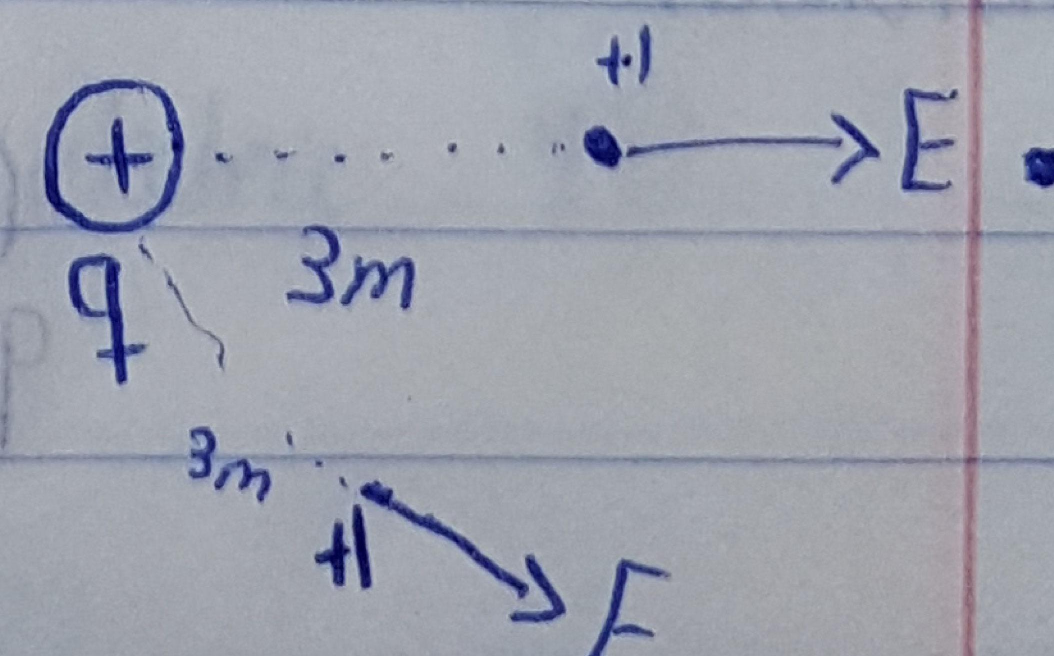
خطوط المجال تدخل في الشحنة السالبة

electric field: is the electric force acting on $+1C$

(e.g) $q = 5\mu C$. Find E at $r = 3m$.

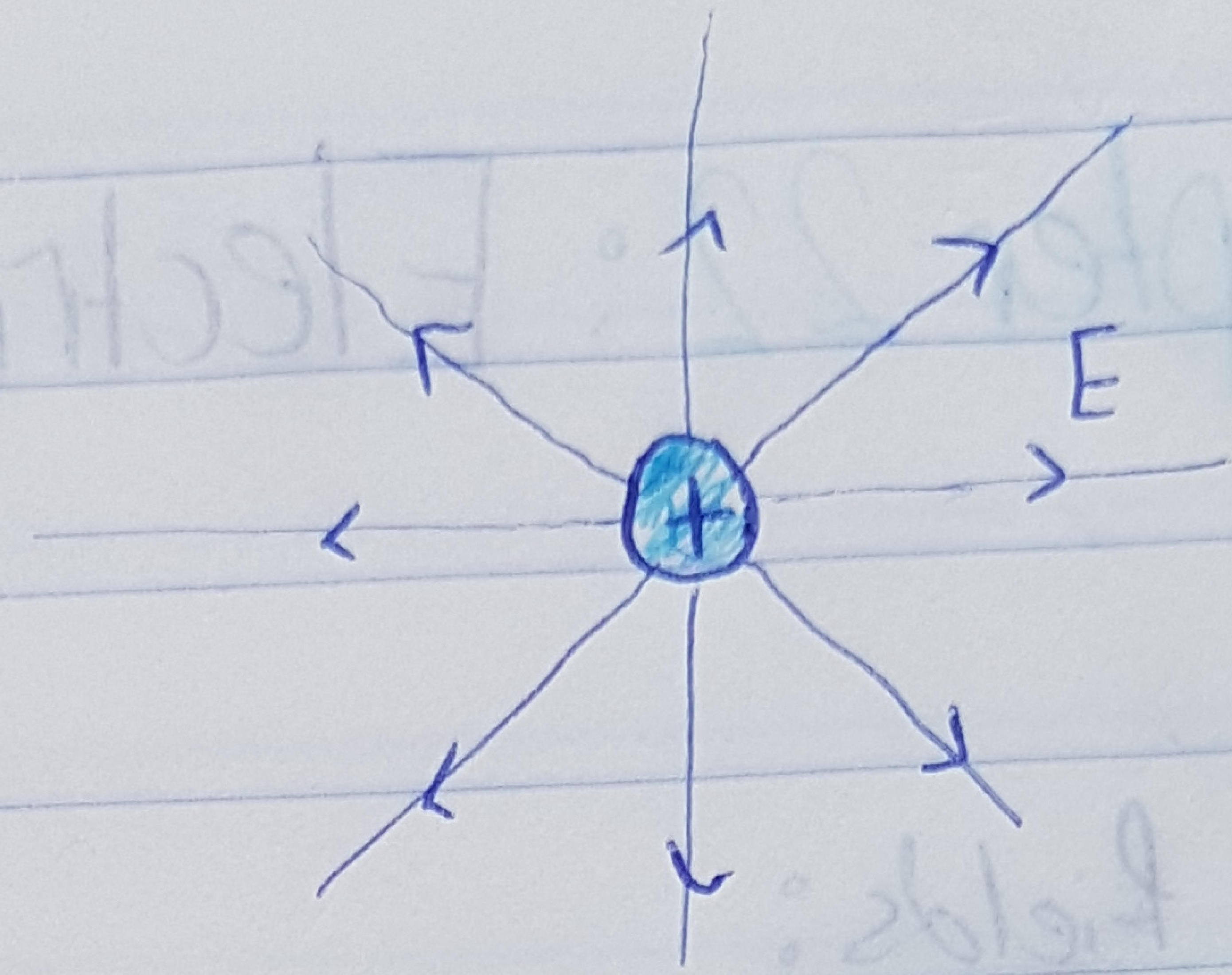
$$E = \frac{9\times10^{9} \times 5\times10^{-6}}{(3)^2}$$

$$\boxed{E = 5\times10^{3} \; N/C}$$

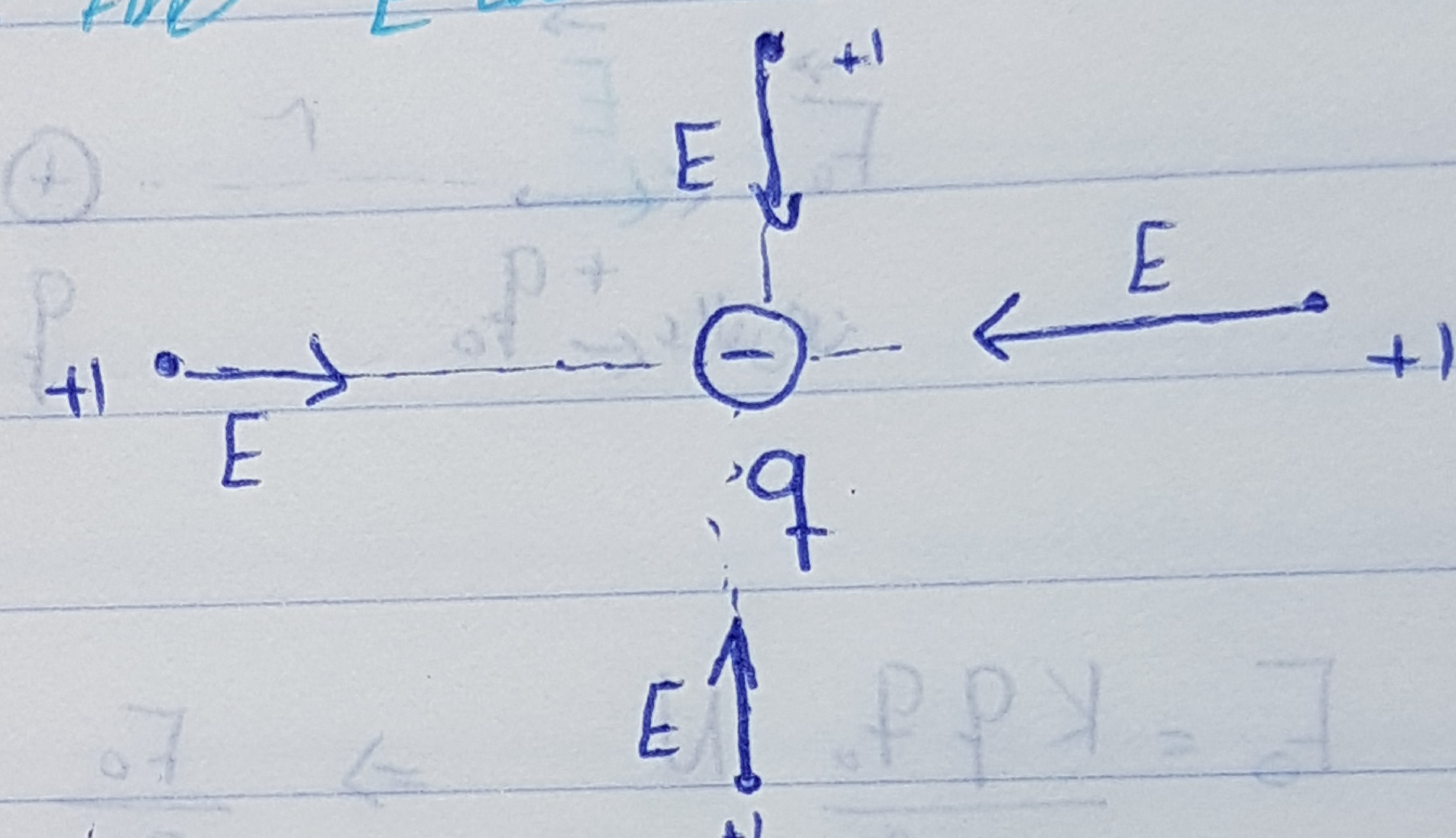


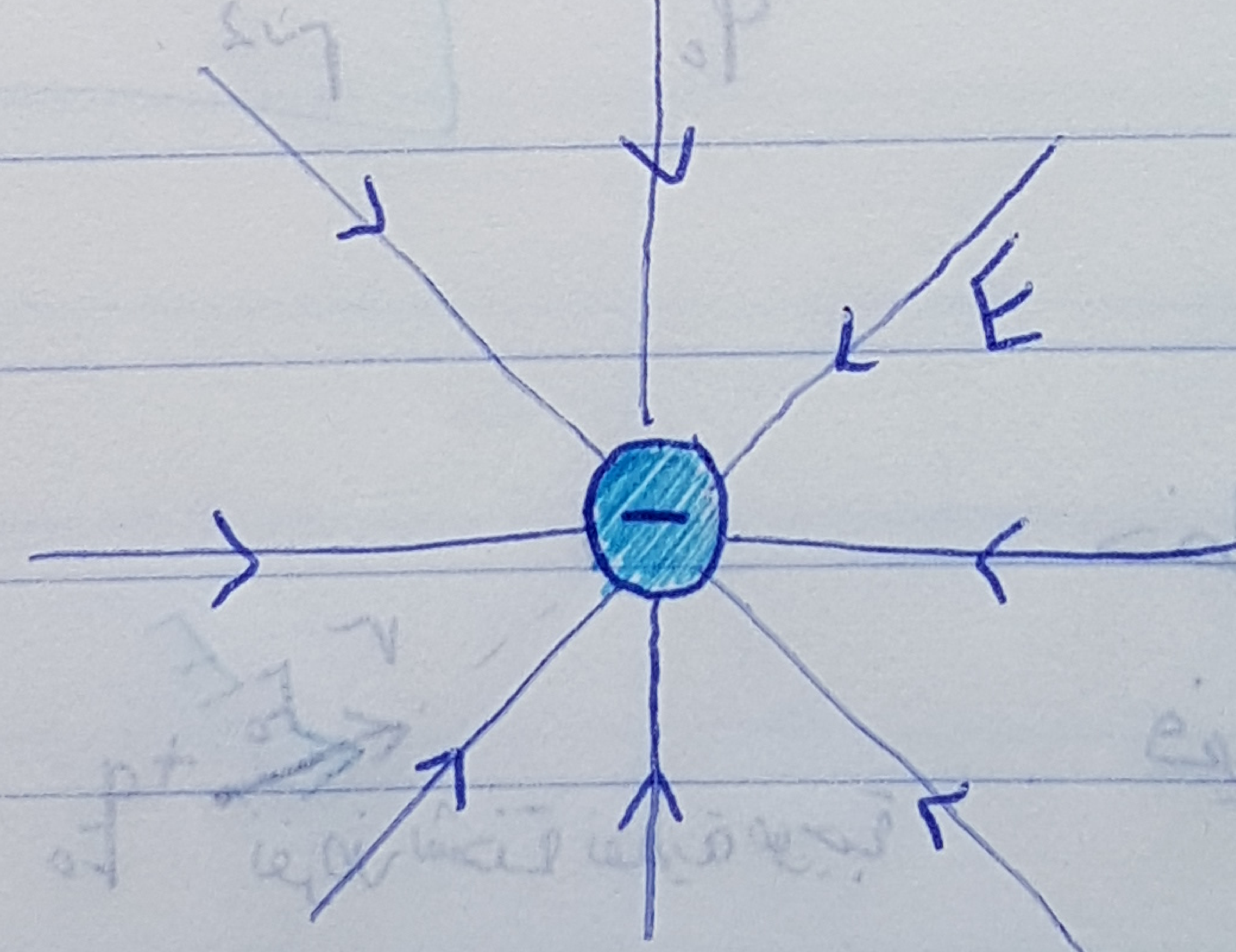
كل النقاط التي تبعد 3m عن الشحنة فيها نفس E بالمقدار وتختلف بالاتجاه

non uniform



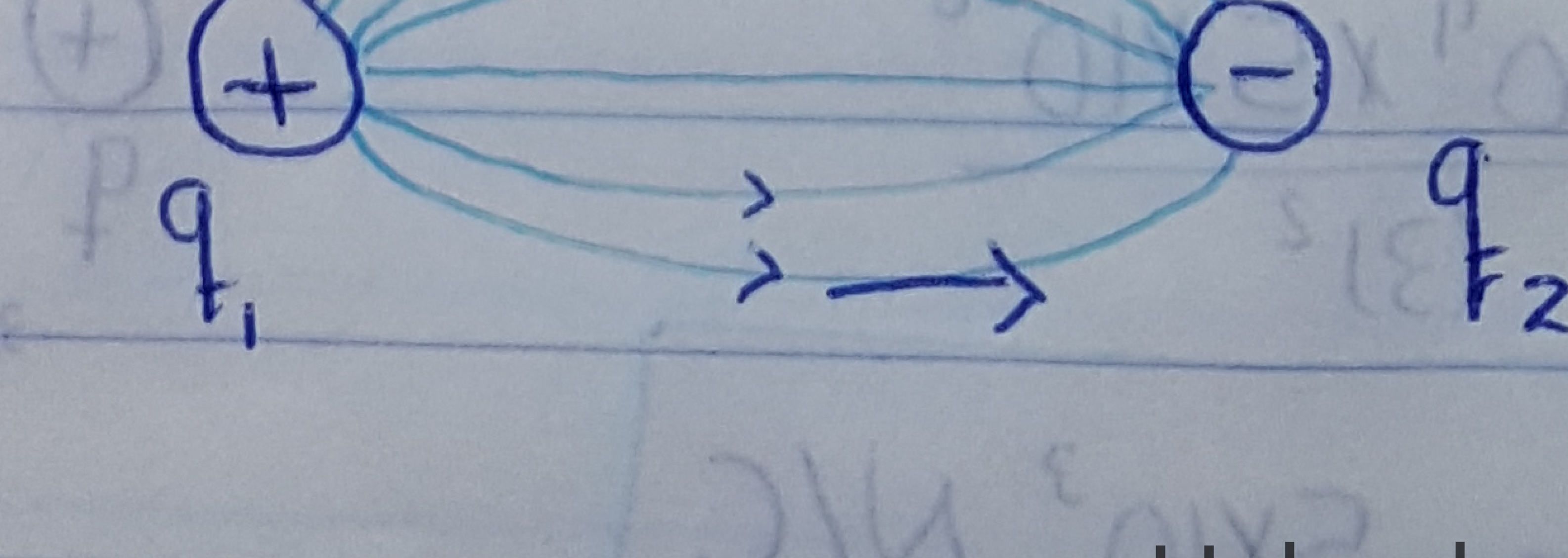
eg $q = -5 \mu C$ find E at r = 3m.





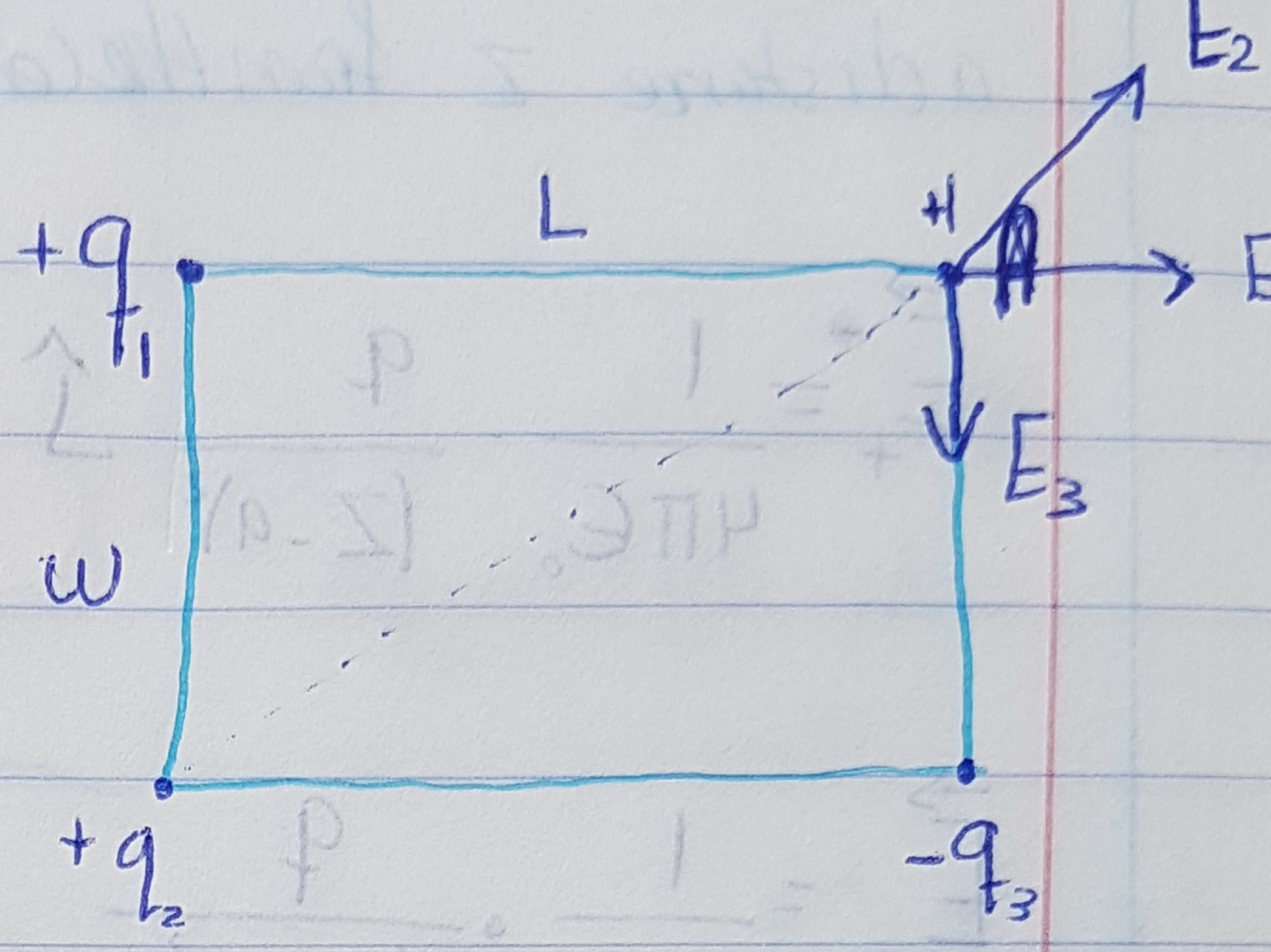
non uniform electric field.

non uniform



* $\vec{E}$ due to a set of point charges:

* find E at point A
  from: q_1, q_2, q_3:



$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{L^2}$$

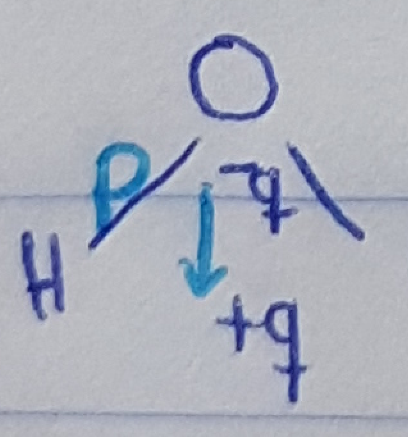
$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{L^2+w^2}$$

$$E_3 = \frac{1}{4\pi\epsilon_0}\frac{q_3}{w^2}$$

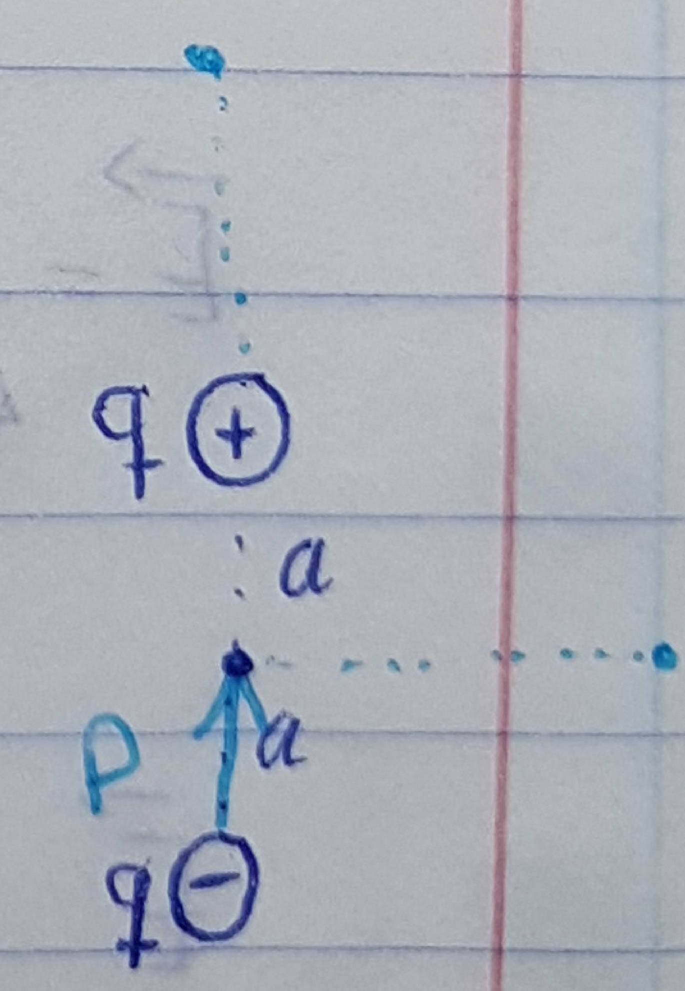
! Solve Sample problem page 583

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

* $\vec{E}$ due to an electric Dipole:



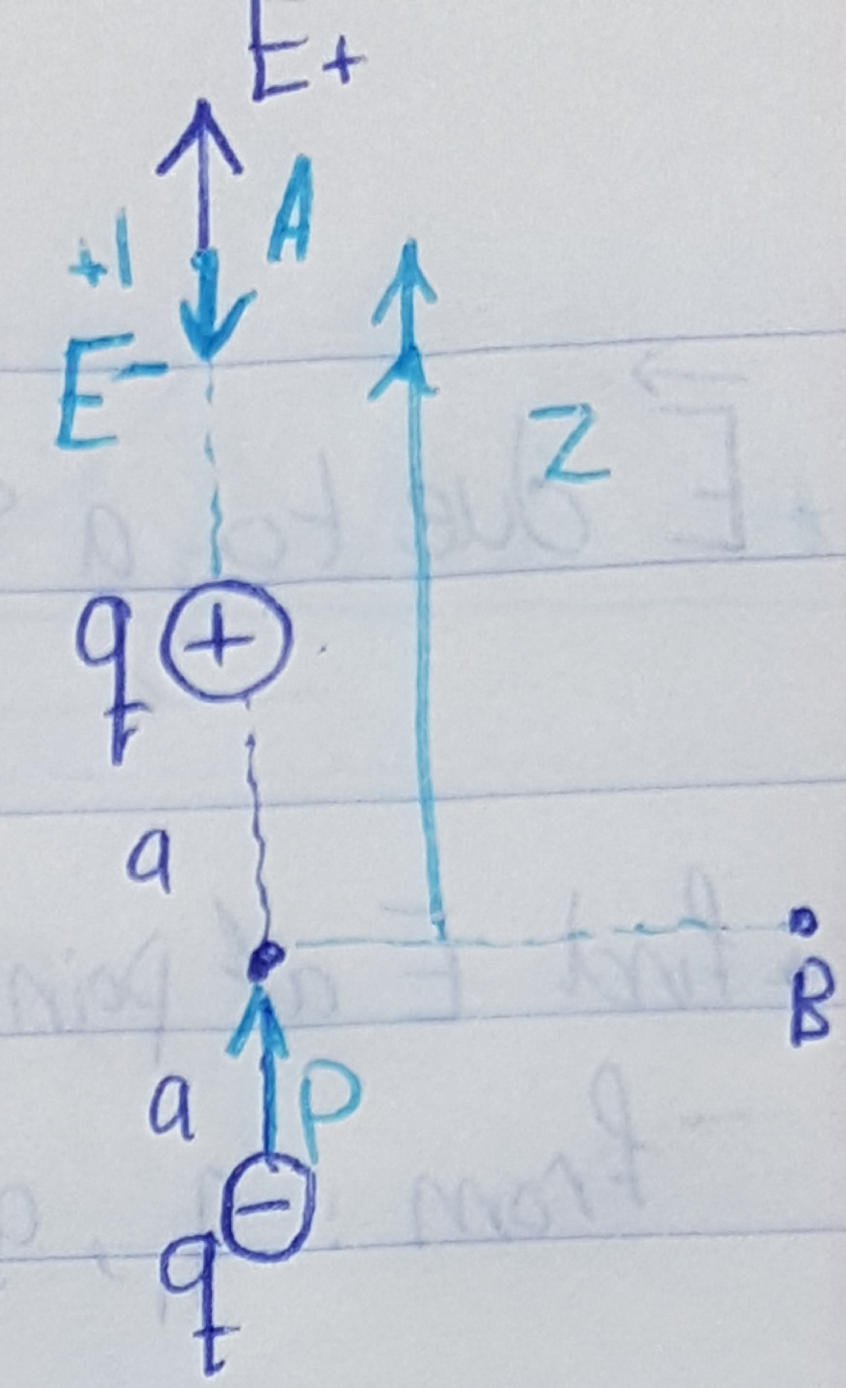
تقريب



for each electric Dipole there's an electric Dipole moment = $\vec{P}$

$$\vec{P} = q\,(2\vec{a})\ \ C.m$$

* find E at point A
a distance z from the center.



$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{(z-a)^2}\,\hat{j}$$

$$\vec{E}_- = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{(z+a)^2}\,(-\hat{j})$$

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(z-a)^2} - \frac{1}{(z+a)^2}\right]\hat{j}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{(z+a)^2-(z-a)^2}{[(z-a)(z+a)]^2}\right]\hat{j}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{4za}{(z^2-a^2)^2}\right]\hat{j}$$

$$\vec{E} = \frac{q(2a)z\,(2)}{4\pi\epsilon_0\,[z^2-a^2]^2}\,\hat{j} = \frac{Pz}{2\pi\epsilon_0\,[z^2-a^2]^2}\,\hat{j}$$

find E when $z \gg a$

$$\vec{E} = \frac{Pz}{2\pi\epsilon_0\left[z^2\left(1-\frac{a^2}{z^2}\right)\right]^2} \qquad \cdot\ \frac{a^2}{z^2} \rightsquigarrow 0$$

$$\vec{E} = \frac{Pz}{2\pi\epsilon_0 z^4}\hat{\imath}$$

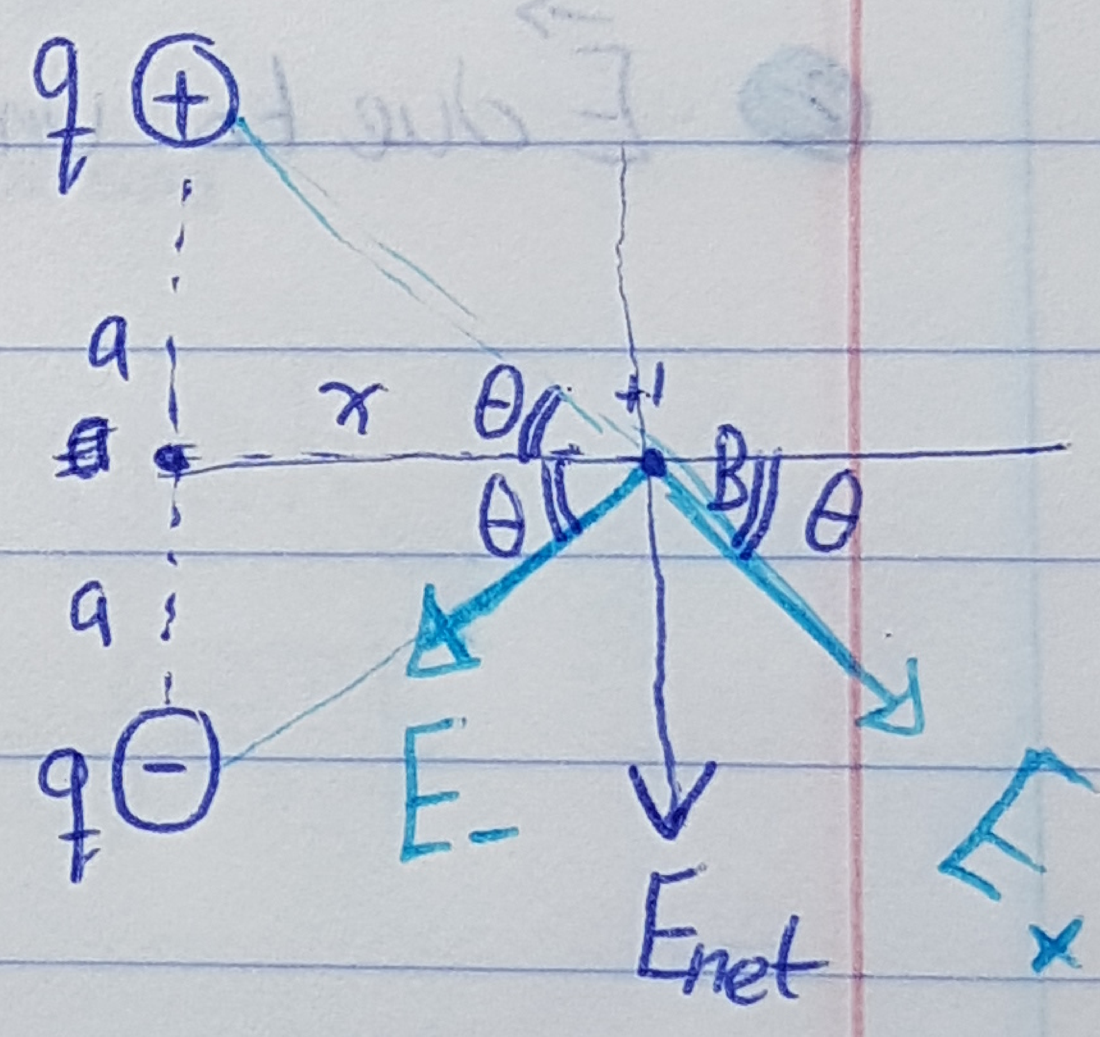
$$\vec{E} = \frac{P}{2\pi\epsilon_0 z^3}\hat{\imath}$$

problem 90:

$r^2 = a^2 + x^2$

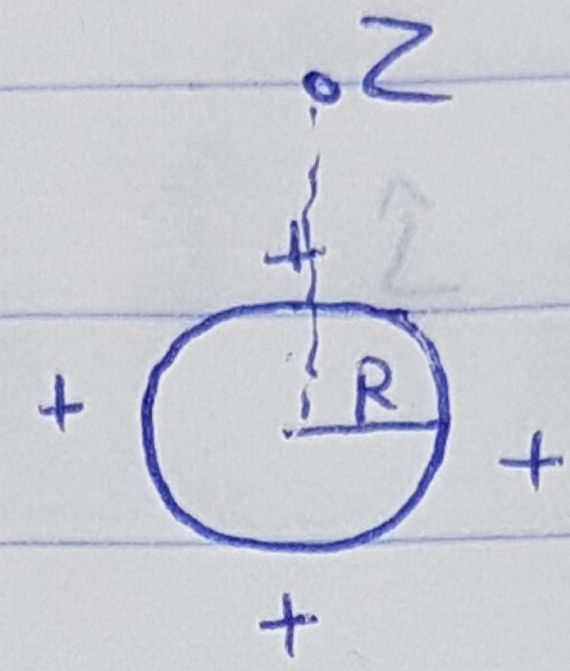
① find E at point B

② find E at point B when $x \gg a$

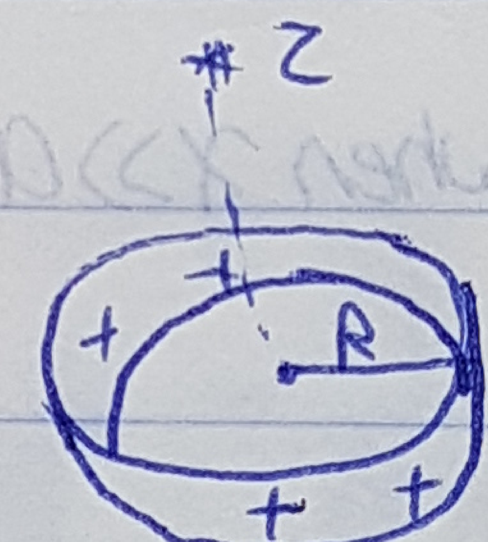


* $\vec{E}$ due to a continous charge distribution

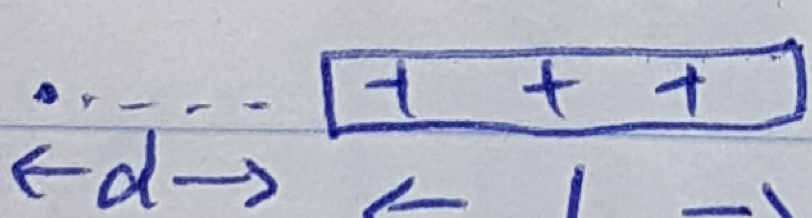
① $\vec{E}$ due to uniformaly Charged Ringe.



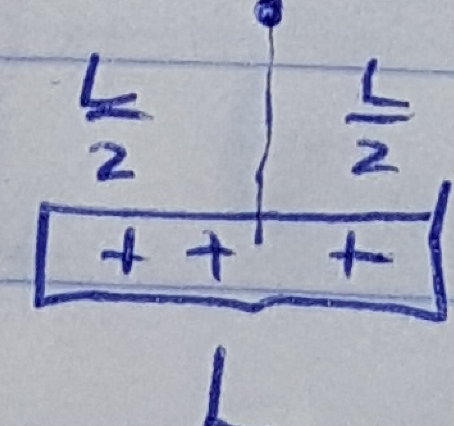
② $\vec{E}$ due to uniformaly charged Disk



③

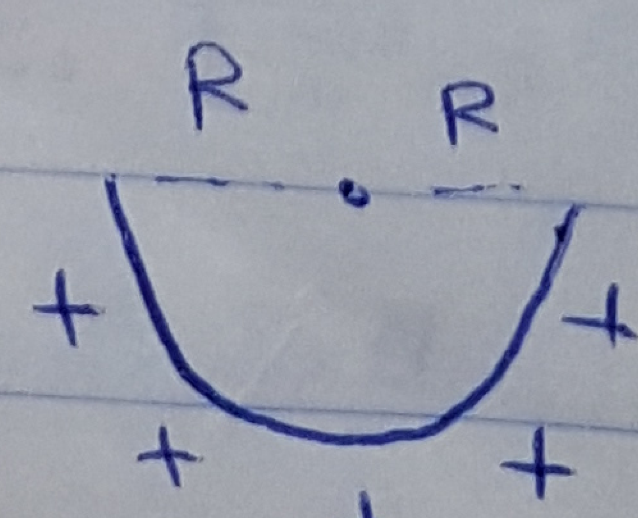


④



$\vec{E}$ due to uniformly Charged rod.

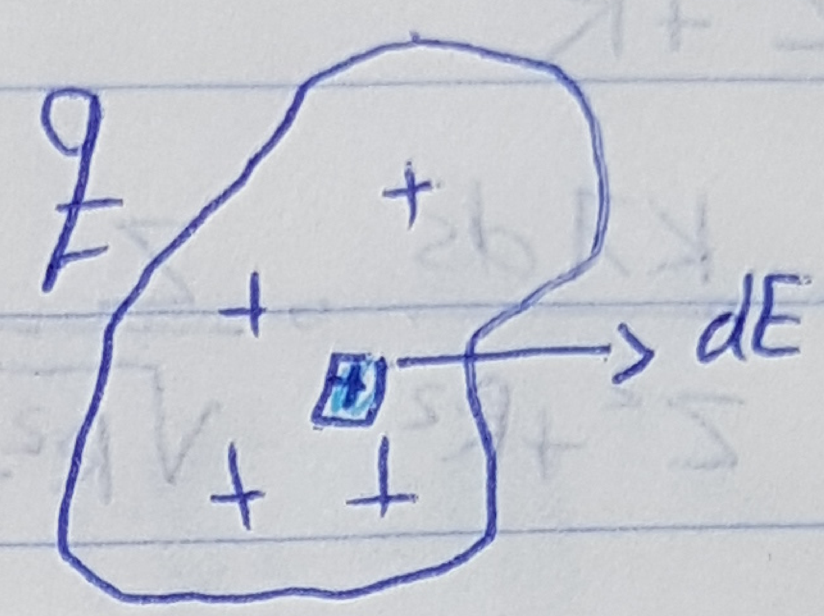
⑤



$\vec{E}$ due to arc

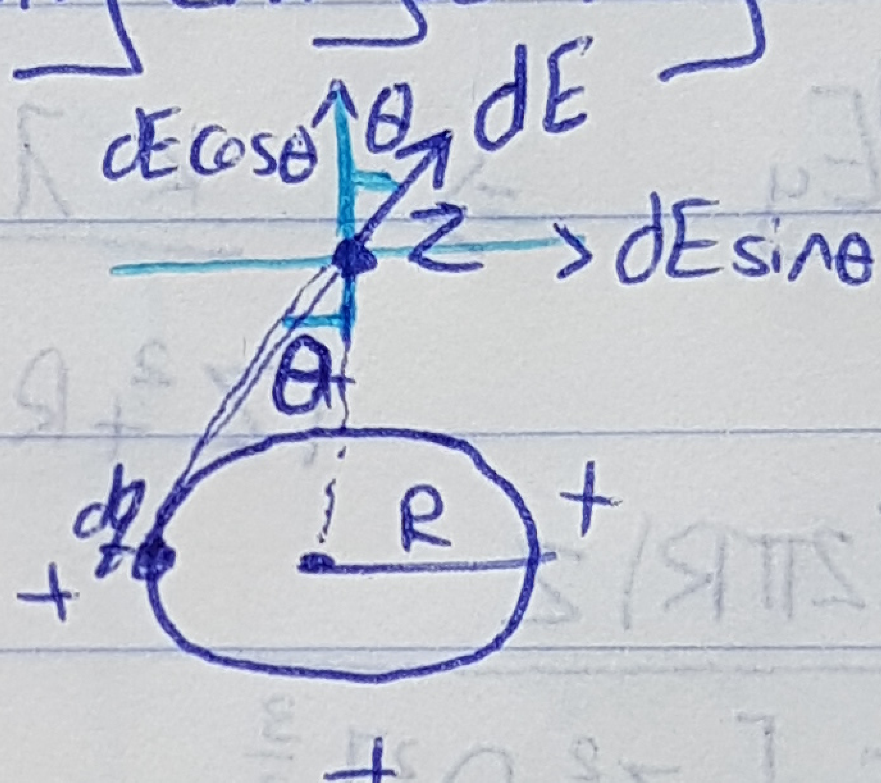
~~① $\vec{E}$ due to~~

$$dE = \frac{k\,dq}{r^2}$$



$$E = \int dE$$

① $\vec{E}$ due to a uniformly charged ring



Charge = q

radius = R

linear charge density $= \dfrac{q}{2\pi R} = \lambda$

* find $\vec{E}$ at z above the center.

$dq = \lambda\, ds$ $\boxed{r}$ ds الطول

$$dE = \frac{k\,dq}{r^2}$$

$dE_x = \dfrac{k\,\lambda\, ds\, \sin\theta}{r^2}$, $E_x = \int dE_x = 0$

from symmetry

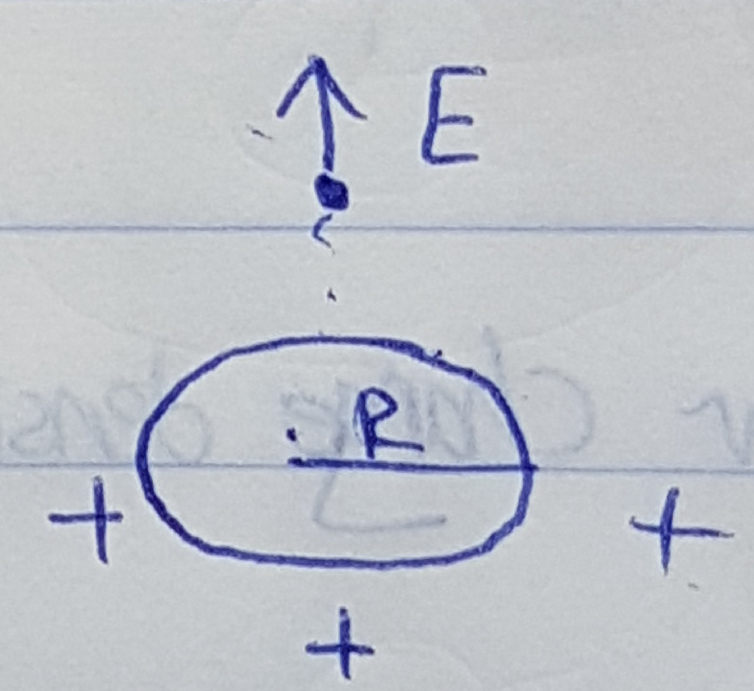
$$dE_y = \frac{K\lambda ds}{Z^2+R^2}\cos\theta$$

$$dE_y = \frac{K\lambda ds}{Z^2+R^2}\cdot\frac{Z}{\sqrt{R^2+Z^2}}$$

$$dE_y = \frac{K\lambda ds\, Z}{[Z^2+R^2]^{\frac{3}{2}}}$$

$$E_y = \int dE_y \Rightarrow \frac{K\lambda Z}{[Z^2+R^2]^{\frac{3}{2}}}\int ds$$

$$E_y = \frac{\lambda(2\pi R)Z}{4\pi\epsilon_0[Z^2+R^2]^{\frac{3}{2}}}$$



$$E_y = \frac{qZ}{4\pi\epsilon_0[Z^2+R^2]^{\frac{3}{2}}}$$

Consider the following 2 cases:

*① let $Z \gg R$, $\frac{R^2}{Z^2} \rightsquigarrow 0$

$$E = \frac{q}{4\pi\epsilon_0 Z^3} \quad \text{(point charge)}$$

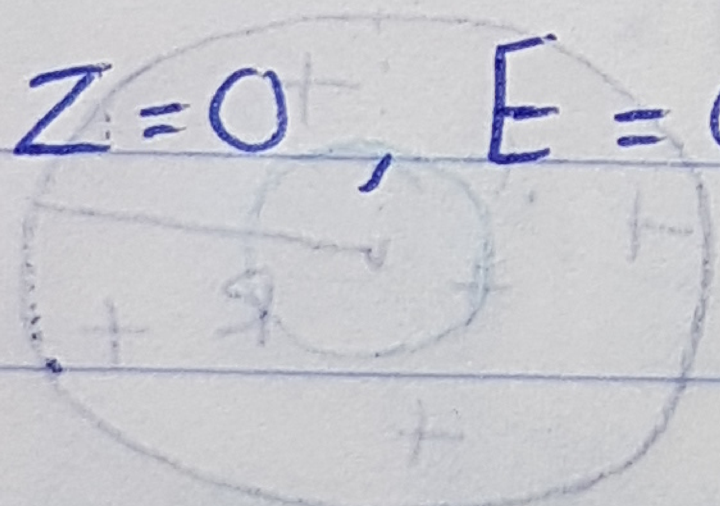
2. let $R \gg z$

$$E_y = \frac{qz}{4\pi\epsilon_0 \left[R^2\left(\frac{z^2}{R^2}+1\right)\right]^{\frac{3}{2}}}$$

$$\frac{z^2}{R^2} \rightsquigarrow 0$$

$$E = \frac{qz}{4\pi\epsilon_0 R^3}$$

3. $z = 0$, $E = 0$



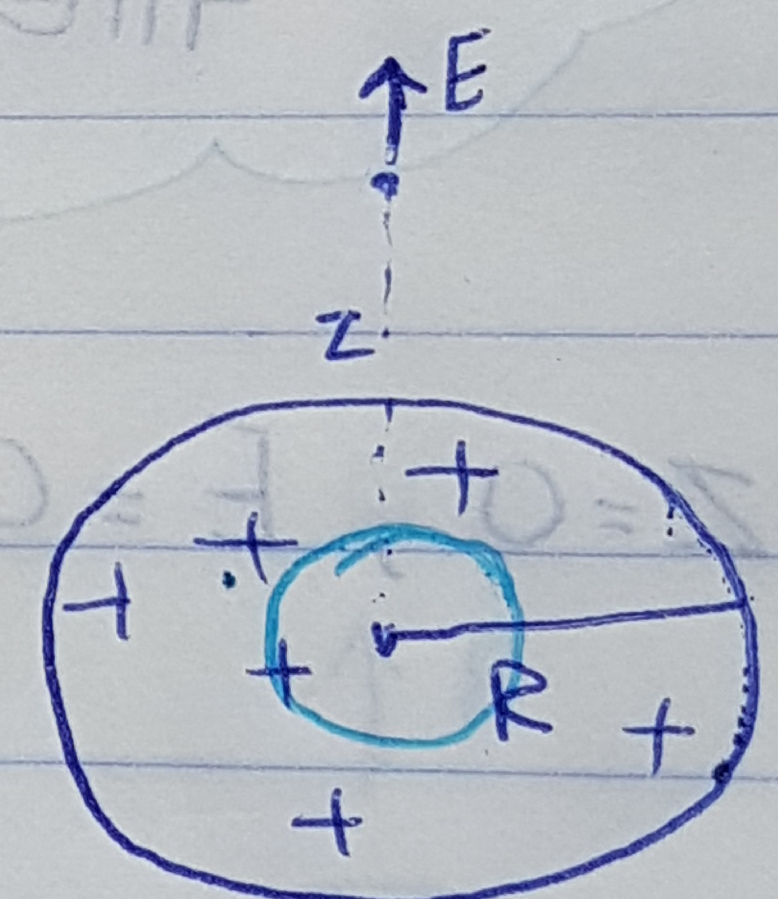
(2) $\vec{E}$ due to a uniformly Charged disk

Radius = R

Charge = q

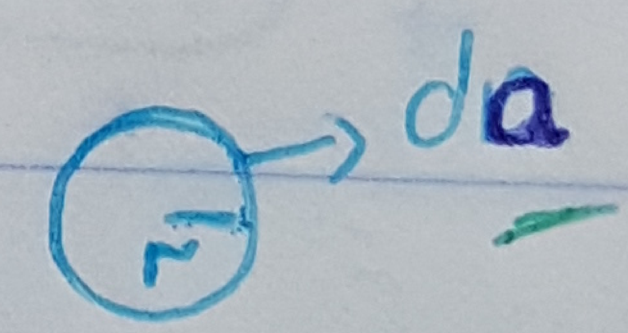
Surface charge density $\sigma = \frac{q}{\pi R^2}$

* find E, Z, above the center



$\vec{E}_{Disk} = \int (dE)_{Ring}$

dq = Charge on a ring of radius = r, width = dr



$dq = \sigma(2\pi r\,dr)$

$dA = 2\pi r\,dr$

$$(dE)_{ring} = \frac{\sigma(2\pi r\,dr)z}{4\pi\epsilon_0\left[z^2+r^2\right]^{\frac{3}{2}}}$$

$$E_{Disk} = \frac{\sigma \pi Z}{4\pi \epsilon_0} \int_0^R \frac{2r\,dr}{[Z^2+r^2]^{\frac{3}{2}}}$$, let $u = Z^2 + r^2$

$du = 2r\,dr$

$$\frac{\sigma Z}{4\epsilon_0} \int u^{-\frac{3}{2}} du = \frac{\sigma Z}{4\epsilon_0} \cdot \frac{u^{-\frac{1}{2}}}{-\frac{1}{2}}$$

$$E_{Disk} = -2\frac{\sigma Z}{4\epsilon_0}\left[\frac{1}{\sqrt{Z^2+r^2}}\right]_0^R$$

$$E_{Disk} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{Z}{\sqrt{R^2+Z^2}}\right]$$

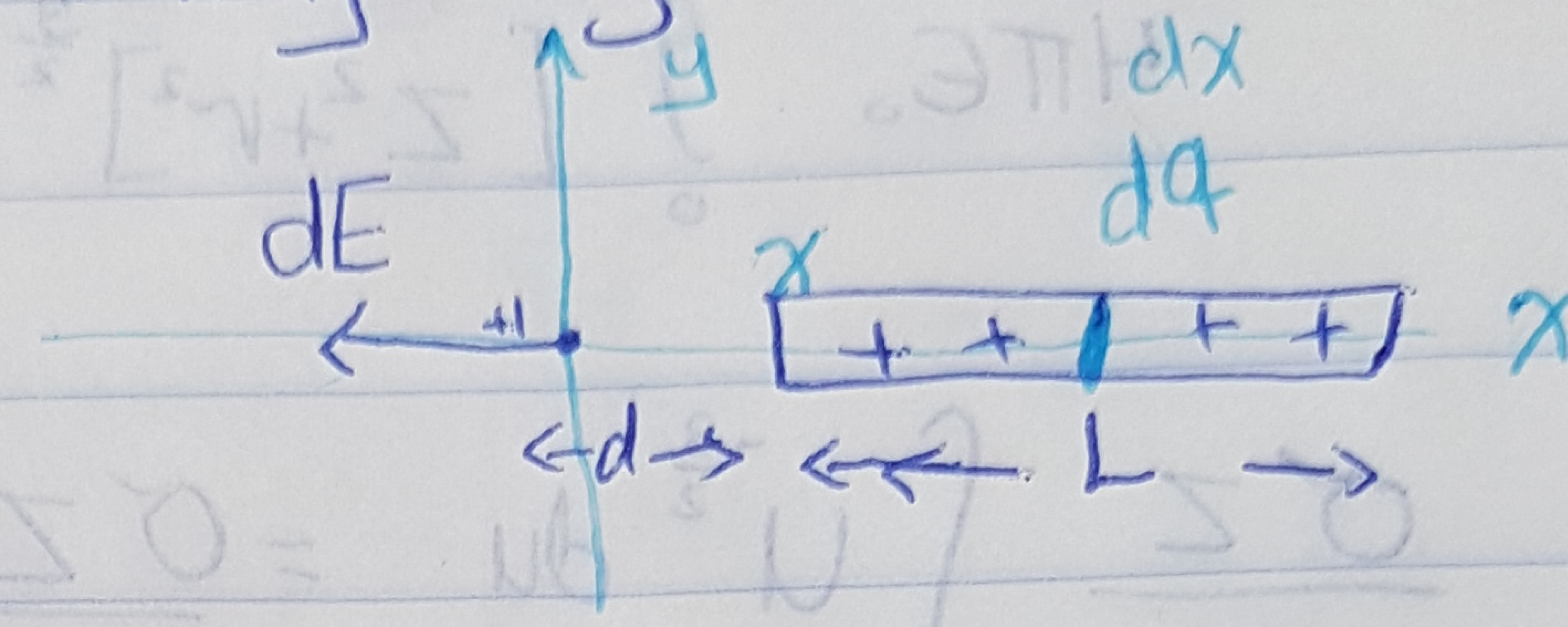
* Consider the following case:

$R \gg Z$ (near the surface) infinite Disk

قريب جدا

$$E = \frac{\sigma}{2\epsilon_0}[1-0] \Rightarrow \boxed{\frac{\sigma}{2\epsilon_0}}$$ constant

$\boxed{3}$ $\vec{E}$ due to a uniformly charged rod:



length = L

charge = q

$\lambda = \frac{q}{L}$

find $\vec{E}$ at a point a distance d from one end

$$dE = \frac{K\,dq}{r^2}, \quad dq = \lambda\,dx, \quad r^2 = x^2$$

$$dE = \frac{K\lambda\,dx}{x^2}$$

$$E = K\lambda \int_d^{L+d} \frac{dx}{x^2} \Rightarrow \cancel{K\lambda \left[\;\right]_d^{L+d}}$$

$$E = K\lambda \left[\frac{-1}{x}\right]_d^{L+d}$$

$$E = \frac{\lambda L}{4\pi\epsilon_0 d(L+d)} = \frac{q}{4\pi\epsilon_0 d(L+d)}$$

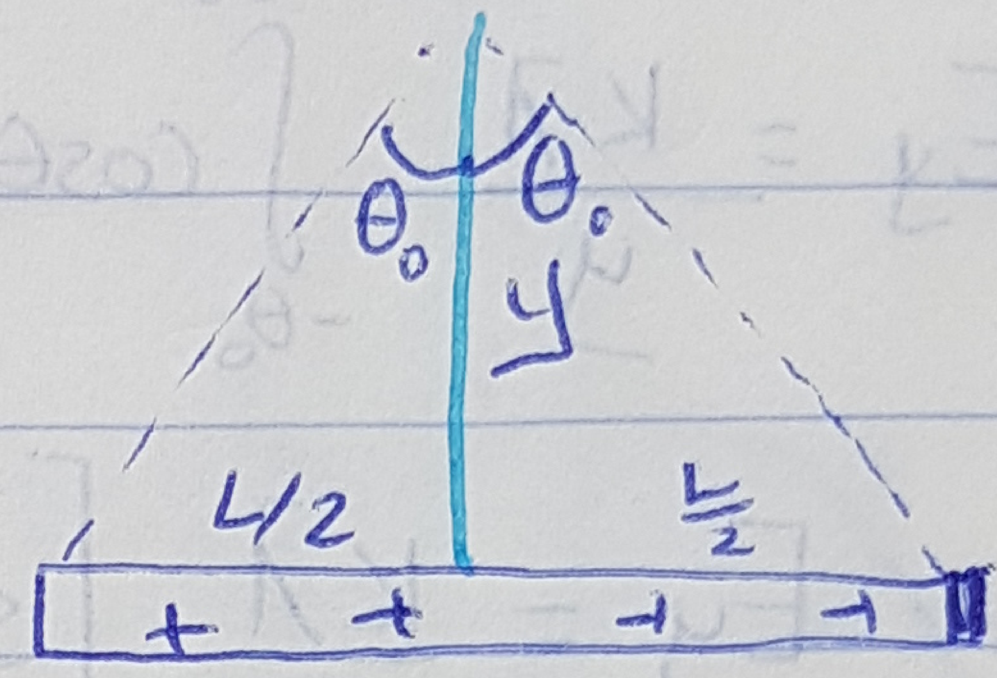
find E for $d \gg L$

$$E = \frac{q}{4\pi\epsilon_0 d^2 (\underbrace{\tfrac{L}{d}}_{\to 0}+1)}$$

$$E = \frac{q}{4\pi\epsilon_0 d^2}, \quad d \gg L$$

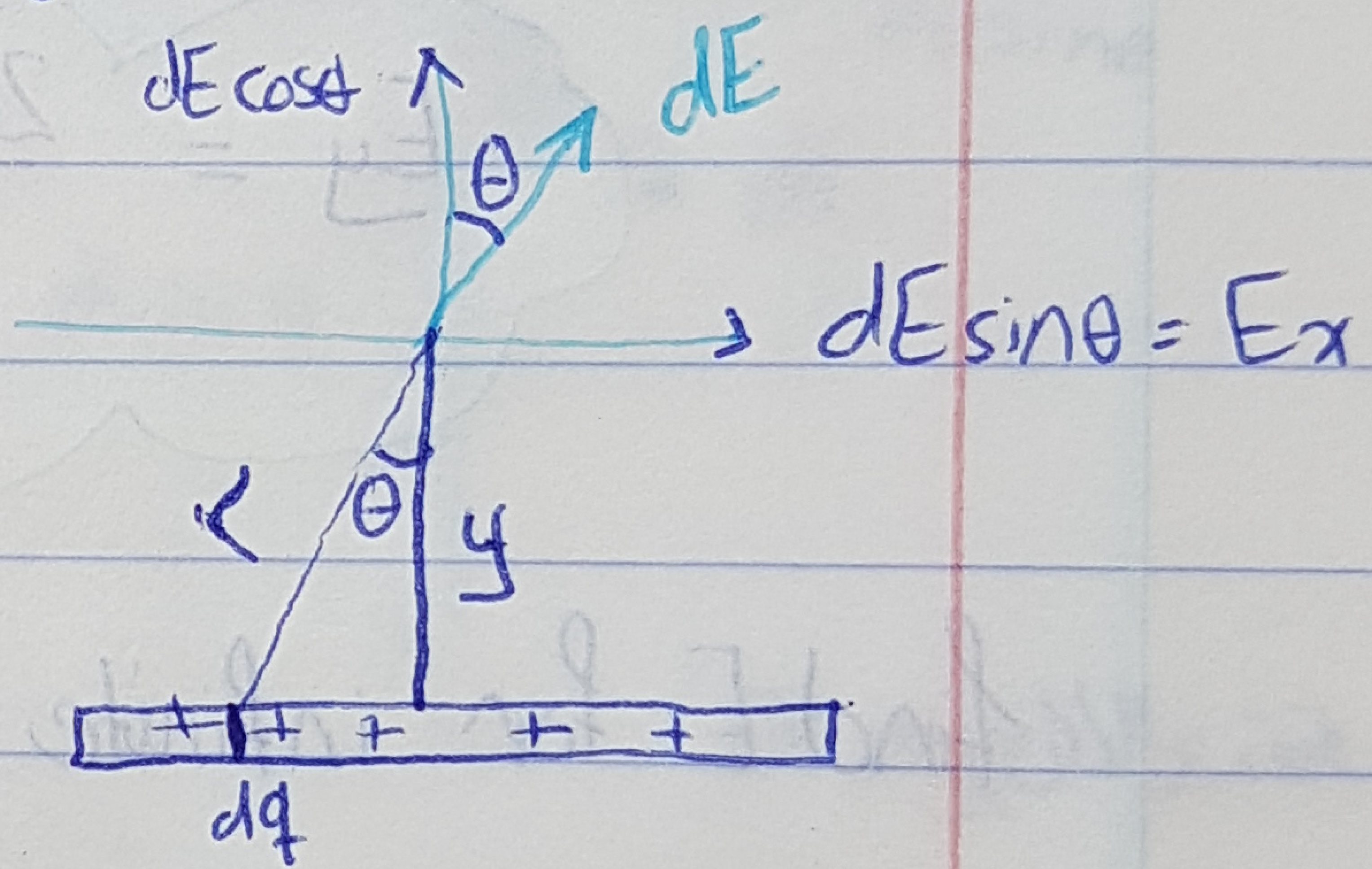
$\boxed{4}$ $\vec{E}$ due to a Uniformly Charged rod.

length $= L$, Charge $= q$



$$\lambda = \frac{q}{L}$$

find E a distance y on the perpendicular bisector . العمود المنصف



$$dE = k\,\frac{dq}{r^2}$$

$$r^2 = x^2 + y^2$$

$$dq = \lambda\, dx$$

$$dE_x = k\,\frac{\lambda\, dx}{r^2}\sin\theta \;,\; E_x = \int dE_x = \boxed{0} \text{ from symmetry}$$

$$dE_y = \frac{k\,\lambda\, dx}{r^2}\cos\theta \Rightarrow E_y = k\lambda \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\cos\theta\, dx}{r^2}$$

$$\tan\theta = \frac{x}{\underbrace{y}_{\to \text{constant}}} \Rightarrow x = y\tan\theta \Rightarrow \boxed{dx = y\sec^2\theta\, d\theta}$$

$$\cos\theta = \frac{y}{r} \;,\; r = \frac{y}{\cos\theta} = y\sec\theta \Rightarrow \boxed{r^2 = y^2\sec^2\theta}$$

$$E_y = K\lambda \int_{-\theta_0}^{+\theta_0} \frac{\cos\theta \cdot y\sec^2\theta \, d\theta}{y^2 \sec^2\theta}$$

$$E_y = \frac{K\lambda}{y} \int_{-\theta_0}^{+\theta_0} \cos\theta \, d\theta$$

$$E_y = \frac{K\lambda}{y} \left[\sin\theta_0 - \sin(-\theta_0)\right]$$

$$\boxed{E_y = \frac{2K\lambda}{y} \sin\theta_0}$$

find E for infinite wire ($L \gg y$)

$$\theta_0 \rightsquigarrow \frac{\pi}{2}$$

$$\sin\frac{\pi}{2} \to 1$$

$$E_y = \frac{2K\lambda}{y} = \frac{\lambda}{2\pi\epsilon_0 y}$$

Solve P $\Rightarrow$ 31, 32, 33, 26, 27

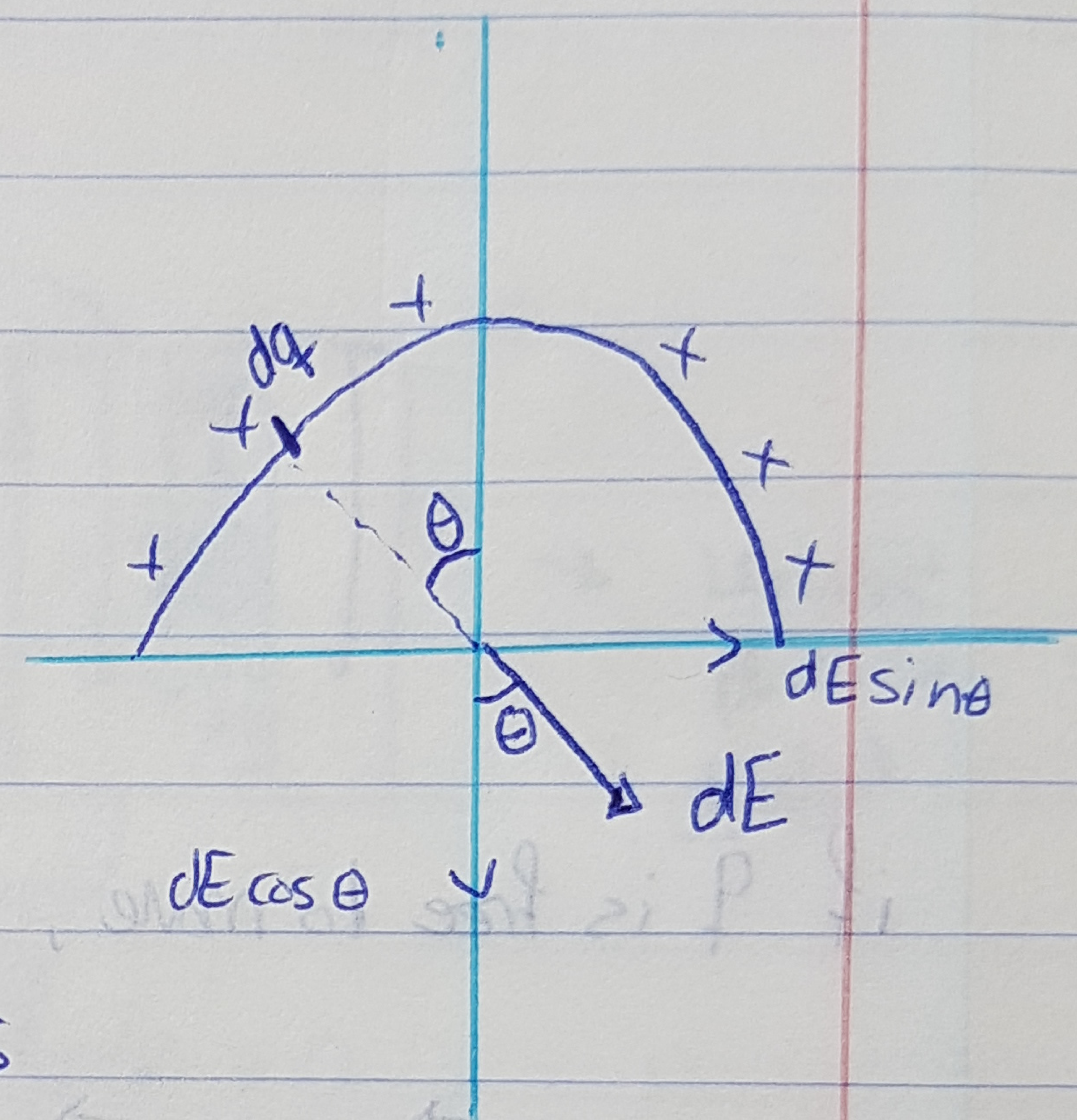
(5) $\vec{E}$ due to a uniformly charged arc:

arc is a semicircle

Radius = R

Charge = q

$$\lambda = \frac{q}{\pi R}$$



* find $\vec{E}$ at the center.

$$dE = \frac{K\,dq}{R^2}, \quad dq = \lambda\,ds$$

$$dq = \lambda(d\theta\,R)$$

$\theta \Rightarrow$ بالتقدير الدائري

$$dE_x = dE\sin\theta$$

$$dE_y = dE\cos\theta$$

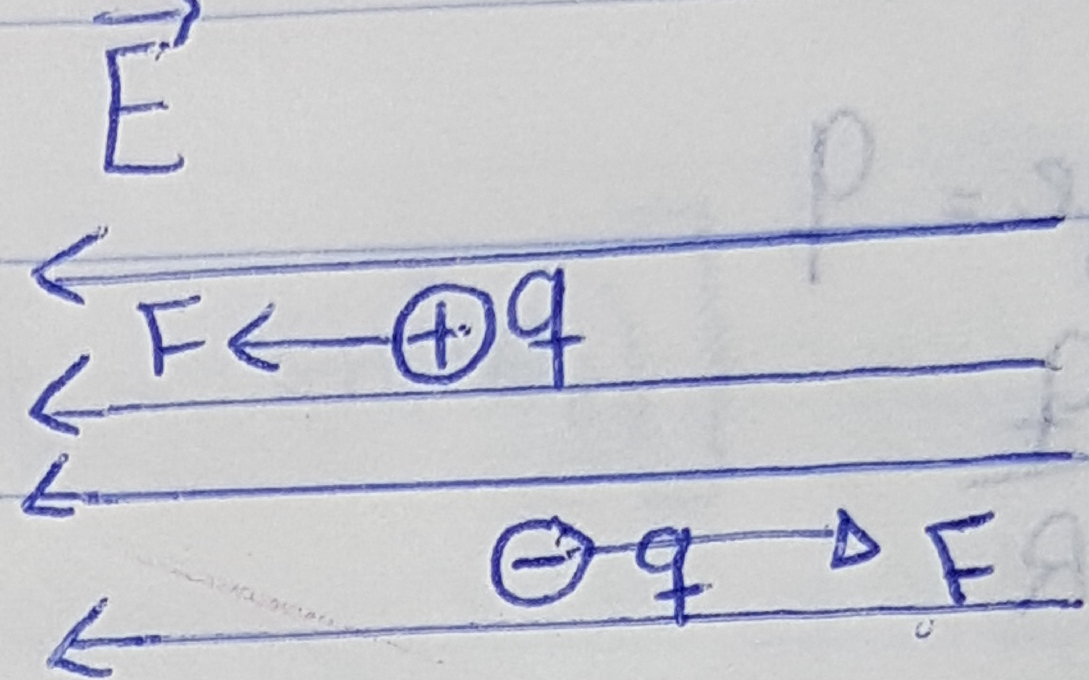
$$E_x = \int dE_x = 0 \text{ from symmetry}$$

$$E_y = \int dE_y = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{KR\lambda\,d\theta}{R^2}\cos\theta$$

$$E_y = \frac{K\lambda}{R}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos\theta\,d\theta = \frac{K\lambda}{R}\left(\sin\theta\right)_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} = \frac{2K\lambda}{R} = \frac{\lambda}{2\pi\epsilon_0 R}$$

A point Charge in a uniform $\vec{E}$:

↳ E constant in magnitude & Direction.



if q is free to move, $m\vec{a} = q\vec{E}$

$$\vec{a} = \frac{q\vec{E}}{m}$$

$\vec{a}$ is constant →

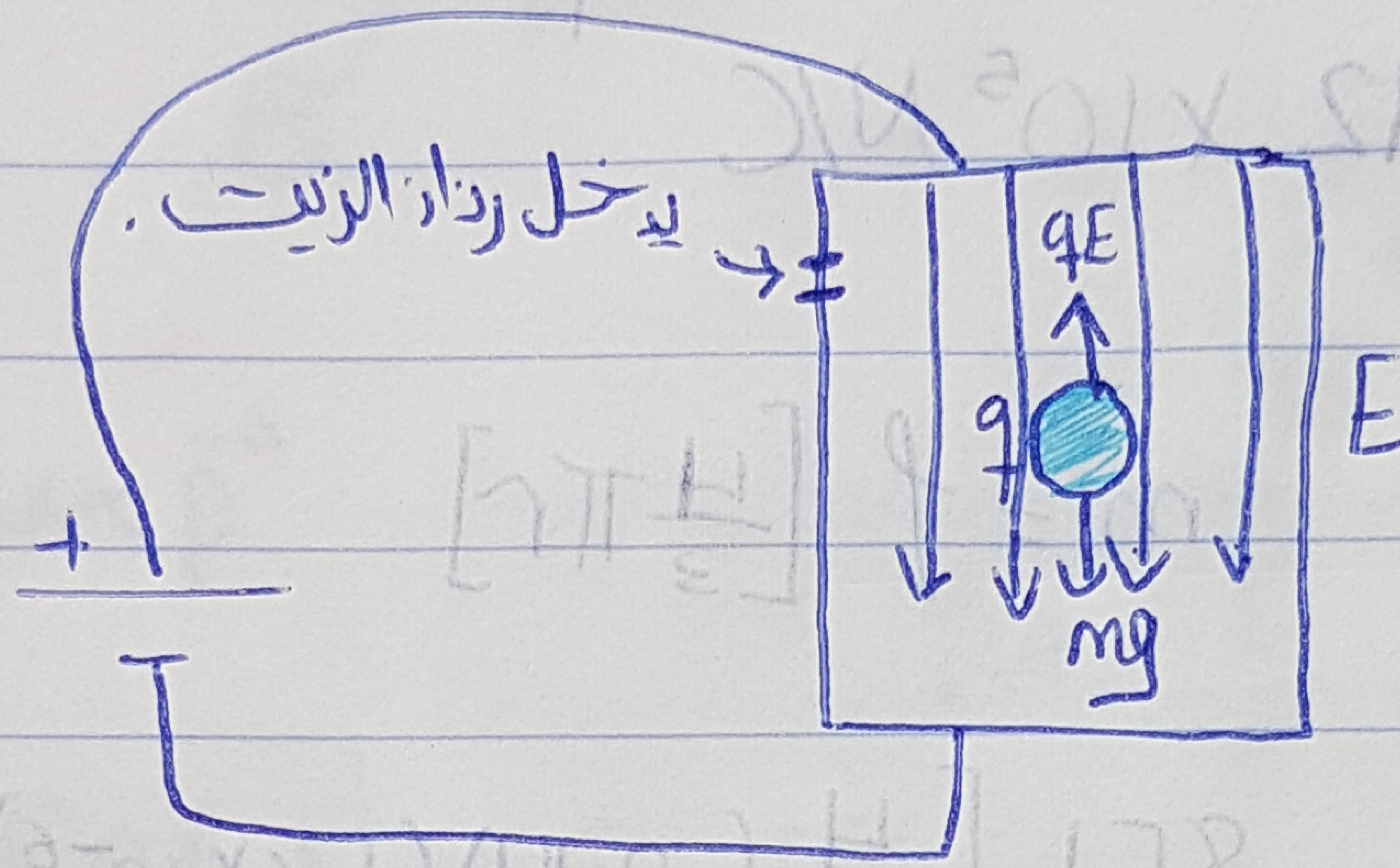
$$V_x = V_{0x}t + a_x t$$

$$V_x^2 = V_{0x}^2 + 2a\,\Delta x$$

$$\Delta x = \frac{V_{0x} + V_x}{2}\,t$$

$$\Delta x = V_0 t + \frac{1}{2} a_x t^2$$

Millikan Oil drop experiment :-



يتم ايجادها باستخدام الفولتميتر

Oil drop at rest ⇒ net force = 0

$$qE - mg = 0$$

$$q = \frac{mg}{E}$$

الكتلة ← من المايكروسكوب
يمكن ايجاد نق القطرة
ومعنا كثافة الزيت .

$$\boxed{q = ne}$$

$\boxed{22-39}$

Oil drop → $r = 1.64\ \mu m$

E must be downward → $\rho = 0.851\ g/cm^3$

$E = 1.92 \times 10^5\ N/C$

q ? $m = \rho \left[\frac{4}{3}\pi r\right]$

$851 \left[\frac{4}{3}(3.14)(1.6 \times 10^{-6})^3\right]$ Kg

$q = \dfrac{mg}{E}$

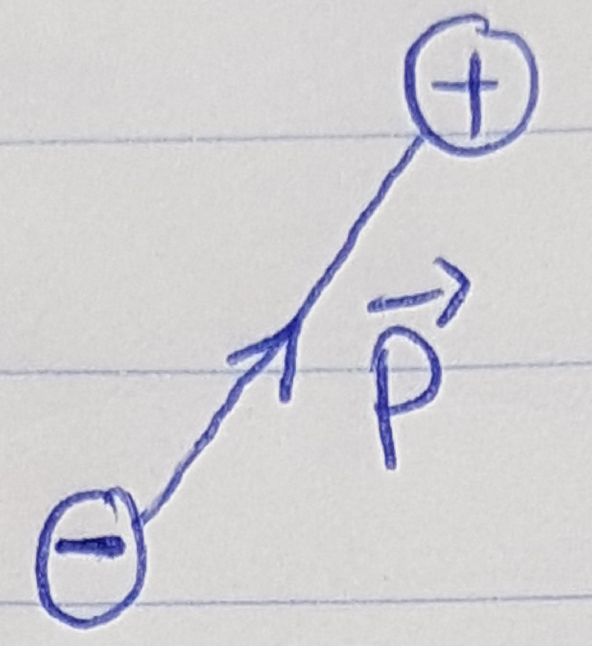
$= 8.026 \times 10^{-19}\ C$

$n = \dfrac{q}{e} = 5$

An Electric Dipole in a Uniform $\vec{E}$

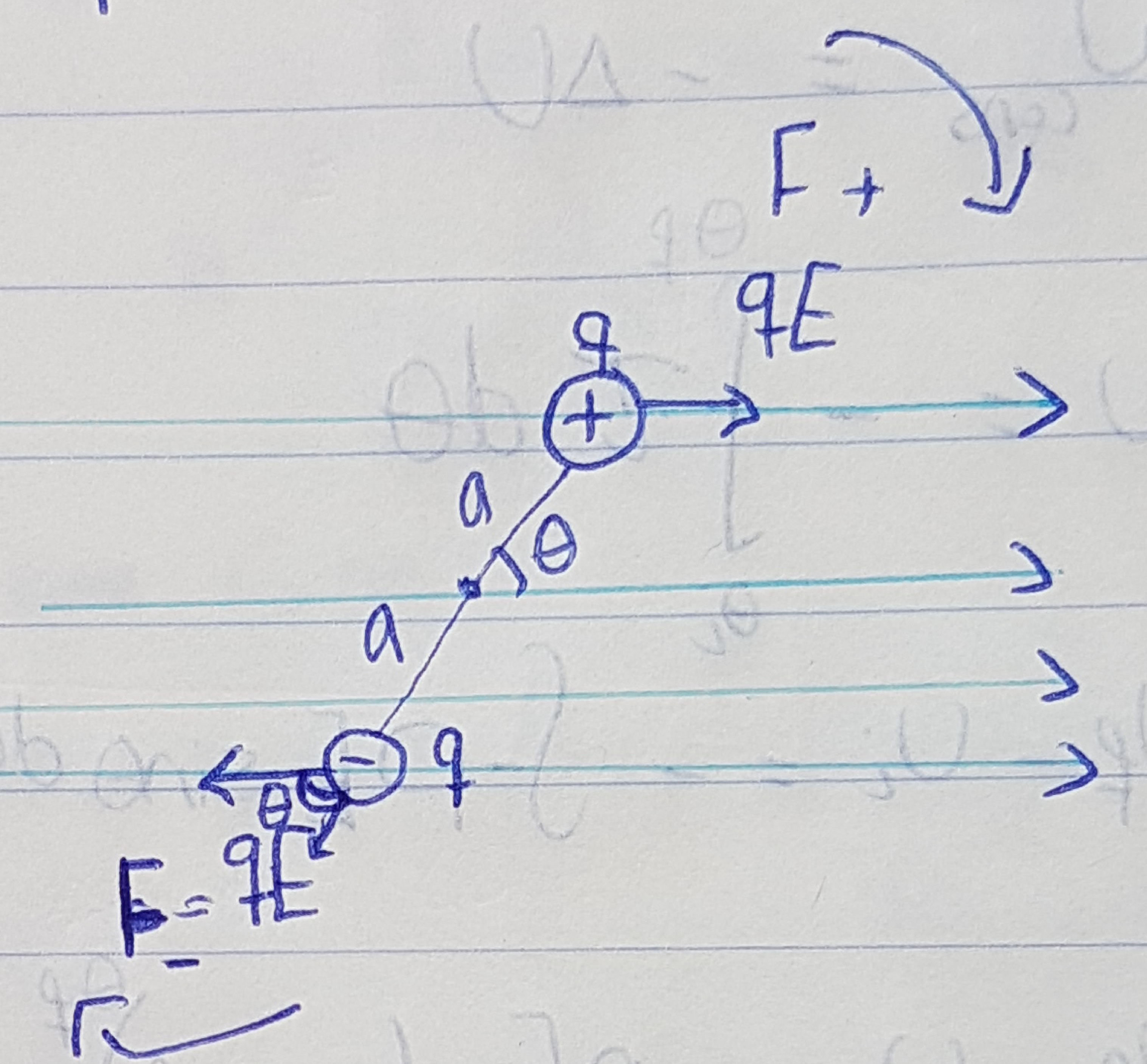
$\vec{p} = q\vec{d}$

$p = 2\vec{a}q$



$\vec{E}$ will rotates $\vec{p}$

$\boxed{\vec{\tau} = \vec{r} \times \vec{F} \ (N.m)}$



اتجاه τ يمكن ايجادها باستخدام قاعدة اليد اليمنى

الأصابع على r باتجاه F

$\tau_+ = -aqE\sin\theta$ (Clock wise)

$\tau_- = -aqE\sin\theta$

$\tau = -q(2a)E\sin\theta$

$\tau = -pE\sin\theta$

$\vec{\tau} = \vec{p} \times \vec{E}$ N.m

Potential Energy of Electric Dipole:

$$W_{cons} = -\Delta U$$

$$\Delta U = -\int_{\theta_i}^{\theta_f} \tau \, d\theta$$

$$U_f - U_i = -\int -pE \sin\theta \, d\theta$$

$$U_f - U_i = pE \left(-\cos\theta\right)\Big|_{\theta_i}^{\theta_f}$$

$$U_f - U_i = -pE\cos\theta_f + pE\cos\theta_i$$

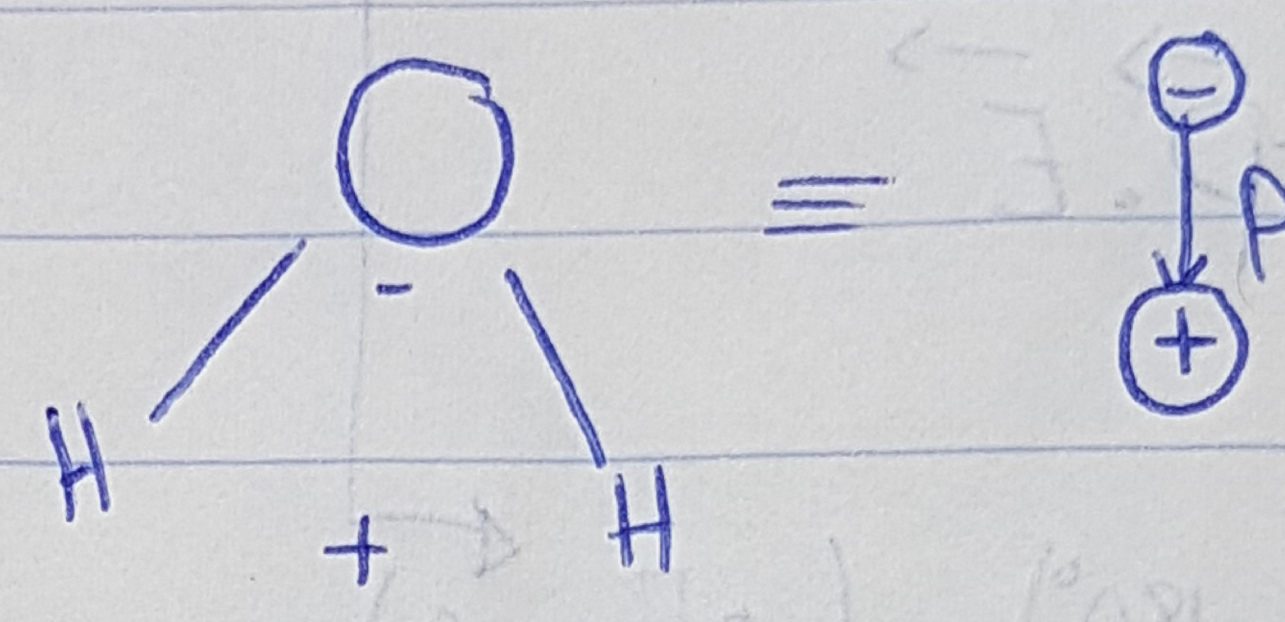
$U = 0$ at $\vec{p} \perp \vec{E}$ $\quad \theta = 90^\circ$

let $\theta_i = 90^\circ$, $U_{90} = 0$

$$U_f = -pE\cos\theta_f$$

$$\boxed{U = -\vec{p}\cdot\vec{E} \ \ J}$$

Sample problem page 596:-



$p = 6.2 \times 10^{-30}$ C.m

(a) d? $p = qd$

10^{-12} = البيكو

$$d = \frac{p}{q} = \frac{6.2 \times 10^{-30}}{10(1.6 \times 10^{-19})}$$

$d = 3.9 \times 10^{-12}$ m

(b) $E = 1.2 \times 10^{4}$ N/C

$\tau_{max} = ?$

$\tau = pE\sin\theta$, $\theta = 90$

$\tau_{max} = (6.2 \times 10^{-30})(1.5 \times 10^{4} \sin 90)$

$\boxed{9.3 \times 10^{-26} \text{ N.m}}$

(c) $W_{external\ agent} = +\Delta U$

$W_a = U_f - U_i$

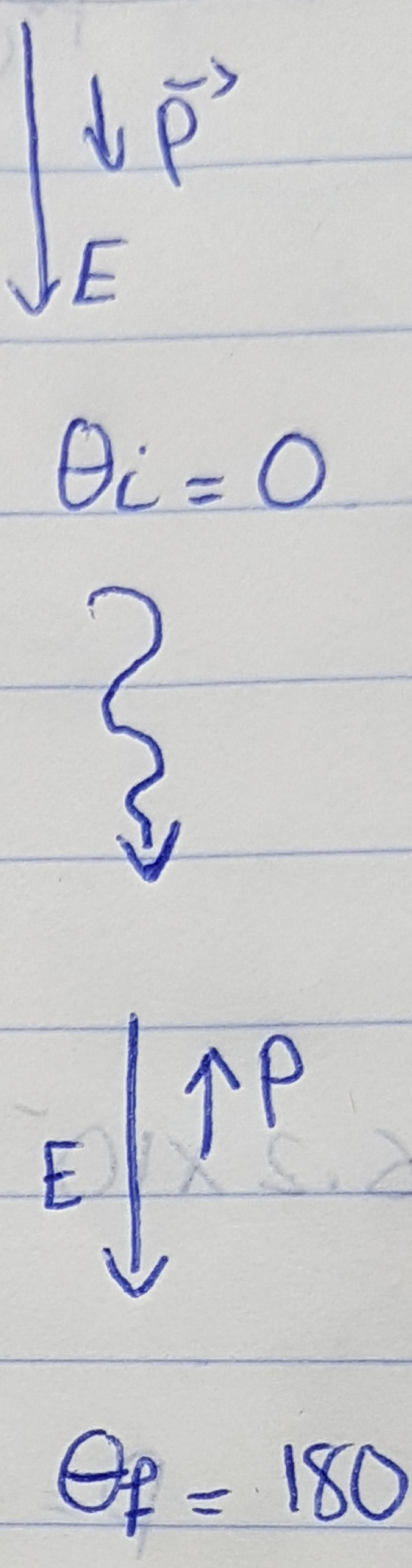
$U = -\vec{p} \cdot \vec{E}$

$(-pE \cos 180°) - (-pE \cos\theta)$

$W_a = pE - (-pE)$

$= 2pE$

$= 1.9 \times 10^{-25}$ J



~~solve~~ $\boxed{p\ 83}$

$\vec{p} = (3\hat{i} + 4\hat{j})(1.24 \times 10^{-30})$ C.m

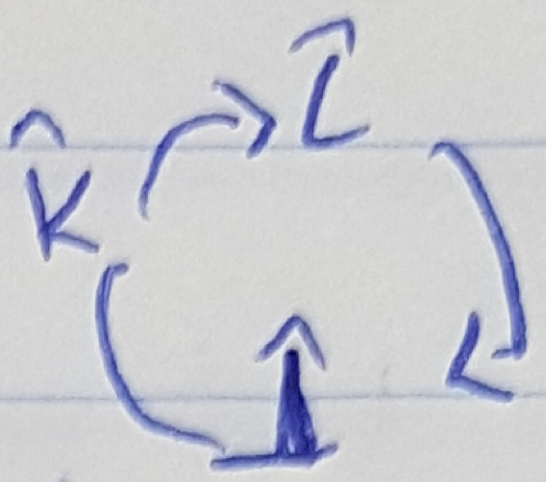
$\vec{E} = 4000\hat{i}$ N/C

(a) $U = ?$ $\quad U = -\vec{p} \cdot \vec{E}$

$-[3(1.24 \times 10^{-30})(4000)]$ J

$(-) 1.488 \times 10^{-26}$ J

② $\vec{\tau} = \vec{p} \times \vec{E}$



$(-\hat{k})(4)(1.24 \times 10^{-30})(4000)$