Principles of Physics (10th edition) 141

(Discussion)

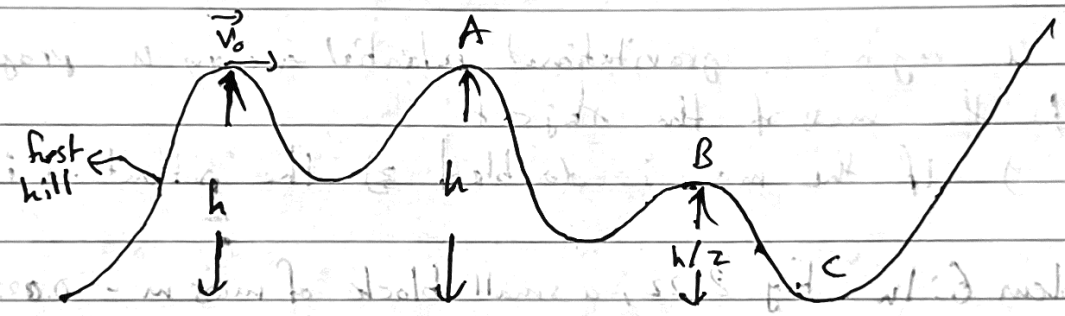
Ch 8: Potential energy and conservation of Energy

Problems: 2, 6, 23, 26, 34, 60, 65

Problem 2: In fig 8-18 a single frictionless roler-coaster car of mass $m = 825$ kg tops the first hill with speed $v_0 = 20.0$ m/s at ~~height~~ height $h = 50$ m. How much work does the gravitational force do on the car from that point to a) point A b) point B and c) point C?

If the gravitational potential energy of the car-earth system is taken to be zero at C, what is its value when the car is at (d) B e) A ?

If ~~the~~ mass m were doubled, would the change in the gravitational potential energy of the system between points A and B increase, decrease or remain the same



Sol: a) There is no change in hight between initial point and A (Δy or ΔH = zero) ⇒ work = zero

b) $W_g = F_g \cdot d = mg\left(\frac{1}{2}h\right)$

$= 825 \times 9.8 \times \frac{1}{2} \times 50$

$= 20225$ J

$= 2.02 \times 10^5$ J

or $W_g = -\Delta U = -mg\Delta h$

$= -825 \times 9.8 \times \left(\frac{h}{2} - h\right)$

$= 2.02 \times 10^5$ J

c) $W_g = -\Delta U = -mg(h_f - h_i)$

$= -825 \times 9.8 (0 - 50)$

$= 404250\ J$

$W_g = 4.04 \times 10^5\ J$

With Reference C h = zero

d) $U_B = mgy = \frac{1}{2} mgh = \frac{1}{2} \times 825 \times 9.8 \times 50$

$= 202125\ J$

$= 2.02 \times 10^5\ J$

e) $U_A = mgy = mgh = 825 \times 9.8 \times 50$

$= 404250\ J$

$= 4.04 \times 10^5\ J$

f) $U = mgh$ gravitational potential energy U proportional to the mass of the object

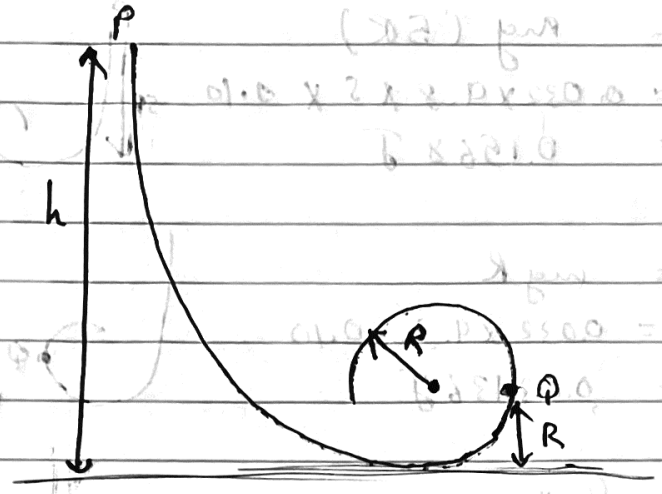
⇒ If the mass is doubled ⇒ the potential is doubled

Problem 6: In Fig 8-22, a small block of mass m = 0.032 kg can slide along the frictionless loop-the-loop, with loop ~~radius~~ radius R = 10 cm. The block is released rest at point P, at height h = 5.0R above the bottom of the loop. How much work does the gravitational force do on the block as the block travels from point P to (a) point Q and (b) the top of the loop.

If the gravitational potential energy of the block-Earth system is taken to be zero at the bottom of the loop, what is that potential energy when the block is c) at point P d) at point Q and e) at the top of the loop?

f) If insted of merely being released, the block is given

Some intial speed down ward along the track, do the answer (a) thrayh (c) increase, decrease or remain the same?



sol: $m = 0.032\ kg$, $R = 10\ cm$, $h = 5R$

a) $W_g = F_g \cdot d = mg\,dh_y = mg\,4R$

$W_g = 0.032 \times 9.8 \times 4 \times 0.10\ m$

$= 0.12544\ m$

$= 0.125\ m$

or $W_g = -\Delta U = -mg\Delta y$

$= -mg(y_f - y_i)$

$= -mg(R - 5R)$

$W_g = mg\,4R$

b) $W_g = mgd = mg(3R)$

or $W_g = -\Delta U = -mg(2R - 5R)$

$= mg\,3R$

$= 0.032 \times 9.8 \times 3 \times 0.10$

$= 0.094\ kg$

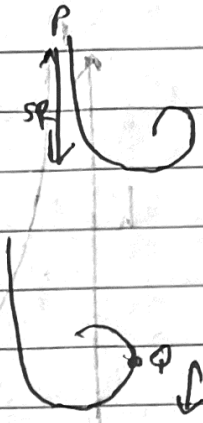
If the bottom of the loop taken to be zero
d) What is the Potential energy

c) at point P

$U_p = mgy = mg(5R)$

$= 0.032 \times 9.8 \times 5 \times 0.10$

$= 0.1568\ J$



d) at point Q

$U_Q = mgy = mgR$

$= 0.032 \times 9.8 \times 0.10$

$= 0.03136\ J$

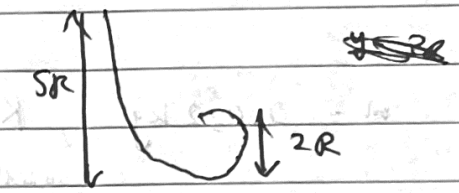
e) at the loop of the loop

$U = mgy$

$= mg[2R]$

$= 0.032 \times 9.8 \times 2 \times 0.10$

$= 0.06272\ J$



f) اذا تغيرت السرعه الابتدائيه فتغيرت من الكره الى شيء معين هل يؤثر ذلك على الاجابات السابقه

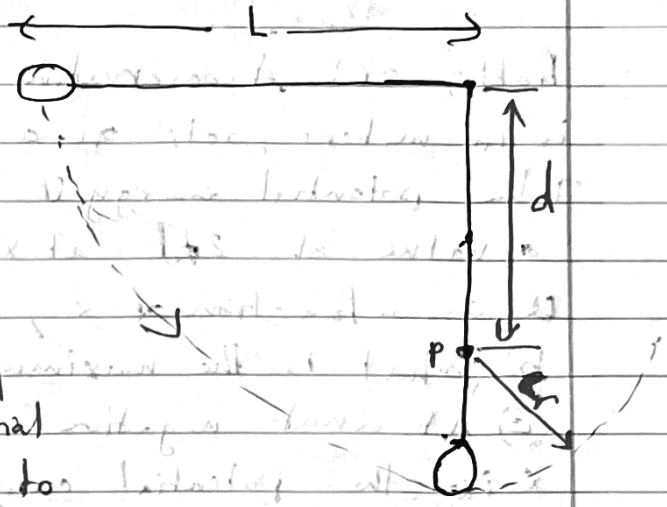
The potential energies and work done are unaffected by the ~~initial~~ initial velocity that the particle might have.

Problem 23: The string in Fig 8-26 is $L = 120$ cm along, has a ball attached to one end and is fixed at its other end. The distance d from the fixed end to a fixed peg at point P is 75.0 cm. When the initially stationary ball is released with the string horizontal as shown, it will swing along the dashed arc. What is its speed when it reaches (a) its lowest point and (b) its highest point after string catches on the peg?

Sol: L = 120 cm = 1.2 m

d = 75 cm = 0.75 m

v_i = zero



a) lowest point

As the string reaches its lowest point, ($y = 0$) its initial potential energy $U = mgL$ is converted to kinetic energy at lower point ⇒ $U = 0$

$K_f = \frac{1}{2} m v^2 = U_i$

$K_i + U_i = U_f + K_f$

$U_i = K_f$

$mgy = \frac{1}{2} m v^2$

$mgL = \frac{1}{2} m v^2$ ⇒ $v^2 = 2gL$

$\Rightarrow v = \sqrt{2gL} = \sqrt{2 \times 9.8 \times 1.2} = 4.85$ m/s

b) at the highest point after string catches on the peg

$E = U_f + K_f$

$E_i = E_f$

$mgL = U_c + K_c$

$mgL = mgy_c + \frac{1}{2} m v_c^2$

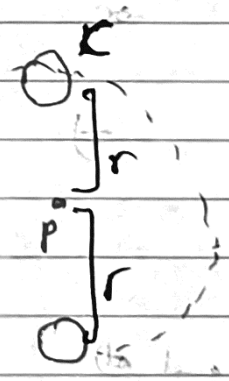
$gL = gy_c + \frac{1}{2} v_c^2$

$gL - gy_c = \frac{1}{2} v_c^2$

$2(gL - gy_c) = v_c^2$

$2(gL - g(2r)) = v_c^2 \Rightarrow v_c = \sqrt{2(gL - 2gr)}$

$v_c = \sqrt{2(9.8 \times 1.2 - 2 \times 9.8 \times 0.45)} = 2.42$ m/s



$r = L - d = 1.2 - 0.75 = 0.45$ m

Problem 26: A conservative force $\vec{F} = (6.0x - 12)\hat{i}$ N. Where x is in meters, acts on a particle moving along an x-axis. The potential energy U associated with this force is assigned a value of 27 J at x = 0. a) Write an expression for U as a function of x, with U in joules and x in meters. b) What is the maximum positive potential energy? c) At what negative value and d) positive value of x is the potential energy equale to zero?

Sol: $U = -\int F(x)\,dx$

$$= -\int_0^x (6.0x - 12)\,dx$$

$$= -\left[\frac{6x^2}{2} - 12x\right]_0^x$$

$$U = U(x) - U(0) = -\left[3x^2 - 12x - 27\right]$$

$$\boxed{U = 27 + 12x - 3x^2}$$

b) we can maximize by $\frac{dU}{dx} = 0$ or treat as equilibrium force $\Rightarrow F = 0$

$$\Rightarrow \quad F = 6x - 12$$

$$0 = 6x - 12$$

$$6x = 12 \Rightarrow \boxed{x = 2\text{ m}}$$

(c) and (d) solve equation

$$U = 27 + 12x - 3x^2$$

$$0 = 3x^2 - 12x - 27$$

$$0 = x^2 - 4x - 9$$

$$a = 1 \quad b = -4 \quad c = -9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

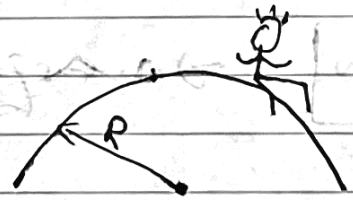
$$= \frac{4 \pm \sqrt{4^2 - 4(1)(-9)}}{2}$$

$$= \frac{4 \pm \sqrt{52}}{2}$$

(c) positive $x = \frac{4 + \sqrt{52}}{2} = 5.6$ m

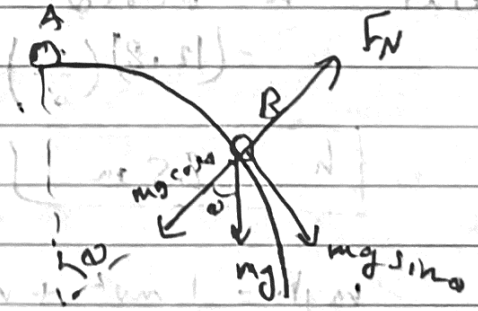
d) negative $x = \frac{4 - \sqrt{52}}{2} = -1.6$ m

Problem 34: A boy is initially seated on the top of hemispherical ice mound of Radius $R = 12.8$ m. He begins to slide down the ice, with negligible intial speed (Fig 8-35). Approximate the ice as being frictionless. At what height does the boy lose contact with the ice



Sol:

$F_N = mg\cos\theta$ قوة الفعل العمودي في حالة الاتزان



$mg\cos\theta - F_N = F_c$

$mg\cos\theta - F_N = \frac{mv^2}{R}$

v: the speed of the boy

* When the boy is leave the ice $\Rightarrow F_N = 0$

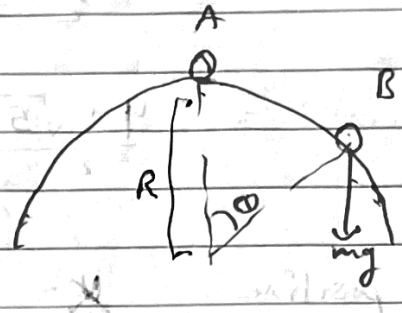
$$mg\cos\theta = \frac{mv^2}{R}$$

(8)

$$g\cos\theta = \frac{v^2}{R}$$

$$\boxed{v^2 = gR\cos\theta} \quad \text{---} \quad (1)$$

Apply the law of conservation of energy



$$E_A = E_B$$
$$K_A + U_A = K_B + U_B$$

$$zero + U_A = K_B + U_B$$
$$mgR = \frac{1}{2}mv^2 + mgh \longrightarrow (3)$$

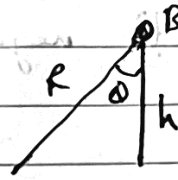
(3) و (2) و (1) نعوض

$$mgR = \frac{1}{2}m(gR\cos\theta) + mgR\cos\theta$$

$$1 = \frac{1}{2}\cos\theta + \cos\theta$$

$$1 = \frac{3}{2}\cos\theta$$

$$\boxed{\frac{2}{3} = \cos\theta} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$



$$\cos\theta = \frac{h}{R}$$

$$\Rightarrow \boxed{h = R\cos\theta} \quad (2)$$

but $h = R\cos\theta$

$$= (12.8)\left(\frac{2}{3}\right)$$

$$\boxed{h = 8.5\ m}$$

وهو المطلوب

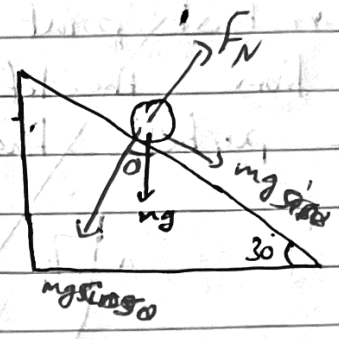
او ممكن بطريقة (3)

$$mgR = \frac{1}{2}mv^2 + mgh$$
$$mgR = \frac{1}{2}mgh + mgh$$

$$R = \frac{3}{2}h \Rightarrow h = \frac{2}{3}R = \frac{2}{3}(12.8) = 8.5\ m$$

Problem 60 : A 4.0 kg bundle starts up at 30° incline with 180 J of kinetic energy. How far will it slide up the incline if the coefficient of kinetic friction between bundle and incline, is 0.36 ?

sol: $F_N = mg\cos\theta$

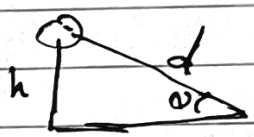


$f_k = \mu_k F_N$

$\boxed{f_k = \mu_k mg\cos\theta}$

$\sin\theta = \frac{h}{d}$

$\Rightarrow \boxed{h = d\sin\theta}$



$W = \Delta E_{mec} + \Delta E_{th}$

$0 = \Delta K + \Delta U + f_k d$

$0 = (K_f - K_i) + (U_f - U_i) + f_k d$

$0 = (0 - K_i) + \cancel{(U_f - U_i)} + f_k d$

$0 = -K_i + mgd\sin\theta + \mu_k mg\cos\theta\, d$

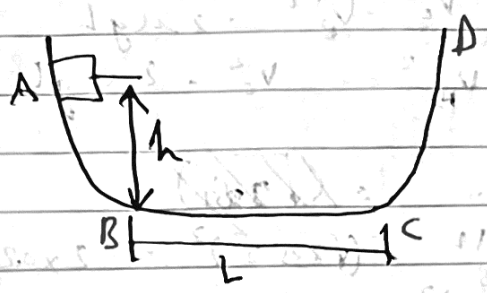
$0 = -K_i + mgd(\sin\theta + \mu_k\cos\theta)$

$\frac{K_i}{mg(\sin\theta + \mu_k\cos\theta)} = d$

$\frac{180}{4\times 9.8(\sin 30 + 0.36\cos 30)} = d$

$\boxed{4.7\,m = d}$

Problem 65: A particle can slide along a track with elevated ends and a flat central part as shown in Fig. The Flat part has length $L = 40$ cm. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is $\mu_k = 0.20$. The particle is released from rest at point A, which is at height $h = L/2$ How far from left edge of the flat part does the particle finally stop?



$L = 40\text{ cm} = 0.4\text{ m}$

$h = \frac{L}{2} = 20\text{ cm}$

sol: * For A to B Energy conservation

loss in PE = Gain in K.E

$mgh = \frac{1}{2}mv_B^2$

$\Rightarrow v_B = \sqrt{2gh} = \sqrt{2\times 9.8\times 20} = \sqrt{392} = 19.8$ cm/s

* from B to C, kinetic energy loss due to frictional track. work

$\Delta K = -W_f$

$\frac{1}{2}m(v_B^2 - v_C^2) = f_k L$

$\frac{1}{2}m(v_B^2 - v_C^2) = \mu_k mgL$

$v_B^2 - v_C^2 = 2\mu_k gL$

$\Rightarrow v_C^2 = v_B^2 - 2\mu_k gL$

$= (\sqrt{392})^2 - 2\times 0.20\times 9.8\times 40$

$v_C^2 = 235.2$

$v_C = \sqrt{235.2}$ cm/s

$= 15.3$ cm/s.

* At point C particle (block) has positive velocity mean it doesn't stop in between B to C in the first time.

L.P.L

* Again, the particle gain PE and loss in KE, and when it came back at point C, it has the same velocity but in the opposit direction.

* C to B, KE loss in frictional work

$$\frac{1}{2}m(V_B'^2 - V_C^2) = -\mu mgL$$

$$\frac{1}{2}m(V_C^2 - V_B'^2) = \mu mgL$$

$$V_C^2 - V_B'^2 = 2\mu gL$$

$$V_B'^2 = V_C^2 - 2\mu gL \quad , \quad V_C = \sqrt{235.2}\ cm/s$$

$$V_B'^2 = (\sqrt{235.2})^2 - 2\times 0.2\times 9.8 \times 40$$

$$V_B'^2 = 78.4$$

$$V_B' = \sqrt{78.4}\ cm/s$$

$$V_B' \simeq 8.6\ cm/s \qquad V_B' > 0$$

* Again it makes up and come down with velocity V_B'

Now from B to C

$$\frac{1}{2}m(V_B'^2 - V_C'^2) = \mu mgL$$

$$V_C'^2 = V_B'^2 - 2\mu gL$$

$$= (\sqrt{78.4})^2 - 2\times 0.2\times 9.8\times 40\ cm$$

$$= 78.4 - 156.8$$

$$V_C'^2 = -78.4$$

$$\Rightarrow V_C' \ngeq 0$$

It mean, Particle stop between B and C

K.E. $= \frac{1}{2}mV_B'^2 = \frac{1}{2}m \times (\sqrt{78.4})^2 = 39.2\ m$

Let x be from point B (left edge)

$$K.E = \mu mgx$$

$$39.2\,m = \mu mgx$$

$$x = \frac{39.2}{\mu g}$$

$$x = \frac{39.2}{0.2 \times 9.8}$$

$$\boxed{x = 20 \text{ cm}}$$

So the particle stops at mid point of B and C.