CHAPTER 33

# Electromagnetic Waves

## 33-1 ELECTROMAGNETIC WAVES

### Learning Objectives

*After reading this module, you should be able to . . .*

**33.01** In the electromagnetic spectrum, identify the relative wavelengths (longer or shorter) of AM radio, FM radio, television, infrared light, visible light, ultraviolet light, x rays, and gamma rays.

**33.02** Describe the transmission of an electromagnetic wave by an $LC$ oscillator and an antenna.

**33.03** For a transmitter with an $LC$ oscillator, apply the relationships between the oscillator's inductance $L$, capacitance $C$, and angular frequency $\omega$, and the emitted wave's frequency $f$ and wavelength $\lambda$.

**33.04** Identify the speed of an electromagnetic wave in vacuum (and approximately in air).

**33.05** Identify that electromagnetic waves do not require a medium and can travel through vacuum.

**33.06** Apply the relationship between the speed of an electromagnetic wave, the straight-line distance traveled by the wave, and the time required for the travel.

**33.07** Apply the relationships between an electromagnetic wave's frequency $f$, wavelength $\lambda$, period $T$, angular frequency $\omega$, and speed $c$.

**33.08** Identify that an electromagnetic wave consists of an electric component and a magnetic component that are (a) perpendicular to the direction of travel, (b) perpendicular to each other, and (c) sinusoidal waves with the same frequency and phase.

**33.09** Apply the sinusoidal equations for the electric and magnetic components of an EM wave, written as functions of position and time.

**33.10** Apply the relationship between the speed of light $c$, the permittivity constant $\varepsilon_0$, and the permeability constant $\mu_0$.

**33.11** For any instant and position, apply the relationship between the electric field magnitude $E$, the magnetic field magnitude $B$, and the speed of light $c$.

**33.12** Describe the derivation of the relationship between the speed of light $c$ and the ratio of the electric field amplitude $E$ to the magnetic field amplitude $B$.

### Key Ideas

- An electromagnetic wave consists of oscillating electric and magnetic fields.
- The various possible frequencies of electromagnetic waves form a spectrum, a small part of which is visible light.
- An electromagnetic wave traveling along an $x$ axis has an electric field $\vec{E}$ and a magnetic field $\vec{B}$ with magnitudes that depend on $x$ and $t$:

$$E = E_m \sin(kx - \omega t)$$

and

$$B = B_m \sin(kx - \omega t),$$

where $E_m$ and $B_m$ are the amplitudes of $\vec{E}$ and $\vec{B}$. The electric field induces the magnetic field and vice versa.

- The speed of any electromagnetic wave in vacuum is $c$, which can be written as

$$c = \frac{E}{B} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}},$$

where $E$ and $B$ are the simultaneous magnitudes of the fields.

## What Is Physics?

The information age in which we live is based almost entirely on the physics of electromagnetic waves. Like it or not, we are now globally connected by television, telephones, and the web. And like it or not, we are constantly immersed in those signals because of television, radio, and telephone transmitters.

Much of this global interconnection of information processors was not imagined by even the most visionary engineers of 40 years ago. The challenge for today's engineers is trying to envision what the global interconnection will be like 40 years from now. The starting point in meeting that challenge is understanding the basic physics of electromagnetic waves, which come in so many different types that they are poetically said to form *Maxwell's rainbow*.

## Maxwell's Rainbow

The crowning achievement of James Clerk Maxwell (see Chapter 32) was to show that a beam of light is a traveling wave of electric and magnetic fields—an **electromagnetic wave**—and thus that optics, the study of visible light, is a branch of electromagnetism. In this chapter we move from one to the other: we conclude our discussion of strictly electrical and magnetic phenomena, and we build a foundation for optics.

In Maxwell's time (the mid 1800s), the visible, infrared, and ultraviolet forms of light were the only electromagnetic waves known. Spurred on by Maxwell's work, however, Heinrich Hertz discovered what we now call radio waves and verified that they move through the laboratory at the same speed as visible light, indicating that they have the same basic nature as visible light.

As Fig. 33-1 shows, we now know a wide *spectrum* (or range) of electromagnetic waves: Maxwell's rainbow. Consider the extent to which we are immersed in electromagnetic waves throughout this spectrum. The Sun, whose radiations define the environment in which we as a species have evolved and adapted, is the dominant source. We are also crisscrossed by radio and television signals. Microwaves from radar systems and from telephone relay systems may reach us. There are electromagnetic waves from lightbulbs, from the heated engine blocks of automobiles, from x-ray machines, from lightning flashes, and from buried radioactive materials. Beyond this, radiation reaches us from stars and other objects in our galaxy and from other galaxies. Electromagnetic waves also travel in the other direction. Television signals, transmitted from Earth since about 1950, have now taken news about us (along with episodes of *I Love Lucy,* albeit *very* faintly) to whatever technically sophisticated inhabitants there may be on whatever planets may encircle the nearest 400 or so stars.

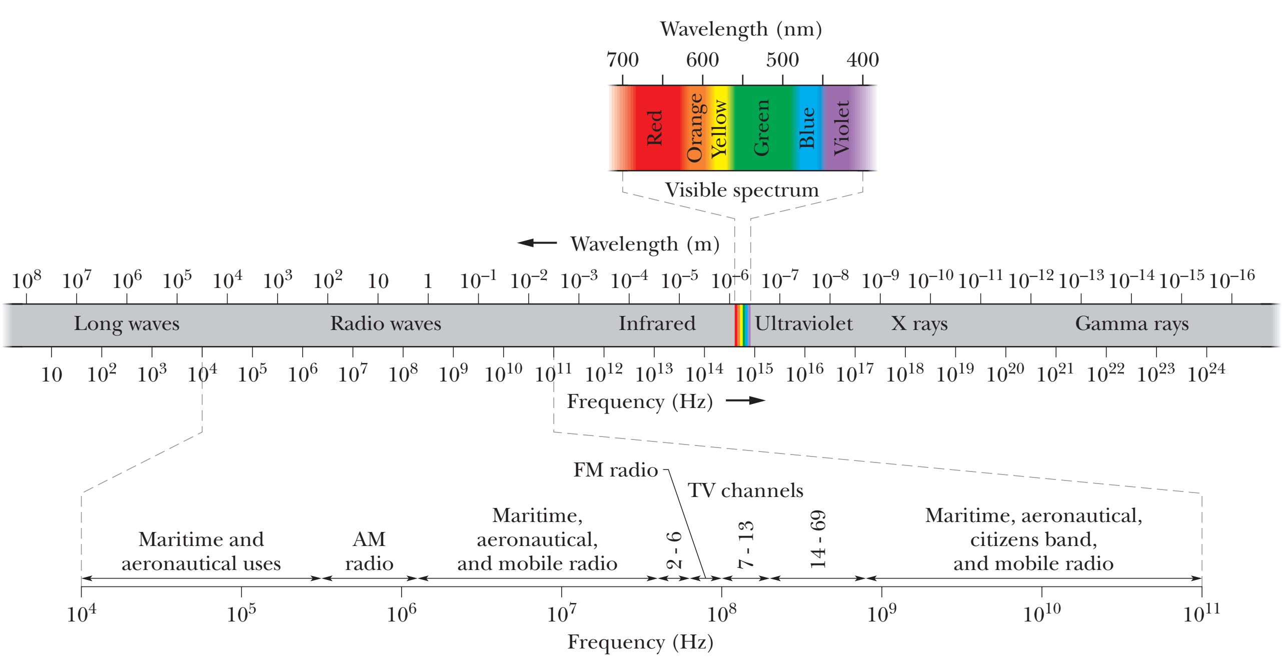



**Figure 33-1** The electromagnetic spectrum.

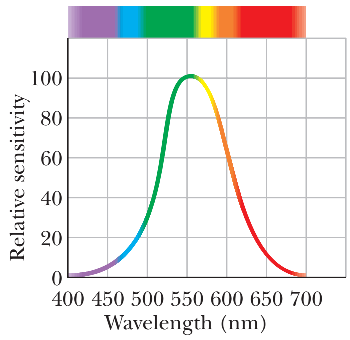



**Figure 33-2** The relative sensitivity of the average human eye to electromagnetic waves at different wavelengths. This portion of the electromagnetic spectrum to which the eye is sensitive is called *visible light.*

In the wavelength scale in Fig. 33-1 (and similarly the corresponding frequency scale), each scale marker represents a change in wavelength (and correspondingly in frequency) by a factor of 10. The scale is open-ended; the wavelengths of electromagnetic waves have no inherent upper or lower bound.

Certain regions of the electromagnetic spectrum in Fig. 33-1 are identified by familiar labels, such as *x rays* and *radio waves.* These labels denote roughly defined wavelength ranges within which certain kinds of sources and detectors of electromagnetic waves are in common use. Other regions of Fig. 33-1, such as those labeled TV channels and AM radio, represent specific wavelength bands assigned by law for certain commercial or other purposes. There are no gaps in the electromagnetic spectrum—and all electromagnetic waves, no matter where they lie in the spectrum, travel through *free space* (vacuum) with the same speed $c$.

The visible region of the spectrum is of course of particular interest to us. Figure 33-2 shows the relative sensitivity of the human eye to light of various wavelengths. The center of the visible region is about 555 nm, which produces the sensation that we call yellow-green.

The limits of this visible spectrum are not well defined because the eye sensitivity curve approaches the zero-sensitivity line asymptotically at both long and short wavelengths. If we take the limits, arbitrarily, as the wavelengths at which eye sensitivity has dropped to 1% of its maximum value, these limits are about 430 and 690 nm; however, the eye can detect electromagnetic waves somewhat beyond these limits if they are intense enough.

## The Traveling Electromagnetic Wave, Qualitatively

Some electromagnetic waves, including x rays, gamma rays, and visible light, are *radiated* (emitted) from sources that are of atomic or nuclear size, where quantum physics rules. Here we discuss how other electromagnetic waves are generated. To simplify matters, we restrict ourselves to that region of the spectrum (wavelength $\lambda \approx 1$ m) in which the source of the *radiation* (the emitted waves) is both macroscopic and of manageable dimensions.

Figure 33-3 shows, in broad outline, the generation of such waves. At its heart is an *LC oscillator,* which establishes an angular frequency $\omega\,(= 1/\sqrt{LC})$. Charges and currents in this circuit vary sinusoidally at this frequency, as depicted in Fig. 31-1. An external source—possibly an ac generator—must be included to supply energy to compensate both for thermal losses in the circuit and for energy carried away by the radiated electromagnetic wave.

The $LC$ oscillator of Fig. 33-3 is coupled by a transformer and a transmission line to an *antenna,* which consists essentially of two thin, solid, conducting rods. Through this coupling, the sinusoidally varying current in the oscillator causes charge to oscillate sinusoidally along the rods of the antenna at the angular frequency $\omega$ of the $LC$ oscillator. The current in the rods associated with this movement of charge also varies sinusoidally, in magnitude and direction, at angular frequency $\omega$. The antenna has the effect of an electric dipole whose electric dipole moment varies sinusoidally in magnitude and direction along the antenna.

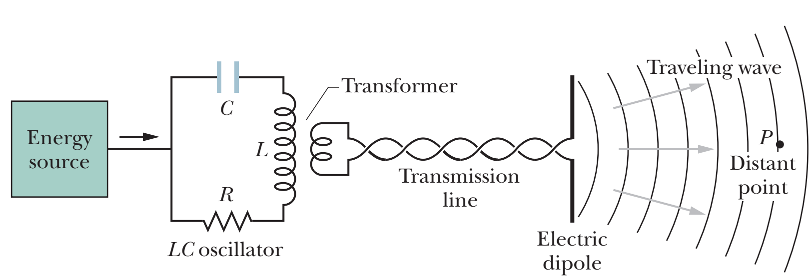



**Figure 33-3** An arrangement for generating a traveling electromagnetic wave in the shortwave radio region of the spectrum: an $LC$ oscillator produces a sinusoidal current in the antenna, which generates the wave. $P$ is a distant point at which a detector can monitor the wave traveling past it.

Because the dipole moment varies in magnitude and direction, the electric field produced by the dipole varies in magnitude and direction. Also, because the current varies, the magnetic field produced by the current varies in magnitude and direction. However, the changes in the electric and magnetic fields do not happen everywhere instantaneously; rather, the changes travel outward from the antenna at the speed of light $c$. Together the changing fields form an electromagnetic wave that travels away from the antenna at speed $c$. The angular frequency of this wave is $\omega$, the same as that of the $LC$ oscillator.

***Electromagnetic Wave.*** Figure 33-4 shows how the electric field $\vec{E}$ and the magnetic field $\vec{B}$ change with time as one wavelength of the wave sweeps past the distant point $P$ of Fig. 33-3; in each part of Fig. 33-4, the wave is traveling directly out of the page. (We choose a distant point so that the curvature of the waves suggested in Fig. 33-3 is small enough to neglect. At such points, the wave is said to be a *plane wave,* and discussion of the wave is much simplified.) Note several key features in Fig. 33-4; they are present regardless of how the wave is created:

1. The electric and magnetic fields $\vec{E}$ and $\vec{B}$ are always perpendicular to the direction in which the wave is traveling. Thus, the wave is a *transverse wave,* as discussed in Chapter 16.
2. The electric field is always perpendicular to the magnetic field.
3. The cross product $\vec{E} \times \vec{B}$ always gives the direction in which the wave travels.
4. The fields always vary sinusoidally, just like the transverse waves discussed in Chapter 16. Moreover, the fields vary with the same frequency and *in phase* (in step) with each other.

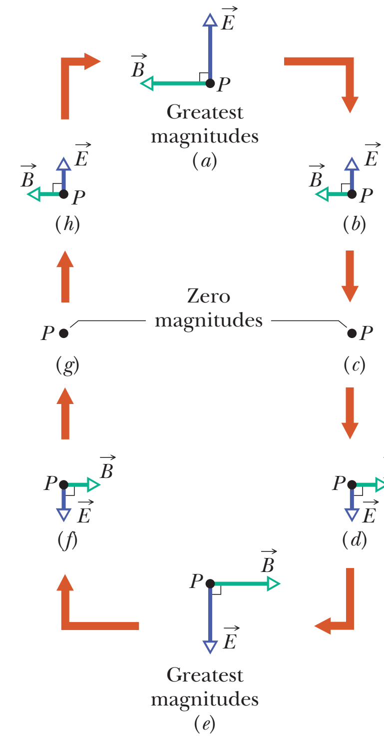



**Figure 33-4** (*a*)–(*h*) The variation in the electric field $\vec{E}$ and the magnetic field $\vec{B}$ at the distant point $P$ of Fig. 33-3 as one wavelength of the electromagnetic wave travels past it. In this perspective, the wave is traveling directly out of the page. The two fields vary sinusoidally in magnitude and direction. Note that they are always perpendicular to each other and to the wave's direction of travel.

In keeping with these features, we can assume that the electromagnetic wave is traveling toward $P$ in the positive direction of an $x$ axis, that the electric field in Fig. 33-4 is oscillating parallel to the $y$ axis, and that the magnetic field is then oscillating parallel to the $z$ axis (using a right-handed coordinate system, of course). Then we can write the electric and magnetic fields as sinusoidal functions of position $x$ (along the path of the wave) and time $t$:

$$E = E_m \sin(kx - \omega t), \tag{33-1}$$

$$B = B_m \sin(kx - \omega t), \tag{33-2}$$

in which $E_m$ and $B_m$ are the amplitudes of the fields and, as in Chapter 16, $\omega$ and $k$ are the angular frequency and angular wave number of the wave, respectively. From these equations, we note that not only do the two fields form the electromagnetic wave but each also forms its own wave. Equation 33-1 gives the *electric wave component* of the electromagnetic wave, and Eq. 33-2 gives the *magnetic wave component.* As we shall discuss below, these two wave components cannot exist independently.

***Wave Speed.*** From Eq. 16-13, we know that the speed of the wave is $\omega/k$. However, because this is an electromagnetic wave, its speed (in vacuum) is given the symbol $c$ rather than $v$. In the next section you will see that $c$ has the value

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \text{(wave speed)}, \tag{33-3}$$

which is about $3.0 \times 10^8$ m/s. In other words,

All electromagnetic waves, including visible light, have the same speed $c$ in vacuum.

You will also see that the wave speed $c$ and the amplitudes of the electric and magnetic fields are related by

$$\frac{E_m}{B_m} = c \quad \text{(amplitude ratio).} \tag{33-4}$$

If we divide Eq. 33-1 by Eq. 33-2 and then substitute with Eq. 33-4, we find that the magnitudes of the fields at every instant and at any point are related by

$$\frac{E}{B} = c \quad \text{(magnitude ratio).} \tag{33-5}$$

***Rays and Wavefronts.*** We can represent the electromagnetic wave as in Fig. 33-5*a*, with a *ray* (a directed line showing the wave's direction of travel) or with *wavefronts* (imaginary surfaces over which the wave has the same magnitude of electric field), or both. The two wavefronts shown in Fig. 33-5*a* are separated by one wavelength $\lambda$ $(= 2\pi/k)$ of the wave. (Waves traveling in approximately the same direction form a *beam,* such as a laser beam, which can also be represented with a ray.)

***Drawing the Wave.*** We can also represent the wave as in Fig. 33-5*b*, which shows the electric and magnetic field vectors in a "snapshot" of the wave at a certain instant. The curves through the tips of the vectors represent the sinusoidal oscillations given by Eqs. 33-1 and 33-2; the wave components $\vec{E}$ and $\vec{B}$ are in phase, perpendicular to each other, and perpendicular to the wave's direction of travel.

Interpretation of Fig. 33-5*b* requires some care. The similar drawings for a transverse wave on a taut string that we discussed in Chapter 16 represented the up and down displacement of sections of the string as the wave passed (*something actually moved*). Figure 33-5*b* is more abstract. At the instant shown, the electric and magnetic fields each have a certain magnitude and direction (but always perpendicular to the $x$ axis) at each point along the $x$ axis. We choose to represent these vector quantities with a pair of arrows for each point, and so we must draw arrows of different lengths for different points, all directed away from the $x$ axis, like thorns on a rose stem. However, the arrows represent field values only at points that are on the $x$ axis. Neither the arrows nor the sinusoidal curves represent a sideways motion of anything, nor do the arrows connect points on the $x$ axis with points off the axis.

***Feedback.*** Drawings like Fig. 33-5 help us visualize what is actually a very complicated situation. First consider the magnetic field. Because it varies sinusoidally, it induces (via Faraday's law of induction) a perpendicular electric field that also varies sinusoidally. However, because that electric field is varying sinusoidally, it induces (via Maxwell's law of induction) a perpendicular magnetic field that also varies sinusoidally. And so on. The two fields continuously create each other via induction, and the resulting sinusoidal variations in the fields travel as a wave—the electromagnetic wave. Without this amazing result, we could not see; indeed, because we need electromagnetic waves

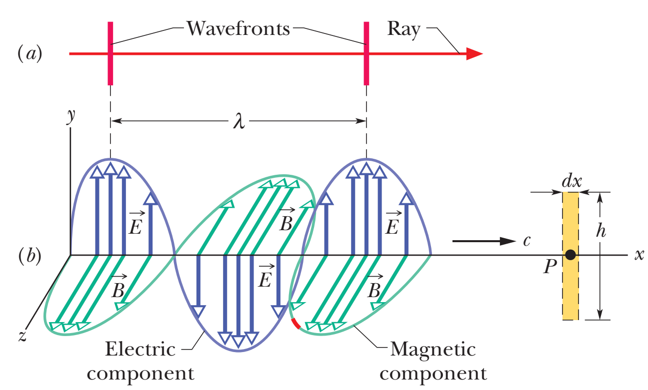



**Figure 33-5** ($a$) An electromagnetic wave represented with a ray and two wavefronts; the wavefronts are separated by one wavelength $\lambda$. ($b$) The same wave represented in a "snapshot" of its electric field $\vec{E}$ and magnetic field $\vec{B}$ at points on the $x$ axis, along which the wave travels at speed $c$. As it travels past point $P$, the fields vary as shown in Fig. 33-4. The electric component of the wave consists of only the electric fields; the magnetic component consists of only the magnetic fields. The dashed rectangle at $P$ is used in Fig. 33-6.

from the Sun to maintain Earth's temperature, without this result we could not even exist.

### A Most Curious Wave

The waves we discussed in Chapters 16 and 17 require a *medium* (some material) through which or along which to travel. We had waves traveling along a string, through Earth, and through the air. However, an electromagnetic wave (let's use the term *light wave* or *light*) is curiously different in that it requires no medium for its travel. It can, indeed, travel through a medium such as air or glass, but it can also travel through the vacuum of space between a star and us.

Once the special theory of relativity became accepted, long after Einstein published it in 1905, the speed of light waves was realized to be special. One reason is that light has the same speed regardless of the frame of reference from which it is measured. If you send a beam of light along an axis and ask several observers to measure its speed while they move at different speeds along that axis, either in the direction of the light or opposite it, they will all measure the *same speed* for the light. This result is an amazing one and quite different from what would have been found if those observers had measured the speed of any other type of wave; for other waves, the speed of the observers relative to the wave would have affected their measurements.

The meter has now been defined so that the speed of light (any electromagnetic wave) in vacuum has the exact value

$$c = 299\,792\,458 \text{ m/s},$$

which can be used as a standard. In fact, if you now measure the travel time of a pulse of light from one point to another, you are not really measuring the speed of the light but rather the distance between those two points.

## The Traveling Electromagnetic Wave, Quantitatively

We shall now derive Eqs. 33-3 and 33-4 and, even more important, explore the dual induction of electric and magnetic fields that gives us light.

### Equation 33-4 and the Induced Electric Field

The dashed rectangle of dimensions $dx$ and $h$ in Fig. 33-6 is fixed at point $P$ on the $x$ axis and in the $xy$ plane (it is shown on the right in Fig. 33-5$b$). As the electromagnetic wave moves rightward past the rectangle, the magnetic flux $\Phi_B$ through the rectangle changes and—according to Faraday's law of induction—induced electric fields appear throughout the region of the rectangle. We take $\vec{E}$ and $\vec{E} + d\vec{E}$ to be the induced fields along the two long sides of the rectangle. These induced electric fields are, in fact, the electrical component of the electromagnetic wave.

Note the small red portion of the magnetic field component curve far from the $y$ axis in Fig. 33-5$b$. Let's consider the induced electric fields at the instant when this red portion of the magnetic component is passing through the rectangle. Just then, the magnetic field through the rectangle points in the positive $z$ direction and is decreasing in magnitude (the magnitude was greater just before the red section arrived). Because the magnetic field is decreasing, the magnetic flux $\Phi_B$ through the rectangle is also decreasing. According to Faraday's law, this change in flux is opposed by induced electric fields, which produce a magnetic field $\vec{B}$ in the positive $z$ direction.

According to Lenz's law, this in turn means that if we imagine the boundary of the rectangle to be a conducting loop, a counterclockwise induced current would have to appear in it. There is, of course, no conducting loop; but this analysis shows that the induced electric field vectors $\vec{E}$ and $\vec{E} + d\vec{E}$ are indeed

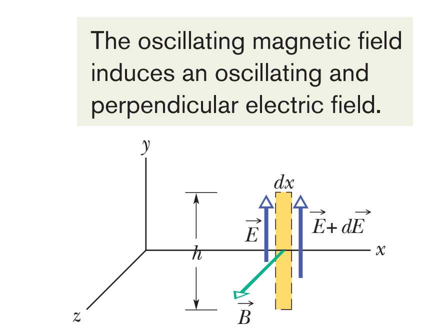



**Figure 33-6** As the electromagnetic wave travels rightward past point $P$ in Fig. 33-5$b$, the sinusoidal variation of the magnetic field $\vec{B}$ through a rectangle centered at $P$ induces electric fields along the rectangle. At the instant shown, $\vec{B}$ is decreasing in magnitude and the induced electric field is therefore greater in magnitude on the right side of the rectangle than on the left.

oriented as shown in Fig. 33-6, with the magnitude of $\vec{E} + d\vec{E}$ greater than that of $\vec{E}$. Otherwise, the net induced electric field would not act counterclockwise around the rectangle.

***Faraday's Law.*** Let us now apply Faraday's law of induction,

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}, \tag{33-6}$$

counterclockwise around the rectangle of Fig. 33-6. There is no contribution to the integral from the top or bottom of the rectangle because $\vec{E}$ and $d\vec{s}$ are perpendicular to each other there. The integral then has the value

$$\oint \vec{E} \cdot d\vec{s} = (E + dE)h - Eh = h\,dE. \tag{33-7}$$

The flux $\Phi_B$ through this rectangle is

$$\Phi_B = (B)(h\,dx), \tag{33-8}$$

where $B$ is the average magnitude of $\vec{B}$ within the rectangle and $h\,dx$ is the area of the rectangle. Differentiating Eq. 33-8 with respect to $t$ gives

$$\frac{d\Phi_B}{dt} = h\,dx\,\frac{dB}{dt}. \tag{33-9}$$

If we substitute Eqs. 33-7 and 33-9 into Eq. 33-6, we find

$$h\,dE = -h\,dx\,\frac{dB}{dt}$$

or

$$\frac{dE}{dx} = -\frac{dB}{dt}. \tag{33-10}$$

Actually, both $B$ and $E$ are functions of *two* variables, coordinate $x$ and time $t$, as Eqs. 33-1 and 33-2 show. However, in evaluating $dE/dx$, we must assume that $t$ is constant because Fig. 33-6 is an "instantaneous snapshot." Also, in evaluating $dB/dt$ we must assume that $x$ is constant (a particular value) because we are dealing with the time rate of change of $B$ at a particular place, the point $P$ shown in Fig. 33-5$b$. The derivatives under these circumstances are *partial derivatives,* and Eq. 33-10 must be written

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}. \tag{33-11}$$

The minus sign in this equation is appropriate and necessary because, although magnitude $E$ is increasing with $x$ at the site of the rectangle in Fig. 33-6, magnitude $B$ is decreasing with $t$.

From Eq. 33-1 we have

$$\frac{\partial E}{\partial x} = kE_m \cos(kx - \omega t)$$

and from Eq. 33-2

$$\frac{\partial B}{\partial t} = -\omega B_m \cos(kx - \omega t).$$

Then Eq. 33-11 reduces to

$$kE_m \cos(kx - \omega t) = \omega B_m \cos(kx - \omega t). \tag{33-12}$$

The ratio $\omega/k$ for a traveling wave is its speed, which we are calling $c$. Equation 33-12 then becomes

$$\frac{E_m}{B_m} = c \quad \text{(amplitude ratio)}, \tag{33-13}$$

which is just Eq. 33-4.

## Equation 33-3 and the Induced Magnetic Field

The oscillating electric field induces an oscillating and perpendicular magnetic field.

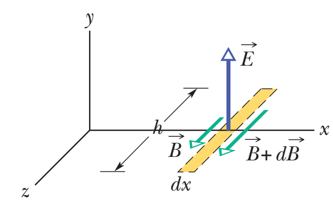



**Figure 33-7** The sinusoidal variation of the electric field through this rectangle, located (but not shown) at point $P$ in Fig. 33-5$b$, induces magnetic fields along the rectangle. The instant shown is that of Fig. 33-6: $\vec{E}$ is decreasing in magnitude, and the magnitude of the induced magnetic field is greater on the right side of the rectangle than on the left.

Figure 33-7 shows another dashed rectangle at point $P$ of Fig. 33-5$b$; this one is in the $xz$ plane. As the electromagnetic wave moves rightward past this new rectangle, the electric flux $\Phi_E$ through the rectangle changes and—according to Maxwell's law of induction—induced magnetic fields appear throughout the region of the rectangle. These induced magnetic fields are, in fact, the magnetic component of the electromagnetic wave.

We see from Fig. 33-5$b$ that at the instant chosen for the magnetic field represented in Fig. 33-6, marked in red on the magnetic component curve, the electric field through the rectangle of Fig. 33-7 is directed as shown. Recall that at the chosen instant, the magnetic field in Fig. 33-6 is decreasing. Because the two fields are in phase, the electric field in Fig. 33-7 must also be decreasing, and so must the electric flux $\Phi_E$ through the rectangle. By applying the same reasoning we applied to Fig. 33-6, we see that the changing flux $\Phi_E$ will induce a magnetic field with vectors $\vec{B}$ and $\vec{B} + d\vec{B}$ oriented as shown in Fig. 33-7, where field $\vec{B} + d\vec{B}$ is greater than field $\vec{B}$.

***Maxwell's Law.*** Let us apply Maxwell's law of induction,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}, \qquad (33\text{-}14)$$

by proceeding counterclockwise around the dashed rectangle of Fig. 33-7. Only the long sides of the rectangle contribute to the integral because the dot product along the short sides is zero. Thus, we can write

$$\oint \vec{B} \cdot d\vec{s} = -(B + dB)h + Bh = -h\,dB. \qquad (33\text{-}15)$$

The flux $\Phi_E$ through the rectangle is

$$\Phi_E = (E)(h\,dx), \qquad (33\text{-}16)$$

where $E$ is the average magnitude of $\vec{E}$ within the rectangle. Differentiating Eq. 33-16 with respect to $t$ gives

$$\frac{d\Phi_E}{dt} = h\,dx \frac{dE}{dt}.$$

If we substitute this and Eq. 33-15 into Eq. 33-14, we find

$$-h\,dB = \mu_0 \varepsilon_0 \left( h\,dx \frac{dE}{dt} \right)$$

or, changing to partial-derivative notation as we did for Eq. 33-11,

$$-\frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}. \qquad (33\text{-}17)$$

Again, the minus sign in this equation is necessary because, although $B$ is increasing with $x$ at point $P$ in the rectangle in Fig. 33-7, $E$ is decreasing with $t$.

Evaluating Eq. 33-17 by using Eqs. 33-1 and 33-2 leads to

$$-kB_m \cos(kx - \omega t) = -\mu_0 \varepsilon_0 \omega E_m \cos(kx - \omega t),$$

which we can write as

$$\frac{E_m}{B_m} = \frac{1}{\mu_0 \varepsilon_0 (\omega/k)} = \frac{1}{\mu_0 \varepsilon_0 c}.$$

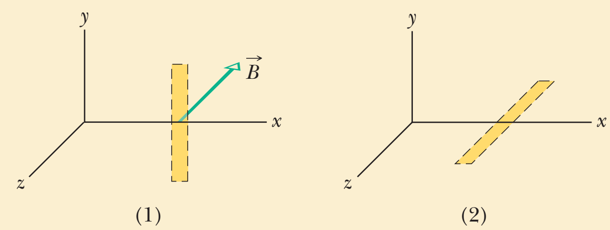
Combining this with Eq. 33-13 leads at once to

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \text{(wave speed)}, \qquad (33\text{-}18)$$

which is exactly Eq. 33-3.

### Checkpoint 1

The magnetic field $\vec{B}$ through the rectangle of Fig. 33-6 is shown at a different instant in part 1 of the figure here; $\vec{B}$ is directed in the $xz$ plane, parallel to the $z$ axis, and its magnitude is increasing. (a) Complete part 1 by drawing the induced electric fields, indicating both directions and relative magnitudes (as in Fig. 33-6). (b) For the same instant, complete part 2 of the figure by drawing the electric field of the electromagnetic wave. Also draw the induced magnetic fields, indicating both directions and relative magnitudes (as in Fig. 33-7).




# 33-2 ENERGY TRANSPORT AND THE POYNTING VECTOR

## Learning Objectives

*After reading this module, you should be able to . . .*

**33.13** Identify that an electromagnetic wave transports energy.

**33.14** For a target, identify that an EM wave's rate of energy transport per unit area is given by the Poynting vector $\vec{S}$, which is related to the cross product of the electric field $\vec{E}$ and magnetic field $\vec{B}$.

**33.15** Determine the direction of travel (and thus energy transport) of an electromagnetic wave by applying the cross product for the corresponding Poynting vector.

**33.16** Calculate the instantaneous rate $S$ of energy flow of an EM wave in terms of the instantaneous electric field magnitude $E$.

**33.17** For the electric field component of an electromagnetic wave, relate the rms value $E_{\text{rms}}$ to the amplitude $E_m$.

**33.18** Identify an EM wave's intensity $I$ in terms of energy transport.

**33.19** Apply the relationships between an EM wave's intensity $I$ and the electric field's rms value $E_{\text{rms}}$ and amplitude $E_m$.

**33.20** Apply the relationship between average power $P_{\text{avg}}$, energy transfer $\Delta E$, and the time $\Delta t$ taken by that transfer, and apply the relationship between the instantaneous power $P$ and the rate of energy transfer $dE/dt$.

**33.21** Identify an isotropic point source of light.

**33.22** For an isotropic point source of light, apply the relationship between the emission power $P$, the distance $r$ to a point of measurement, and the intensity $I$ at that point.

**33.23** In terms of energy conservation, explain why the intensity from an isotropic point source of light decreases as $1/r^2$.

## Key Ideas

- The rate per unit area at which energy is transported via an electromagnetic wave is given by the Poynting vector $\vec{S}$:

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}.$$

The direction of $\vec{S}$ (and thus of the wave's travel and the energy transport) is perpendicular to the directions of both $\vec{E}$ and $\vec{B}$.

- The time-averaged rate per unit area at which energy is transported is $S_{\text{avg}}$, which is called the intensity $I$ of the wave:

$$I = \frac{1}{c\mu_0}E_{\text{rms}}^2,$$

in which $E_{\text{rms}} = E_m/\sqrt{2}$.

- A point source of electromagnetic waves emits the waves isotropically—that is, with equal intensity in all directions. The intensity of the waves at distance $r$ from a point source of power $P_s$ is

$$I = \frac{P_s}{4\pi r^2}.$$

## Energy Transport and the Poynting Vector

All sunbathers know that an electromagnetic wave can transport energy and deliver it to a body on which the wave falls. The rate of energy transport per unit area in such a wave is described by a vector $\vec{S}$, called the **Poynting vector** after physicist John Henry Poynting (1852–1914), who first discussed its properties. This vector is defined as

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \quad \text{(Poynting vector).} \tag{33-19}$$

Its magnitude $S$ is related to the rate at which energy is transported by a wave across a unit area at any instant (inst):

$$S = \left(\frac{\text{energy/time}}{\text{area}}\right)_{\text{inst}} = \left(\frac{\text{power}}{\text{area}}\right)_{\text{inst}}. \tag{33-20}$$

From this we can see that the SI unit for $\vec{S}$ is the watt per square meter ($W/m^2$).

> The direction of the Poynting vector $\vec{S}$ of an electromagnetic wave at any point gives the wave's direction of travel and the direction of energy transport at that point.

Because $\vec{E}$ and $\vec{B}$ are perpendicular to each other in an electromagnetic wave, the magnitude of $\vec{E} \times \vec{B}$ is $EB$. Then the magnitude of $\vec{S}$ is

$$S = \frac{1}{\mu_0}EB, \tag{33-21}$$

in which $S$, $E$, and $B$ are instantaneous values. The magnitudes $E$ and $B$ are so closely coupled to each other that we need to deal with only one of them; we choose $E$, largely because most instruments for detecting electromagnetic waves deal with the electric component of the wave rather than the magnetic component. Using $B = E/c$ from Eq. 33-5, we can rewrite Eq. 33-21 in terms of just the electric component as

$$S = \frac{1}{c\mu_0}E^2 \quad \text{(instantaneous energy flow rate).} \tag{33-22}$$

***Intensity.*** By substituting $E = E_m \sin(kx - \omega t)$ into Eq. 33-22, we could obtain an equation for the energy transport rate as a function of time. More useful in practice, however, is the average energy transported over time; for that, we need to find the time-averaged value of $S$, written $S_{\text{avg}}$ and also called the **intensity** $I$ of the wave. Thus from Eq. 33-20, the intensity $I$ is

$$I = S_{\text{avg}} = \left(\frac{\text{energy/time}}{\text{area}}\right)_{\text{avg}} = \left(\frac{\text{power}}{\text{area}}\right)_{\text{avg}}. \tag{33-23}$$

From Eq. 33-22, we find

$$I = S_{\text{avg}} = \frac{1}{c\mu_0}[E^2]_{\text{avg}} = \frac{1}{c\mu_0}[E_m^2 \sin^2(kx - \omega t)]_{\text{avg}}. \tag{33-24}$$

Over a full cycle, the average value of $\sin^2\theta$, for any angular variable $\theta$, is $\frac{1}{2}$ (see Fig. 31-17). In addition, we define a new quantity $E_{\text{rms}}$, the *root-mean-square* value of the electric field, as

$$E_{\text{rms}} = \frac{E_m}{\sqrt{2}}. \tag{33-25}$$

The energy emitted by light source $S$ must pass through the sphere of radius $r$.

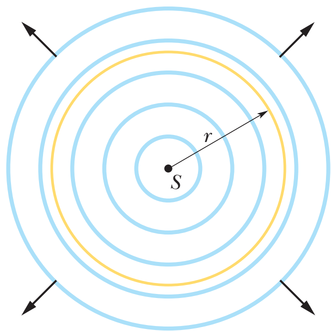



**Figure 33-8** A point source $S$ emits electromagnetic waves uniformly in all directions. The spherical wavefronts pass through an imaginary sphere of radius $r$ that is centered on $S$.

We can then rewrite Eq. 33-24 as

$$I = \frac{1}{c\mu_0} E_{\text{rms}}^2. \tag{33-26}$$

Because $E = cB$ and $c$ is such a very large number, you might conclude that the energy associated with the electric field is much greater than that associated with the magnetic field. That conclusion is incorrect; the two energies are exactly equal. To show this, we start with Eq. 25-25, which gives the energy density $u\ (= \frac{1}{2}\varepsilon_0 E^2)$ within an electric field, and substitute $cB$ for $E$; then we can write

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0 (cB)^2.$$

If we now substitute for $c$ with Eq. 33-3, we get

$$u_E = \tfrac{1}{2}\varepsilon_0 \frac{1}{\mu_0 \varepsilon_0} B^2 = \frac{B^2}{2\mu_0}.$$

However, Eq. 30-55 tells us that $B^2/2\mu_0$ is the energy density $u_B$ of a magnetic field $\vec{B}$; so we see that $u_E = u_B$ everywhere along an electromagnetic wave.

### Variation of Intensity with Distance

How intensity varies with distance from a real source of electromagnetic radiation is often complex—especially when the source (like a searchlight at a movie premier) beams the radiation in a particular direction. However, in some situations we can assume that the source is a *point source* that emits the light *isotropically*—that is, with equal intensity in all directions. The spherical wavefronts spreading from such an isotropic point source $S$ at a particular instant are shown in cross section in Fig. 33-8.

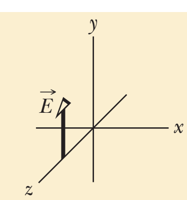
Let us assume that the energy of the waves is conserved as they spread from this source. Let us also center an imaginary sphere of radius $r$ on the source, as shown in Fig. 33-8. All the energy emitted by the source must pass through the sphere. Thus, the rate at which energy passes through the sphere via the radiation must equal the rate at which energy is emitted by the source—that is, the source power $P_s$. The intensity $I$ (power per unit area) measured at the sphere must then be, from Eq. 33-23,

$$I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2}, \tag{33-27}$$

where $4\pi r^2$ is the area of the sphere. Equation 33-27 tells us that the intensity of the electromagnetic radiation from an isotropic point source decreases with the square of the distance $r$ from the source.

### Checkpoint 2

The figure here gives the electric field of an electromagnetic wave at a certain point and a certain instant. The wave is transporting energy in the negative $z$ direction. What is the direction of the magnetic field of the wave at that point and instant?




### Sample Problem 33.01 Light wave: rms values of the electric and magnetic fields

When you look at the North Star (Polaris), you intercept light from a star at a distance of 431 ly and emitting energy at a rate of $2.2 \times 10^3$ times that of our Sun ($P_{\text{sun}} = 3.90 \times 10^{26}$ W). Neglecting any atmospheric absorption, find the rms values of the electric and magnetic fields when the starlight reaches you.

## KEY IDEAS

1. The rms value $E_{rms}$ of the electric field in light is related to the intensity $I$ of the light via Eq. 33-26 ($I = E_{rms}^2/c\mu_0$).
2. Because the source is so far away and emits light with equal intensity in all directions, the intensity $I$ at any distance $r$ from the source is related to the source's power $P_s$ via Eq. 33-27 ($I = P_s/4\pi r^2$).
3. The magnitudes of the electric field and magnetic field of an electromagnetic wave at any instant and at any point in the wave are related by the speed of light $c$ according to Eq. 33-5 ($E/B = c$). Thus, the rms values of those fields are also related by Eq. 33-5.

***Electric field:*** Putting the first two ideas together gives us

$$I = \frac{P_s}{4\pi r^2} = \frac{E_{rms}^2}{c\mu_0}$$

and
$$E_{rms} = \sqrt{\frac{P_s c\mu_0}{4\pi r^2}}.$$

By substituting $P_s = (2.2 \times 10^3)(3.90 \times 10^{26}$ W), $r = 431$ ly $= 4.08 \times 10^{18}$ m, and values for the constants, we find

$$E_{rms} = 1.24 \times 10^{-3} \text{ V/m} \approx 1.2 \text{ mV/m}. \quad \text{(Answer)}$$

***Magnetic field:*** From Eq. 33-5, we write

$$B_{rms} = \frac{E_{rms}}{c} = \frac{1.24 \times 10^{-3} \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 4.1 \times 10^{-12} \text{ T} = 4.1 \text{ pT}.$$

***Cannot compare the fields:*** Note that $E_{rms}$ (= 1.2 mV/m) is small as judged by ordinary laboratory standards, but $B_{rms}$ (= 4.1 pT) is quite small. This difference helps to explain why most instruments used for the detection and measurement of electromagnetic waves are designed to respond to the electric component. It is wrong, however, to say that the electric component of an electromagnetic wave is "stronger" than the magnetic component. You cannot compare quantities that are measured in different units. However, these electric and magnetic components are on an equal basis because their average energies, which *can* be compared, are equal.



# 33-3 RADIATION PRESSURE

### Learning Objectives

*After reading this module, you should be able to . . .*

**33.24** Distinguish between force and pressure.
**33.25** Identify that an electromagnetic wave transports momentum and can exert a force and a pressure on a target.
**33.26** For a uniform electromagnetic beam that is perpendicular to a target area, apply the relationships between that area, the wave's intensity, and the force on the target, for both total absorption and total backward reflection.
**33.27** For a uniform electromagnetic beam that is perpendicular to a target area, apply the relationships between the wave's intensity and the pressure on the target, for both total absorption and total backward reflection.

### Key Ideas

- When a surface intercepts electromagnetic radiation, a force and a pressure are exerted on the surface.
- If the radiation is totally absorbed by the surface, the force is

$$F = \frac{IA}{c} \quad \text{(total absorption)},$$

in which $I$ is the intensity of the radiation and $A$ is the area of the surface perpendicular to the path of the radiation.

- If the radiation is totally reflected back along its original path, the force is

$$F = \frac{2IA}{c} \quad \text{(total reflection back along path)}.$$

- The radiation pressure $p_r$ is the force per unit area:

$$p_r = \frac{I}{c} \quad \text{(total absorption)}$$

and
$$p_r = \frac{2I}{c} \quad \text{(total reflection back along path)}.$$

## Radiation Pressure

Electromagnetic waves have linear momentum and thus can exert a pressure on an object when shining on it. However, the pressure must be very small because, for example, you do not feel a punch during a camera flash.

To find an expression for the pressure, let us shine a beam of electromagnetic radiation—light, for example—on an object for a time interval $\Delta t$. Further, let us assume that the object is free to move and that the radiation is entirely **absorbed** (taken up) by the object. This means that during the interval $\Delta t$, the object gains an energy $\Delta U$ from the radiation. Maxwell showed that the object also gains linear momentum. The magnitude $\Delta p$ of the momentum change of the object is related to the energy change $\Delta U$ by

$$\Delta p = \frac{\Delta U}{c} \quad \text{(total absorption)}, \tag{33-28}$$

where $c$ is the speed of light. The direction of the momentum change of the object is the direction of the *incident* (incoming) beam that the object absorbs.

Instead of being absorbed, the radiation can be **reflected** by the object; that is, the radiation can be sent off in a new direction as if it bounced off the object. If the radiation is entirely reflected back along its original path, the magnitude of the momentum change of the object is twice that given above, or

$$\Delta p = \frac{2\,\Delta U}{c} \quad \text{(total reflection back along path)}. \tag{33-29}$$

In the same way, an object undergoes twice as much momentum change when a perfectly elastic tennis ball is bounced from it as when it is struck by a perfectly inelastic ball (a lump of wet putty, say) of the same mass and velocity. If the incident radiation is partly absorbed and partly reflected, the momentum change of the object is between $\Delta U/c$ and $2\,\Delta U/c$.

***Force.*** From Newton's second law in its linear momentum form (Module 9-3), we know that a change in momentum is related to a force by

$$F = \frac{\Delta p}{\Delta t}. \tag{33-30}$$

To find expressions for the force exerted by radiation in terms of the intensity $I$ of the radiation, we first note that intensity is

$$I = \frac{\text{power}}{\text{area}} = \frac{\text{energy/time}}{\text{area}}.$$

Next, suppose that a flat surface of area $A$, perpendicular to the path of the radiation, intercepts the radiation. In time interval $\Delta t$, the energy intercepted by area $A$ is

$$\Delta U = IA\,\Delta t. \tag{33-31}$$

If the energy is completely absorbed, then Eq. 33-28 tells us that $\Delta p = IA\,\Delta t/c$, and, from Eq. 33-30, the magnitude of the force on the area $A$ is

$$F = \frac{IA}{c} \quad \text{(total absorption)}. \tag{33-32}$$

Similarly, if the radiation is totally reflected back along its original path, Eq. 33-29 tells us that $\Delta p = 2IA\,\Delta t/c$ and, from Eq. 33-30,

$$F = \frac{2IA}{c} \quad \text{(total reflection back along path)}. \tag{33-33}$$

If the radiation is partly absorbed and partly reflected, the magnitude of the force on area $A$ is between the values of $IA/c$ and $2IA/c$.

***Pressure.*** The force per unit area on an object due to radiation is the radiation pressure $p_r$. We can find it for the situations of Eqs. 33-32 and 33-33 by dividing both sides of each equation by $A$. We obtain

$$p_r = \frac{I}{c} \quad \text{(total absorption)} \tag{33-34}$$

and $$p_r = \frac{2I}{c} \quad \text{(total reflection back along path).} \tag{33-35}$$

Be careful not to confuse the symbol $p_r$ for radiation pressure with the symbol $p$ for momentum. Just as with fluid pressure in Chapter 14, the SI unit of radiation pressure is the newton per square meter ($N/m^2$), which is called the pascal (Pa).

The development of laser technology has permitted researchers to achieve radiation pressures much greater than, say, that due to a camera flashlamp. This comes about because a beam of laser light—unlike a beam of light from a small lamp filament—can be focused to a tiny spot. This permits the delivery of great amounts of energy to small objects placed at that spot.

**Checkpoint 3**

Light of uniform intensity shines perpendicularly on a totally absorbing surface, fully illuminating the surface. If the area of the surface is decreased, do (a) the radiation pressure and (b) the radiation force on the surface increase, decrease, or stay the same?

# 33-4 POLARIZATION

## Learning Objectives

*After reading this module, you should be able to . . .*

**33.28** Distinguish between polarized light and unpolarized light.

**33.29** For a light beam headed toward you, sketch representations of polarized light and unpolarized light.

**33.30** When a beam is sent into a polarizing sheet, explain the function of the sheet in terms of its polarizing direction (or axis) and the electric field component that is absorbed and the component that is transmitted.

**33.31** For light that emerges from a polarizing sheet, identify its polarization relative to the sheet's polarizing direction.

**33.32** For a light beam incident perpendicularly on a polarizing sheet, apply the one-half rule and the cosine-squared rule, distinguishing their uses.

**33.33** Distinguish between a polarizer and an analyzer.

**33.34** Explain what is meant if two sheets are crossed.

**33.35** When a beam is sent into a system of polarizing sheets, work through the sheets one by one, finding the transmitted intensity and polarization.

## Key Ideas

- Electromagnetic waves are polarized if their electric field vectors are all in a single plane, called the plane of oscillation. Light waves from common sources are not polarized; that is, they are unpolarized, or polarized randomly.
- When a polarizing sheet is placed in the path of light, only electric field components of the light parallel to the sheet's polarizing direction are transmitted by the sheet; components perpendicular to the polarizing direction are absorbed. The light that emerges from a polarizing sheet is polarized parallel to the polarizing direction of the sheet.
- If the original light is initially unpolarized, the transmitted intensity $I$ is half the original intensity $I_0$:
$$I = \tfrac{1}{2} I_0.$$
- If the original light is initially polarized, the transmitted intensity depends on the angle $\theta$ between the polarization direction of the original light and the polarizing direction of the sheet:
$$I = I_0 \cos^2 \theta.$$

## Polarization

VHF (very high frequency) television antennas in England are oriented vertically, but those in North America are horizontal. The difference is due to the direction of oscillation of the electromagnetic waves carrying the TV signal. In England, the transmitting equipment is designed to produce waves that are **polarized** vertically; that is, their electric field oscillates vertically. Thus, for the

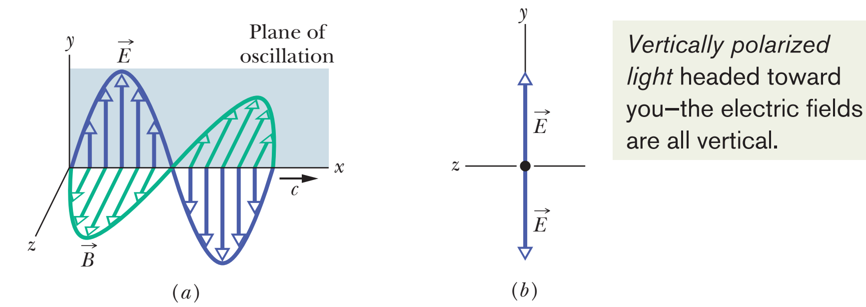


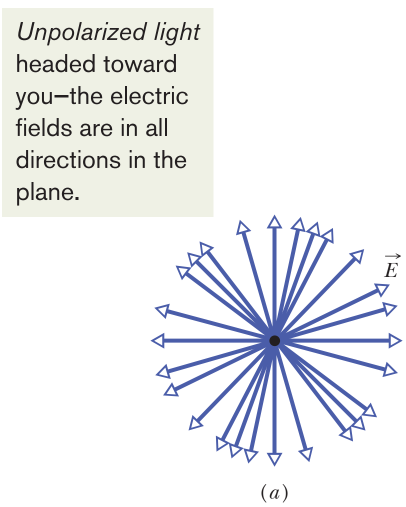

**Figure 33-9** (*a*) The plane of oscillation of a polarized electromagnetic wave. (*b*) To represent the polarization, we view the plane of oscillation head-on and indicate the directions of the oscillating electric field with a double arrow.

electric field of the incident television waves to drive a current along an antenna (and provide a signal to a television set), the antenna must be vertical. In North America, the waves are polarized horizontally.

Figure 33-9*a* shows an electromagnetic wave with its electric field oscillating parallel to the vertical $y$ axis. The plane containing the $\vec{E}$ vectors is called the **plane of oscillation** of the wave (hence, the wave is said to be *plane-polarized* in the $y$ direction). We can represent the wave's *polarization* (state of being polarized) by showing the directions of the electric field oscillations in a head-on view of the plane of oscillation, as in Fig. 33-9*b*. The vertical double arrow in that figure indicates that as the wave travels past us, its electric field oscillates vertically—it continuously changes between being directed up and down the $y$ axis.

## Polarized Light

The electromagnetic waves emitted by a television station all have the same polarization, but the electromagnetic waves emitted by any common source of light (such as the Sun or a bulb) are **polarized randomly,** or **unpolarized** (the two terms mean the same thing). That is, the electric field at any given point is always perpendicular to the direction of travel of the waves but changes directions randomly. Thus, if we try to represent a head-on view of the oscillations over some time period, we do not have a simple drawing with a single double arrow like that of Fig. 33-9*b*; instead we have a mess of double arrows like that in Fig. 33-10*a*.




In principle, we can simplify the mess by resolving each electric field of Fig. 33-10*a* into $y$ and $z$ components. Then as the wave travels past us, the net $y$ component oscillates parallel to the $y$ axis and the net $z$ component oscillates parallel to the $z$ axis. We can then represent the unpolarized light with a pair of double arrows as shown in Fig. 33-10*b*. The double arrow along the $y$ axis represents the oscillations of the net $y$ component of the electric field. The double arrow along the $z$ axis represents the oscillations of the net $z$ component of the electric field. In doing all this, we effectively change unpolarized light into the superposition of two polarized waves whose planes of oscillation are perpendicular to each other—one plane contains the $y$ axis and the other contains the $z$ axis. One reason to make this change is that drawing Fig. 33-10*b* is a lot easier than drawing Fig. 33-10*a*.

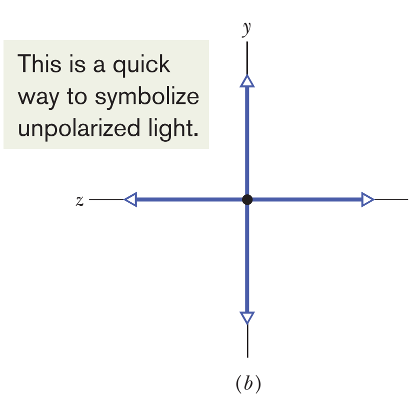



**Figure 33-10** (*a*) Unpolarized light consists of waves with randomly directed electric fields. Here the waves are all traveling along the same axis, directly out of the page, and all have the same amplitude $E$. (*b*) A second way of representing unpolarized light—the light is the superposition of two polarized waves whose planes of oscillation are perpendicular to each other.

We can draw similar figures to represent light that is **partially polarized** (its field oscillations are not completely random as in Fig. 33-10*a*, nor are they parallel to a single axis as in Fig. 33-9*b*). For this situation, we draw one of the double arrows in a perpendicular pair of double arrows longer than the other one.

***Polarizing Direction.*** We can transform unpolarized visible light into polarized light by sending it through a *polarizing sheet,* as is shown in Fig. 33-11. Such sheets, commercially known as Polaroids or Polaroid filters, were invented in 1932 by Edwin Land while he was an undergraduate student. A polarizing sheet consists of certain long molecules embedded in plastic. When the sheet is manufactured, it is stretched to align the molecules in parallel rows, like rows in a plowed field. When light is then sent through the sheet, electric field components along one direction pass through the sheet, while components perpendicular to that direction are absorbed by the molecules and disappear.

We shall not dwell on the molecules but, instead, shall assign to the sheet a *polarizing direction,* along which electric field components are passed:

An electric field component parallel to the polarizing direction is passed (*transmitted*) by a polarizing sheet; a component perpendicular to it is absorbed.

Thus, the electric field of the light emerging from the sheet consists of only the components that are parallel to the polarizing direction of the sheet; hence the light is polarized in that direction. In Fig. 33-11, the vertical electric field components are transmitted by the sheet; the horizontal components are absorbed. The transmitted waves are then vertically polarized.

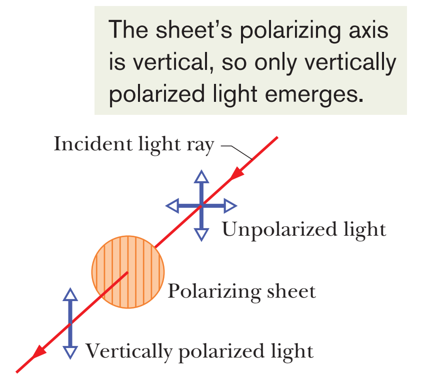



**Figure 33-11** Unpolarized light becomes polarized when it is sent through a polarizing sheet. Its direction of polarization is then parallel to the polarizing direction of the sheet, which is represented here by the vertical lines drawn in the sheet.

## Intensity of Transmitted Polarized Light

We now consider the intensity of light transmitted by a polarizing sheet. We start with unpolarized light, whose electric field oscillations we can resolve into $y$ and $z$ components as represented in Fig. 33-10*b*. Further, we can arrange for the $y$ axis to be parallel to the polarizing direction of the sheet. Then only the $y$ components of the light's electric field are passed by the sheet; the $z$ components are absorbed. As suggested by Fig. 33-10*b*, if the original waves are randomly oriented, the sum of the $y$ components and the sum of the $z$ components are equal. When the $z$ components are absorbed, half the intensity $I_0$ of the original light is lost. The intensity $I$ of the emerging polarized light is then

$$I = \tfrac{1}{2} I_0 \quad \text{(one-half rule).} \tag{33-36}$$

Let us call this the *one-half rule;* we can use it *only* when the light reaching a polarizing sheet is unpolarized.

Suppose now that the light reaching a polarizing sheet is already polarized. Figure 33-12 shows a polarizing sheet in the plane of the page and the electric field $\vec{E}$ of such a polarized light wave traveling toward the sheet (and thus prior to any absorption). We can resolve $\vec{E}$ into two components relative to the polarizing direction of the sheet: parallel component $E_y$ is transmitted by the sheet, and perpendicular component $E_z$ is absorbed. Since $\theta$ is the angle between $\vec{E}$ and the polarizing direction of the sheet, the transmitted parallel component is

$$E_y = E \cos \theta. \tag{33-37}$$

Recall that the intensity of an electromagnetic wave (such as our light wave) is proportional to the square of the electric field's magnitude (Eq. 33-26, $I = E_{\text{rms}}^2/c\mu_0$). In our present case then, the intensity $I$ of the emerging wave is proportional to $E_y^2$ and the intensity $I_0$ of the original wave is proportional to $E^2$. Hence, from Eq. 33-37 we can write $I/I_0 = \cos^2 \theta$, or

$$I = I_0 \cos^2 \theta \quad \text{(cosine-squared rule).} \tag{33-38}$$

Let us call this the *cosine-squared rule;* we can use it *only* when the light reaching a polarizing sheet is already polarized. Then the transmitted intensity $I$ is a maximum and is equal to the original intensity $I_0$ when the original wave is polarized parallel to the polarizing direction of the sheet (when $\theta$ in Eq. 33-38 is 0° or 180°). The transmitted intensity is zero when the original wave is polarized perpendicular to the polarizing direction of the sheet (when $\theta$ is 90°).

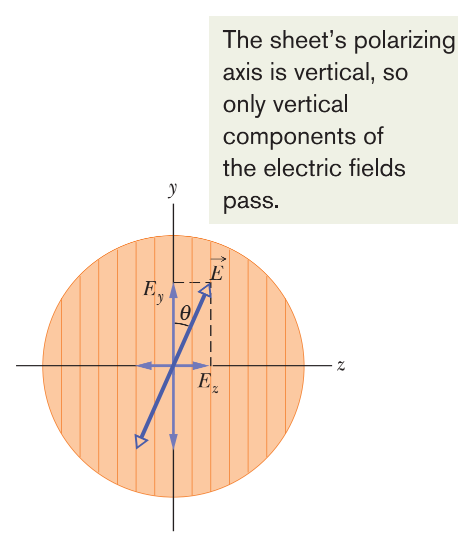



**Figure 33-12** Polarized light approaching a polarizing sheet. The electric field $\vec{E}$ of the light can be resolved into components $E_y$ (parallel to the polarizing direction of the sheet) and $E_z$ (perpendicular to that direction). Component $E_y$ will be transmitted by the sheet; component $E_z$ will be absorbed.

**_Two Polarizing Sheets._** Figure 33-13 shows an arrangement in which initially unpolarized light is sent through two polarizing sheets $P_1$ and $P_2$. (Often, the first sheet is called the *polarizer,* and the second the *analyzer.*) Because the polarizing direction of $P_1$ is vertical, the light transmitted by $P_1$ to $P_2$ is polarized vertically. If the polarizing direction of $P_2$ is also vertical, then all the light transmitted by $P_1$ is transmitted by $P_2$. If the polarizing direction of $P_2$ is horizontal, none of the light transmitted by $P_1$ is transmitted by $P_2$. We reach the same conclusions by considering only the *relative* orientations of the two sheets: If their polarizing directions are parallel, all the light passed by the first sheet is passed by the second sheet (Fig. 33-14*a*). If those directions are perpendicular (the sheets are said to be *crossed*), no light is passed by the second sheet (Fig. 33-14*b*). Finally, if the two polarizing directions of Fig. 33-13 make an angle between 0° and 90°, some of the light transmitted by $P_1$ will be transmitted by $P_2$, as set by Eq. 33-38.

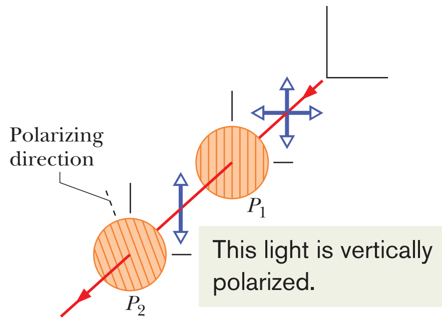
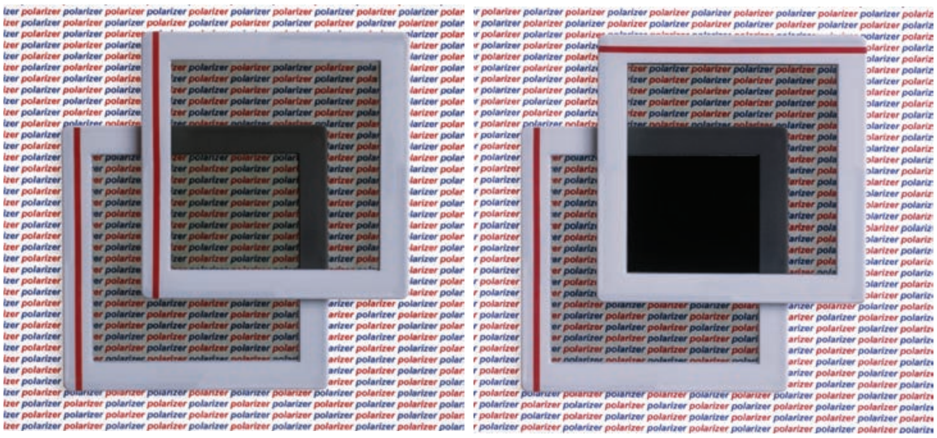



**Figure 33-13** The light transmitted by polarizing sheet $P_1$ is vertically polarized, as represented by the vertical double arrow. The amount of that light that is then transmitted by polarizing sheet $P_2$ depends on the angle between the polarization direction of that light and the polarizing direction of $P_2$ (indicated by the lines drawn in the sheet and by the dashed line).

**_Other Means._** Light can be polarized by means other than polarizing sheets, such as by reflection (discussed in Module 33-7) and by scattering from atoms or molecules. In *scattering,* light that is intercepted by an object, such as a molecule, is sent off in many, perhaps random, directions. An example is the scattering of sunlight by molecules in the atmosphere, which gives the sky its general glow.

Although direct sunlight is unpolarized, light from much of the sky is at least partially polarized by such scattering. Bees use the polarization of sky light in navigating to and from their hives. Similarly, the Vikings used it to navigate across the North Sea when the daytime Sun was below the horizon (because of the high latitude of the North Sea). These early seafarers had discovered certain crystals (now called cordierite) that changed color when rotated in polarized light. By looking at the sky through such a crystal while rotating it about their line of sight, they could locate the hidden Sun and thus determine which way was south.



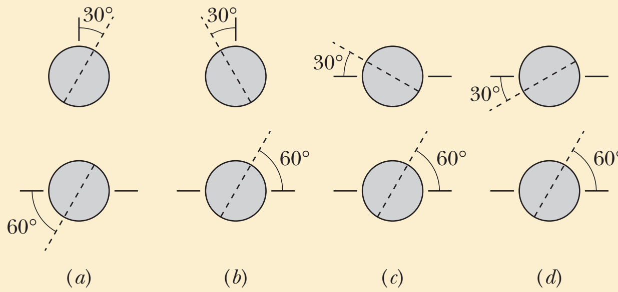

Richard Megna/Fundamental Photographs

(*a*) (*b*)

**Figure 33-14** (*a*) Overlapping polarizing sheets transmit light fairly well when their polarizing directions have the same orientation, but (*b*) they block most of the light when they are crossed.

## Checkpoint 4

The figure shows four pairs of polarizing sheets, seen face-on. Each pair is mounted in the path of initially unpolarized light. The polarizing direction of each sheet (indicated by the dashed line) is referenced to either a horizontal $x$ axis or a vertical $y$ axis. Rank the pairs according to the fraction of the initial intensity that they pass, greatest first.



## Sample Problem 33.02 Polarization and intensity with three polarizing sheets

Figure 33-15*a*, drawn in perspective, shows a system of three polarizing sheets in the path of initially unpolarized light. The polarizing direction of the first sheet is parallel to the $y$ axis, that of the second sheet is at an angle of 60° counterclockwise from the $y$ axis, and that of the third sheet is parallel to the $x$ axis. What fraction of the initial intensity $I_0$ of the light emerges from the three-sheet system, and in which direction is that emerging light polarized?

### KEY IDEAS

1. We work through the system sheet by sheet, from the first one encountered by the light to the last one.
2. To find the intensity transmitted by any sheet, we apply either the one-half rule or the cosine-squared rule, depending on whether the light reaching the sheet is unpolarized or already polarized.
3. The light that is transmitted by a polarizing sheet is always polarized parallel to the polarizing direction of the sheet.

***First sheet:*** The original light wave is represented in Fig. 33-15*b*, using the head-on, double-arrow representation of Fig. 33-10*b*. Because the light is initially unpolarized, the intensity $I_1$ of the light transmitted by the first sheet is given by the one-half rule (Eq. 33-36):

$$I_1 = \tfrac{1}{2}I_0.$$

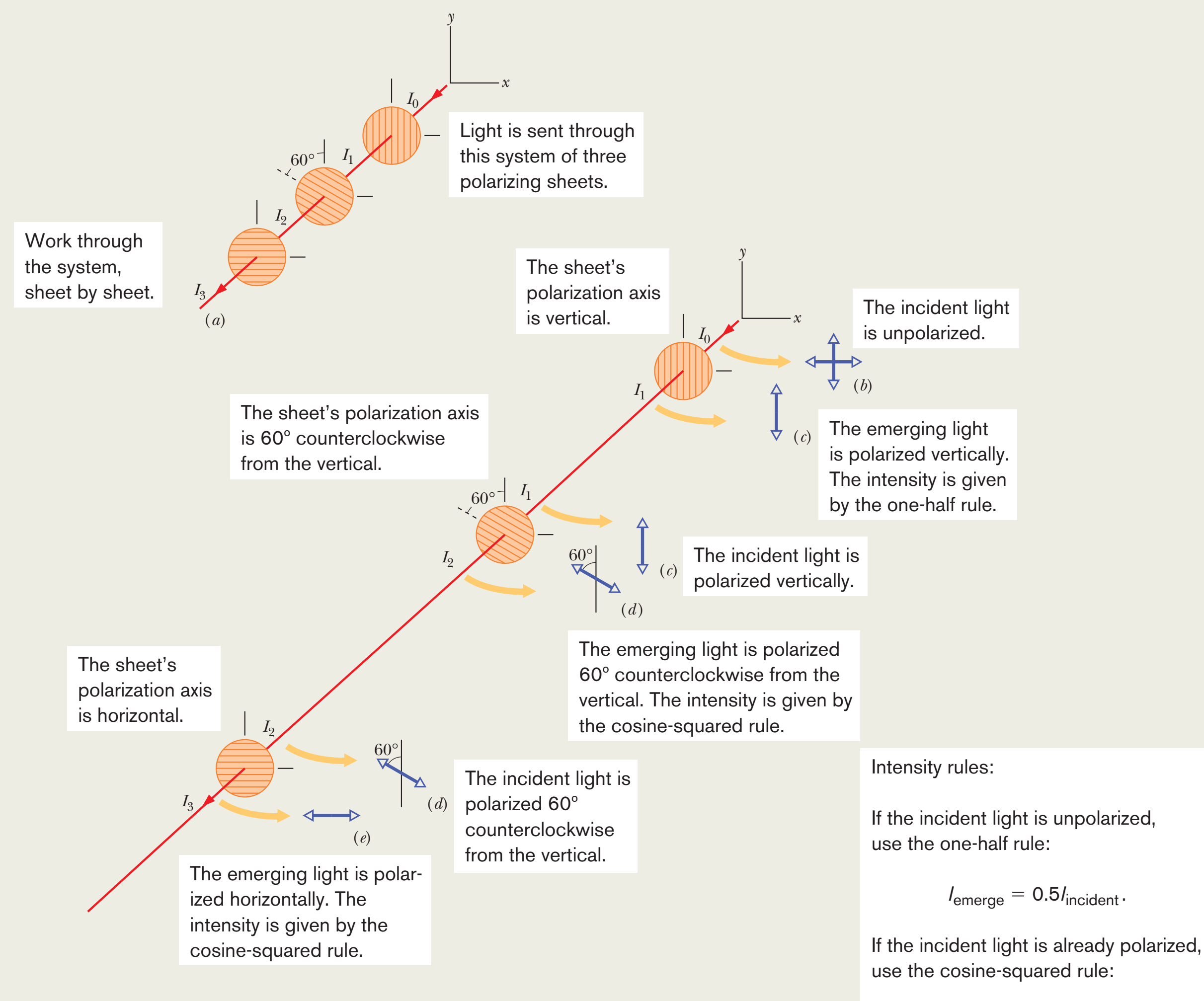



**Figure 33-15** (*a*) Initially unpolarized light of intensity $I_0$ is sent into a system of three polarizing sheets. The intensities $I_1$, $I_2$, and $I_3$ of the light transmitted by the sheets are labeled. Shown also are the polarizations, from head-on views, of (*b*) the initial light and the light transmitted by (*c*) the first sheet, (*d*) the second sheet, and (*e*) the third sheet.

Because the polarizing direction of the first sheet is parallel to the $y$ axis, the polarization of the light transmitted by it is also, as shown in the head-on view of Fig. 33-15*c*.

***Second sheet:*** Because the light reaching the second sheet is polarized, the intensity $I_2$ of the light transmitted by that sheet is given by the cosine-squared rule (Eq. 33-38). The angle $\theta$ in the rule is the angle between the polarization direction of the entering light (parallel to the $y$ axis) and the polarizing direction of the second sheet (60° counterclockwise from the $y$ axis), and so $\theta$ is 60°. (The larger angle between the two directions, namely 120°, can also be used.) We have

$$I_2 = I_1 \cos^2 60°.$$

The polarization of this transmitted light is parallel to the polarizing direction of the sheet transmitting it—that is, 60° counterclockwise from the $y$ axis, as shown in the head-on view of Fig. 33-15*d*.

***Third sheet:*** Because the light reaching the third sheet is polarized, the intensity $I_3$ of the light transmitted by that sheet is given by the cosine-squared rule. The angle $\theta$ is now the angle between the polarization direction of the entering light (Fig. 33-15*d*) and the polarizing direction of the third sheet (parallel to the $x$ axis), and so $\theta = 30°$. Thus,

$$I_3 = I_2 \cos^2 30°.$$

This final transmitted light is polarized parallel to the $x$ axis (Fig. 33-15*e*). We find its intensity by substituting first for $I_2$ and then for $I_1$ in the equation above:

$$I_3 = I_2 \cos^2 30° = (I_1 \cos^2 60°) \cos^2 30°$$
$$= (\tfrac{1}{2} I_0) \cos^2 60° \cos^2 30° = 0.094 I_0.$$

Thus, $$\frac{I_3}{I_0} = 0.094. \qquad \text{(Answer)}$$

That is to say, 9.4% of the initial intensity emerges from the three-sheet system. (If we now remove the second sheet, what fraction of the initial intensity emerges from the system?)

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# 33-5 REFLECTION AND REFRACTION

## Learning Objectives

*After reading this module, you should be able to . . .*

**33.36** With a sketch, show the reflection of a light ray from an interface and identify the incident ray, the reflected ray, the normal, the angle of incidence, and the angle of reflection.

**33.37** For a reflection, relate the angle of incidence and the angle of reflection.

**33.38** With a sketch, show the refraction of a light ray at an interface and identify the incident ray, the refracted ray, the normal on each side of the interface, the angle of incidence, and the angle of refraction.

**33.39** For refraction of light, apply Snell's law to relate the index of refraction and the angle of the ray on one side of the interface to those quantities on the other side.

**33.40** In a sketch and using a line along the undeflected direction, show the refraction of light from one material into a second material that has a greater index, a smaller index, and the same index, and, for each situation, describe the refraction in terms of the ray being bent toward the normal, away from the normal, or not at all.

**33.41** Identify that refraction occurs only at an interface and not in the interior of a material.

**33.42** Identify chromatic dispersion.

**33.43** For a beam of red and blue light (or other colors) refracting at an interface, identify which color has the greater bending and which has the greater angle of refraction when they enter a material with a lower index than the initial material and a greater index.

**33.44** Describe how the primary and secondary rainbows are formed and explain why they are circular arcs.

## Key Ideas

- Geometrical optics is an approximate treatment of light in which light waves are represented as straight-line rays.

- When a light ray encounters a boundary between two transparent media, a reflected ray and a refracted ray generally appear. Both rays remain in the plane of incidence. The angle of reflection is equal to the angle of incidence, and the angle of refraction is related to the angle of incidence by Snell's law,

$$n_2 \sin \theta_2 = n_1 \sin \theta_1 \qquad \text{(refraction)},$$

where $n_1$ and $n_2$ are the indexes of refraction of the media in which the incident and refracted rays travel.

## Reflection and Refraction

Although a light wave spreads as it moves away from its source, we can often approximate its travel as being in a straight line; we did so for the light wave in Fig. 33-5*a*. The study of the properties of light waves under that approximation is called *geometrical optics.* For the rest of this chapter and all of Chapter 34, we shall discuss the geometrical optics of visible light.

The photograph in Fig. 33-16*a* shows an example of light waves traveling in approximately straight lines. A narrow beam of light (the *incident* beam), angled downward from the left and traveling through air, encounters a *plane* (flat) water surface. Part of the light is **reflected** by the surface, forming a beam directed upward toward the right, traveling as if the original beam had bounced from the surface. The rest of the light travels through the surface and into the water, forming a beam directed downward to the right. Because light can travel through it, the water is said to be *transparent;* that is, we can see through it. (In this chapter we shall consider only transparent materials and not opaque materials, through which light cannot travel.)

The travel of light through a surface (or *interface*) that separates two media is called **refraction,** and the light is said to be *refracted.* Unless an incident beam of light is perpendicular to the surface, refraction changes the light's direction of travel. For this reason, the beam is said to be "bent" by the refraction. Note in Fig. 33-16*a* that the bending occurs only at the surface; within the water, the light travels in a straight line.

In Figure 33-16*b*, the beams of light in the photograph are represented with an *incident ray,* a *reflected ray,* and a *refracted ray* (and wavefronts). Each ray is oriented with respect to a line, called the *normal,* that is perpendicular to the surface at the point of reflection and refraction. In Fig. 33-16*b*, the **angle of incidence** is $\theta_1$, the **angle of reflection** is $\theta_1'$, and the **angle of refraction** is $\theta_2$, all measured *relative to the normal.* The plane containing the incident ray and the normal is the *plane of incidence,* which is in the plane of the page in Fig. 33-16*b*.

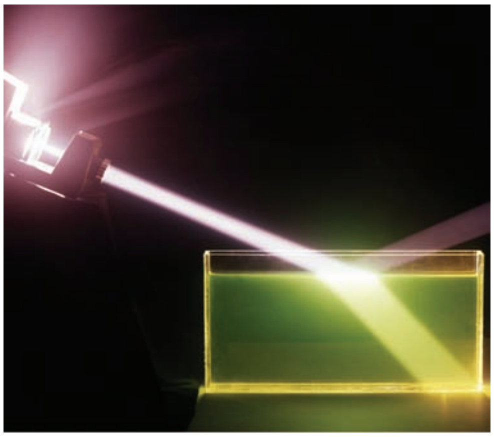
Experiment shows that reflection and refraction are governed by two laws:

**Law of reflection:** A reflected ray lies in the plane of incidence and has an angle of reflection equal to the angle of incidence (both relative to the normal). In Fig. 33-16*b*, this means that

$$\theta_1' = \theta_1 \quad \text{(reflection).} \tag{33-39}$$

(We shall now usually drop the prime on the angle of reflection.)

(*a*)





(*b*)

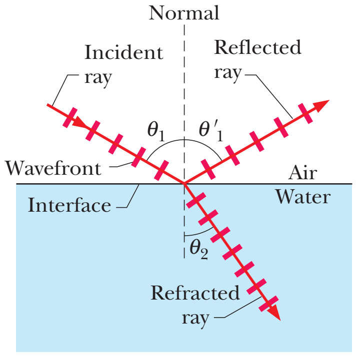



**Figure 33-16** (*a*) A photograph showing an incident beam of light reflected and refracted by a horizontal water surface. (*b*) A ray representation of (*a*). The angles of incidence ($\theta_1$), reflection ($\theta_1'$), and refraction ($\theta_2$) are marked.

**Table 33-1 Some Indexes of Refraction**[a]

| Medium | Index | Medium | Index |
|---|---|---|---|
| Vacuum | Exactly 1 | Typical crown glass | 1.52 |
| Air (STP)[b] | 1.00029 | Sodium chloride | 1.54 |
| Water (20°C) | 1.33 | Polystyrene | 1.55 |
| Acetone | 1.36 | Carbon disulfide | 1.63 |
| Ethyl alcohol | 1.36 | Heavy flint glass | 1.65 |
| Sugar solution (30%) | 1.38 | Sapphire | 1.77 |
| Fused quartz | 1.46 | Heaviest flint glass | 1.89 |
| Sugar solution (80%) | 1.49 | Diamond | 2.42 |

[a]For a wavelength of 589 nm (yellow sodium light).

[b]STP means "standard temperature (0°C) and pressure (1 atm)."

**Law of refraction:** A refracted ray lies in the plane of incidence and has an angle of refraction $\theta_2$ that is related to the angle of incidence $\theta_1$ by

$$n_2 \sin \theta_2 = n_1 \sin \theta_1 \quad \text{(refraction).} \tag{33-40}$$

Here each of the symbols $n_1$ and $n_2$ is a dimensionless constant, called the **index of refraction,** that is associated with a medium involved in the refraction. We derive this equation, called **Snell's law**, in Chapter 35. As we shall discuss there, the index of refraction of a medium is equal to $c/v$, where $v$ is the speed of light in that medium and $c$ is its speed in vacuum.

Table 33-1 gives the indexes of refraction of vacuum and some common substances. For vacuum, $n$ is defined to be exactly 1; for air, $n$ is very close to 1.0 (an approximation we shall often make). Nothing has an index of refraction below 1.

We can rearrange Eq. 33-40 as

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 \tag{33-41}$$

to compare the angle of refraction $\theta_2$ with the angle of incidence $\theta_1$. We can then see that the relative value of $\theta_2$ depends on the relative values of $n_2$ and $n_1$:

**1.** If $n_2$ is equal to $n_1$, then $\theta_2$ is equal to $\theta_1$ and refraction does not bend the light beam, which continues in the *undeflected direction,* as in Fig. 33-17*a*.

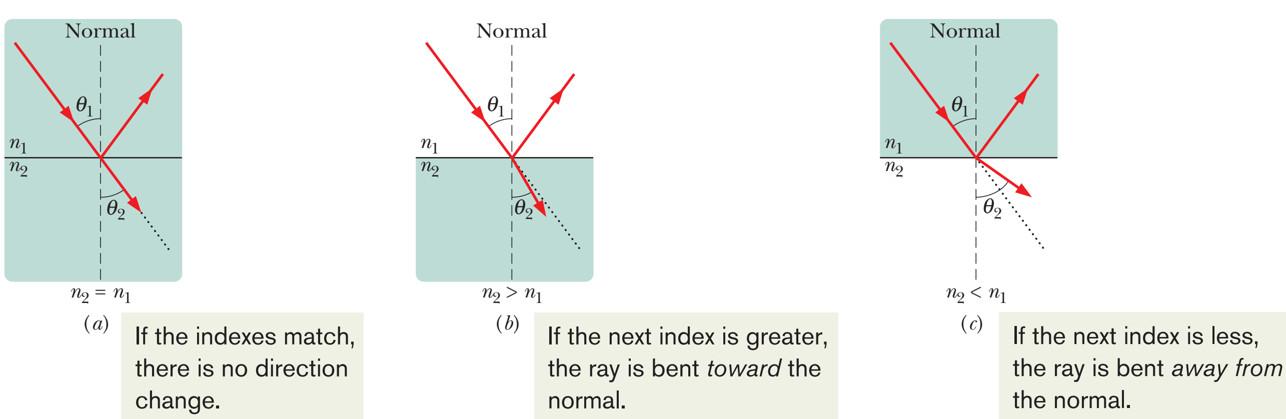



**Figure 33-17** Refraction of light traveling from a medium with an index of refraction $n_1$ into a medium with an index of refraction $n_2$. (*a*) The beam does not bend when $n_2 = n_1$; the refracted light then travels in the *undeflected direction* (the dotted line), which is the same as the direction of the incident beam. The beam bends (*b*) toward the normal when $n_2 > n_1$ and (*c*) away from the normal when $n_2 < n_1$.

2. If $n_2$ is greater than $n_1$, then $\theta_2$ is less than $\theta_1$. In this case, refraction bends the light beam away from the undeflected direction and toward the normal, as in Fig. 33-17*b*.
3. If $n_2$ is less than $n_1$, then $\theta_2$ is greater than $\theta_1$. In this case, refraction bends the light beam away from the undeflected direction and away from the normal, as in Fig. 33-17*c*.

Refraction *cannot* bend a beam so much that the refracted ray is on the same side of the normal as the incident ray.

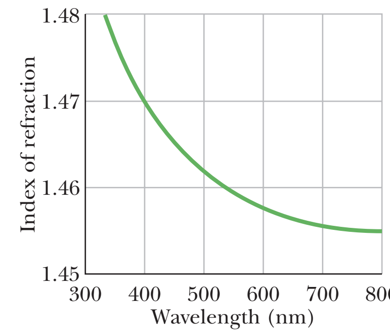


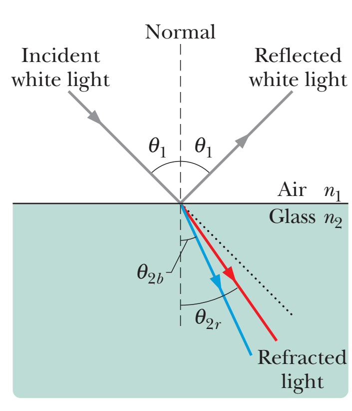

**Figure 33-18** The index of refraction as a function of wavelength for fused quartz. The graph indicates that a beam of short-wavelength light, for which the index of refraction is higher, is bent more upon entering or leaving quartz than a beam of long-wavelength light.

## Chromatic Dispersion

The index of refraction $n$ encountered by light in any medium except vacuum depends on the wavelength of the light. The dependence of $n$ on wavelength implies that when a light beam consists of rays of different wavelengths, the rays will be refracted at different angles by a surface; that is, the light will be spread out by the refraction. This spreading of light is called **chromatic dispersion,** in which "chromatic" refers to the colors associated with the individual wavelengths and "dispersion" refers to the spreading of the light according to its wavelengths or colors. The refractions of Figs. 33-16 and 33-17 do not show chromatic dispersion because the beams are *monochromatic* (of a single wavelength or color).

Generally, the index of refraction of a given medium is *greater* for a shorter wavelength (corresponding to, say, blue light) than for a longer wavelength (say, red light). As an example, Fig. 33-18 shows how the index of refraction of fused quartz depends on the wavelength of light. Such dependence means that when a beam made up of waves of both blue and red light is refracted through a surface, such as from air into quartz or vice versa, the blue *component* (the ray corresponding to the wave of blue light) bends more than the red component.

A beam of *white light* consists of components of all (or nearly all) the colors in the visible spectrum with approximately uniform intensities. When you see such a beam, you perceive white rather than the individual colors. In Fig. 33-19*a*, a beam of white light in air is incident on a glass surface. (Because the pages of this book are white, a beam of white light is represented with a gray ray here. Also, a beam of monochromatic light is generally represented with a red ray.) Of the refracted light in Fig. 33-19*a*, only the red and blue components are shown. Because the blue component is bent more than the red component, the angle of refraction $\theta_{2b}$ for the blue component is *smaller* than the angle of refraction $\theta_{2r}$ for the red component. (Remember, angles are measured relative to the normal.) In Fig. 33-19*b*, a ray of white light in glass is incident on a glass–air interface. Again, the blue component is bent more than the red component, but now $\theta_{2b}$ is greater than $\theta_{2r}$.



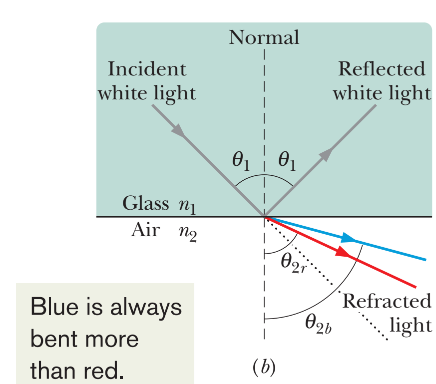
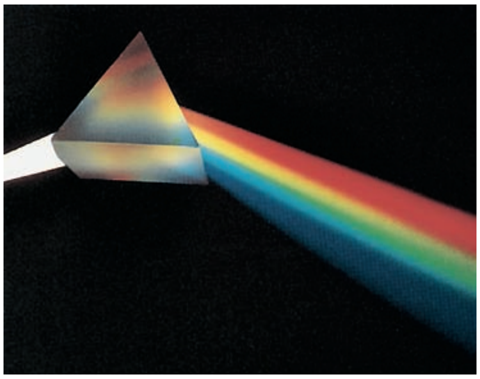
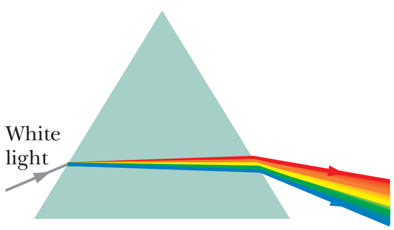




**Figure 33-19** Chromatic dispersion of white light. The blue component is bent more than the red component. (*a*) Passing from air to glass, the blue component ends up with the smaller angle of refraction. (*b*) Passing from glass to air, the blue component ends up with the greater angle of refraction. Each dotted line represents the direction in which the light would continue to travel if it were not bent by the refraction.



Courtesy Bausch & Lomb

(a)




(b)

**Figure 33-20** (a) A triangular prism separating white light into its component colors. (b) Chromatic dispersion occurs at the first surface and is increased at the second surface.

To increase the color separation, we can use a solid glass prism with a triangular cross section, as in Fig. 33-20a. The dispersion at the first surface (on the left in Figs. 33-20a, b) is then enhanced by the dispersion at the second surface.

## Rainbows

The most charming example of chromatic dispersion is a rainbow. When sunlight (which consists of all visible colors) is intercepted by a falling raindrop, some of the light refracts into the drop, reflects once from the drop's inner surface, and then refracts out of the drop. Figure 33-21a shows the situation when the Sun is on the horizon at the left (and thus when the rays of sunlight are horizontal). The first refraction separates the sunlight into its component colors, and the second refraction increases the separation. (Only the red and blue rays are shown in the figure.) If many falling drops are brightly illuminated, you can see the separated colors they produce when the drops are at an angle of 42° from the direction of the *antisolar point* $A$, the point directly opposite the Sun in your view.

To locate the drops, face away from the Sun and point both arms directly away from the Sun, toward the shadow of your head. Then move your right arm directly up, directly rightward, or in any intermediate direction until the angle between your arms is 42°. If illuminated drops happen to be in the direction of your right arm, you see color in that direction.

Because any drop at an angle of 42° in any direction from $A$ can contribute to the rainbow, the rainbow is always a 42° circular arc around $A$ (Fig. 33-21b) and the top of a rainbow is never more than 42° above the horizon. When the Sun is above the horizon, the direction of $A$ is below the horizon, and only a shorter, lower rainbow arc is possible (Fig. 33-21c).

Because rainbows formed in this way involve one reflection of light inside each drop, they are often called *primary rainbows.* A *secondary rainbow* involves two reflections inside a drop, as shown in Fig. 33-21d. Colors appear in the secondary rainbow at an angle of 52° from the direction of $A$. A secondary rainbow

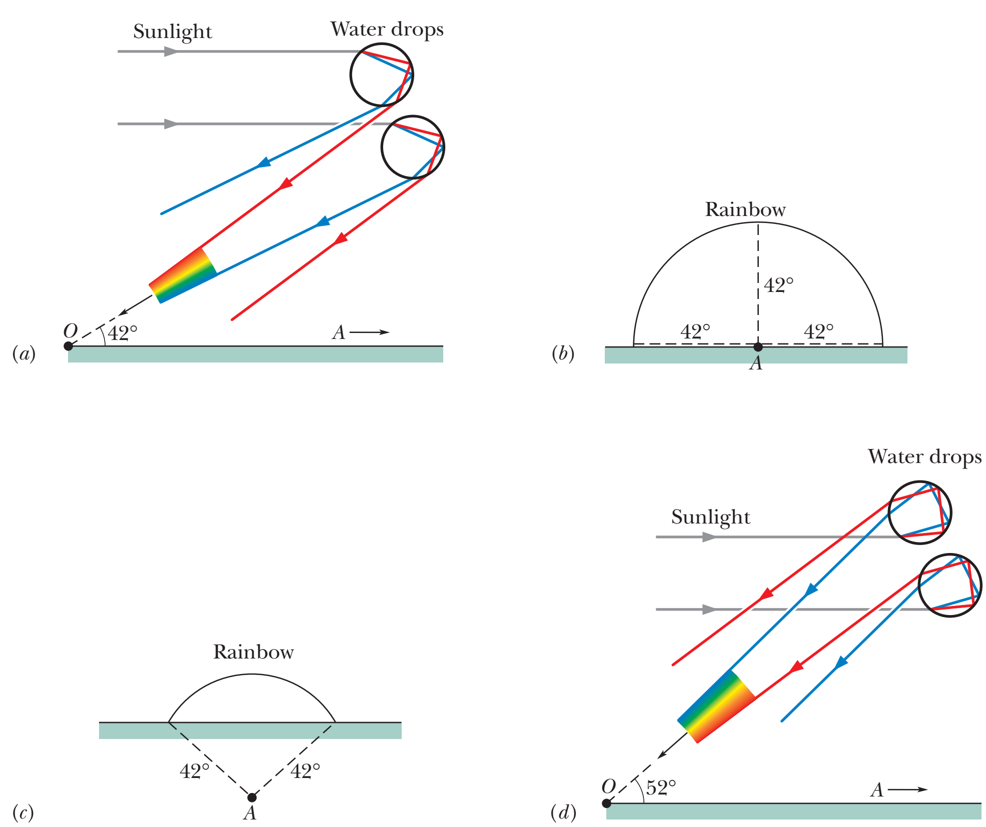


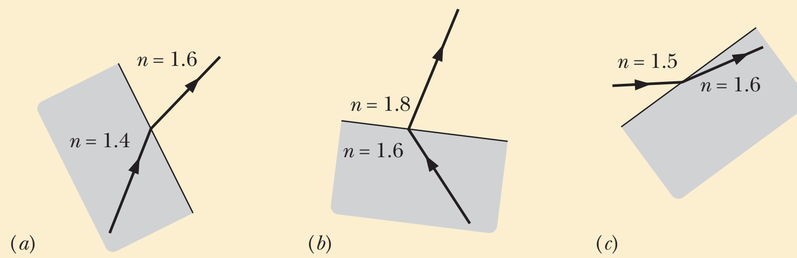

**Figure 33-21** (a) The separation of colors when sunlight refracts into and out of falling raindrops leads to a primary rainbow. The antisolar point $A$ is on the horizon at the right. The rainbow colors appear at an angle of 42° from the direction of $A$. (b) Drops at 42° from $A$ in any direction can contribute to the rainbow. (c) The rainbow arc when the Sun is higher (and thus $A$ is lower). (d) The separation of colors leading to a secondary rainbow.

is wider and dimmer than a primary rainbow and thus is more difficult to see. Also, the order of colors in a secondary rainbow is reversed from the order in a primary rainbow, as you can see by comparing parts *a* and *d* of Fig. 33-21.

Rainbows involving three or four reflections occur in the direction of the Sun and cannot be seen against the glare of sunshine in that part of the sky but have been photographed with special techniques.

### Checkpoint 5

Which of the three drawings here (if any) show physically possible refraction?




## Sample Problem 33.03 Reflection and refraction of a monochromatic beam

(a) In Fig. 33-22*a*, a beam of monochromatic light reflects and refracts at point $A$ on the interface between material 1 with index of refraction $n_1 = 1.33$ and material 2 with index of refraction $n_2 = 1.77$. The incident beam makes an angle of 50° with the interface. What is the angle of reflection at point $A$? What is the angle of refraction there?

### KEY IDEAS

(1) The angle of reflection is equal to the angle of incidence, and both angles are measured relative to the normal to the surface at the point of reflection. (2) When light reaches the interface between two materials with different indexes of refraction (call them $n_1$ and $n_2$), part of the light can be refracted by the interface according to Snell's law, Eq. 33-40:

$$n_2 \sin \theta_2 = n_1 \sin \theta_1, \qquad (33\text{-}42)$$

where both angles are measured relative to the normal at the point of refraction.

***Calculations:*** In Fig. 33-22*a*, the normal at point $A$ is drawn as a dashed line through the point. Note that the angle of incidence $\theta_1$ is not the given 50° but is $90° - 50° = 40°$. Thus, the angle of reflection is

$$\theta_1' = \theta_1 = 40°. \qquad \text{(Answer)}$$

The light that passes from material 1 into material 2 undergoes refraction at point $A$ on the interface between the two materials. Again we measure angles between light rays and a normal, here at the point of refraction. Thus, in Fig. 33-22*a*, the angle of refraction is the angle marked $\theta_2$. Solving Eq. 33-42 for $\theta_2$ gives us

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.33}{1.77}\sin 40°\right)$$

$$= 28.88° \approx 29°. \qquad \text{(Answer)}$$

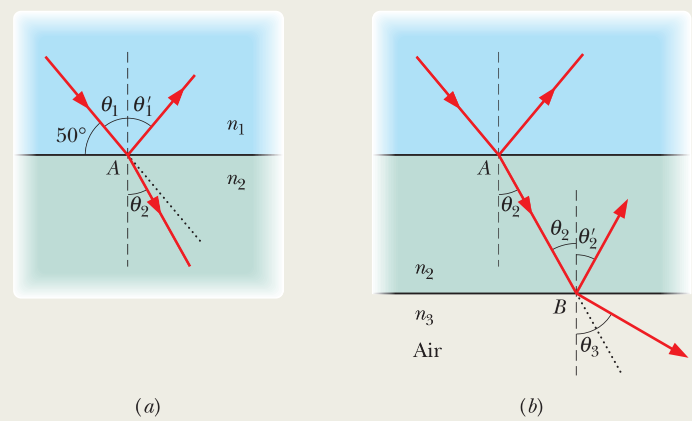



**Figure 33-22** (*a*) Light reflects and refracts at point $A$ on the interface between materials 1 and 2. (*b*) The light that passes through material 2 reflects and refracts at point $B$ on the interface between materials 2 and 3 (air). Each dashed line is a normal. Each dotted line gives the incident direction of travel.

This result means that the beam swings toward the normal (it was at 40° to the normal and is now at 29°). The reason is that when the light travels across the interface, it moves into a material with a greater index of refraction. *Caution:* Note that the beam does *not* swing through the normal so that it appears on the left side of Fig. 33-22*a*.

(b) The light that enters material 2 at point $A$ then reaches point $B$ on the interface between material 2 and material 3, which is air, as shown in Fig. 33-22*b*. The interface through $B$ is parallel to that through $A$. At $B$, some of the light reflects and the rest enters the air. What is the angle of reflection? What is the angle of refraction into the air?

***Calculations:*** We first need to relate one of the angles at point $B$ with a known angle at point $A$. Because the interface through point $B$ is parallel to that through point $A$, the incident angle at $B$ must be equal to the angle of refraction $\theta_2$, as shown in Fig. 33-22*b*. Then for reflection, we again use the law of reflection. Thus, the angle of reflection at $B$ is

$$\theta_2' = \theta_2 = 28.88° \approx 29°. \quad \text{(Answer)}$$

Next, the light that passes from material 2 into the air undergoes refraction at point $B$, with refraction angle $\theta_3$. Thus, we again apply Snell's law of refraction, but this time we write Eq. 33-40 as

$$n_3 \sin \theta_3 = n_2 \sin \theta_2. \quad (33\text{-}43)$$

Solving for $\theta_3$ then leads to

$$\theta_3 = \sin^{-1}\left(\frac{n_2}{n_3}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.77}{1.00}\sin 28.88°\right)$$

$$= 58.75° \approx 59°. \quad \text{(Answer)}$$

Thus, the beam swings away from the normal (it was at 29° to the normal and is now at 59°) because it moves into a material (air) with a lower index of refraction.



# 33-6 TOTAL INTERNAL REFLECTION

## Learning Objectives

*After reading this module, you should be able to . . .*

**33.45** With sketches, explain total internal reflection and include the angle of incidence, the critical angle, and the relative values of the indexes of refraction on the two sides of the interface.

**33.46** Identify the angle of refraction for incidence at a critical angle.

**33.47** For a given pair of indexes of refraction, calculate the critical angle.

## Key Idea

- A wave encountering a boundary across which the index of refraction decreases will experience total internal reflection if the angle of incidence exceeds a critical angle $\theta_c$, where

$$\theta_c = \sin^{-1}\frac{n_2}{n_1} \quad \text{(critical angle).}$$

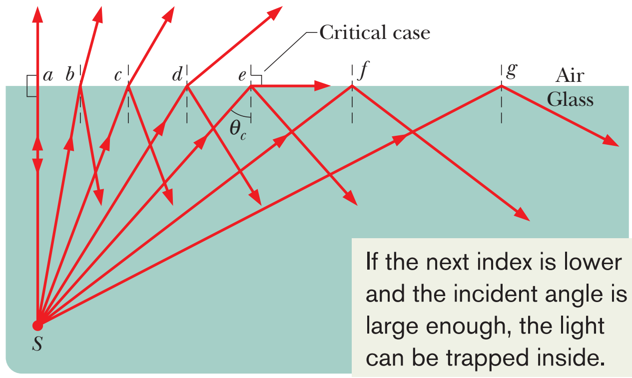
## Total Internal Reflection

Figure 33-23*a* shows rays of monochromatic light from a point source $S$ in glass incident on the interface between the glass and air. For ray $a$, which is perpendicular to the interface, part of the light reflects at the interface and the rest travels through it with no change in direction.

For rays $b$ through $e$, which have progressively larger angles of incidence at the interface, there are also both reflection and refraction at the interface. As the angle of incidence increases, the angle of refraction increases; for ray $e$ it is 90°, which means that the refracted ray points directly along the interface. The angle of incidence giving this situation is called the **critical angle** $\theta_c$. For angles of incidence larger than $\theta_c$, such as for rays $f$ and $g$, there is no refracted ray and *all* the light is reflected; this effect is called **total internal reflection** because all the light remains inside the glass.

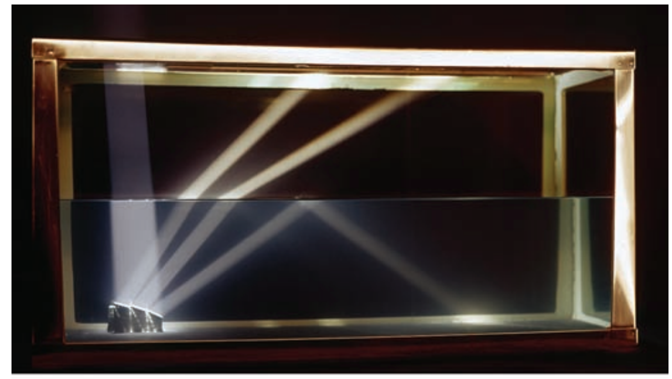
To find $\theta_c$, we use Eq. 33-40; we arbitrarily associate subscript 1 with the glass and subscript 2 with the air, and then we substitute $\theta_c$ for $\theta_1$ and 90° for $\theta_2$, which leads to

$$n_1 \sin \theta_c = n_2 \sin 90°, \quad (33\text{-}44)$$




(a)

(b)



Ken Kay/Fundamental Photographs

**Figure 33-23** (*a*) Total internal reflection of light from a point source *S* in glass occurs for all angles of incidence greater than the critical angle $\theta_c$. At the critical angle, the refracted ray points along the air–glass interface. (*b*) A source in a tank of water.

which gives us

$$\theta_c = \sin^{-1}\frac{n_2}{n_1} \quad \text{(critical angle).} \tag{33-45}$$

Because the sine of an angle cannot exceed unity, $n_2$ cannot exceed $n_1$ in this equation. This restriction tells us that total internal reflection cannot occur when the incident light is in the medium of lower index of refraction. If source *S* were in the air in Fig. 33-23*a*, all its rays that are incident on the air–glass interface (including *f* and *g*) would be both reflected *and* refracted at the interface.

Total internal reflection has found many applications in medical technology. For example, a physician can view the interior of an artery of a patient by running two thin bundles of *optical fibers* through the chest wall and into an artery (Fig. 33-24). Light introduced at the outer end of one bundle undergoes repeated total internal reflection within the fibers so that, even though the bundle provides a curved path, most of the light ends up exiting the other end and illuminating the interior of the artery. Some of the light reflected from the interior then comes back up the second bundle in a similar way, to be detected and converted to an image on a monitor's screen for the physician to view. The physician can then perform a surgical procedure, such as the placement of a stent.

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**Figure 33-24** An endoscope used to inspect an artery.

# 33-7 POLARIZATION BY REFLECTION

## Learning Objectives

*After reading this module, you should be able to . . .*

**33.48** With sketches, explain how unpolarized light can be converted to polarized light by reflection from an interface.

**33.49** Identify Brewster's angle.

**33.50** Apply the relationship between Brewster's angle and the indexes of refraction on the two sides of an interface.

**33.51** Explain the function of polarizing sunglasses.

## Key Idea

- A reflected wave will be fully polarized, with its $\vec{E}$ vectors perpendicular to the plane of incidence, if it strikes a boundary at the Brewster angle $\theta_B$, where

$$\theta_B = \tan^{-1}\frac{n_2}{n_1} \quad \text{(Brewster angle).}$$

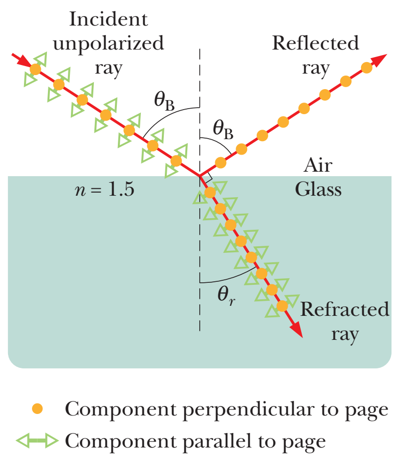


**Figure 33-25** A ray of unpolarized light in air is incident on a glass surface at the Brewster angle $\theta_B$. The electric fields along that ray have been resolved into components perpendicular to the page (the plane of incidence, reflection, and refraction) and components parallel to the page. The reflected light consists only of components perpendicular to the page and is thus polarized in that direction. The refracted light consists of the original components parallel to the page and weaker components perpendicular to the page; this light is partially polarized.

## Polarization by Reflection

You can vary the glare you see in sunlight that has been reflected from, say, water by looking through a polarizing sheet (such as a polarizing sunglass lens) and then rotating the sheet's polarizing axis around your line of sight. You can do so because any light that is reflected from a surface is either fully or partially polarized by the reflection.

Figure 33-25 shows a ray of unpolarized light incident on a glass surface. Let us resolve the electric field vectors of the light into two components. The *perpendicular components* are perpendicular to the plane of incidence and thus also to the page in Fig. 33-25; these components are represented with dots (as if we see the tips of the vectors). The *parallel components* are parallel to the plane of incidence and the page; they are represented with double-headed arrows. Because the light is unpolarized, these two components are of equal magnitude.

In general, the reflected light also has both components but with unequal magnitudes. This means that the reflected light is partially polarized—the electric fields oscillating along one direction have greater amplitudes than those oscillating along other directions. However, when the light is incident at a particular incident angle, called the *Brewster angle* $\theta_B$, the reflected light has only perpendicular components, as shown in Fig. 33-25. The reflected light is then fully polarized perpendicular to the plane of incidence. The parallel components of the incident light do not disappear but (along with perpendicular components) refract into the glass.

***Polarizing Sunglasses.*** Glass, water, and the other dielectric materials discussed in Module 25-5 can partially and fully polarize light by reflection. When you intercept sunlight reflected from such a surface, you see a bright spot (the glare) on the surface where the reflection takes place. If the surface is horizontal as in Fig. 33-25, the reflected light is partially or fully polarized horizontally. To eliminate such glare from horizontal surfaces, the lenses in polarizing sunglasses are mounted with their polarizing direction vertical.

### Brewster's Law

For light incident at the Brewster angle $\theta_B$, we find experimentally that the reflected and refracted rays are perpendicular to each other. Because the reflected ray is reflected at the angle $\theta_B$ in Fig. 33-25 and the refracted ray is at an angle $\theta_r$, we have

$$\theta_B + \theta_r = 90°. \tag{33-46}$$

These two angles can also be related with Eq. 33-40. Arbitrarily assigning subscript 1 in Eq. 33-40 to the material through which the incident and reflected rays travel, we have, from that equation,

$$n_1 \sin \theta_B = n_2 \sin \theta_r. \tag{33-47}$$

Combining these equations leads to

$$n_1 \sin \theta_B = n_2 \sin(90° - \theta_B) = n_2 \cos \theta_B, \tag{33-48}$$

which gives us

$$\theta_B = \tan^{-1} \frac{n_2}{n_1} \quad \text{(Brewster angle).} \tag{33-49}$$

(Note carefully that the subscripts in Eq. 33-49 are *not* arbitrary because of our decision as to their meanings.) If the incident and reflected rays travel *in air,* we can approximate $n_1$ as unity and let $n$ represent $n_2$ in order to write Eq. 33-49 as

$$\theta_B = \tan^{-1} n \quad \text{(Brewster's law).} \tag{33-50}$$

This simplified version of Eq. 33-49 is known as **Brewster's law.** Like $\theta_B$, it is named after Sir David Brewster, who found both experimentally in 1812.

# Review & Summary

**Electromagnetic Waves** An electromagnetic wave consists of oscillating electric and magnetic fields. The various possible frequencies of electromagnetic waves form a *spectrum,* a small part of which is visible light. An electromagnetic wave traveling along an $x$ axis has an electric field $\vec{E}$ and a magnetic field $\vec{B}$ with magnitudes that depend on $x$ and $t$:

$$E = E_m \sin(kx - \omega t)$$

and
$$B = B_m \sin(kx - \omega t), \qquad (33\text{-}1, 33\text{-}2)$$

where $E_m$ and $B_m$ are the amplitudes of $\vec{E}$ and $\vec{B}$. The oscillating electric field induces the magnetic field, and the oscillating magnetic field induces the electric field. The speed of any electromagnetic wave in vacuum is $c$, which can be written as

$$c = \frac{E}{B} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}, \qquad (33\text{-}5, 33\text{-}3)$$

where $E$ and $B$ are the simultaneous (but nonzero) magnitudes of the two fields.

**Energy Flow** The rate per unit area at which energy is transported via an electromagnetic wave is given by the Poynting vector $\vec{S}$:

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}. \qquad (33\text{-}19)$$

The direction of $\vec{S}$ (and thus of the wave's travel and the energy transport) is perpendicular to the directions of both $\vec{E}$ and $\vec{B}$. The time-averaged rate per unit area at which energy is transported is $S_{avg}$, which is called the *intensity I* of the wave:

$$I = \frac{1}{c\mu_0} E_{rms}^2, \qquad (33\text{-}26)$$

in which $E_{rms} = E_m/\sqrt{2}$. A *point source* of electromagnetic waves emits the waves *isotropically*—that is, with equal intensity in all directions. The intensity of the waves at distance $r$ from a point source of power $P_s$ is

$$I = \frac{P_s}{4\pi r^2}. \qquad (33\text{-}27)$$

**Radiation Pressure** When a surface intercepts electromagnetic radiation, a force and a pressure are exerted on the surface. If the radiation is totally absorbed by the surface, the force is

$$F = \frac{IA}{c} \quad \text{(total absorption)}, \qquad (33\text{-}32)$$

in which $I$ is the intensity of the radiation and $A$ is the area of the surface perpendicular to the path of the radiation. If the radiation is totally reflected back along its original path, the force is

$$F = \frac{2IA}{c} \quad \text{(total reflection back along path)}. \qquad (33\text{-}33)$$

The radiation pressure $p_r$ is the force per unit area:

$$p_r = \frac{I}{c} \quad \text{(total absorption)} \qquad (33\text{-}34)$$

and
$$p_r = \frac{2I}{c} \quad \text{(total reflection back along path)}. \qquad (33\text{-}35)$$

**Polarization** Electromagnetic waves are **polarized** if their electric field vectors are all in a single plane, called the *plane of oscillation.* From a head-on view, the field vectors oscillate parallel to a single axis perpendicular to the path taken by the waves. Light waves from common sources are not polarized; that is, they are **unpolarized**, or **polarized randomly**. From a head-on view, the vectors oscillate parallel to every possible axis that is perpendicular to the path taken by the waves.

**Polarizing Sheets** When a polarizing sheet is placed in the path of light, only electric field components of the light parallel to the sheet's **polarizing direction** are *transmitted* by the sheet; components perpendicular to the polarizing direction are absorbed. The light that emerges from a polarizing sheet is polarized parallel to the polarizing direction of the sheet.

If the original light is initially unpolarized, the transmitted intensity $I$ is half the original intensity $I_0$:

$$I = \tfrac{1}{2} I_0. \qquad (33\text{-}36)$$

If the original light is initially polarized, the transmitted intensity depends on the angle $\theta$ between the polarization direction of the original light (the axis along which the fields oscillate) and the polarizing direction of the sheet:

$$I = I_0 \cos^2 \theta. \qquad (33\text{-}38)$$

**Geometrical Optics** *Geometrical optics* is an approximate treatment of light in which light waves are represented as straight-line rays.

**Reflection and Refraction** When a light ray encounters a boundary between two transparent media, a **reflected** ray and a **refracted** ray generally appear. Both rays remain in the plane of incidence. The **angle of reflection** is equal to the angle of incidence, and the **angle of refraction** is related to the angle of incidence by Snell's law,

$$n_2 \sin \theta_2 = n_1 \sin \theta_1 \quad \text{(refraction)}, \qquad (33\text{-}40)$$

where $n_1$ and $n_2$ are the indexes of refraction of the media in which the incident and refracted rays travel.

**Total Internal Reflection** A wave encountering a boundary across which the index of refraction decreases will experience **total internal reflection** if the angle of incidence exceeds a **critical angle** $\theta_c$, where

$$\theta_c = \sin^{-1} \frac{n_2}{n_1} \quad \text{(critical angle)}. \qquad (33\text{-}45)$$

**Polarization by Reflection** A reflected wave will be fully **polarized,** with its $\vec{E}$ vectors perpendicular to the plane of incidence, if the incident, unpolarized wave strikes a boundary at the **Brewster angle** $\theta_B$, where

$$\theta_B = \tan^{-1} \frac{n_2}{n_1} \quad \text{(Brewster angle)}. \qquad (33\text{-}49)$$

# Problems

**1** A beam of polarized light is sent into a system of two polarizing sheets. Relative to the polarization direction of that incident light, the polarizing directions of the sheets are at angles $\theta$ for the first sheet and 90° for the second sheet. (a) If 0.20 of the incident intensity is transmitted by the two sheets, what is $\theta$? (b) What percentage of the incident intensity is transmitted if the first sheet's angle is reduced to 0°?

**2** In Fig. 33-26, initially unpolarized light is sent into a system of three polarizing sheets whose polarizing directions make angles of $\theta_1 = \theta_2 = \theta_3 = 40°$ with the direction of the $y$ axis. (a) What percentage of the initial intensity is transmitted by the system? (*Hint:* Be careful with the angles.) (b) At what angle to the $y$ axis is the emerging light polarized? (Give the angle and indicate either clockwise or counterclockwise.)

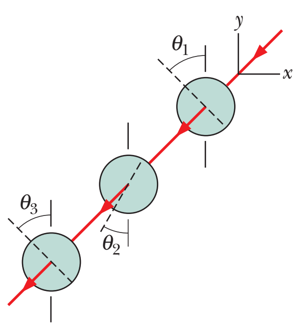


**Figure 33-26** Problems 2 and 3.

**3** In Fig. 33-26, initially unpolarized light is sent into a system of three polarizing sheets whose polarizing directions make angles of $\theta_1 = 40°$, $\theta_2 = 10°$, and $\theta_3 = 40°$ with the direction of the $y$ axis. (a) What percentage of the light's initial intensity is transmitted by the system? (*Hint:* Be careful with the angles.) (b) At what angle to the $y$ axis is the emerging light polarized? (Give the angle and indicate either clockwise or counterclockwise.)

**4** In Fig. 33-27, a beam of unpolarized light, with intensity 43 W/m$^2$, is sent into a system of two polarizing sheets with polarizing directions at angles $\theta_1 = 60°$ and $\theta_2 = 90°$ to the $y$ axis. What is the intensity of the light transmitted by the system?

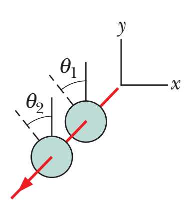


**Figure 33-27** Problems 4, 5, and 6.

**5** In Fig. 33-27, a beam of light, with intensity 43 W/m$^2$ and polarization parallel to a $y$ axis, is sent into a system of two polarizing sheets with polarizing directions at angles of $\theta_1 = 70°$ and $\theta_2 = 90°$ to the $y$ axis. (a) What is the intensity of the light transmitted by the two-sheet system? (b) What is the transmitted intensity if, instead, the initial polarization is parallel to the $x$ axis?

**6** In Fig. 33-27, unpolarized light is sent into a system of two polarizing sheets. The angles $\theta_1$ and $\theta_2$ of the polarizing directions of the sheets are measured counterclockwise from the positive direction of the $y$ axis (they are not drawn to scale in the figure). Angle $\theta_1$ is fixed but angle $\theta_2$ can be varied. Figure 33-28 gives the intensity of the light emerging from sheet 2 as a function of $\theta_2$. (The scale of the intensity axis is not indicated.) What percentage of the light's initial intensity is transmitted by the two-sheet system when $\theta_2 = 110°$?

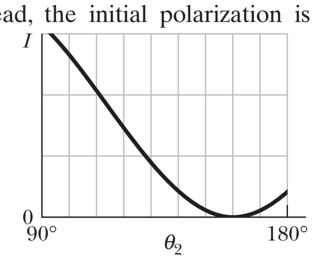


**Figure 33-28** Problem 6.

**7** In Fig. 33-29, light enters a 90° triangular prism at point $P$ with incident angle $\theta$, and then some of it refracts at point $Q$ with an angle of refraction of 90°. (a) What is the index of refraction of the prism in terms of $\theta$? (b) What, numerically, is the maximum value that the index of refraction can have? Does light emerge at $Q$ if the incident angle at $P$ is (c) increased slightly and (d) decreased slightly? (e) What is the incident angle if the index of refraction is 1.30?

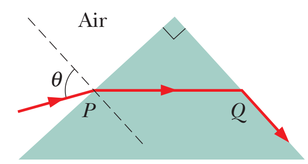


**Figure 33-29** Problem 7.

**8** In Fig. 33-30, unpolarized light is sent into a system of three polarizing sheets. The angles $\theta_1$, $\theta_2$, and $\theta_3$ of the polarizing directions are measured counterclockwise from the positive direction of the $y$ axis (they are not drawn to scale). Angles $\theta_1$ and $\theta_3$ are fixed, but angle $\theta_2$ can be varied. Figure 33-31 gives the intensity of the light emerging from sheet 3 as a function of $\theta_2$. (The scale of the intensity axis is not indicated.) What percentage of the light's initial intensity is transmitted by the system when $\theta_2 = 35°$?

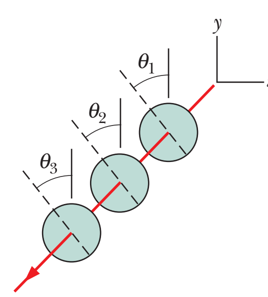


**Figure 33-30** Problems 8, 10, and 12.

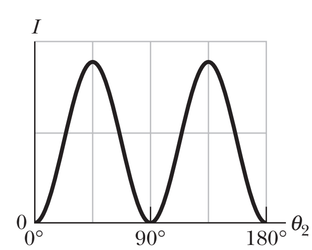


**Figure 33-31** Problem 8.

**9** When the rectangular metal tank in Fig. 33-32 is filled to the top with an unknown liquid, observer $O$, with eyes level with the top of the tank, can just see corner $E$. A ray that refracts toward $O$ at the top surface of the liquid is shown. If $D = 85.0$ cm and $L = 0.680$ m, what is the index of refraction of the liquid?

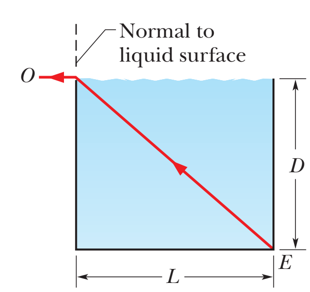


**Figure 33-32** Problem 9.

**10** In Fig. 33-30, unpolarized light is sent into a system of three polarizing sheets. The angles $\theta_1$, $\theta_2$, and $\theta_3$ of the polarizing directions are measured counterclockwise from the positive direction of the $y$ axis (they are not drawn to scale). Angles $\theta_1$ and $\theta_3$ are fixed, but angle $\theta_2$ can be varied. Figure 33-33 gives the intensity of the light emerging from sheet 3 as a function of $\theta_2$. (The scale of the intensity axis is not indicated.) What percentage of the light's initial intensity is transmitted by the three-sheet system when $\theta_2 = 110°$?

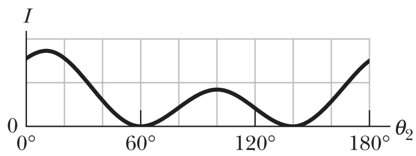


**Figure 33-33** Problem 10.

**11** We want to rotate the direction of polarization of a beam of polarized light through 90° by sending the beam through one or more polarizing sheets. (a) What is the minimum number of sheets required? (b) What is the minimum number of sheets required if the transmitted intensity is to be more than 65% of the original intensity?

**12** In Fig. 33-30, unpolarized light is sent into a system of three polarizing sheets, which transmits 0.0450 of the initial light intensity. The polarizing directions of the first and third sheets are at angles $\theta_1 = 0°$ and $\theta_3 = 90°$. What are the (a) smaller and (b) larger possible values of angle $\theta_2$ ($< 90°$) for the polarizing direction of sheet 2?

**13** In Fig. 33-34, a ray is incident on one face of a triangular glass prism in air. The angle of incidence $\theta$ is chosen so that the emerging ray also makes the same angle $\theta$ with the normal to the other face. Show that the index of refraction $n$ of the glass prism is given by

$$n = \frac{\sin \frac{1}{2}(\psi + \phi)}{\sin \frac{1}{2}\phi},$$

where $\phi$ is the vertex angle of the prism and $\psi$ is the *deviation angle,* the total angle through which the beam is turned in passing through the prism. (Under these conditions the deviation angle $\psi$ has the smallest possible value, which is called the *angle of minimum deviation.*)

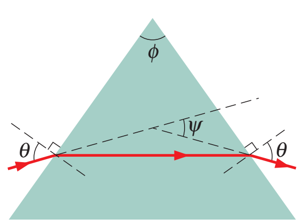


**Figure 33-34** Problems 13 and 14.

**14** Suppose the prism of Fig. 33-34 has apex angle $\phi = 60.0°$ and index of refraction $n = 1.56$. (a) What is the smallest angle of incidence $\theta$ for which a ray can enter the left face of the prism and exit the right face? (b) What angle of incidence $\theta$ is required for the ray to exit the prism with an identical angle $\theta$ for its refraction, as it does in Fig. 33-53?

**15** As a comet swings around the Sun, ice on the comet's surface vaporizes, releasing trapped dust particles and ions. The ions, because they are electrically charged, are forced by the electrically charged *solar wind* into a straight *ion tail* that points radially away from the Sun (Fig. 33-35). The (electrically neutral) dust particles are pushed radially outward from the Sun by the radiation force on them from sunlight. Assume that the dust particles are spherical, have density $3.0 \times 10^3$ kg/m$^3$, and are totally absorbing. (a) What radius must a particle have in order to follow a straight path, like path 2 in the figure? (b) If its radius is larger, does its path curve away from the Sun (like path 1) or toward the Sun (like path 3)?

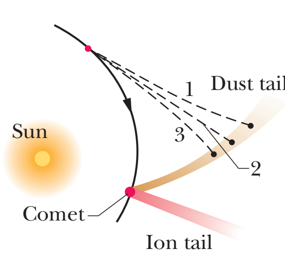


**Figure 33-35** Problem 15.

**16** At a beach the light is generally partially polarized due to reflections off sand and water. At a particular beach on a particular day near sundown, the horizontal component of the electric field vector is 2.0 times the vertical component. A standing sunbather puts on polarizing sunglasses; the glasses eliminate the horizontal field component. (a) What fraction of the light intensity received before the glasses were put on now reaches the sunbather's eyes? (b) The sunbather, still wearing the glasses, lies on his side. What fraction of the light intensity received before the glasses were put on now reaches his eyes?

**17** A beam of partially polarized light can be considered to be a mixture of polarized and unpolarized light. Suppose we send such a beam through a polarizing filter and then rotate the filter through 360° while keeping it perpendicular to the beam. If the transmitted intensity varies by a factor of 4.0 during the rotation, what fraction of the intensity of the original beam is associated with the beam's polarized light?

**18** *Rainbows from square drops.* Suppose that, on some surreal world, raindrops had a square cross section and always fell with one face horizontal. Figure 33-36 shows such a falling drop, with a white beam of sunlight incident at $\theta = 65.0°$ at point $P$. The part of the light that enters the drop then travels to point $A$, where some of it refracts out into the air and the rest reflects. That reflected light then travels to point $B$, where again some of the light refracts out into the air and the rest reflects. What is the difference in the angles of the red light ($n = 1.331$) and the blue light ($n = 1.343$) that emerge at (a) point $A$ and (b) point $B$? (This angular difference in the light emerging at, say, point $A$ would be the rainbow's angular width.)

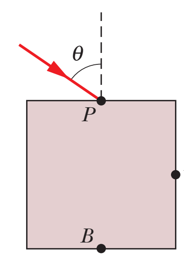


**Figure 33-36** Problem 18.

**19** In Fig. 33-37, a 2.50 m long vertical pole extends from the bottom of a swimming pool to a point 50.0 cm above the water. Sunlight is incident at angle $\theta = 55.0°$. What is the length of the shadow of the pole on the level bottom of the pool?

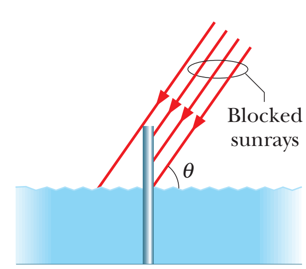


**Figure 33-37** Problem 19.

**20** In Fig. 33-38$a$, a beam of light in material 1 is incident on a boundary at an angle $\theta_1 = 40°$. Some of the light travels through material 2, and then some of it emerges into material 3. The two boundaries between the three materials are parallel. The final direction of the beam depends, in part, on the index of refraction $n_3$ of the third material. Figure 33-38$b$ gives the angle of refraction $\theta_3$ in that material versus $n_3$ for a range of possible $n_3$ values. The vertical axis scale is set by $\theta_{3a} = 30.0°$ and $\theta_{3b} = 50.0°$. (a) What is the index of refraction of material 1, or is the index impossible to calculate without more information? (b) What is the index of refraction of material 2, or is the index impossible to calculate without more information? (c) If $\theta_1$ is changed to 75° and the index of refraction of material 3 is 2.4, what is $\theta_3$?

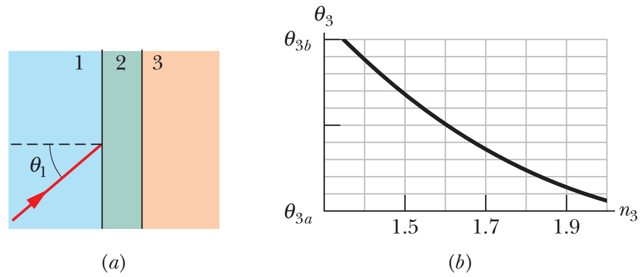


**Figure 33-38** Problem 20.

**21** Light in vacuum is incident on the surface of a glass slab. In the vacuum the beam makes an angle of 32.0° with the normal to the surface, while in the glass it makes an angle of 16.0° with the normal. What is the index of refraction of the glass?

**22** A certain type of glass has an index of refraction of 1.62 for red light. (a) At what angle will light (in air) be completely polarized when reflected by the glass? (b) Will the reflection angle for complete polarization of blue light be larger or smaller?

**23** In Fig. 33-39, light is incident at angle $\theta_1 = 40.1°$ on a boundary between two transparent materials. Some of the light travels down through the next three layers of transparent materials, while some of it reflects upward and then escapes into the air. If $n_1 = 1.30$, $n_2 = 1.40$, $n_3 = 1.32$, and $n_4 = 1.75$, what is the value of (a) $\theta_5$ in the air and (b) $\theta_4$ in the bottom material?

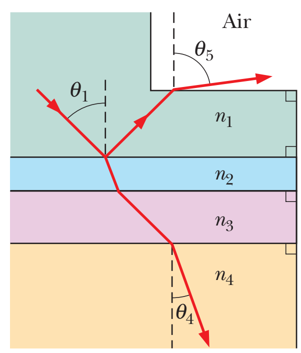


**Figure 33-39** Problem 23.

**24** An isotropic point source emits light at wavelength 500 nm, at the rate of 300 W. A light detector is positioned 400 m from the source. What is the maximum rate $\partial B/\partial t$ at which the magnetic component of the light changes with time at the detector's location?

**25** In Fig. 33-40, a ray of light is perpendicular to the face *ab* of a glass prism ($n = 1.81$). Find the largest value for the angle $\phi$ so that the ray is totally reflected at face *ac* if the prism is immersed (a) in air and (b) in water.

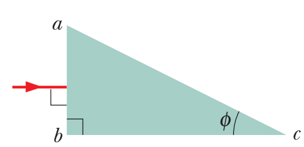


**Figure 33-40** Problem 25.

**26** *Dispersion in a window pane.* In Fig. 33-41, a beam of white light is incident at angle $\theta = 40°$ on a common window pane (shown in cross section). For the pane's type of glass, the index of refraction for visible light ranges from 1.524 at the blue end of the spectrum to 1.509 at the red end. The two sides of the pane are parallel. What is the angular spread of the colors in the beam (a) when the light enters the pane and (b) when it emerges from the opposite side? (*Hint:* When you look at an object through a window pane, are the colors in the light from the object dispersed as shown in, say, Fig. 33-20?)

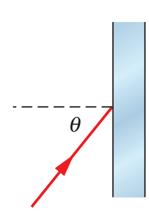


**Figure 33-41** Problem 26.

**27** The maximum electric field 27 m from an isotropic point source of light is 17 V/m. What are (a) the maximum value of the magnetic field and (b) the average intensity of the light there? (c) What is the power of the source?

**28** Radiation from the Sun reaching Earth (just outside the atmosphere) has an intensity of 1.4 kW/m$^2$. (a) Assuming that Earth (and its atmosphere) behaves like a flat disk perpendicular to the Sun's rays and that all the incident energy is absorbed, calculate the force on Earth due to radiation pressure. (b) For comparison, calculate the ratio of that force to the force due to the Sun's gravitational attraction.

**29** Figure 33-42 shows light reflecting from two perpendicular reflecting surfaces *A* and *B*. Find the angle between the incoming ray *i* and the outgoing ray *r'*.

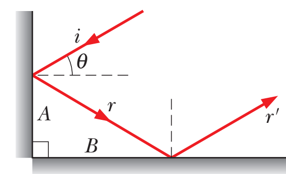


**Figure 33-42** Problem 29.

**30** In Fig. 33-43*a*, a beam of light in material 1 is incident on a boundary at an angle of $\theta_1 = 30°$. The extent of refraction of the light into material 2 depends, in part, on the index of refraction $n_2$ of material 2. Figure 33-43*b* gives the angle of refraction $\theta_2$ versus $n_2$ for a range of possible $n_2$ values. The vertical axis scale is set by $\theta_{2a} = 20.0°$ and $\theta_{2b} = 40.0°$. (a) What is the index of refraction of material 1? (b) If the incident angle is changed to 65° and material 2 has $n_2 = 2.4$, then what is angle $\theta_2$?

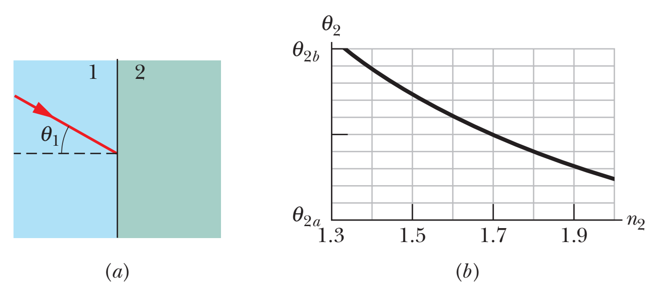


**Figure 33-43** Problem 30.

**31** In Fig. 33-44, light initially in material 1 refracts into material 2, crosses that material, and is then incident at the critical angle on the interface between materials 2 and 3. The indexes of refraction are $n_1 = 1.80$, $n_2 = 1.40$, and $n_3 = 1.20$. (a) What is angle $\theta$? (b) If $\theta$ is increased, is there refraction of light into material 3?

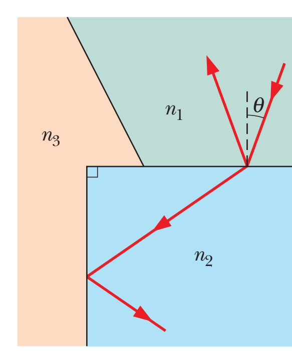


**Figure 33-44** Problem 31.

**32** In a plane radio wave the maximum value of the electric field component is 8.00 V/m. Calculate (a) the maximum value of the magnetic field component and (b) the wave intensity.

**33** A point source of light is 1.20 m below the surface of a body of water. Find the diameter of the circle at the surface through which light emerges from the water.

**34** About how far apart must you hold your hands for them to be separated by 2.0 nano-light-second (the distance light travels in 2.0 ns)?

**35** A plane electromagnetic wave, with wavelength 5.0 m, travels in vacuum in the positive direction of an $x$ axis. The electric field, of amplitude 215 V/m, oscillates parallel to the $y$ axis. What are the (a) frequency, (b) angular frequency, and (c) angular wave number of the wave? (d) What is the amplitude of the magnetic field component? (e) Parallel to which axis does the magnetic field oscillate? (f) What is the time-averaged rate of energy flow in watts per square meter associated with this wave? The wave uniformly illuminates a surface of area 2.0 m$^2$. If the surface totally absorbs the wave, what are (g) the rate at which momentum is transferred to the surface and (h) the radiation pressure on the surface?

**36** A black, totally absorbing piece of cardboard of area $A = 2.0\ \text{cm}^2$ intercepts light with an intensity of 20 W/m$^2$ from a camera strobe light. What radiation pressure is produced on the cardboard by the light?

**37** What is the intensity of a traveling plane electromagnetic wave if $B_m$ is $3.0 \times 10^{-4}$ T?

**38** The index of refraction of benzene is 1.8. What is the critical angle for a light ray traveling in benzene toward a flat layer of water above the benzene?

**39** Figure 33-45 depicts a simplistic optical fiber: a plastic core ($n_1 = 1.58$) is surrounded by a plastic sheath ($n_2 = 1.46$). A light ray is incident on one end of the fiber at angle $\theta$. The ray is to undergo total internal reflection at point $A$, where it encounters the core–sheath boundary. (Thus there is no loss of light through that boundary.) What is the maximum value of $\theta$ that allows total internal reflection at $A$?

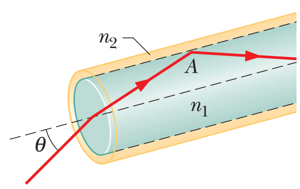


**Figure 33-45** Problem 39.

**40** A catfish is 1.50 m below the surface of a smooth lake. (a) What is the diameter of the circle on the surface through which the fish can see the world outside the water? (b) If the fish descends, does the diameter of the circle increase, decrease, or remain the same?

**41** Prove, for a plane electromagnetic wave that is normally incident on a flat surface, that the radiation pressure on the surface is equal to the energy density in the incident beam. (This relation between pressure and energy density holds no matter what fraction of the incident energy is reflected.)

**42** Frank D. Drake, an investigator in the SETI (Search for Extra-Terrestrial Intelligence) program, once said that the large radio telescope in Arecibo, Puerto Rico (Fig. 33-46), "can detect a signal which lays down on the entire surface of the earth a power of only one picowatt." (a) What is the power that would be received by the Arecibo antenna for such a signal? The antenna diameter is 300 m. (b) What would be the power of an isotropic source 9000 ly away that could provide such a signal? A light-year (ly) is the distance light travels in one year.

Courtesy SRI International, USRA, UMET

**Figure 33-46** Problem 42. Radio telescope at Arecibo.

**43** From Fig. 33-2, approximate the (a) smaller and (b) larger wavelength at which the eye of a standard observer has half the eye's maximum sensitivity. What are the (c) wavelength, (d) frequency, and (e) period of the light at which the eye is the most sensitive?

**44** A small laser emits light at power 5.00 mW and wavelength 633 nm. The laser beam is focused (narrowed) until its diameter matches the 1206 nm diameter of a sphere placed in its path. The sphere is perfectly absorbing and has density $5.00 \times 10^3$ kg/m$^3$. What are (a) the beam intensity at the sphere's location, (b) the radiation pressure on the sphere, (c) the magnitude of the corresponding force, and (d) the magnitude of the acceleration that force alone would give the sphere?

**45** Sunlight just outside Earth's atmosphere has an intensity of 1.40 kW/m$^2$. Calculate (a) $E_m$, (b) $B_m$, (c) $E_{\text{rms}}$, and (d) $B_{\text{rms}}$ for sunlight there, assuming it to be a plane wave.

**46** It has been proposed that a spaceship might be propelled in the solar system by radiation pressure, using a large sail made of foil. How large must the surface area of the sail be if the radiation force is to be equal in magnitude to the Sun's gravitational attraction? Assume that the mass of the ship + sail is 1800 kg, that the sail is perfectly reflecting, and that the sail is oriented perpendicular to the Sun's rays. See Appendix C for needed data. (With a larger sail, the ship is continuously driven away from the Sun.)

**47** Light that is traveling in water (with an index of refraction of 1.33) is incident on a plate of glass (with index of refraction 1.71). At what angle of incidence does the reflected light end up fully polarized?

**48** Assume (unrealistically) that a TV station acts as a point source broadcasting isotropically at 3.0 MW. What is the intensity of the transmitted signal reaching Proxima Centauri, the star nearest our solar system, 4.3 ly away? (An alien civilization at that distance might be able to watch *X Files*.) A light-year (ly) is the distance light travels in one year.

**49** In the ray diagram of Fig. 33-47, where the angles are not drawn to scale, the ray is incident at the critical angle on the interface between materials 2 and 3. Angle $\phi = 71.0°$, and two of the indexes of refraction are $n_1 = 1.70$ and $n_2 = 1.60$. Find (a) index of refraction $n_3$ and (b) angle $\theta$. (c) If $\theta$ is decreased, does light refract into material 3?

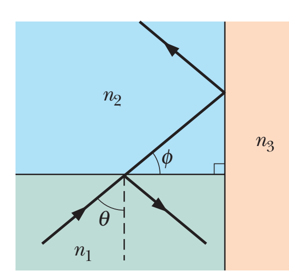


**Figure 33-47** Problem 49.

**50** A plane electromagnetic wave has a maximum electric field magnitude of $1.80 \times 10^{-4}$ V/m. Find the magnetic field amplitude.

**51** High-power lasers are used to compress a plasma (a gas of charged particles) by radiation pressure. A laser generating radiation pulses with peak power $4.5 \times 10^3$ MW is focused onto $0.80\ \text{mm}^2$ of high-electron-density plasma. Find the pressure exerted on the plasma if the plasma reflects all the light beams directly back along their paths.

**52** Project Seafarer was an ambitious program to construct an enormous antenna, buried underground on a site about $10\,000\ \text{km}^2$ in area. Its purpose was to transmit signals to submarines while they were deeply submerged. If the effective wavelength were $2.0 \times 10^4$ Earth radii, what would be the (a) frequency and (b) period of the radiations emitted? Ordinarily, electromagnetic radiations do not penetrate very far into conductors such as seawater, and so normal signals cannot reach the submarines.

**53** A small spaceship with a mass of only $1.5 \times 10^3$ kg (including an astronaut) is drifting in outer space with negligible gravitational forces acting on it. If the astronaut turns on a 25 kW laser beam, what speed will the ship attain in 45.0 day because of the momentum carried away by the beam?

**54** In Fig. 33-48, a light ray in air is incident at angle $\theta_1$ on a block of transparent plastic with an index of refraction of 1.56. The dimensions indicated are $H = 2.00$ cm and $W = 3.00$ cm. The light passes through the block to one of its sides and there undergoes reflection (inside the block) and possibly refraction (out into the air). This is the point of *first reflection*. The reflected light then passes through the block to another of its sides—a point of *second reflection*. If $\theta_1 = 40°$, on which side is the point of (a) first reflection and (b) second reflection? If there is refraction at the point of (c) first reflection and (d) second reflection, give the angle of refraction; if not, answer "none." If $\theta_1 = 75°$, on which side is the point of (e) first reflection and (f) second reflection? If there is refraction at the point of (g) first reflection and (h) second reflection, give the angle of refraction; if not, answer "none."

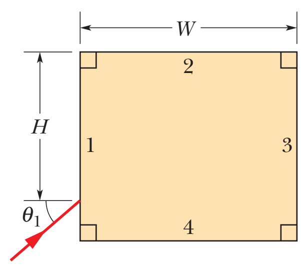

**Figure 33-48** Problem 54.

**55** What inductance must be connected to a 25 pF capacitor in an oscillator capable of generating 410 nm (i.e., visible) electromagnetic waves? Comment on your answer.

**56** In Fig. 33-49, light from ray $A$ refracts from material 1 ($n_1 = 1.50$) into a thin layer of material 2 ($n_2 = 1.80$), crosses that layer, and is then incident at the critical angle on the interface between materials 2 and 3 ($n_3 = 1.30$). (a) What is the value of incident angle $\theta_A$? (b) If $\theta_A$ is decreased, does part of the light refract into material 3?

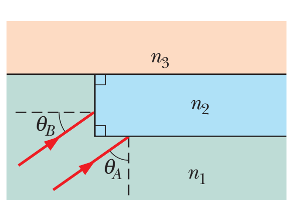

**Figure 33-49** Problem 56.

Light from ray $B$ refracts from material 1 into the thin layer, crosses that layer, and is then incident at the critical angle on the interface between materials 2 and 3. (c) What is the value of incident angle $\theta_B$? (d) If $\theta_B$ is decreased, does part of the light refract into material 3?

**57** What is the radiation pressure 2.7 m away from a 315 W lightbulb? Assume that the surface on which the pressure is exerted faces the bulb and is perfectly absorbing and that the bulb radiates uniformly in all directions.

**58** The intensity $I$ of light from an isotropic point source is determined as a function of distance $r$ from the source. Figure 33-50 gives intensity $I$ versus the inverse square $r^{-2}$ of that distance. The vertical axis scale is set by $I_s = 400\ \text{W/m}^2$, and the horizontal axis scale is set by $r_s^{-2} = 8.0\ \text{m}^{-2}$. What is the power of the source?

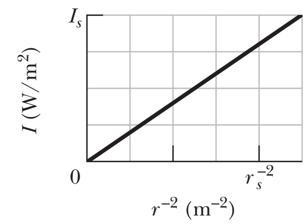

**Figure 33-50** Problem 58.

**59** An airplane flying at a distance of 10 km from a radio transmitter receives a signal of intensity 28 $\mu\text{W/m}^2$. What is the amplitude of the (a) electric and (b) magnetic component of the signal at the airplane? (c) If the transmitter radiates uniformly over a hemisphere, what is the transmission power?

**60** In Fig. 33-51$a$, a light ray in water is incident at angle $\theta_1$ on a boundary with an underlying material, into which some of the light refracts. There are two choices of underlying material. For each, the angle of refraction $\theta_2$ versus the incident angle $\theta_1$ is given in Fig. 33-51$b$. The vertical axis scale is set by $\theta_{2s} = 80°$. Without calculation, determine whether the index of refraction of (a) material 1 and (b) material 2 is greater or less than the index of water ($n = 1.33$). What is the index of refraction of (c) material 1 and (d) material 2?

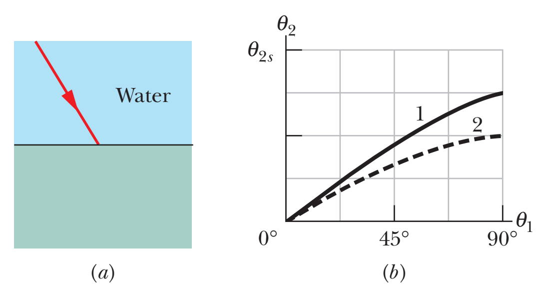

**Figure 33-51** Problem 60.

**61** A certain helium–neon laser emits red light in a narrow band of wavelengths centered at 632.8 nm and with a "wavelength width" (such as on the scale of Fig. 33-1) of 5.00 pm. What is the corresponding "frequency width" for the emission?

**62** In Fig. 33-52, a light ray in air is incident on a flat layer of material 2 that has an index of refraction $n_2 = 1.7$. Beneath material 2 is material 3 with an index of refraction $n_3$. The ray is incident on the air–material 2 interface at the Brewster angle for that interface. The ray of light refracted into material 3 happens to be incident on the material 2–material 3 interface at the Brewster angle for that interface. What is the value of $n_3$?

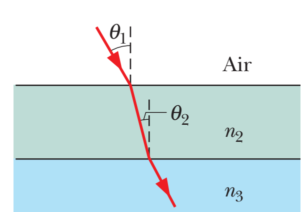

**Figure 33-52** Problem 62.

**63** Someone plans to float a small, totally absorbing sphere 0.200 m above an isotropic point source of light, so that the upward radiation force from the light matches the downward gravitational force on the sphere. The sphere's density is 19.0 g/cm$^3$, and its radius is 0.500 mm. (a) What power would be required of the light source? (b) Even if such a source were made, why would the support of the sphere be unstable?

**64** The average intensity of the solar radiation that strikes normally on a surface just outside Earth's atmosphere is 1.4 kW/m$^2$. (a) What radiation pressure $p_r$ is exerted on this surface, assuming complete absorption? (b) For comparison, find the ratio of $p_r$ to Earth's sea-level atmospheric pressure, which is $1.0 \times 10^5$ Pa.

**65** A plane electromagnetic wave traveling in the positive direction of an $x$ axis in vacuum has components $E_x = E_y = 0$ and $E_z = (4.0 \text{ V/m}) \cos[(\pi \times 10^{15} \text{ s}^{-1})(t - x/c)]$. (a) What is the amplitude of the magnetic field component? (b) Parallel to which axis does the magnetic field oscillate? (c) When the electric field component is in the positive direction of the $z$ axis at a certain point $P$, what is the direction of the magnetic field component there? (d) In what direction is the wave moving?

**66** What is the wavelength of the electromagnetic wave emitted by the oscillator–antenna system of Fig. 33-3 if $L = 0.253\ \mu$H and $C = 30.0$ pF?

**67** Unpolarized light of intensity 6.5 mW/m$^2$ is sent into a polarizing sheet as in Fig. 33-11. What are (a) the amplitude of the electric field component of the transmitted light and (b) the radiation pressure on the sheet due to its absorbing some of the light?

**68** In Fig. 33-53, a laser beam of power 4.60 W and diameter $D = 3.00$ mm is directed upward at one circular face (of diameter $d < 2.60$ mm) of a perfectly reflecting cylinder. The cylinder is levitated because the upward radiation force matches the downward gravitational force. If the cylinder's density is 1.20 g/cm$^3$, what is its height $H$?

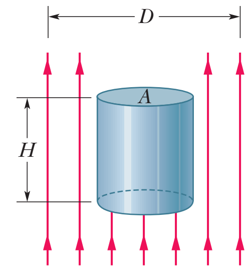


**Figure 33-53** Problem 68.

**69** Some neodymium–glass lasers can provide 100 TW of power in 2.2 ns pulses at a wavelength of 0.26 $\mu$m. How much energy is contained in a single pulse?

**70** In Fig. 33-54$a$, a light ray in an underlying material is incident at angle $\theta_1$ on a boundary with water, and some of the light refracts into the water. There are two choices of underlying material. For each, the angle of refraction $\theta_2$ versus the incident angle $\theta_1$ is given in Fig. 33-54$b$. The horizontal axis scale is set by $\theta_{1s} = 80°$. Without calculation, determine whether the index of refraction of (a) material 1 and (b) material 2 is greater or less than the index of water ($n = 1.33$). What is the index of refraction of (c) material 1 and (d) material 2?

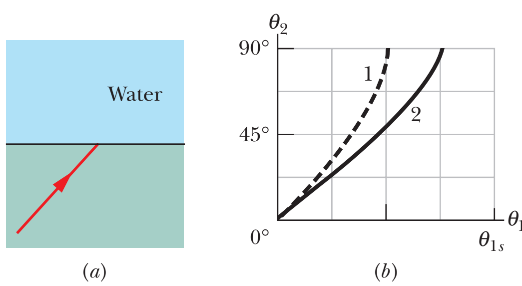


**Figure 33-54** Problem 70.