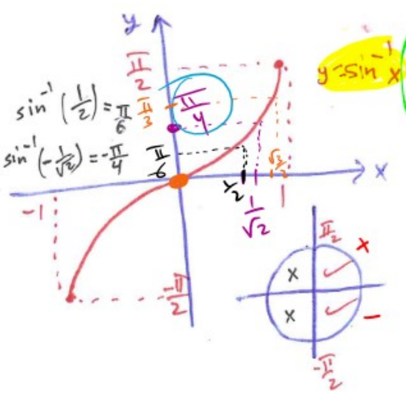
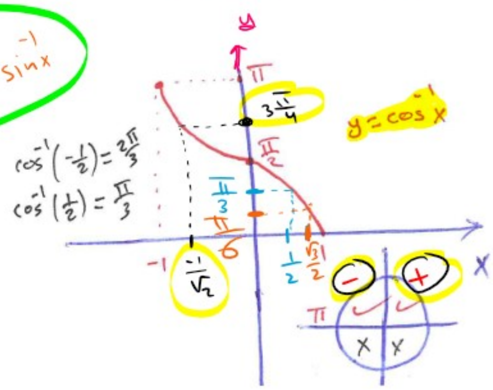
Discussion

lecture

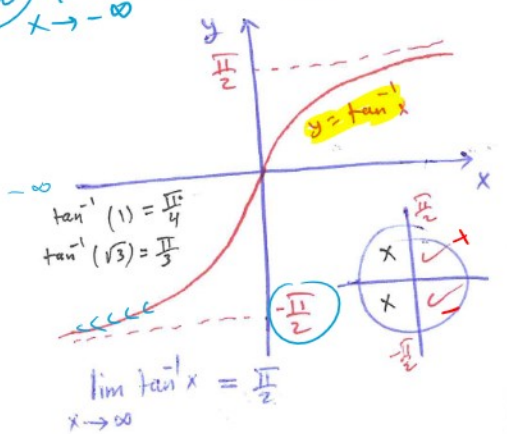
7.6	34, 64, 69, 74	2, 5, 6, 12, 16, 17, 21, 25, 33, 46, 50, 59, 63, 70, 75, 85

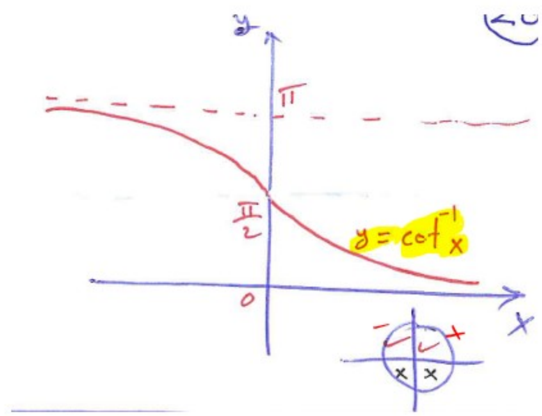


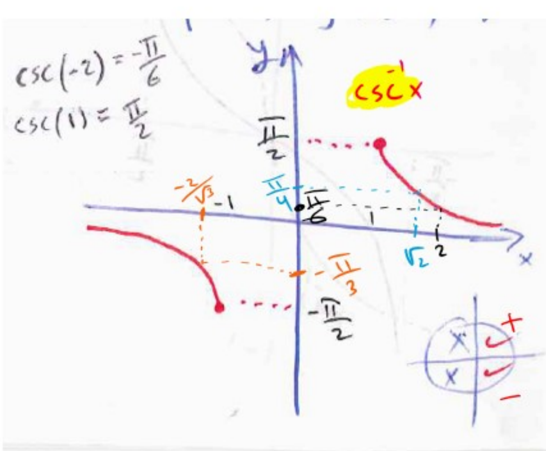
odd
$\sin^{-1}(-x) = -\sin^{-1} x$

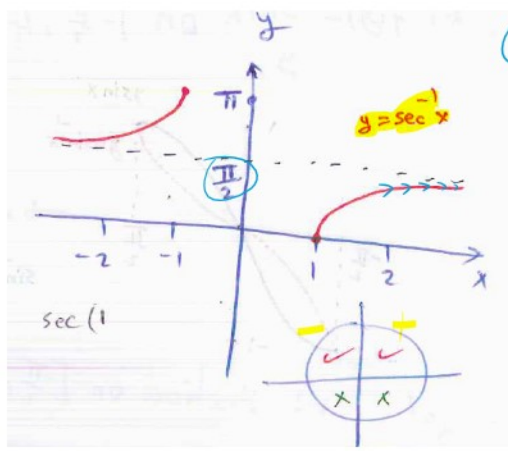


(16) $\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$









(17) $\lim_{x \to \infty} \sec^{-1} x = \frac{\pi}{2}$

$$\boxed{\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}, \quad |u| < 1}$$

$$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}\left(\frac{du}{dx}\right), \quad |u|<1$$

$$\frac{d}{dx}(\cos^{-1} u) = \frac{-1}{\sqrt{1-u^2}}\left(\frac{du}{dx}\right), \quad |u|<1$$

$$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2}\left(\frac{du}{dx}\right)$$

$$\frac{d}{dx}(\cot^{-1} u) = \frac{-1}{1+u^2}\left(\frac{du}{dx}\right)$$

$$\frac{d}{dx}(\sec^{-1} u) = \frac{1}{|u|\sqrt{u^2-1}}\left(\frac{du}{dx}\right), \quad |u|>1$$

$$\frac{d}{dx}(\csc^{-1} u) = \frac{-1}{|u|\sqrt{u^2-1}}\left(\frac{du}{dx}\right), \quad |u|>1$$

(85) $\displaystyle\int \frac{dy}{\tan^{-1}y\,(1+y^2)} = \int \frac{du}{u}$

$u = \tan^{-1} y$

$du = \dfrac{dy}{1+y^2}$

$= \ln|u| + C$

$= \ln|\tan^{-1} y| + C$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \boxed{\sin^{-1}\left(\frac{u}{a}\right) + C}, \quad u^2 < a^2$$

$a = 3$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C, \quad |u|>a>0$$

(50) $\displaystyle\int_0^{\frac{3\sqrt{2}}{4}} \frac{ds}{\sqrt{9-4s^2}}$

u^2

$u = 2s$

$du = 2\,ds$

$\dfrac{du}{2} = ds$

$s = 0 \Rightarrow u = 0$

$s = \dfrac{3\sqrt{2}}{4} \Rightarrow u = \dfrac{3\sqrt{2}}{2}$

$\displaystyle\int_0^{\frac{3\sqrt{2}}{2}} \frac{\frac{du}{2}}{\sqrt{9-u^2}}$

$\displaystyle = \frac{1}{2}\int_0^{\frac{3\sqrt{2}}{2}} \frac{du}{\sqrt{3^2-u^2}} = \frac{1}{2}\sin^{-1}\frac{u}{3}\Big|_0^{\frac{3\sqrt{2}}{2}}$

$= \frac{1}{2}\sin^{-1}\frac{\sqrt{2}}{2} - \frac{1}{2}\sin^{-1} 0$

$\sin^{-1} 0 \to 0$

$\dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{\sqrt{2}\sqrt{2}}$

$= \dfrac{1}{\sqrt{2}}$

$= \frac{1}{2}\boxed{\sin^{-1}\frac{1}{\sqrt{2}}}$

$= \frac{1}{2}\boxed{\frac{\pi}{4}}$

$= \boxed{\frac{\pi}{8}}$

Note That

(16) $\csc^{-1}(-2) = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{x}\right)$

$\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1}{x}\right)$

$\cos^{-1}\theta + \sin^{-1}\theta = \frac{\pi}{2}$

$\theta = \frac{1}{x}$

$$\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1+u^2}\,\frac{du}{dx}$$

(46) $$\int \frac{dx}{9+3x^2} = \frac{1}{3}\int \frac{dx}{3+x^2} = \frac{1}{3}\int \frac{dx}{(\sqrt{3})^2+x^2} = \frac{1}{3}\,\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}+c$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right)+C \quad , \quad u^2<a^2$$

$$\int \frac{du}{a^2+u^2} = \boxed{\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C}$$

$a=\sqrt{3}$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right|+C \quad , \quad |u|>a>0$$

$$\frac{d}{dx}\left(\sec^{-1} u\right) = \frac{1}{|u|\sqrt{u^2-1}}\,\frac{du}{dx} \quad , \quad |u|>1$$

(25) $$y = \sec^{-1}\underbrace{(2s+1)}_{u} \Rightarrow y' = \frac{2}{|2s+1|\sqrt{(2s+1)^2-1}}$$

$$= \frac{2}{|2s+1|\sqrt{4s^2+4s+\cancel{1}-\cancel{1}}} = \frac{\cancel{2}}{|2s+1|\sqrt{\cancel{4}(s^2+s)}}$$

$$= \frac{1}{|2s+1|\sqrt{s^2+s}}$$

$$\frac{d}{dx}\left(\cos^{-1} u\right) = \frac{-1}{\sqrt{1-u^2}}\,\frac{du}{dx} \quad , \quad |u|<1$$

(21) $$y = \cos^{-1} x^2 \Rightarrow y' = \frac{-\overset{u'}{2x}}{\ldots}$$

(21) $y = \cos^{-1} \underbrace{x^2}_{u} \Rightarrow y' = \dfrac{-(2x)}{\sqrt{1 - x^4}}$

Note That

مثال $\csc^{-1}(-2) = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{x}\right)$

$\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1}{x}\right)$

(6)
- $\csc^{-1}\sqrt{2} = \boxed{\sin^{-1}\frac{1}{\sqrt{2}}} = \frac{\pi}{4}$
- $\csc^{-1}\frac{-2}{\sqrt{3}} = \sin^{-1}\frac{-\sqrt{3}}{2} = -\sin^{-1}\frac{\sqrt{3}}{2} = -\frac{\pi}{3}$
- $\csc^{-1} 2 = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

(5)
- $\cos^{-1}\frac{1}{2} = \theta$

$\cancel{\cos}\left(\cancel{\cos^{-1}}\frac{1}{2}\right) = \cos\theta$

$\frac{1}{2} = \cos\theta$

$\theta = \frac{\pi}{3}$

$\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$

زاوية

- $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{\pi}{4} + \boxed{\frac{\pi}{2}}$

$= \frac{3\pi}{4}$

الربع الاول

$\cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$

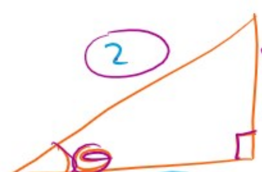
زاوية

$\theta = \cos^{-1}\frac{\sqrt{3}}{2}$

$\Leftrightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}$

$\sin\theta = \frac{1}{2}$



$\boxed{?? = 1}$

$4 = 3 + ?^2$

$1 = ?^2$

$\theta = \cos^{-1}\frac{\sqrt{3}}{2}$

$\cos\theta = \cancel{\cos}\left(\cancel{\cos^{-1}}\frac{\sqrt{3}}{2}\right)$

$\cos\theta = \cancel{\cos}\left(\cancel{\cos^{-1}} \frac{\sqrt{3}}{2}\right)$

$\cos\theta = \frac{\sqrt{3}}{2}$

$1 = ?^2$

$1 = ?$