

الجذر

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شك كسر

Ex 1 Simplify the following expressions

$$\textcircled{1} \left(2^{\frac{2}{3}} \cdot 4 \right)^{\frac{2}{3}} = \left(2 \times 2 \times 2 \cdot 4 \right)^{\frac{2}{3}} = (8 \cdot 4)^{\frac{2}{3}} = 32^{\frac{2}{3}} = \sqrt[3]{32^2}$$

$$= \sqrt[3]{32 \times 32} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \cdot 2^5}$$

2	32
2	16
2	8
2	4
2	2

$$= 2 \sqrt[3]{2 \times 2 \cdot 2 \times 2 \times 2 \times 2 \times 2}$$

$$= 2 \times 2 \sqrt[3]{2 \times 2 \times 2 \times 2}$$

$$= 2 \times 2 \times 2 \sqrt[3]{2}$$

$$= 8 \sqrt[3]{2}$$

$$\textcircled{2} 3^{-\frac{1}{2}} \cdot 3^{-\frac{3}{2}} = 3^{-\frac{1}{2} + \frac{-3}{2}} = 3^{-\frac{4}{2}} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$\textcircled{3} -8^{-\frac{2}{3}} = -\frac{1}{8^{\frac{2}{3}}} = -\frac{1}{\sqrt[3]{8^2}}$$

$$(-8)^{-\frac{2}{3}} = \frac{1}{(-8)^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{8 \times 8}} = \frac{1}{\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2}}$$

$$\vdots \quad \left| \quad \sqrt[5]{8 \times 8} \quad \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2} \right. \\ = \frac{-1}{2 \times 2} = \frac{-1}{4}$$

(4) $\sqrt{4} = \sqrt[2]{4} = \sqrt[2]{2 \times 2} = 2$

~~$\sqrt{-4} = \pm 2$~~
 $\sqrt{-4}$ undefined

~~$\sqrt[2]{-2} = \pm 2$~~

$$\sqrt[4]{-8} =$$

$$\sqrt[10]{-1000} =$$

$\sqrt[3]{-27}$	a) 3
$\sqrt[3]{-3 \times -3 \times -3}$	b) -3
$= -3$	c) ± 3
	d) undefined

$\sqrt[6]{-1}$ undefined

$\sqrt[5]{-1} = -1$ ✓

Operations with Radicals

Operations with Radicals

- $\sqrt[n]{a^n} = a^{\frac{n}{n}} = a^1 = a$

Exp

$$\sqrt[3]{2^3} = 2$$

- $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$

$$\sqrt[3]{2} \sqrt[3]{4} =$$

$$\sqrt[3]{2 \times 4} = \sqrt[3]{8}$$

$$= \sqrt[3]{2 \times 2 \times 2}$$

$$= 2$$

- $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}, b \neq 0$

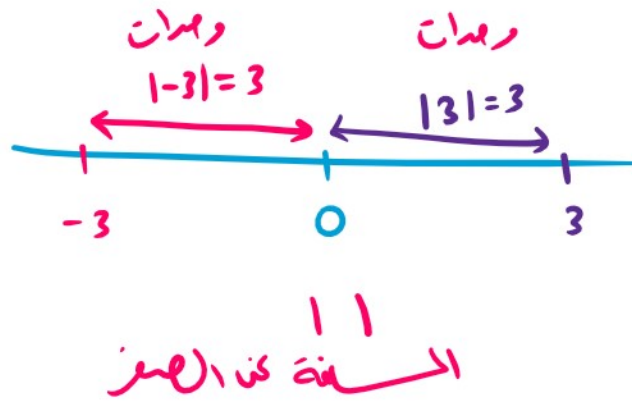
Exp

$$\frac{\sqrt[2]{32}}{\sqrt[2]{2}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4$$

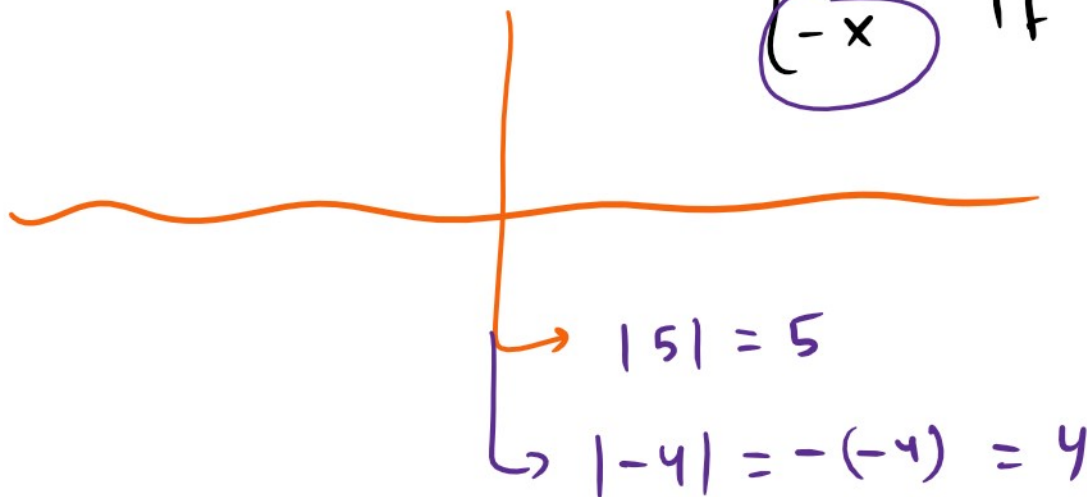
- $|3| = 3$

- $|-3| = 3$

$$|a| = \begin{cases} a & \text{if } a > 0 \\ -a & \text{if } a < 0 \end{cases}$$



$$\sqrt{x^2} = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



Exp Simplify the following radicals

$$(1) \sqrt{48x^5y^6} = \sqrt[2]{2 \times 2 \times 2 \times 2 \times 3 \times x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}$$

$$\sqrt[2]{48}$$

$$\sqrt[2]{2x}$$

2	18
2	24
2	12
2	6
3	3

$$= 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y \cdot y \sqrt[3]{3x}$$

$$= 4 x^2 y^3 \sqrt{3x}$$



$$(2) \sqrt[3]{72 a^3 b^4} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b}$$

$$= 2 \cdot a \cdot b \sqrt[3]{3 \cdot 3 \cdot b}$$

$$= 2ab \sqrt[3]{9b}$$

2	72
2	36
2	18
3	9
3	3

$$(3) \sqrt[3]{16 x^2 y} \sqrt[3]{3 x^2 y} = \sqrt[3]{16 x^2 y \cdot 3 x^2 y}$$

$$= \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot 3}$$

$$= 2 \cdot x \sqrt[3]{2 \cdot 3 \cdot x \cdot y \cdot y}$$

$$= 2x \sqrt[3]{6xy^2}$$

2	16
2	8
2	4
2	2

$$\begin{array}{r|l} 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$= 2 \times \sqrt[3]{6xy^2}$$

$$(4) \frac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\frac{32}{4}} = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2} = 2$$

Exp Express the following with **no radical** in the **denominator** بدون جذر في المقام

$$(1) \frac{3}{\sqrt{x}} \cdot \left[\frac{\sqrt{x}}{\sqrt{x}} \right]^1 = \frac{3\sqrt{x}}{\sqrt{x^2}} = \frac{3\sqrt{x}}{1 \times 1}$$

$$(2) \frac{3x}{\sqrt[2]{18xy}} \cdot \frac{\sqrt[2]{18xy}}{\sqrt[2]{18xy}} = \frac{3x\sqrt[2]{18xy}}{\sqrt[2]{18 \cdot 18 \cdot x \cdot x \cdot y \cdot y}}$$

$$= \frac{3x\sqrt{18xy}}{18xy}$$

$$\left(\frac{3}{\sqrt{18xy}} \right) \neq \frac{3}{(\sqrt{18xy})^2} = \left(\frac{9}{x} \right)$$

$$\left(\frac{3}{\sqrt{x}} \right) \neq \frac{3}{(\sqrt{x})^2} = \left(\frac{3}{x} \right)$$

$\neq$

$$\left(\frac{3}{\sqrt{x}} \right)^2 \neq \left(\frac{3}{x} \right)^2$$

$$\frac{9}{x} \neq \frac{81}{x^2}$$

$$\left(\frac{3}{\sqrt{x}} \right)^2 = \left(\frac{3\sqrt{x}}{x} \right)^2$$

$$\frac{9}{x} = \frac{9\cancel{x}}{\cancel{x}x}$$

$$\frac{x^1}{x^2} = x^{1-2} = x^{-1} = \frac{1}{x}$$

$$\boxed{\frac{9}{x} = \frac{9}{x}} \quad \checkmark$$

Ex: Express the following with **no radicals in the numerator**
بدون جذور في البسط

$$\textcircled{1} \quad \frac{\sqrt{3xy}}{\sqrt{x}} = \frac{\sqrt[2]{3 \cdot 3 \cdot x \cdot x \cdot y \cdot y}}{\sqrt[2]{x \cdot 3 \cdot x \cdot y}}$$

$$\frac{1}{\sqrt{x}} \cdot \sqrt{\frac{3xy}{x}} = \sqrt[2]{x \cdot 3 \cdot x \cdot y}$$

$$= \frac{3 \cancel{x} y}{\cancel{x} \sqrt{3y}} = \frac{3y}{\sqrt{3y}}$$

$$\begin{array}{l|l} \sqrt{2} \sqrt{2} \neq 1 & \sqrt{2} \sqrt{2} = \sqrt[2]{2 \cdot 2} \\ & = \sqrt{4} \\ & = 2 \end{array}$$

$$\begin{array}{l} \sqrt{3} \sqrt{3} = 3 \\ \sqrt{x} \sqrt{x} = x, \quad x > 0 \end{array}$$