مراجعة مساق
تحليل الدوائر الكهربائية

Circuit Analysis

ENEE2304

First Exam

Summer Semester

2024 / 2025

By :

Mohammed Saada.

Chapter 1 — Chapter 3

$$V = \frac{dW}{dq} \quad \text{Volt.}$$

$$I = \frac{dq}{dt} \quad A.$$

$$P = \frac{dW}{dt} = IV = I^2R = \frac{V^2}{R} \quad \text{watt.}$$

$$G \text{ (conductance)} = \frac{1}{R} \quad \mho$$

Ohm's Law $V = IR$

Kirchhoff's Voltage Law (KVL): The algebraic sum of all the voltages around any closed path (loop) in a circuit equals zero

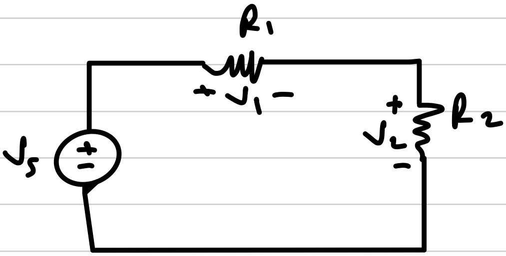
Kirchhoff's current Law (KCL): The algebraic sum of the currents at any node in a circuit equals zero.

Resistors in Series: $R_{eq} = R_1 + R_2 + \cdots + R_n$

Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

2 Resistors in parallel: $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Voltage divider rule :



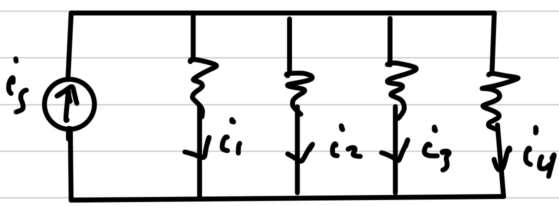
$$V_1 = \frac{V_s R_1}{R_1 + R_2}$$

$$V_2 = \frac{V_s R_2}{R_1 + R_2}$$

Current divider rule:

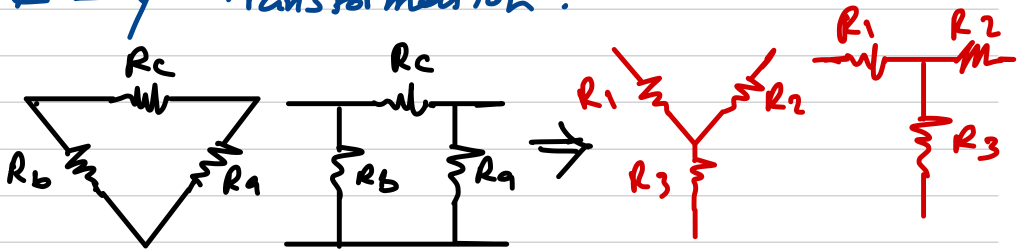
$$i_1 = \frac{i_s R_2}{R_1 + R_2}$$

$$i_2 = \frac{i_s R_1}{R_1 + R_2}$$



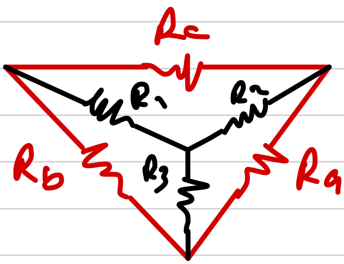
$$i_j = \frac{i_s R_{eq}}{R_j}$$

Δ – Y Transformation:



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_a R_c}{R_a + R_b + R_c}, \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Y – Δ Transformation:



$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

For the Balance Case where

$R_a = R_b = R_c = R_\Delta$, $R_1 = R_2 = R_3 = R_Y$

$R_\Delta = 3R_Y$, $R_Y = \frac{1}{3} R_\Delta$

Chapter 4 : Techniques of circuit analysis

* Node-Voltage method.
* Mesh-current method.
* Source Transformation.
* Super position.
* Thevenin and Norton.
* Maximum Power Transfer.

R_L will recieve a maximum power, when $R_L = R_{th}$

$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

Chapter 9: Sinusoidal Steady-state Analysis

$$f = \frac{1}{T} \text{ Hz}, \quad \omega = 2\pi f \text{ rad/s}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}, \quad I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$\cos(\alpha) = \sin(\alpha + 90^\circ)$$
$$\sin(\alpha) = -\cos(\alpha + 90^\circ)$$

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$
$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\sin(\omega t \pm 180^\circ) = -\sin(\omega t)$$
$$\cos(\omega t \pm 180^\circ) = -\cos(\omega t)$$

$$\sin(\omega t \pm 90^\circ) = \pm\cos(\omega t)$$
$$\cos(\omega t \pm 90^\circ) = \mp\sin(\omega t)$$

Impedance

$$Z_R = R = R + j0$$

$$Z_L = j\omega L = 0 + j\omega L$$

$$Z_C = \frac{-j}{\omega C} = 0 + j\frac{-1}{\omega C}$$

$Z = a + jb$

a: Resistive

b: Reactance (X)

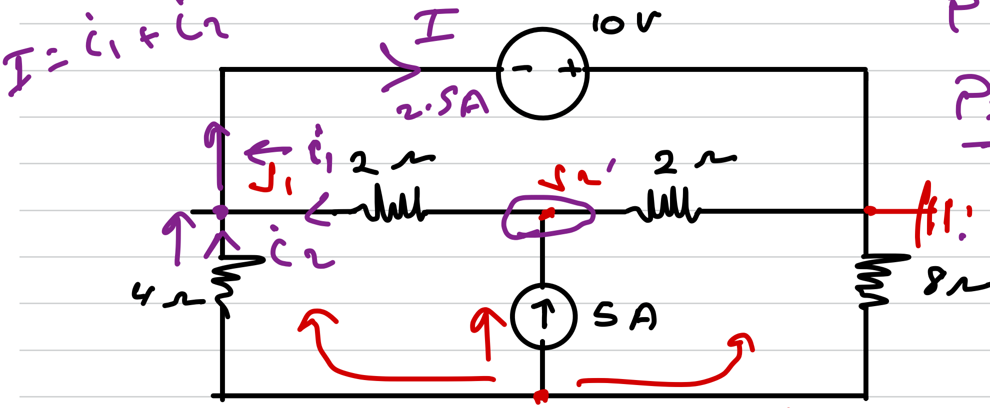
$$X_R = 0, \quad X_L = \omega L, \quad X_C = \frac{-1}{\omega C}$$

Admitance $Y = \frac{1}{Z}$

Q1: Use nodal analysis to find the power of the Voltage Source (P_{10V})

$P = I(V)$

$P = -IV$



$v_2 = 0 \qquad\qquad v_3$

$v_1 - 0 = -10$

$0 - V_1 = 10$

$v_3 - v_1 = 10 \text{ Volt} \rightarrow v_3 = 10 + V_1$

for $(V_1 - V_3)$:

$$\frac{v_1}{4} + \frac{v_1 - v_2}{2} + \frac{v_3 - v_2}{2} + \frac{v_3}{8} = 0$$

$$2V_1 + 4V_1 - 4V_2 + 4V_3 - 4V_2 + v_3 = 0$$

$$\boxed{6V_1 - 8V_2 + 5V_3 = 0}$$

$$6V_1 - 8V_2 + 5(10 + V_1) = 0$$

$$6V_1 - 8V_2 + 50 + 5V_1 = 0$$

$$\boxed{11V_1 - 8V_2 = -50} \quad (1)$$

$$V_1 = -10 \text{ Volt}$$

$$V_2 = 0 \text{ Volt}$$

$$\frac{V_3 - V_1}{4} + \frac{V_3}{8} + 5 = 0$$

$$2V_3 - 2V_1 + V_3 + 40 = 0$$

$$3V_3 - 2V_1 = -40$$

$$3V_3 + 20 = -40$$

$$3V_3 = -60$$

$$V_3 = -20 \text{ Volt}$$

$$I = i_1 + i_2$$

$$= \frac{V_2 - V_1}{2} + \frac{V_3 - V_1}{4}$$

$$= \frac{10}{2} + \frac{-20 - -10}{4}$$

$$= 5 + -2.5$$

$$= 2.5 \text{ A}$$

$$P = -IV = (-2.5)(10) = -25 \text{ Watt}$$

For V_2

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{2} - 5 = 0$$

$$V_2 - V_1 + V_2 - V_3 = 10$$

$$-V_1 + 2V_2 - V_3 = 10 \qquad V_3 = 10 + V_1$$

$$-V_1 + 2V_2 - (10 + V_1) = 10$$

$$-V_1 + 2V_2 - 10 - V_1 = 10$$

$$-2V_1 + 2V_2 = 20 \quad \text{(2)}$$

$$11V_1 - 8V_2 = -50$$

$$+\ 4(-2V_1 + 2V_2 = 20)$$

$$3V_1 = 30$$

$$V_1 = 10 \text{ Volt}$$

$$-2(10) + 2V_2 = 20 \rightarrow 2V_2 = 40$$

$$V_2 = 20 \text{ Volt}$$

$$V_3 = 10 + V_1 \Rightarrow V_3 = 20 \text{ Volt}$$

$$I = i_1 + i_2$$

$$= \frac{V_3 - V_2}{2} + \frac{V_3}{8}$$

$$= \frac{20 - 20}{2} + \frac{20}{8} = 2.5 \text{ A}$$

$$P = -I V$$

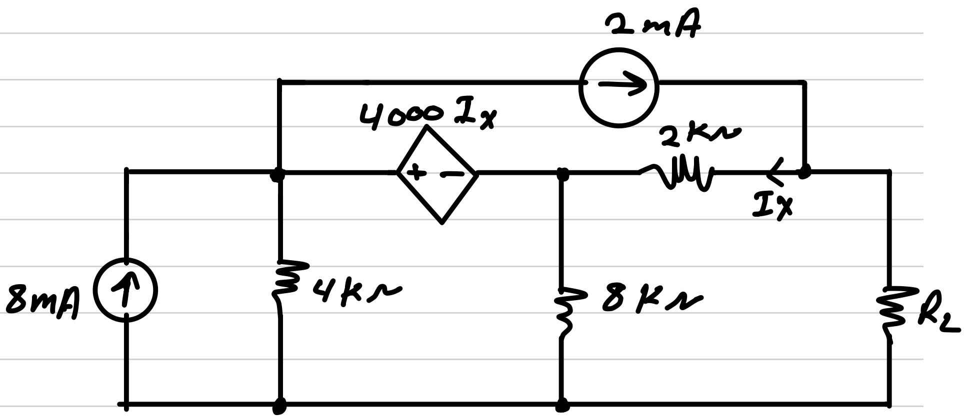
$$= (-2.5)(10)$$

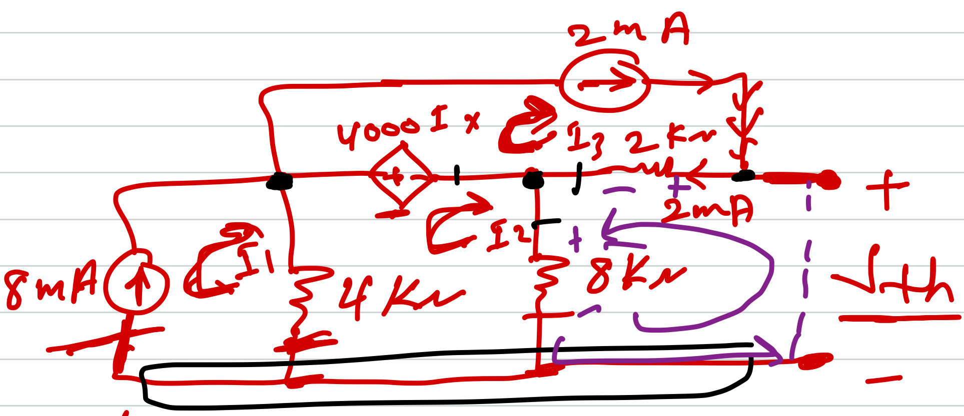
$$= -25 \text{ Watt.}$$ Supplied power

Q2. For the following circuit:

ⓐ Find the Value of R_L for max Power transfer.

ⓑ Calculate the maximum power transferred to R_L





mesh

$I_1 = 8mA$, $I_3 = 2mA$

For mesh ②

$$(4+8) I_2 - 4 I_1 + 4000 I_x = 0$$

but $(I_x = 2mA)$

$$12 I_2 - 4(8) + 4\times10^{3} \times 2\times10^{-3} = 0$$

$$12 I_2 - 32 + 8 = 0$$

$$12 I_2 = 24$$

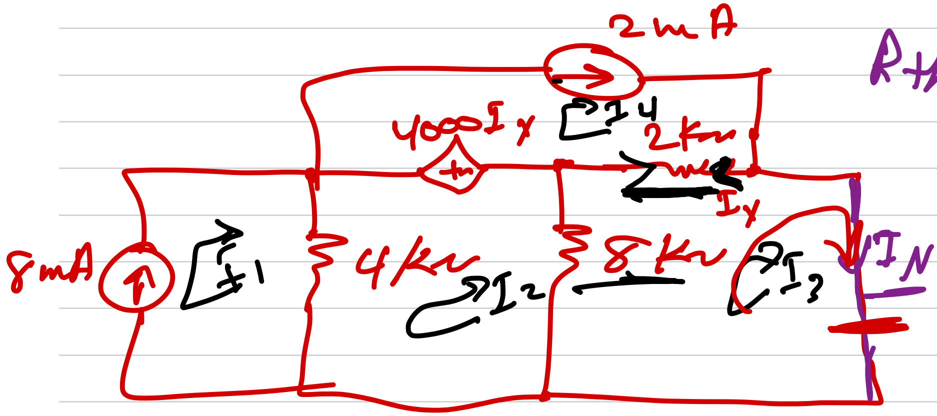
$$\boxed{I_2 = 2mA}$$

KVL

$$-V_{th} + 4 + 16 = 0$$

$$\boxed{V_{th} = 20 \text{ Volt}}$$

$$R_{th} = \frac{V_{th}}{I_N}$$



$$I_N = I_3$$

$I_1 = 8mA \quad , \quad I_4 = 2mA$

for mesh ②

$$(4+8)I_2 - 4I_1 - 8I_3 + 4000 I_x = 0$$

$$(I_x = I_4 - I_3)$$

$$12I_2 - 4I_1 - 8I_3 + 4(I_4 - I_3) = 0$$

$$12I_2 - 4I_1 - 8I_3 + 4I_4 - 4I_3 = 0$$

$$12I_2 - 4(8) - 12I_3 + 4(2) = 0$$

$$\boxed{12I_2 - 12I_3 = 24} \quad \text{—— ①}$$

for mesh ③

$$(8+2)I_3 - 8I_2 - 2I_4 = 0$$

$$10I_3 - 8I_2 - 2(2) = 0$$

$$\boxed{-8I_2 + 10I_3 = 4} \quad \text{—— ②}$$

① ÷ 3 / ② ÷ 2

$\cancel{4I_2} - 4I_3 = 8$

$+ \cancel{-4I_2} + 5I_3 = 2$

$$\boxed{I_3 = 10\,mA}$$

$I_N = I_3 = 10\,mA$

$R_{th} = \dfrac{V_{th}}{I_N} = \dfrac{20}{10} = \boxed{2\,K\Omega}$

for max power tranfer

$$\boxed{R_L = R_{th} = 2\,K\Omega}$$

$P_{max} = \dfrac{V_{th}^2}{4R_{th}} = \dfrac{(20)^2}{4(2)\times10^3}$

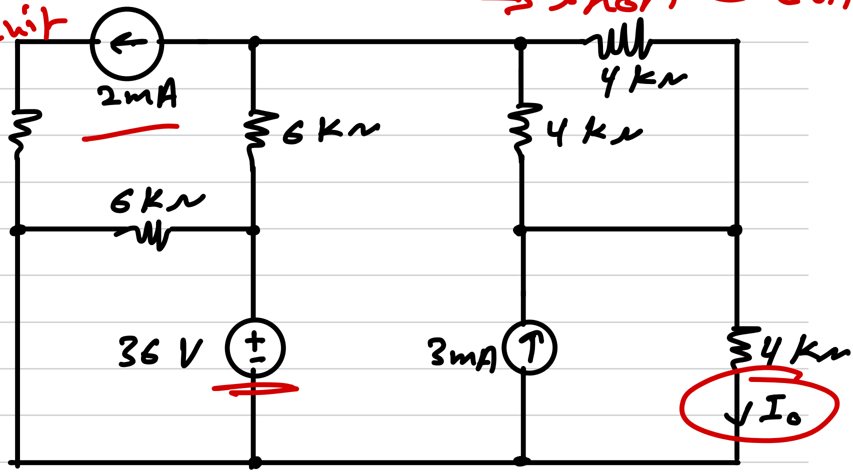
$= 50 \times 10^{-3}$

$= 50\,m\,Watt.$

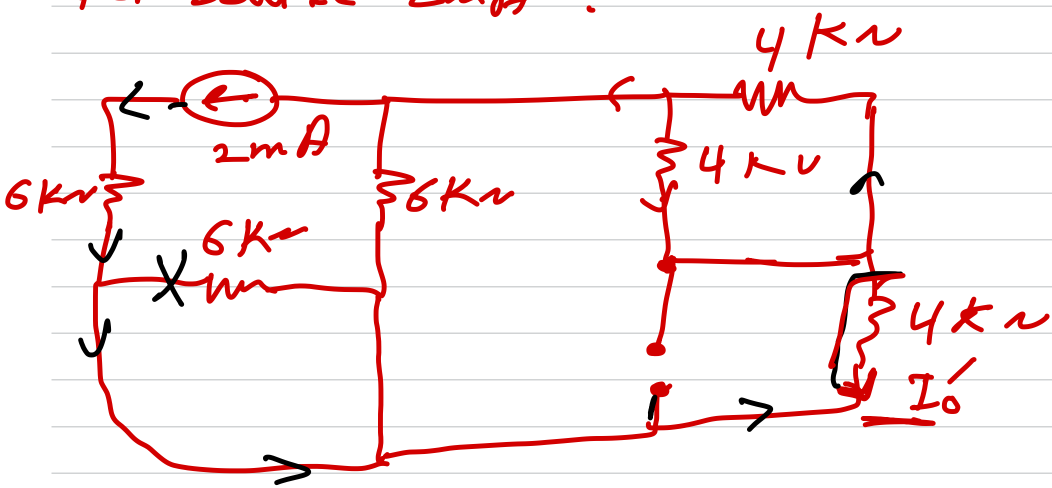
Q3. Use the Superposition theorem to find the output current I_o

Current source ⇒ open circuit

Voltage source ⇒ short circuit

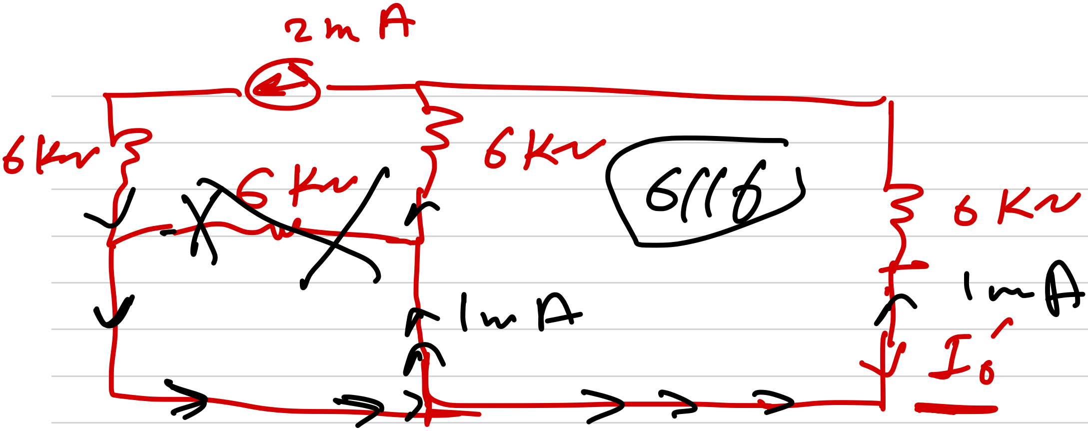


For source 2mA :



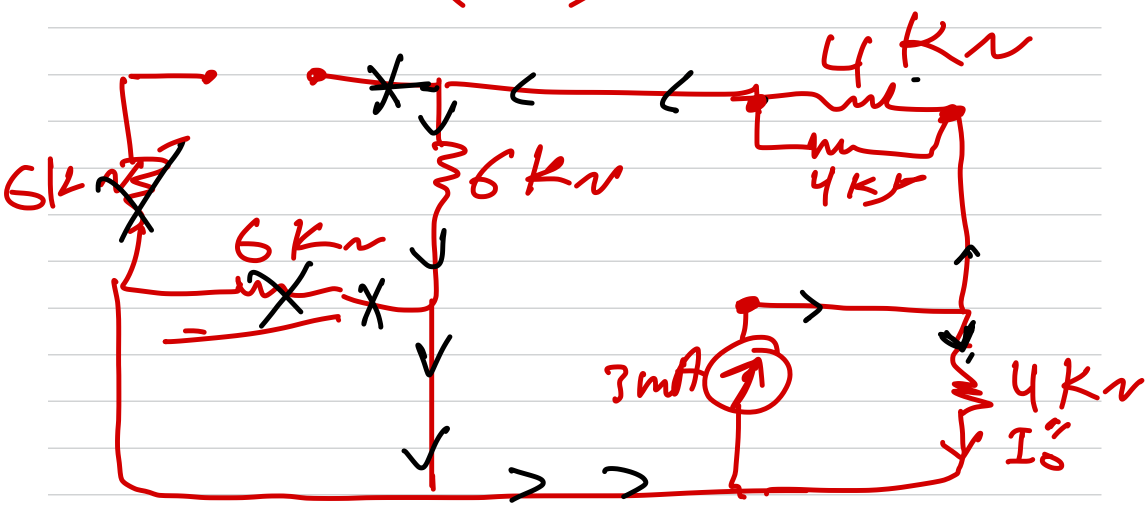
$(4 // 4) + 4 =$

$2 + 4 = 6\ k\Omega$

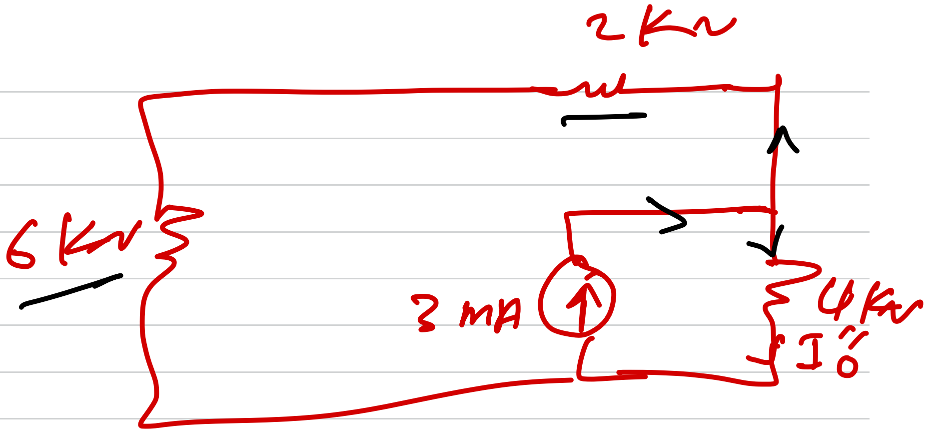


$$I_o' = -1\,mA$$

For source (3mA)

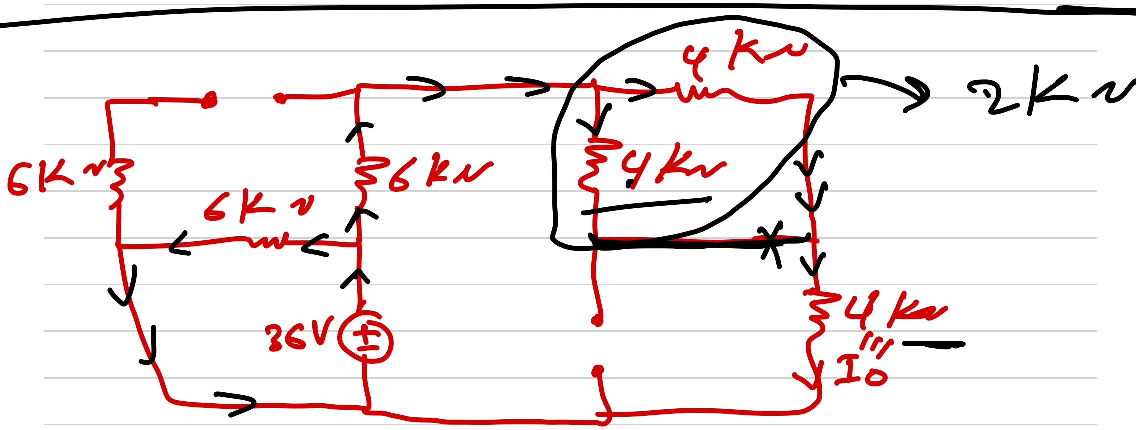


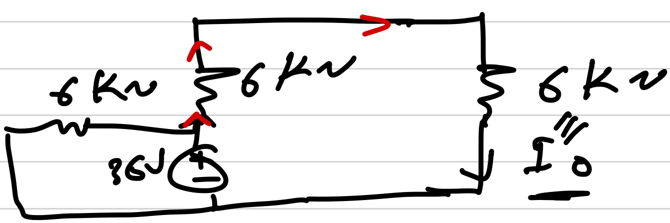
$(4 // 4) \rightarrow R = 2\,k\Omega$

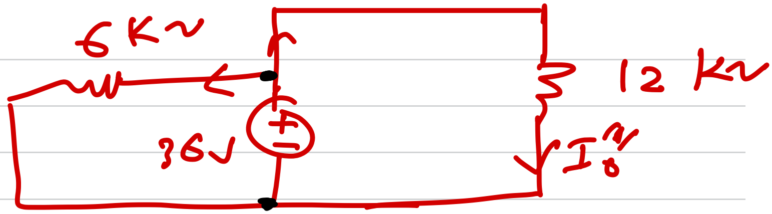


$$I_o'' = \frac{3(8)}{8+4} = \frac{24}{12}$$

$$\boxed{I_o'' = 2\,mA}$$







$$V_{12k\Omega} = 36\,V$$

$$IR = 36\,V$$

$$I_o'''(12\,k) = 36\,V$$

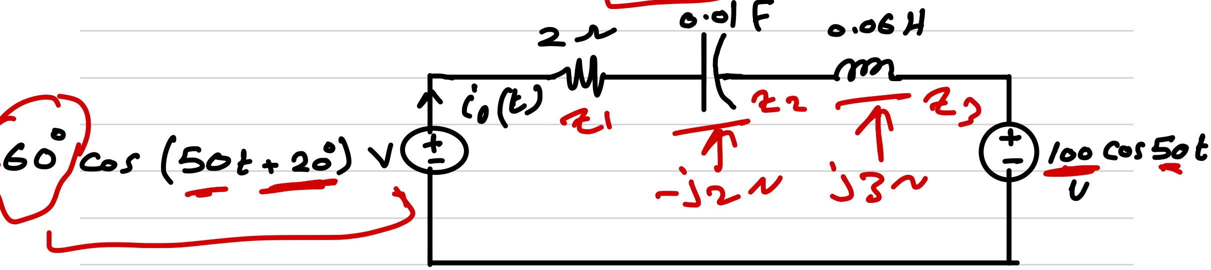
$$\boxed{I_o''' = 3\,mA}$$

$$I_o = I_o' + I_o'' + I_o'''$$

$$= -1 + 2 + 3$$

$$\boxed{= 4\,mA}$$

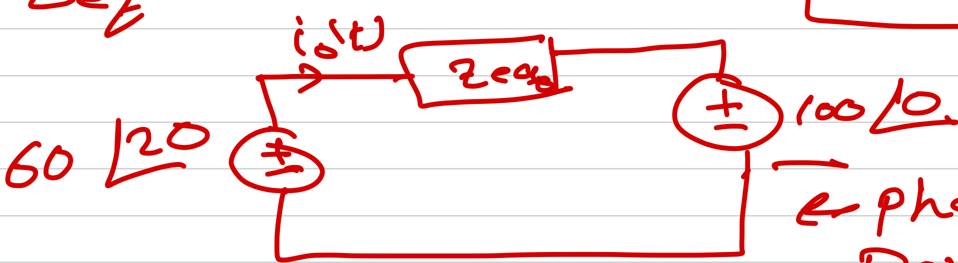
Q4. For the following circuit, Find the steady state output current $I_o(t)$



$$Z_C = \frac{-j}{\omega C} = \frac{-j}{(50)(0.01)} = -j2\ \Omega$$

$$Z_L = j\omega L = j(50)(0.06) = j3\ \Omega$$

$$Z_{eq} = 2 + -j2 + j3 = 2 + j\ \Omega$$



← phasor Domain

$$i_o(t) = \frac{60\angle 20 - 100\angle 0}{2+j}$$ ← Polar

Rect

$$60\angle 20 = 56.38 + j20.52$$
$$-\ 100\angle 0 = 100 + j0$$
$$= -43.62 + j20.52$$

$$i_o(t) = \frac{-43.62 + j20.52}{2 + j}$$

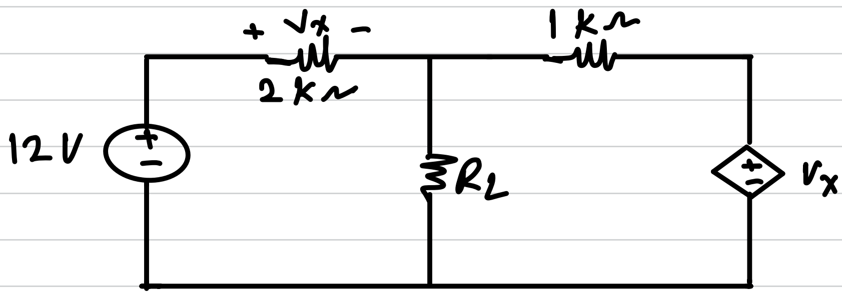
$$= \frac{48.2\angle 154.8}{2.236\angle 26.56}$$

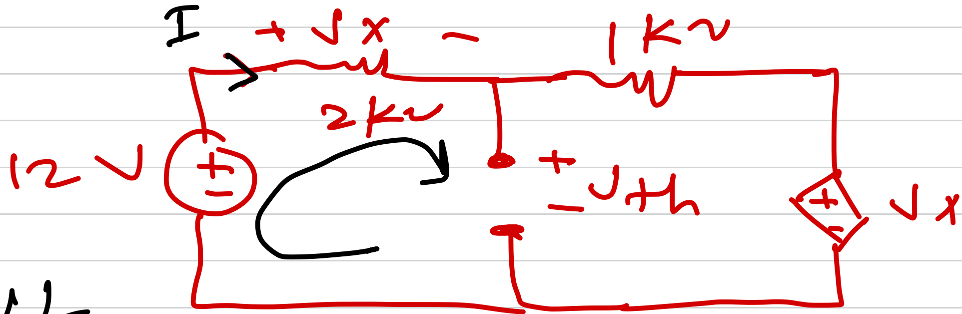
$$i_o(t) = 21.55\angle 128.24\ A$$

Phasor Domain

$$I_o(t) = 21.55\cos(50t + 128.24)\ A$$

Q5. For the circuit shown, Find the maximum power that can be delivered to the load R_L





KVL

$$-12 + V_x + I + V_x = 0$$

$$-12 + 2V_x + I = 0$$

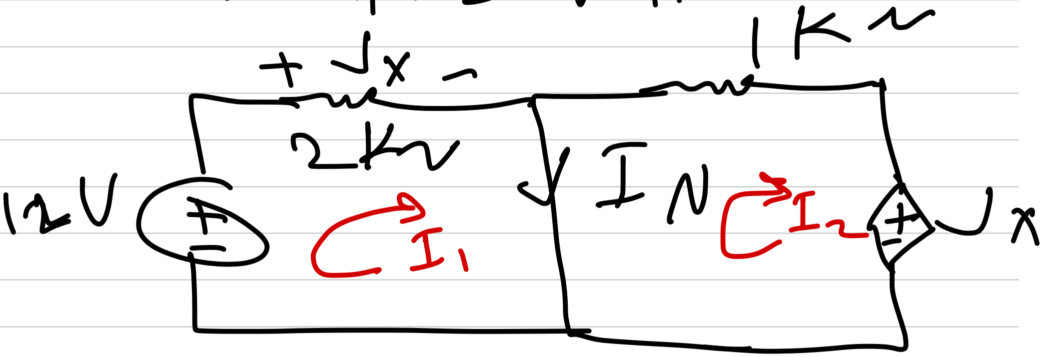
$$-12 + 4I + I = 0 \qquad \boxed{V_x = 2I}$$

$$5I = 12 \rightarrow I = \frac{12}{5} \text{ mA}$$

$$= 2.4 \text{ mA}$$

$$-12 + 2I + V_{th} = 0$$

$$V_{th} = 12 - 2(2.4)$$
$$= 7.2 \text{ Volt.}$$



mesh ①:

$$-12 + 2I_1 = 0$$

$$2I_1 = 12 \rightarrow I_1 = 6\,mA$$

mesh ②:

$$V_x + I_2 = 0 \qquad V_x = 2I_1$$
$$= 12$$

$$12 + I_2 = 0$$

$$I_2 = -12\,mA$$

$$I_N = I_1 - I_2 = 6 - -12 = 18\,mA$$

$$R_{th} = \frac{V_{th}}{I_N} = \frac{7.2}{18\,mA}$$

$$= 0.4\ K\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

$$= \frac{(7.2)^2}{(4)(0.4)\times 10^3}$$

$$= 32.4 \times 10^{-3}$$

$$= 32.4\ mWatt.$$