

APPENDIX C

Answers to Selected Exercises

CHAPTER 1

- 1.1** (a) $\{x : x = 0, 1, 2, 3, 4\};$
 $\{x : x = 2\}.$
 (b) $\{x : 0 < x < 3\};$
 $\{x : 1 \leq x < 2\}.$
1.2 (a) $\{x : 0 < x < \frac{5}{8}\}.$
1.7 (a) $\{x : 0 < x < 3\}.$
 (b) $\{(x, y) : 0 < x^2$
 $+ y^2 < 4\}.$
1.8 (a) $\{x : x = 2\}.$
 (b) Null set.
 (c) $\{(x, y) : x^2 + y^2 = 0\}.$
1.9 $\frac{80}{81}; 1.$
1.10 $\frac{11}{16}; 0; 1.$
1.11 $\frac{8}{3}; 0; \pi/2.$
1.12 $\frac{1}{2}; 0; \frac{2}{9}.$
1.13 $\frac{1}{6}; 0.$
1.15 10.
1.18 $\frac{1}{4}; \frac{1}{13}; \frac{1}{52}; \frac{4}{13}.$
1.19 $\frac{31}{32}; \frac{3}{64}; \frac{1}{32}; \frac{63}{64}.$
1.20 0.3.
1.21 $e^{-4}; 1 - e^{-4}; 1.$
1.22 $\frac{1}{2}.$
1.26 (a) $\binom{6}{4} / \binom{16}{4}.$
 (b) $\binom{10}{4} / \binom{16}{4}.$
1.27 $1 - \binom{990}{5} / \binom{1000}{5}.$
1.29 (b) $1 - \binom{10}{3} / \binom{20}{3}.$
1.34 (a) $\frac{1}{7}.$ (b) $\frac{5}{56}.$
 (c) $\left[\binom{3}{x} / \binom{8}{x} \right] [5/(8-x)].$
1.37 $\frac{1}{3}.$
1.38 $\frac{9}{20}; \frac{2}{3}.$
1.39 $\frac{5}{14}.$
1.40 $\frac{3}{7}, \frac{4}{7}.$
1.42 (a) 0.18. (b) 0.72.
 (c) 0.88.

1.45 0.1029 for (a), (b), (c), (d).
(e) 0.4116.

1.46 $\frac{1}{4}, \frac{3}{4}$.

1.47 $\frac{9}{13}, \frac{1}{13}, \frac{1}{13}, \frac{1}{13}, \frac{1}{13}$.

1.48 (a) $\frac{1}{2}$. (b) $\frac{1}{21}$.

1.49 $\frac{1}{5}, \frac{1}{5}, \frac{1}{5}$.

1.51 (a) $\frac{\binom{13}{x}\binom{39}{5-x}}{\binom{52}{5}}$,

$x=0, 1, \dots, 5$.

(b) $\frac{\binom{39}{5} + \binom{13}{1}\binom{39}{4}}{\binom{52}{5}}$.

1.54 (a) $\frac{1}{10}$, $x=1, 2, \dots, 9$.

(b) $\frac{4}{10}$.

1.56 $\frac{6}{36}$, $x=0$;

$\frac{12-2x}{36}$, $x=1, 2, 3, 4, 5$.

1.59 $\frac{3}{4}$.

1.61 $\frac{5}{8}, \frac{7}{8}, \frac{3}{8}$.

1.63 $e^{-2} - e^{-3}$.

1.64 $\frac{1}{27}$, 1; $\frac{2}{9}$, $\frac{25}{36}$.

1.66 (a) 1. (b) $\frac{2}{3}$. (c) 2.

1.69 (a) 0, $x < 0$; $1 - (1-x)^3$,
 $0 \leq x < 1$; 1, $1 \leq x$;
 $1 - \sqrt[3]{\frac{3}{4}}$, $1 - \sqrt[3]{\frac{1}{2}}$.

1.71 (a) $\frac{1}{4}$. (b) 0. (c) $\frac{1}{4}$. (d) 0.

1.72 0, $y < 0$; y^2 , $0 \leq y < 1$; 1,
 $1 \leq y$, $2y$, $0 < y < 1$;
0 elsewhere.

1.74 $\frac{1}{2}$; $\frac{1}{4}$.

1.76 0, $x < 0$; $1 - e^{-x/2}$, $0 \leq x$.
 $\frac{1}{2}e^{-x/2}$, $0 < x$; 0 elsewhere.

1.79 $1/3\sqrt{y}$, $0 < y < 1$; $1/6\sqrt{y}$,
 $1 < y < 4$; 0 elsewhere.

1.80 2; 86.4; -160.8.

1.81 3; 11; 27.

1.83 (a) $\frac{3}{4}$.

(b) $\frac{1}{4}, \frac{1}{2}$.

1.85 \$7.80.

1.88 $\frac{7}{3}$.

1.89 (a) 1.5, 0.75. (b) 0.5, 0.05.
(c) 2; does not exist.

1.90 $e^t/(2-e^t)$, $t < \ln 2$; 2; 2.

1.99 10; 0; 2; -30.

1.101 $-\frac{2\sqrt{2}}{5}, \frac{2\sqrt{2}}{5}$.

1.103 $1/2p$; $\frac{3}{2}$; $\frac{5}{2}$; 5; 50.

1.105 $\frac{31}{12}, \frac{167}{144}$.

1.110 $\frac{5}{8}, \frac{37}{192}$.

1.114 0.84.

CHAPTER 2

2.1 $\frac{15}{64}$; 0; $\frac{1}{2}$; $\frac{1}{2}$.

2.2 $\frac{1}{4}$.

2.6 ze^{-z} , $0 < z < \infty$;
0 elsewhere.

2.7 $-\ln z$, $0 < z < 1$;
0 elsewhere.

2.10 $5x_2^4$, $0 < x_2 < 1$;
0 elsewhere.

2.11 $(3x_1 + 2)/(6x_1 + 3)$;
 $(6x_1^2 + 6x_1 + 1)/(2)(6x_1 + 3)^2$.

2.13 $3x_2/4$; $3x_2^2/80$.

2.18 (b) $1/e$.

2.20 (a) 1. (b) -1. (c) 0.

2.21 (a) $7/\sqrt{804}$.

2.31 $\frac{5}{81}$.

2.32 $\frac{7}{8}$.

2.36 $\frac{1}{2}$.

2.38 (a) $\frac{1}{8}$, 0.

2.39 $1 - (1-y)^{12}$, $0 \leq y < 1$;
 $12(1-y)^{11}$, $0 < y < 1$.

2.40 $g(y) = [y^3 - (y-1)^3]/6^3$,
 $y=1, 2, 3, 4, 5, 6$.

2.42 $b_2 = \sigma_1(\rho_{12} - \rho_{13}\rho_{23})/$
 $[\sigma_2(1 - \rho_{23}^2)]$;
 $b_3 = \sigma_1(\rho_{13} - \rho_{12}\rho_{23})/$
 $[\sigma_3(1 - \rho_{23}^2)]$.

CHAPTER 3

- 3.1 $\frac{40}{81}$.
 3.4 $\frac{147}{512}$.
 3.6 5.
 3.8 $\frac{3}{16}$.
 3.10 $\frac{65}{81}$.
 3.13 $(\frac{1}{3})(\frac{2}{3})^{x-3}$, $x = 3, 4, 5, \dots$.
 3.14 $\frac{5}{72}$.
 3.17 $\frac{1}{6}$.
 3.18 $\frac{24}{625}$.
 3.20 $\frac{11}{6}$; $x_1/2$; $\frac{11}{6}$.
 3.21 $\frac{25}{4}$.
 3.22 0.09.
 3.25 $4^x e^{-4}/x!$, $x = 0, 1, 2, \dots$.
 3.26 0.84.
 3.31 2.
 3.33 (a) $\exp[-2 + e^{1/2}(1 + e^{1/2})]$.
 (b) $\mu_1 = 1$, $\mu_2 = 2$,
 $\sigma_1^2 = 1$, $\sigma_2^2 = 2$,
 $\rho = \sqrt{2}/2$.
 (c) $y/2$.
 3.34 0.05.
 3.35 0.831, 12.8.
 3.36 0.90.
 3.37 $\chi^2(4)$.
 3.39 $3e^{-3y}$, $0 < y < \infty$.
 3.40 2, 0.95.
 3.45 $\frac{11}{16}$.
 3.46 $\chi^2(2)$.
 3.49 0.067; 0.685.
 3.51 71.3, 189.7.
 3.52 $\sqrt{\ln 2/\pi}$.
 3.57 0.774.
 3.58 $\sqrt{2/\pi}$; $(\pi - 2)/\pi$.
 3.59 0.90.
 3.60 0.477.
 3.61 0.461.
 3.62 $N(0, 1)$.
 3.63 0.433.
 3.64 0; 3.
 3.69 $N(0, 2)$.
 3.70 (a) 0.574.
 (b) 0.735.

- 3.71 (a) 0.264. (b) 0.440.
 (c) 0.433. (d) 0.642.
 3.73 $\rho = \frac{4}{5}$.
 3.74 (38.2, 43.4).

CHAPTER 4

- 4.2 $\frac{405}{1024}$.
 4.3 0.405.
 4.6 $\frac{16}{15}$.
 4.7 $\frac{1}{8}$.
 4.9 $(n+1)/2$; $(n^2-1)/12$.
 4.10 $a + b\bar{x}$; $b^2 s_x^2$.
 4.11 $\chi^2(2)$.
 4.14 $\frac{1}{2}$, $0 < y < 1$;
 $1/2y^2$, $1 < y < \infty$.
 4.15 y^{15} , $0 \leq y < 1$; $15y^{14}$,
 $0 < y < 1$.
 4.16 $\frac{4}{7}$.
 4.17 $\frac{1}{3}$, $y = 3, 5, 7$.
 4.19 $(\frac{1}{2})^{\sqrt{y}}$, $y = 1, 8, 27, \dots$.
 4.20

y_1	$g_1(y_1)$
1	$\frac{1}{36}$
2	$\frac{4}{36}$
3	$\frac{6}{36}$
4	$\frac{4}{36}$
6	$\frac{12}{36}$
9	$\frac{9}{36}$
- 4.25 $\frac{1}{27}$, $0 < y < 27$.
 4.32 $y_1 e^{-y_1}$, $0 < y_1 < \infty$.
 4.34 $(2y_1)(4y_2^3)$, $0 < y_1 < 1$,
 $0 < y_2 < 1$.
 4.35 $\alpha/(\alpha + \beta)$;
 $\alpha\beta/[(\alpha + \beta + 1)(\alpha + \beta)^2]$.
 4.36 (a) 20. (b) 1260. (c) 495.
 4.37 $\frac{10}{243}$.
 4.40 0.05.
 4.43 $1/4.74$, 3.33.
 4.48 $(1/\sqrt{2\pi})^3 y_1^2 e^{-y_1^2/2} \sin y_3$,
 $0 \leq y_1 < \infty$, $0 \leq y_2 < 2\pi$,
 $0 \leq y_3 \leq \pi$.

4.49 $y_2 y_3^2 e^{-y_3}$, $0 < y_1 < 1$,
 $0 < y_2 < 1$, $0 < y_3 < \infty$.

4.53 $1/(2\sqrt{y})$, $0 < y < 1$.

4.54 $e^{-y_1/2}/(2\pi\sqrt{y_1 - y_2^2})$,
 $-\sqrt{y_1} < y_2 < \sqrt{y_1}$,
 $0 < y_1 < \infty$.

4.56 $1 - (1 - e^{-3})^4$.

4.57 $\frac{1}{8}$.

4.62 $\frac{5}{16}$.

4.63 $48z_1 z_2^3 z_3^5$, $0 < z_1 < 1$,
 $0 < z_2 < 1$, $0 < z_3 < 1$.

4.64 $\frac{7}{12}$.

4.69 $\frac{1}{4}$.

4.70 $6uv(u+v)$,
 $0 < u < v < 1$.

4.75

y	$g(y)$
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$

4.76 0.24.

4.79 0.159.

4.82 0.159.

4.88 0.818.

4.91 (b) -1 or 1 .

(c) $Z_i = \sigma_i Y_i + \mu_i$.

4.92 $\sum_{i=1}^n a_i b_i = 0$.

4.94 6.41.

4.95 $n = 16$.

4.97 $(n-1)\sigma^2/n$;
 $2(n-1)\sigma^4/n^2$.

4.98 0.90.

4.100 0.945.

4.102 0.618.

4.103 0.78.

4.104 $\frac{8}{3}; \frac{2}{9}$.

4.105 7.

4.107 2.5; 0.25.

4.109 -5 ; $60 - 12\sqrt{6}$.

4.110 $\sigma_1/\sqrt{\sigma_1^2 + \sigma_2^2}$.

4.113 0.265.

4.115 22.5, $\frac{261}{4}$.

4.116 $r_2 > 4$.

4.118 $\mu_2 \sigma_1 / \sqrt{\sigma_1^2 \sigma_2^2 + \mu_1^2 \sigma_2^2 + \mu_2^2 \sigma_1^2}$.

4.121 $5/\sqrt{39}$.

4.125 $e^{\mu + \sigma^2/2}$; $e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$.

CHAPTER 5

5.1 Degenerate at μ .

5.2 Gamma ($\alpha = 1$, $\beta = 1$).

5.3 Gamma ($\alpha = 1$, $\beta = 1$).

5.4 Gamma ($\alpha = 2$; $\beta = 1$).

5.13 0.682.

5.14 (b) 0.815.

5.17 Degenerate at μ_2
 $+(\sigma_2/\sigma_1)(x - \mu_1)$.

5.18 (b) $N(0, 1)$.

5.19 (b) $N(0, 1)$.

5.21 0.954.

5.23 0.840.

5.26 0.08.

5.28 0.267.

5.29 0.682.

5.35 $N(0, 1)$.

CHAPTER 6

6.1 (a) $\bar{X}$.

(b) $-n/\ln(X_1 X_2 \cdots X_n)$.

(c) $\bar{X}$. (d) The median.

(e) The first order statistic.

6.2 The first order statistic Y_1 ,

$$\sum_{i=1}^n (X_i - Y_1)/n.$$

- 6.4 $\frac{4}{23}, \frac{11}{23}, \frac{7}{23}$.
 6.5 $Y_1 = \min(X_i)$;
 $n/\ln[(X_1 X_2 \cdots X_n)/Y_1^n]$.
 6.7 (b) $\bar{X}/(1 - \bar{X})$. (d) $\bar{X}$.
 (e) $\bar{X} - 1$.
 6.9 $1 - e^{-2/\bar{X}}$.
 6.10 Multiply by $n/(n-1)$.
 6.12 $(Y_1 + Y_n)/2$, $(Y_n - Y_1)/2$;
 $E[(Y_n - Y_1)/2] = \rho(n-1)/(n+1)$.
 6.14 (77.28, 85.12).
 6.15 24 or 25.
 6.16 (3.7, 5.7).
 6.17 160.
 6.23 $(5\bar{x}/6, 5\bar{x}/4)$.
 6.25 1692.
 6.26 3.19 to 3.61.
 6.28 3.92 to 31.50.
 6.30 $(-3.6, 2.0)$.
 6.35 135 or 136.
 6.38 $\frac{1}{4} + \frac{3}{4} \ln \frac{3}{4}$; $\frac{7}{16} + \frac{9}{8} \ln \frac{3}{4}$.
 6.39 $\frac{11}{24}$; $(31)3^8/4^9$.
 6.42 $n=19$ or 20 .
 6.43 $K(\frac{1}{2})=0.062$;
 $K(\frac{1}{12})=0.920$.
 6.44 $n \approx 73$, $c \approx 42$.
 6.46 (a) Reject.
 (b) p -value ≈ 0.005 .
 6.49 (c) p -value ≈ 0.005 .
 6.51 23.3.
 6.52 2.91.
 6.53 $q_3 = \frac{176}{21} > 7.81$,
 reject H_0 .
 6.55 $b \leq 8$ or $32 \leq b$.
 6.56 $q_3 = \frac{22}{9} < 11.3$,
 accept H_0 .
 6.57 $6.4 < 9.49$, accept H_0 .
 6.59 $\hat{p} = (X_1 + X_2/2)/$
 $(X_1 + X_2 + X_3)$.

CHAPTER 7

- 7.4 $\frac{1}{3}, \frac{2}{3}$.
 7.5 $\delta_1(y)$.

- 7.6 $b=0$; does not exist.
 7.7 Does not exist.
 7.17 $\prod_{i=1}^n [X_i(1 - X_i)]$.
 7.19 $60y_3^2(y_3 - y_3)/\theta^5$; $6y_3/5$;
 $\theta^2/7$; $\theta^2/35$.
 7.20 $(1/\theta^2)e^{-y_1/\theta}$,
 $0 < y_2 < y_1 < \infty$;
 $y_1/2$; $\theta^2/2$.
 7.22 $\sum X_i^2/n$; $\sum X_i/n$; $(n+1)Y_n/n$.
 7.24 X ; X .
 7.25 Y_1/n .
 7.27 $Y_1 - 1/n$.
 7.29 $Y_1 = \sum_{i=1}^n X_i$; $Y_1/4n$; yes.
 7.37 $\bar{x}$.
 7.40 $\bar{X}^2 - 1/n$.
 7.43 $\left(\frac{n-1}{n}\right)^Y \left(1 + \frac{Y}{n-1}\right)$.
 7.51 $\frac{Y_1 + Y_n}{2}$, $\frac{(n+1)(Y_n - Y_1)}{2(n-1)}$.
 7.55 Y_1 , $\sum (Y_i - Y_1)/n$.

CHAPTER 8

- 8.2 $[y\tau^2 + \mu\sigma^2/n]/(\tau^2 + \sigma^2/n)$.
 8.3 $\beta(y + \alpha)/(n\beta + 1)$.
 8.9 $\sqrt[6]{2}$ if $y_4 < 1$,
 $\sqrt[6]{2} y_4$ if $1 \leq y_4$.
 8.13 θ^2/n ; $\theta^2/n(n+2)$.
 8.15 (a) $4/\theta^2$.
 8.17 (d) $\text{var}(\hat{\theta}) = \frac{1}{nI(\theta)} = \frac{\theta^2}{5n}$.
 8.22 2.17; 2.44.
 8.25 2.20.

CHAPTER 9

- 9.4 $\sum_{i=1}^{10} x_i^2 \geq 18.3$; yes; yes.

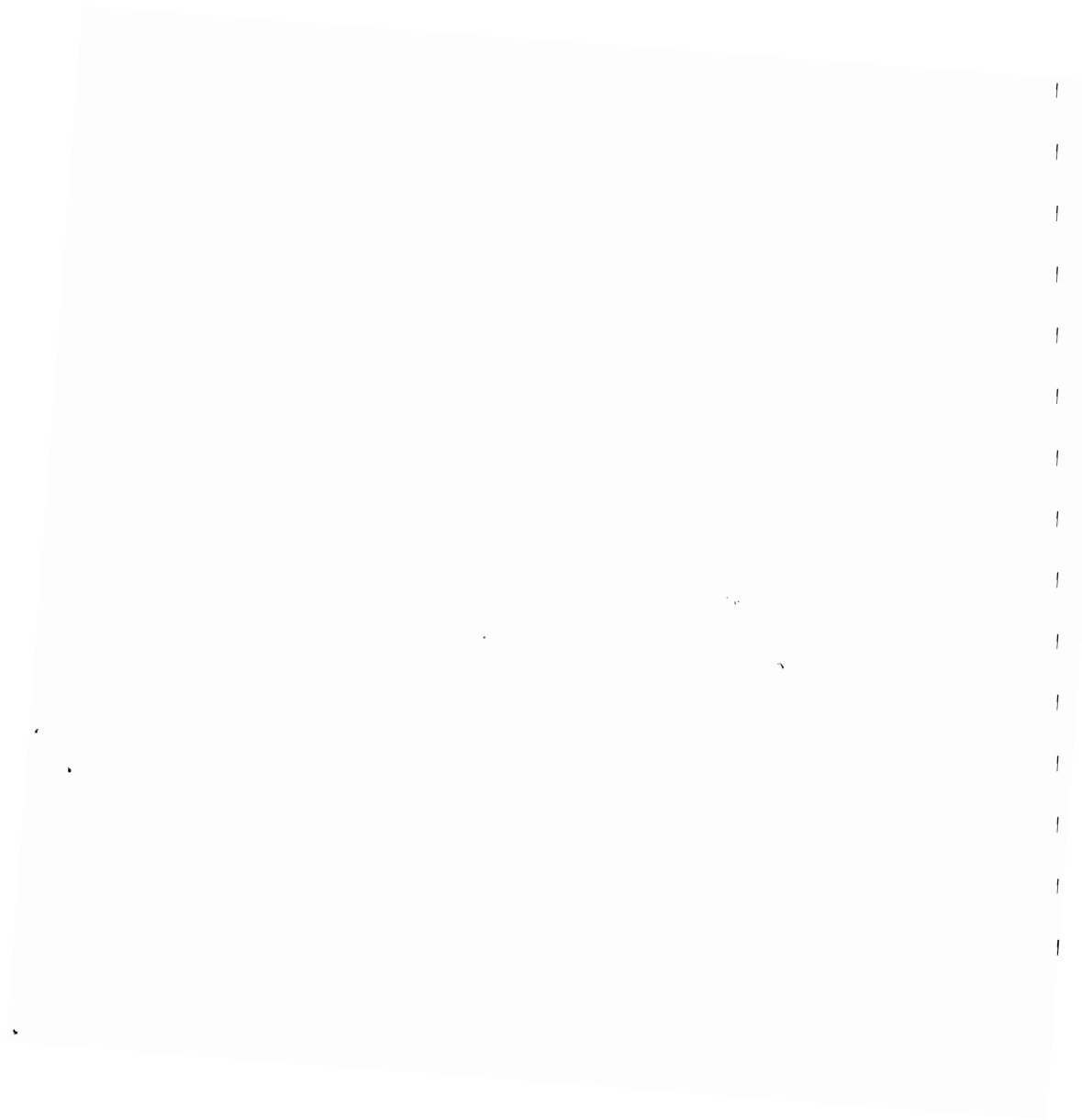
- 9.6 $3 \sum_1^{10} x_i^2 + 2 \sum_1^{10} x_i \geq c.$
- 9.7 95 or 96; 76.7.
- 9.9 38 or 39; 15.
- 9.10 0.08; 0.875.
- 9.11 $(1 - \theta)^9(1 + 9\theta).$
- 9.12 $1, 0 < \theta \leq \frac{1}{2}; 1/(16\theta^4),$
 $\frac{1}{2} < \theta < 1; 1 - 15/(16\theta^4),$
 $1 \leq \theta.$
- 9.14 53 or 54, 5.6.
- 9.17 Reject H_0 if $\bar{x} \geq 77.564.$
- 9.18 26 or 27;
 reject H_0 if $\bar{x} \leq 24.$
- 9.19 220 or 221;
 reject H_0 if $y \geq 17.$
- 9.23 $t = 3 > 2.262$, reject $H_0.$
- 9.24 $|t| = 2.27 > 2.145,$
 reject $H_0.$
- 9.37 $c_0(n) = (14.4)$
 $\times (n \ln 1.5 - \ln 9.5);$
 $c_1(n) = (14.4)$
 $\times (n \ln 1.5 + \ln 18).$
- 9.38 $c_0(n) = (0.05n - \ln 8)/\ln 3.5;$
 $c_1(n) = (0.05n - \ln 4.5)/\ln 3.5.$
- 9.41 (b) $c = 0.18; 0.64,$
 (c) $c = 0.5; 0.16; 0.84.$
 (d) $c = 0.23; 0.06; 0.68.$
- 9.44 $(9y - 20x)/30 \leq c.$

CHAPTER 10

- 10.9 6.39.
- 10.12 $r + \theta, 2r + 4\theta.$
- 10.13 $r_2(\theta + r_1)/[r_1(r_2 - 2)].$
- 10.23 7.00, 9.98.
- 10.25 4.79, 22.82, 30.73.
- 10.26 (a) $4.483x + 6.483.$
- 10.28 $\hat{\beta} = \sum (X_i / nc_i),$
 $\sum [(X_i - \hat{\beta}c_i)^2 / nc_i^2].$
- 10.32 Reject $H_0.$
- 10.44 $a_i = 0, i = 1, 2, 3, 4.$
- 10.45 $\sum_{j=1}^n a_{ij} = 0, i = 1, 2, \dots, n.$

CHAPTER 11

- 11.2 (a) $\frac{15}{16}.$ (b) $675/1024;$
 (c) $(0.8)^4.$
- 11.4 8.
- 11.6 0.954; 0.92; 0.788.
- 11.9 8.
- 11.12 (a) Beta $(n - j + 1, j).$
 (b) Beta $(n - j + i - 1,$
 $j - i + 2).$
- 11.15 0.067.
- 11.18 Reject $H_0.$
- 11.25 0; $4(4^n - 1)/3$; no.
- 11.37 $\frac{2}{99}.$
- 11.44 $98; \frac{686}{3}.$



Index

- Absolute-error loss function, 311, 367
Adaptive methods, 536, 542
Algebra of sets, 4
Analysis of variance, 466
Ancillary statistic, 347, 353
Andrews, D. F., 393
Approximate distribution, 248, 251, 381, 392, 525
 chi-square, 295, 422
 normal for binomial, 249, 499
 normal for chi-square, 244
 normal for Poisson, 246
 Poisson for binomial, 244
Arc sine transformation, 252, 273
Asymptotically efficient, 379
- Basu, D., 354
Bayes' formula, 23, 364
Bayesian methods, 363, 437
Bernoulli trials, 116
Bernstein, S., 112
Best critical region, 396, 399, 402
Beta distribution, 180, 504
Biased estimator, 263
Binary statistic, 514
Binomial distribution, 118, 244, 249, 254, 498, 506
Bivariate normal distribution, 146, 212, 226, 346, 385, 439, 478
Boole's inequality, 465
Borel measurable function, 29, 156
Box-Muller transformation, 177
Burr distribution, 372
- Cauchy distribution, 175, 257, 387
Censoring, 49
Central limit theorem, 246, 511
Change of variable, 163, 168, 186
Characteristic function, 64
Characterization, 202, 214
Chebyshev's inequality, 68, 120, 222, 240
Chi-square distribution, 134, 144, 210, 294, 447, 482, 489, 491
Chi-square test, 293, 424
Classification, 439, 496
Cochran's theorem, 490
Column effect, 467, 470
Complement of a set, 7
Complete sufficient statistics, 332, 335, 343, 353, 537
Completeness, 329, 343
Composite hypothesis, 284, 288, 406, 413
Compounding, 372
Conditional distribution, 82, 148
Conditional expectation, 84, 110
Conditional mean, 85, 93, 123, 148, 357, 367
Conditional probability, 83
Conditional p.d.f., 83, 109, 148, 364
Conditional variance, 85, 95, 148, 357
Confidence coefficient, 270
Confidence interval, 268, 289, 462
 for difference of means, 276
 for means, 268, 462
 for p , 272
 for quantiles, 497
 for ratio of variances, 280
 for regression parameters, 473
 for variances, 276