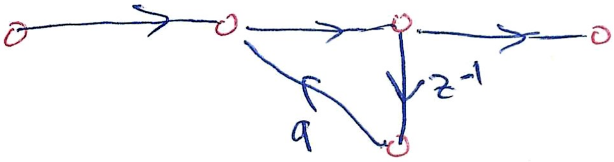
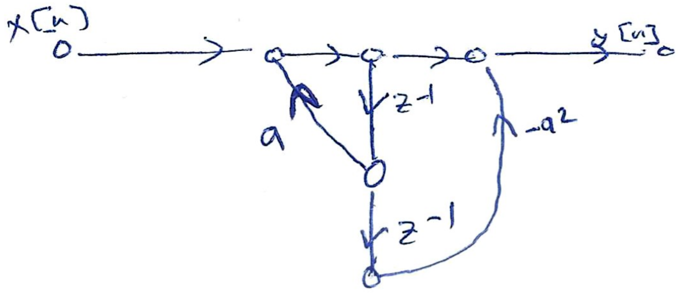
* Feedback in IIR systems:-

Examples:- $y[n] = a\,y[n-1] + x[n]$



Examples:- $H(z) = \dfrac{1 - a^2 z^{-2}}{1 - a z^{-1}} = \dfrac{(1 - a z^{-1})(1 + a z^{-1})}{(1 - a z^{-1})} = 1 + a z^{-1}$

$h[n] = \delta[n] + a\,\delta[n-1]$



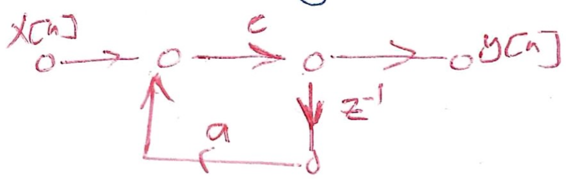
6.4
* Transposed form

1- Reverse direction of all branches.

2- Interchange Input and output.

3- flip the structure to make the input to the left and the output to the right.

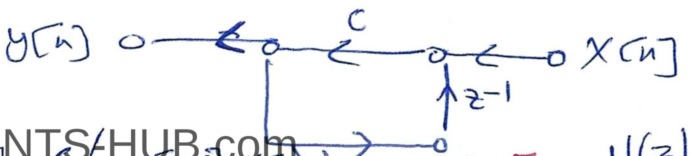
Example :-



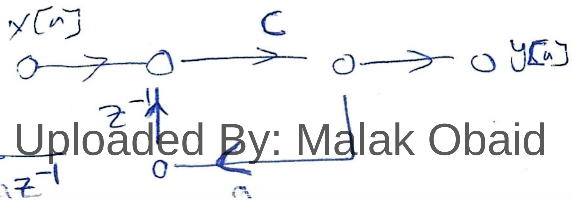
$y[n] = c\left[x[n] + a y[n-1]\right]$

$H(z) = \dfrac{c}{1 - c a z^{-1}}$

1, 2



3

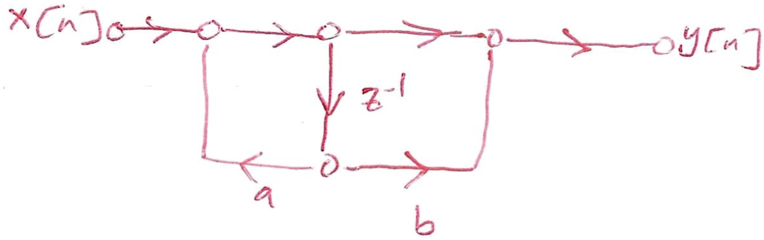


$y[n] = c(a y[n-1] + x[n])$; a

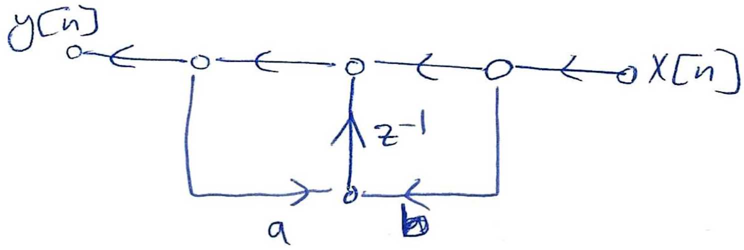
127

$H(z) = \dfrac{c}{1 - c a z^{-1}}$

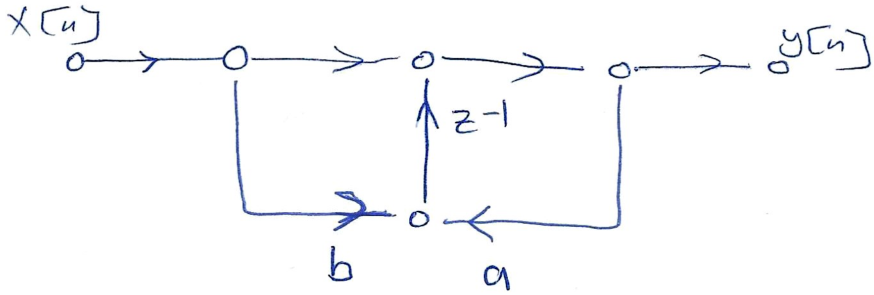
Example:-



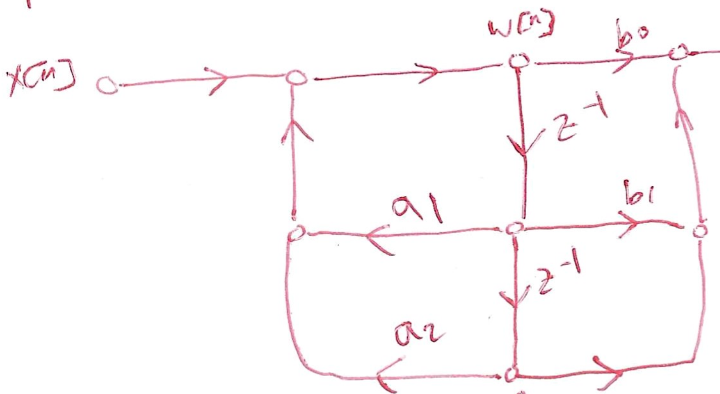
1, 2)



3)



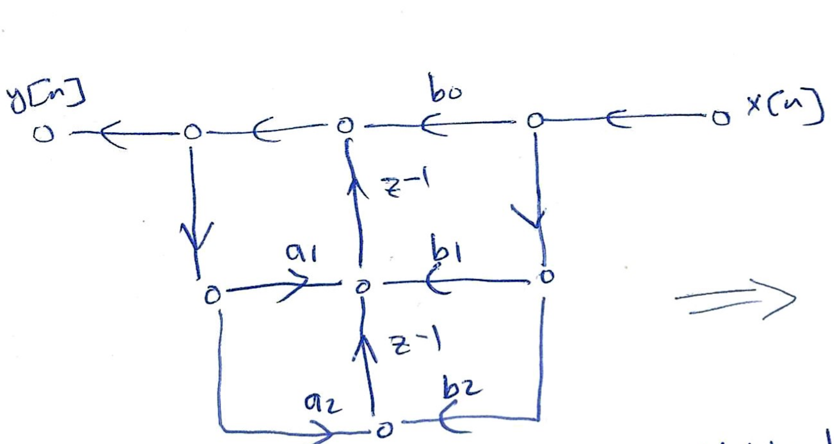
Example:- second-order IIR system



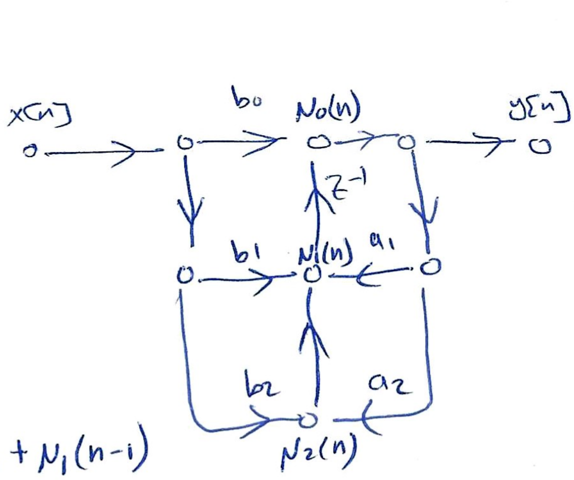
$w(n) = a\, w(n-1) + a_2 w(n-2) + x[n]$

$y[n] = b_0 w[n] + b_1 w(n-1) + b_2 w(n-2)$

Draw the transposed form?



$\Longrightarrow$



$N_0(n) = b_0 x[n] + N_1(n-1)$

$N_1(n) = a_1 N_0[n] + b_1 x(n) + N_2(n-1)$

$N_2(n) = a_2 N_0[n] + b_2 x[n]$

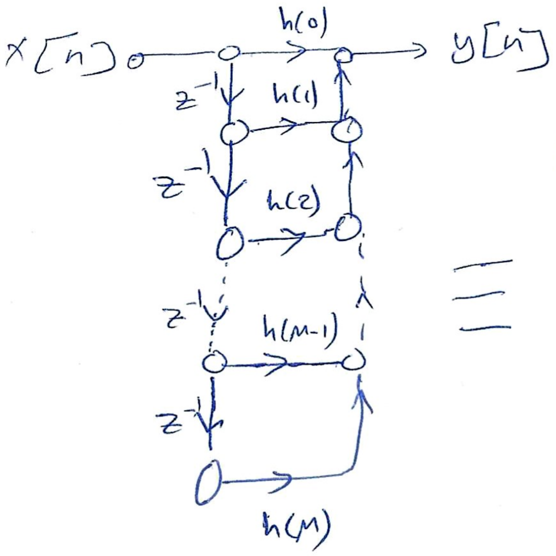
6.6

* Basic structures for FIR systems

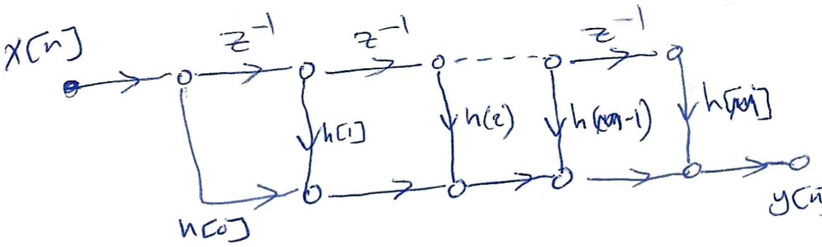
Causal FIR systems have only zeros (except at $z=0$)

$$y(n) = \sum_{k=0}^{M} b_k\, x(n-k) \equiv \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

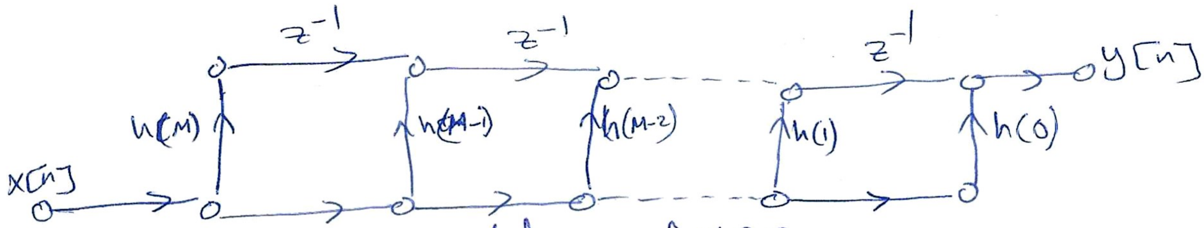
$$h(n) = \begin{cases} b_n, & n=0,1,2,\dots M-1 \\ 0, & \text{otherwise} \end{cases}$$



$\equiv$



Direct form I FIR



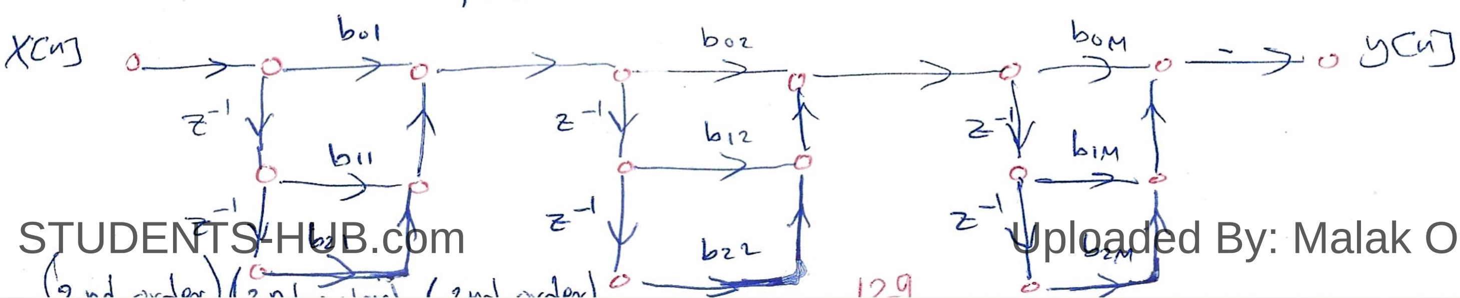
Transposed form of FIR

* Cascade form

* FIR always stable
* FIR : easy to implement a linear phase system

$$H(z) = \sum_{n=0}^{M} h(n)\, z^{-1}$$

polynomial function (products of subsystems)



(2nd order)(2nd order) (2nd order)

Implementation of linear phase FIR systems:-

- to have linear phase FIR, $h[n]$ must satisfy symmetry condition or antisymmetry

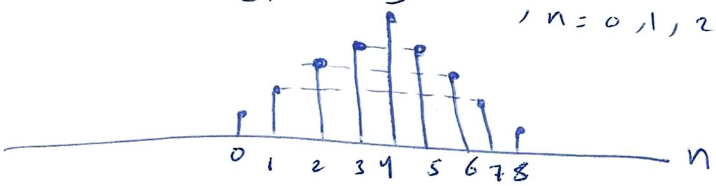
$$h[n] = h[M-n] \quad , n = 0, 1, 2, ---, M$$

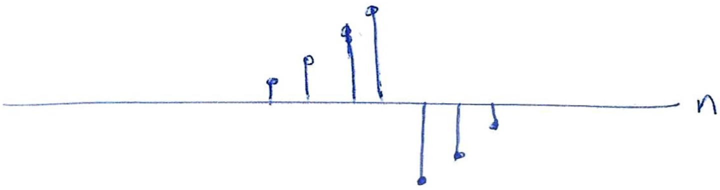
$$h[n] = -h[M-n] \quad , n = 0, 1, 2, ---- M$$

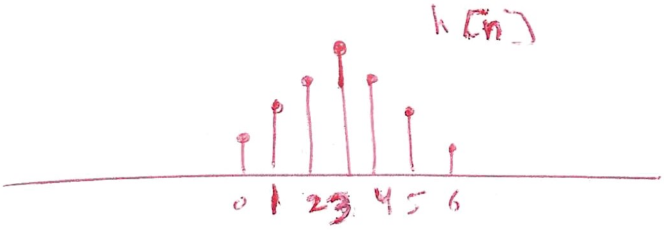
let $M = 8$

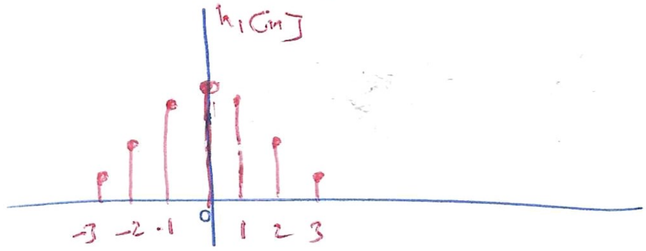
$h(0) = h(8)$

$h(1) = h(7)$









$$h[n] = h_1\left[n + \frac{M-1}{2}\right]$$

$$h[n] = h_1\left(n + \frac{M-1}{2}\right)$$

$$H(e^{jw}) = e^{jw\left(\frac{M-1}{2}\right)} H_1(e^{jw})$$

$\hookrightarrow$ even and real function of w

$\Longrightarrow$ phase $= 0$

$$\measuredangle H(e^{jw}) = w\left(\frac{M-1}{2}\right)$$

which is linear

Example $h[n] = \{0, 1, 2, 3, 2, 1, 0\}$

$$h[n] = x(n-1) + 2x(n-2) + 3x(n-3) + 2x(n-4) + x(n-5)$$

$$h[n] = 1\big(x(n-1) + x(n-5)\big) + 2\big(x(n-2) + x(n-4)\big) + 3x(n-3)$$

$$H(z) = \sum_{n=0}^{M} h(n) z^{-n}$$

assum M even

same coefficient

$$H(z) = \sum_{n=0}^{M/2-1} h(n) z^{-n} + h\left(\frac{M}{2}\right) z^{-\frac{M}{2}} + \sum_{n=M/2+1}^{n=M} h(n) z^{-n}$$

$r = M - n$ if $n = M$ $r = 0$ if $n = 0$ $r = M$

$n = m - r$ if $n = M/2 + 1$ $r = M - \frac{M}{2} - 1 = \frac{M}{2} - 1$

$$= \sum_{n=0}^{M/2-1} h(n) z^{-n} + h\left(\frac{M}{2}\right) z^{-\frac{M}{2}} + \sum_{r=0}^{r=M/2-1} \underbrace{h(m-r)}_{h(r)} z^{-(m-r)}$$

$$H(z) = \sum_{n=0}^{\frac{M}{2}-1} h[n]\left[z^{-n} + z^{-(M-n)}\right] + h(M/2) z^{-M/2}$$

$$y[n] = \left[\sum_{k=0}^{M/2-1} h[k]\big(x(n-k) + x(n-M+k)\big)\right] + h(M/2)\, x(n - M/2)$$

assum $M = 8$