Asy.

H. Asy | V. Asy | O. Asy Oblique

H. Asy

$\lim_{x\to\infty} f(x) = (\text{رقم}) = b$
$-\infty$

$\boxed{y = b}$

V. Asy

نفحص أصفار المقام

O. Asy (Oblique)

$f(x) = \frac{1}{x}$ $\quad \frac{x^0}{x^1}$

موجود فقط

موجود اذا كانت درجة البسط أكبر أو تساوي درجة المقام

بدرجة واحدة لا يوم

$\frac{x^4}{x^2}$ ✗

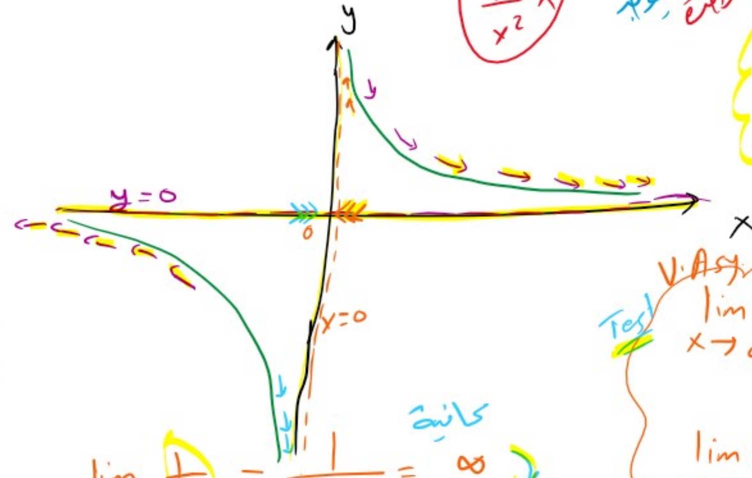
$f(x) = \frac{1}{x}$
check
$\boxed{x = 0}$

Exp $f(x) = \frac{1}{x}$

Find Asy.

$\lim_{x\to\infty} \frac{1}{x} = 0 \Rightarrow \boxed{y = 0}$ H. Asy

$\lim_{x\to-\infty} \frac{1}{x} = 0$



كانية

$\lim_{x\to 0^+} \frac{1}{x} = \frac{1}{\text{small}+} = \infty$

$\boxed{\text{Yes } \quad x = 0 \text{ is V. Asy}}$

$\lim_{x\to 0^-} \frac{1}{x} = \frac{1}{\text{small}-} = -\infty$

V. Asy Test

$\lim_{x\to 0^+} f(x) = \infty$ or $-\infty$

$\lim_{x\to 0^-} f(x) = \infty$ or $-\infty$

if $f(x)$ has O. Asy $\Rightarrow$ $f(x)$ has no H. Asy
if $f(x)$ has H. Asy $\Rightarrow$ $f(x)$ has no O. Asy.

درجة واحدة لصالح البسط

Exp $f(x) = \frac{x^3+1}{x^2-1}$ $\Rightarrow$ $f(x)$ has O. Asy $\quad$ $y = x$

درجة البسط ٣
درجة المقام ٢

$\Rightarrow$ $f(x)$ has no H. Asy

$\lim_{x\to\infty} \frac{x^3+1}{x^2-1} = \infty$

O. Asy

ثانيا O. Asy $y = x$

O. Asy.

long Division

$x \to \infty \quad x \to$

$$\begin{array}{r} x \\ x^2-1 \overline{\big) \cancel{x^3}+1} \\ \cancel{-x^3+x} \\ x+1 \end{array}$$

ناتج القسمة

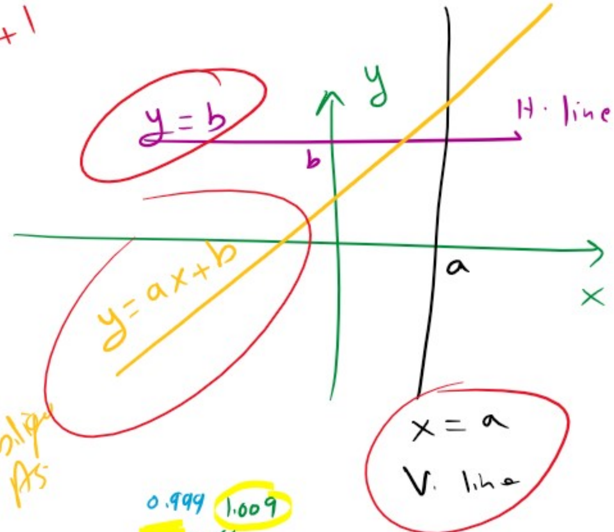
$x \rightarrow$ O. Asy $\quad y = x$

$$f(x) = \frac{x^3+1}{x^2-1} = x + \frac{x+1}{x^2-1}$$

الباقي

المقسوم عليه

$y = x$



V. Asy $\quad f(x) = \dfrac{x^3+1}{x^2-1}$

$x^2-1=0$

$x^2=1$

$x = \pm 1 \Rightarrow$ Check $\boxed{x=1}$ $\boxed{x=-1}$

0.999 $\quad$ 1.009

$x = 1$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{x^3+1}{x^2-1} = \frac{2}{\text{small}+} = \infty$$

$\Rightarrow \boxed{x=1}$ is V. Asy.

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x^3+1}{x^2-1} = \frac{2}{\text{small}-} = -\infty$$

$x = -1$

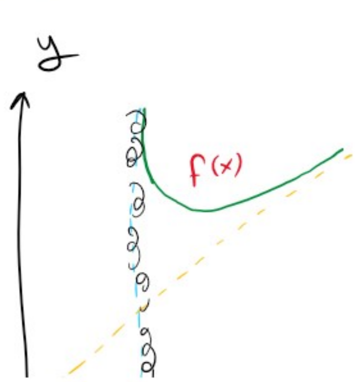
$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \frac{x^3+1}{x^2-1} \quad \frac{0}{0}$$

$$= \lim_{x \to -1^+} \frac{3x^2}{2x} = \frac{3}{-2}$$

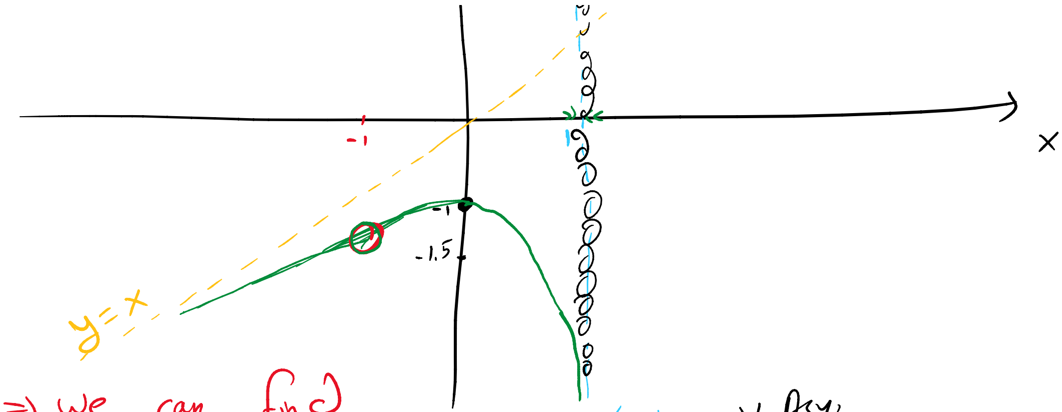
$$\lim_{x \to -1^-} \frac{x^3+1}{x^2-1} = \frac{3}{-2} = -1.5$$

$\boxed{x=-1}$ is not V. Asy $\Rightarrow$ $x=-1$ is Removable

$$f(x) = \frac{x^3+1}{x^2-1}$$



$f(x) = \frac{\quad}{x^2-1}$



Key point $(0,-1)$

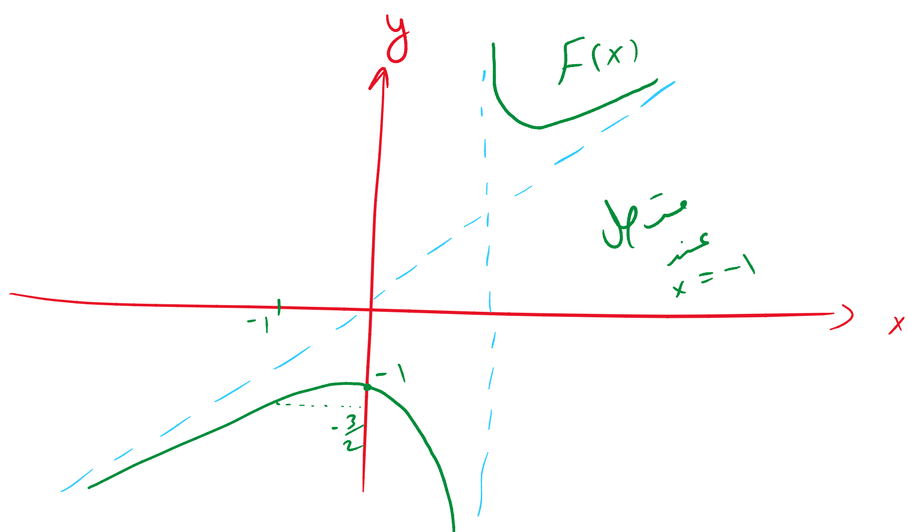
$x=0 \Rightarrow f(0)=-1$

V. Asy.

$x=-1$ removable $\Rightarrow$ we can find cont. extension $F(x) \Rightarrow$

$$F(x) = \begin{cases} f(x) & \text{if } x \neq -1 \\ \lim\limits_{x\to -1} f(x) & \text{if } x = -1 \end{cases}$$

$$= \begin{cases} \dfrac{x^3+1}{x^2-1} & \text{if } x \neq -1 \\ -\frac{3}{2} & \text{if } x = -1 \end{cases}$$



Q4 $\quad h(t) = \dfrac{t^2+3t-10}{t-2}$ Find cont. Extension

$H(t) = \begin{cases} h(t) & \text{if } t \neq 2 \end{cases}$ $\quad \Big| \lim\limits_{t\to 2} \dfrac{t^2+3t-10}{t-2}$

$$H(t) = \begin{cases} h(t) & \text{if } t \neq 2 \\ \lim_{t\to 2} h(t) & \text{if } t = 2 \end{cases}$$

exist

$$\lim_{t\to 2} \frac{t^2+3t-10}{t-2}$$

$$\lim_{t\to 2} \frac{\cancel{(t-2)}(t+5)}{\cancel{(t-2)}}$$

$$\lim_{t\to 2} (t+5) = 7$$

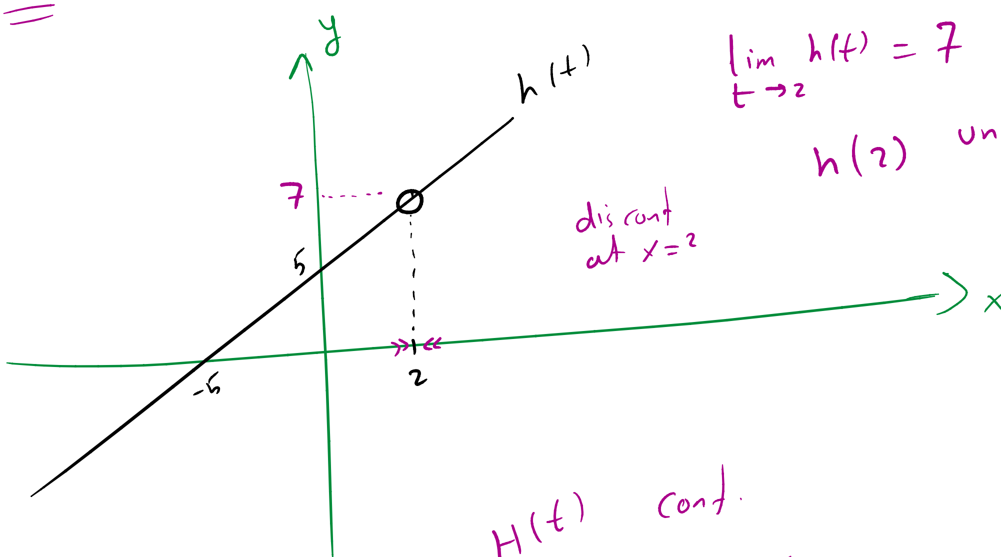
$$= \begin{cases} \dfrac{t^2+3t-10}{t-2} & \text{if } t \neq 2 \\ 7 & \text{if } t = 2 \end{cases}$$

$h(t)$, $H(t)$

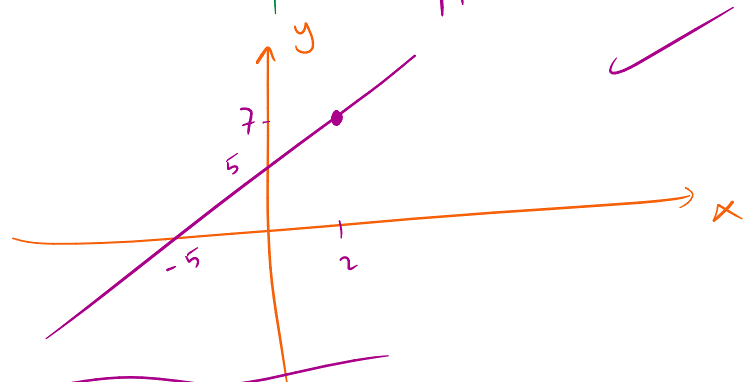
$$h(t) = \frac{t^2+3t-10}{t-2} = \frac{(t-2)(t+5)}{t-2}$$

$$\lim_{t\to 2} h(t) = 7$$

$h(2)$ undefined



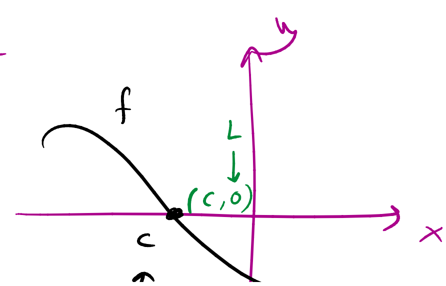
$H(t)$ cont.



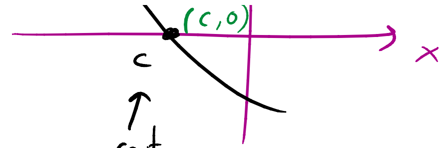
Q5 $f(x) = x^3 - 2x^2 + 2$

Show f has root $\Rightarrow L = 0$

IVT



Show f has (roots) ...



root
zero of f
x-intercept

f cont. , poly on $\mathbb{R}$ $\Rightarrow$ f cont $[a, b]$

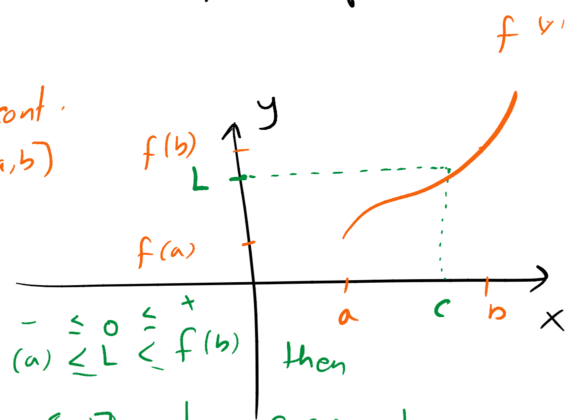
$f(a)<0$ $\quad$ $f(b)>0$

$[a,b] = [-1, 0]$

$f(-1) = (-1)^3 - 2(-1)^2 + 2 = -1 - 2 + 2 = -1 < 0$

$f(0) = 0 - 0 + 2 = 2 > 0$

① f cont. on $[a,b]$



② if $f(a) \le L \le f(b)$ then $\exists\, c \in [a,b]$ s.t $f(c) = L$

f cont. on $[-1, 0]$

$L = 0 \Rightarrow f(-1) \le L \le f(0)$

$-1 \le 0 \le 2$

$\Downarrow$

by IVT $\exists\, c \in [-1, 0]$ s.t $f(c) = 0$

$\Downarrow$

f has root on $[-1, 0]$

