Chapter 3.

— Gate level Minimization. —

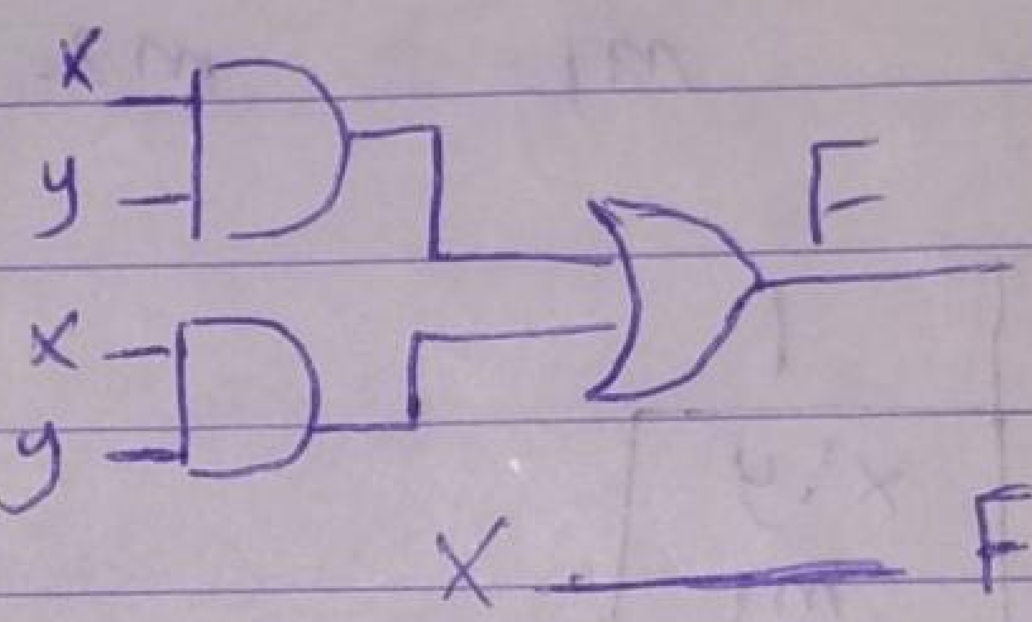
- How to make cost Reduction for any digital circit.

- The complexity of digital logic gates is directly proportional to the boolen expression from which the Function implemented.

ex $F = x.y + x.y'$

$F = x(\cancel{y+y'})^{1}$.

More simplify $F = X$.



"Boolean Algebra".

ch2. غير مباشر

* Minimize the Function using algebra is awakword approach, So we need another way for minimization.

طريقة مباشرة is a straight forword approach for minimization.

* Map Method for Minimization.

Note :- Truth table is unique.

$F = x.y + x.y' \equiv F = X$

$X = 0$

$F = 0$

$x = 1$

$F = 1$

Mathmatical expression is not unique.

X	Y	F
0	0	0. 0.0+0.0
0	1	0 0.1+0.1
1	0	1 1.0+1.0
1	1	1 1.1+1.1

K – map method for minimization.

Diagram mad of squares, each square represent one minterm.

ex Two variables maps.

2 variables $\Rightarrow 2^2 = 4$ minterms.

$x'y'$	$x'y$	xy'	xy
m_0	m_1	m_2	m_3

x \ y	0	1
0	$x'.y'$ m_0	$x'.y$ m_1
1	$x.y'$ m_2	$x.y$ m_3

ex Minimize the Following Function.

$F(x.y) = x'.y + x.y' + x.y$

Sum of minterms.

$= m_1 + m_2 + m_3 = \Sigma(1, 3, 3).$

Algebra

$F(x.y) = x'y + x(\cancelto{1}{y+y'}).$

$= x'y + x = (\cancelto{1}{x+x'}).(x+y).$

$= x+y.$

x \ y	0	1
0		1
1	1	1

one variable أي مربعين بختلفن ب

فهم جيران.

ex $F(x,y) = \overset{m_0}{x'.y'} + \overset{m_1}{x'.y} + \overset{m_3}{x.y}$ — Algebra

$= x'(y'+y) + x.y$

$= x' + x.y$

$= (\cancel{x'+x})' . (x'+y) = x'+y$

K- map :-

$x \backslash y$	0	1
0	1 (m_0)	1 (m_1)
1		1 (m_3)

x' ← ال y تغيرت
وال x ثابتة على 0.

y ← ال x تغيرت
وال y ثابتة على 1.

ex $F = x'.y' + x.y$

$= m_0 + m_3$

$F = x'.y' + x.y$

$x \backslash y$	0	1
0	1	
1		1

$x'.y'$

$x.y$

✱ Three variable maps:

3 variable ⟹ max 8 minterm.
= 8 squares.

x \ yz	00	01	11	10
0	$x'y'z'$ m_0	$x'y'z$ m_1	$x'yz$ m_3	$x'yz'$ m_2
1	$xy'z'$ m_4	$xy'z$ m_5	xyz m_7	xyz' m_6

ex Minimize the following function using k-maps

$F(x, y, z) = \sum(0, 1, 3, 7)$.

x \ yz	00	01	11	10
0	1	1	1	
1			1	

$x'.y'$ ثابتة ثابتة

$y.z$

$$x'.y' + y.z$$

ex $F(A,B,C) = \sum(0,1,2,3)$

A \ BC	00	01	11	10
0	1	1	1	1
1				

A'

ex $F(A,B,C) = \sum(0,1,2,3,4,6)$

A \ BC	00	01	11	10
0	1	1	1	1
1	1			1

A'

C'

$A' + C'$

ex $F(A,B,C) = \sum(0,2,4,5,6,7)$.

A \ BC	00	01	11	10
0	1			1
1	1	1	1	1

C'

$A + C'$

A (ثابتة كل 1).

ex $F(A,B,C) = \Sigma\, 3, 5, 6, 7$

A \ BC	00	01	11	10
0			1	
1		1	1	1

A·B

AC B·C

$AC + BC + AB$.

* Four variables maps :—

$F(A,B,C,D)$ = 4 variables.

16 minterms = 16 squares.

AB \ CD	00	01	11	10
00	$A'B'C'D'$ m0	$A'B'C'D$ m1	$A'B'CD$ m3	$A'B'CD'$ m2
01	$A'BC'D'$ m4	$A'BC'D$ m5	$A'BCD$ m7	$A'BCD'$ m6
11	$ABC'D'$ 1100 m12	$ABC'D$ m13	$ABCD$ m15	$ABCD'$ m14
10	$AB'C'D'$ m8	$AB'C'D$ m9	$AB'CD$ m11	$AB'CD'$ m10

ex Minimize using K-map.

$F(A,B,C,D) = \Sigma(0,1,4,5,7)$

AB \ CD	00	01	11	10
00	1	1		
01	1	1	1	
11				
10				

$\bar{A}\bar{C}$ ← ; → $\bar{A}BD$

$\bar{A}\bar{C} + \bar{A}BD$.

ex Minimize using K-map.

$F(A,B,C,D) = \Sigma\ 0, 2, 4, 6, 8, 10.$

$\bar{A}\bar{D}$

AB \ CD	00	01	11	10
00	1			1
01	1			1
11				
10	1			1

→ $\bar{B}\bar{D}$

$\bar{A}\bar{D} + \bar{B}\bar{D}$.

ex $F(A,B,C,D) = \sum(0,1,2,3,5,7,13,15,9,11)$.

AB \ CD	00	01	11	10
00	1	1	1	1
01		1	1	
11		1	1	
10		1	1	

$\to \bar{A}\bar{B}$

D

$D + \bar{A}\bar{B}$.

ex $F(A,B,C,D) = \sum(0,2,4,6,8,10,12,14)$.

AB \ CD	00	01	11	10
00	1			1
01	1			1
11	1			1
10	1			1

$\bar{D}$.

* Five variables Maps = 32 minterms.

Note :- $F(A,B,C) = \Sigma\, 0, 2, 4, 6.$

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

A = 0

A = 1

A = 0

B	C	F
0	0	1
0	1	0
1	0	1
1	1	0

A = 1

B	C	F
0	0	1
0	1	0
1	0	1
1	1	0

AB = 00

C	F
0	1
1	0

AB = 01

C	F
0	1
1	0

AB = 10

AB = 11

ex Minimize $F(A,B,C,D,E) = \sum(0,2,4,6,9,13,21,23,25,29,31)$

A = 0

BC \ DE	00	01	11	10
00	$\bar A\bar B\bar C\bar D\bar E$ m0	m1	m3	$\bar A\bar B\bar C D\bar E$ m2
01	m4	m5	m7	m6
11	m12	m13	ABCDE m15	m14
10	m8	m9	m11	m10

A = 1

BC \ DE	00	01	11	10
00	m16	m17	m19	m18
01	m20	m21	m23	m22
11	m28	m29	m31	m30
10	m24	m25	m27	m26

$\bar A\bar B\bar E$

BC \ DE	00	01	11	10
00	1			1
01	1			1
11		1		
10		1		

BC \ DE	00	01	11	10
00				
01		1	1	
11		1	1	
10		1		

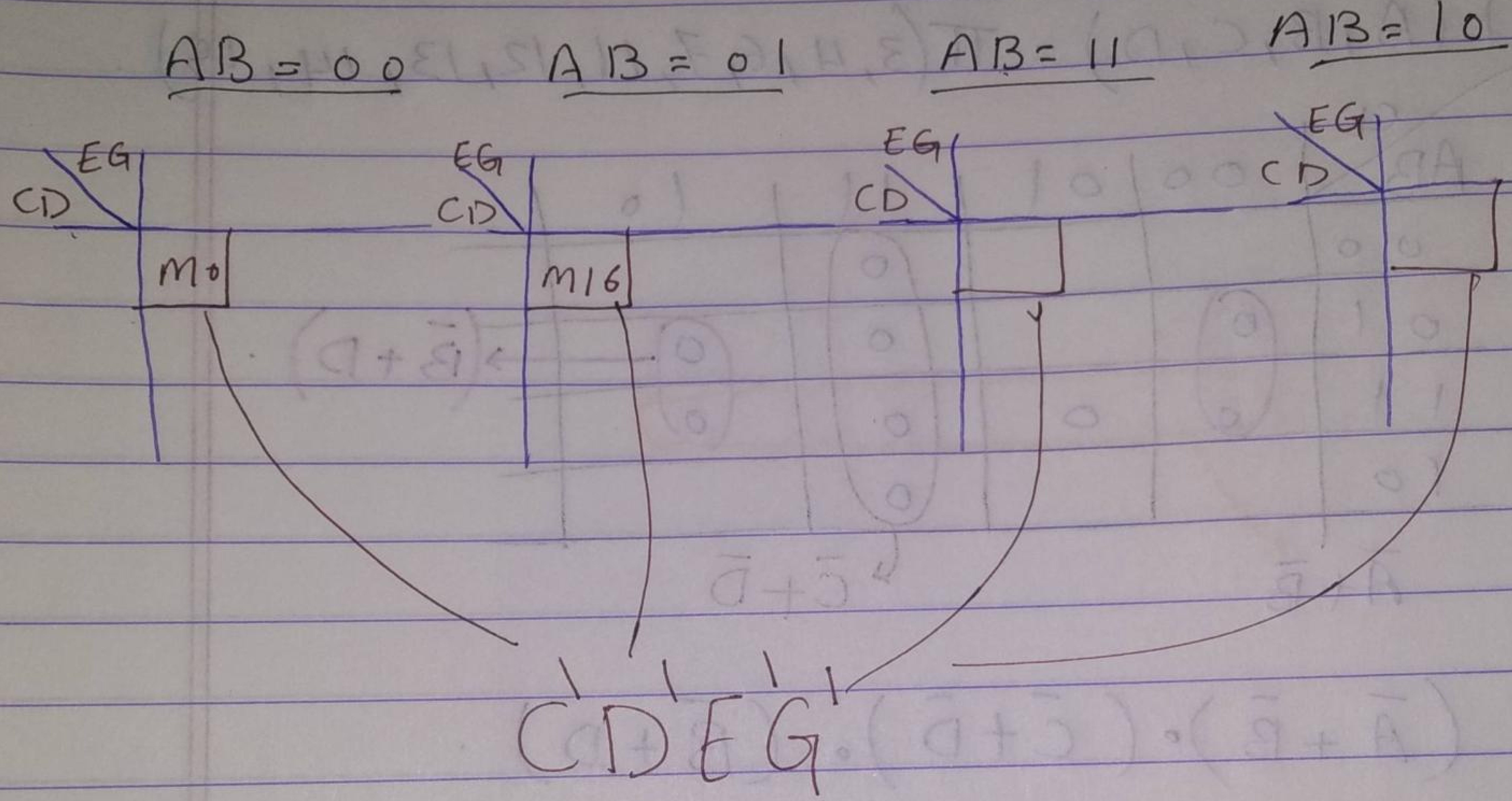
ACE

$B\bar D E$

$$\bar A\bar B\bar E + A\bar D E + ACE.$$

* 6 variable maps. (مش مطلوبة)

$F(A, B, C, D, E, G) = \Sigma$



* product of maxterm :-

Simplify the following Function.

$F(A,B,C,D) = \Pi(3,4,6,7,11,12,13,14,15)$.

AB \ CD	00	01	11	10
00			0	
01	0		0	0
11	0	0	0	0
10			0	

$(\bar{B} + D)$.

$\bar{A} + \bar{B}$ $\bar{C} + \bar{D}$

$(\bar{A} + \bar{B}) \cdot (\bar{C} + \bar{D}) \cdot (\bar{B} + D)$.

* Don't Care Condition.

Function that have unspecified outputs for some inputs.

- The unspecified minterm of a function are don't care condition denoted by x.

ex Simplify the boolean Function.

$F(w, x, y, z) = \sum(1, 3, 7, 11, 15)$.

that has don't care condition.

$d(w, x, y, z) = \sum 0, 2, 5$

w	x	y	z	F
0	0	0	0	x
0	0	0	1	1
0	0	1	0	x
0	0	1	1	1
0	1	0	0	0
0	1	0	1	x
0	1	1	0	0
0	1	1	1	1

$w'z$ yz

wx \ yz	00	01	11	10
00	X	1	1	X
01		1X	1	
11			1	
10			1	

$w'z + yz$.

* أو بنقدر نهمل الصف الأول واحدات
ونوخذ الصف الأول بتصير :

$yz + w'x'$.

* Prime Implicant (PI).

ex $F(A,B,C,D) = \sum(0,2,5,7,8,10,13,15)$

AB \ CD	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

0, 2, 8, 10

→ $B'D'$ [essential PI].

[" term].

→ BD [term essential]

5, 7, 13, 15 [essential PI]

$F(A,B,C,D) = \sum(0,2,5,7,8,10,13,14,15)$.

AB \ CD	00	01	11	10
00	1			1
01		1	1	1
11		1	1	1
10	1			1

→ $B'D'$ [EPI].

→ BC [PI] ①

→ CD' [PI] ②

not essential:

(لأنو بنقدر نحلو بطريقتين).

BD [EPI].

① - $BD + B'D' + CD'$

② - $BD + B'D' + BC$.

ex $F(A,B,C,D) = \sum(0,2,5,7,8,9,10,11,13,15)$.

AB \ CD	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1	1	1	1

$\to B'D'$ [EPI]. لأنو ال 0,2 ما بنقدر نكتبهم الا بطريقة واحدة .

$\to BD$ [EPI].

AB'

AD

$F = B'D' + BD + AD$.

$= B'D' + BD + AB'$.

ex $F(A,B,C,D) = \sum(0,2,5,7,8,9,10,11,13,14,15,6)$.

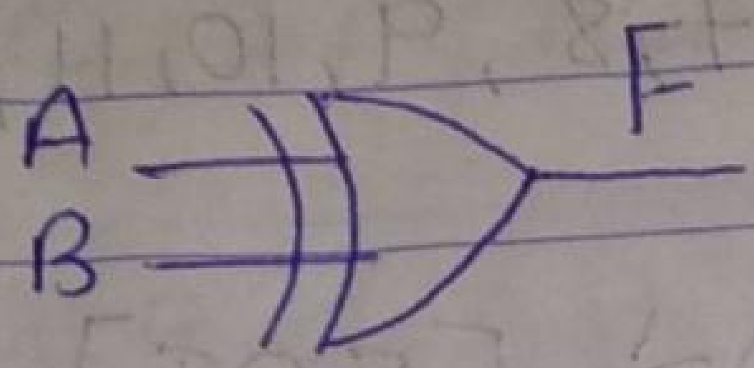
AB \ CD	00	01	11	10
00	1			1
01		1	1	1
11		1	1	1
10	1	1	1	1

$\to B'D'$ (EPI).

$\to BC$ (PI).

BD (EPI).

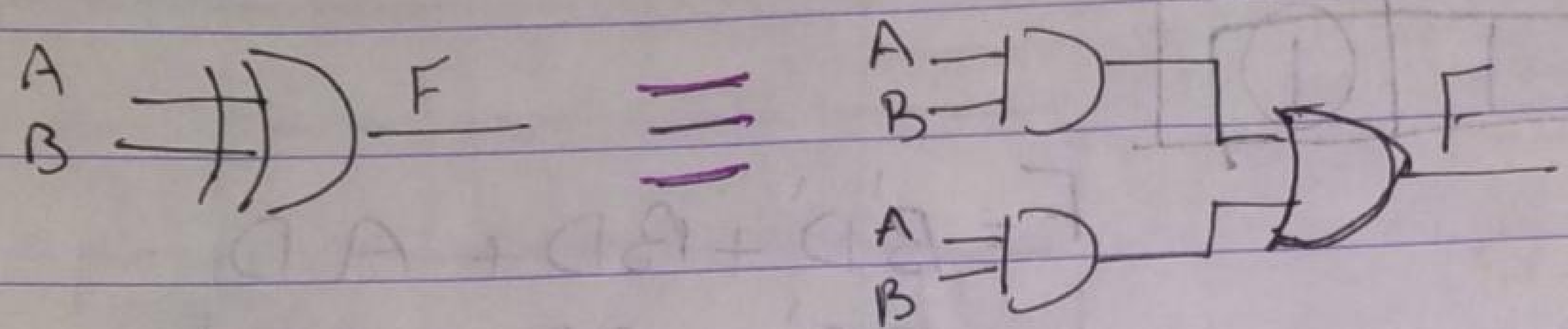
AD (PI)

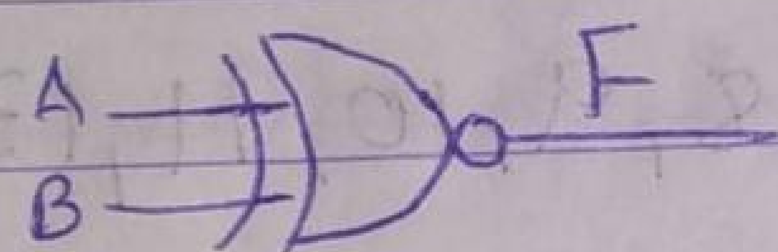
XOR 

A	B	F	
0	0	0	
0	1	1	$\leftarrow A'B$
1	0	1	$\leftarrow AB'$
1	1	0	

$F = m_1 + m_2$

$= A'B + AB'$.



XNOR 

A	B	F	
0	0	1	$\leftarrow A'B'$
0	1	0	
1	0	0	
1	1	1	$\leftarrow AB$

$F = \overline{A \oplus B}$

$F = m_0 + m_3$

$= A'B' + AB$.

$(A'B + AB')' \overset{???}{\equiv} A'B' + AB$.

$XNOR = (A \oplus B)' = [A'B + AB']'$

$= (A'B)' \cdot (AB')'$

$= (A + B') \cdot (A' + B)$.

$= \cancel{AA'} + AB + A'B' + \cancel{B'B}$.

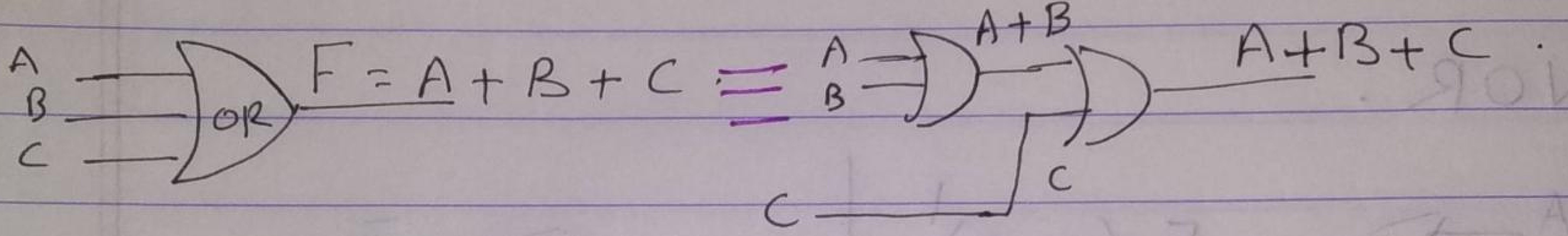
$0 \qquad\qquad\qquad\qquad\qquad\qquad 0$

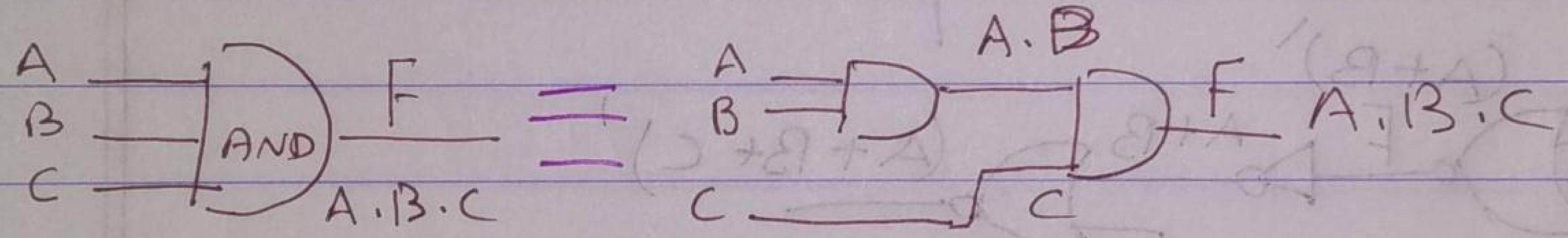
$= AB + A'B'$ ✓

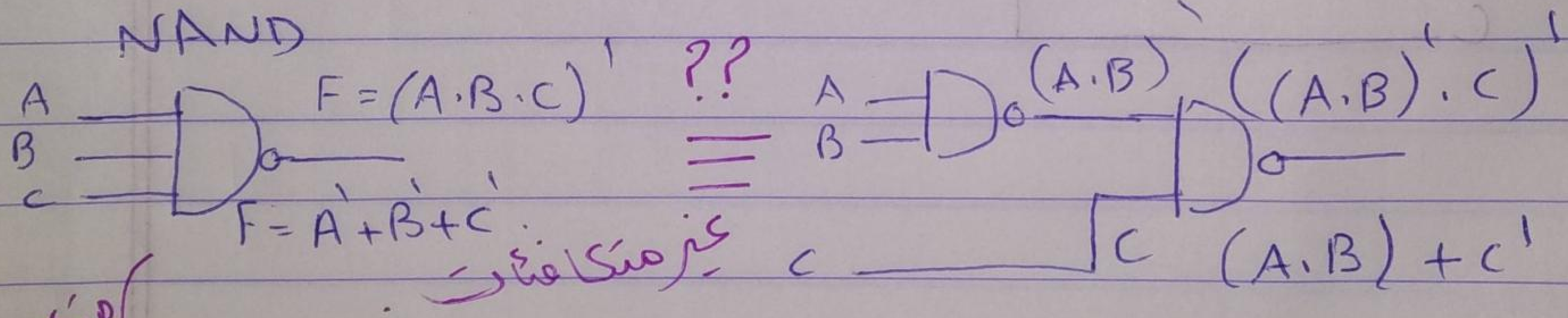
Note All gates are associated except

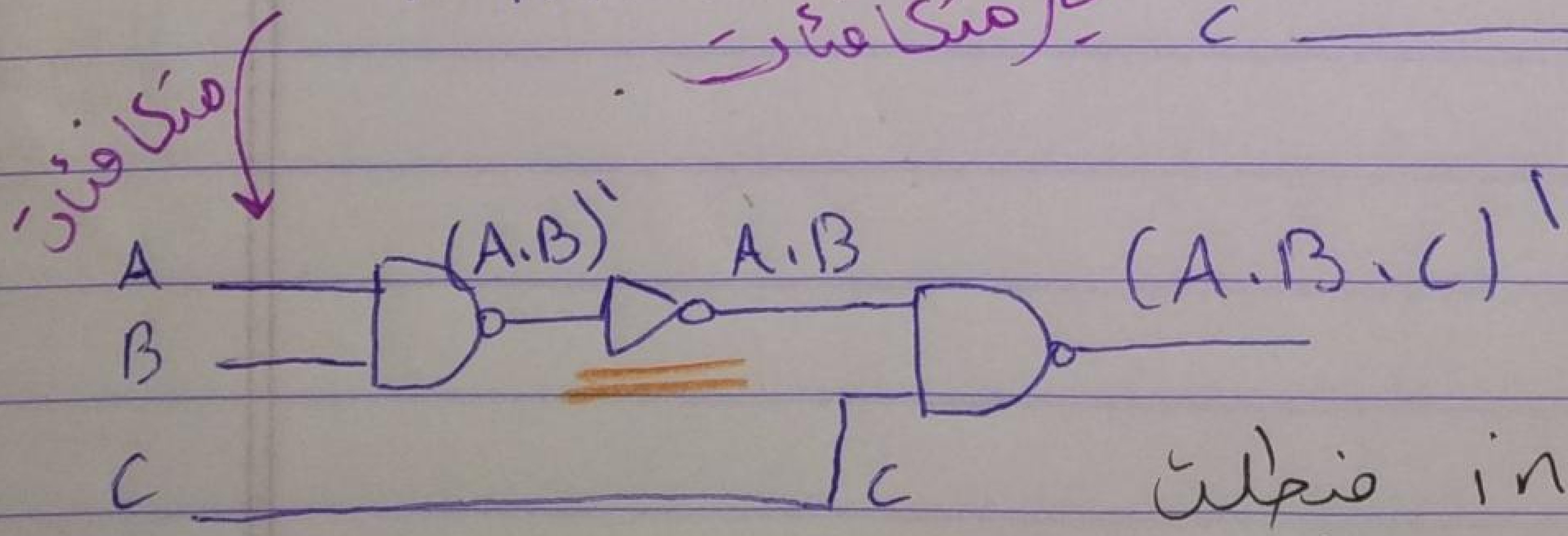
NAND
NOR

$(A + B + C)$ OR $= (A + B) + C$.



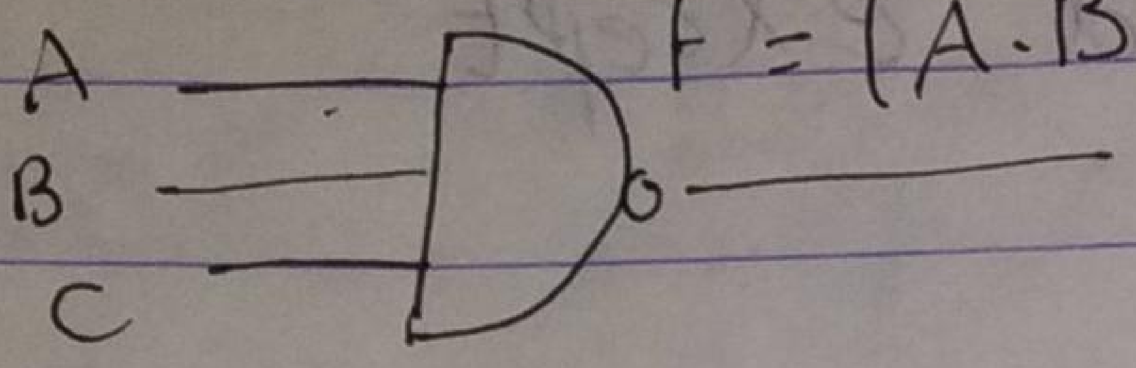




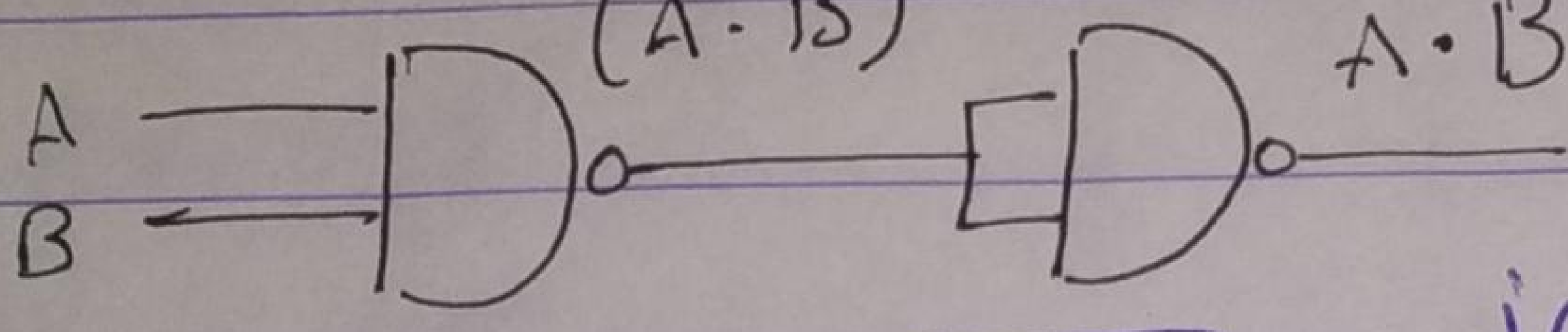


نضيف inverter منشان associated.

NAND



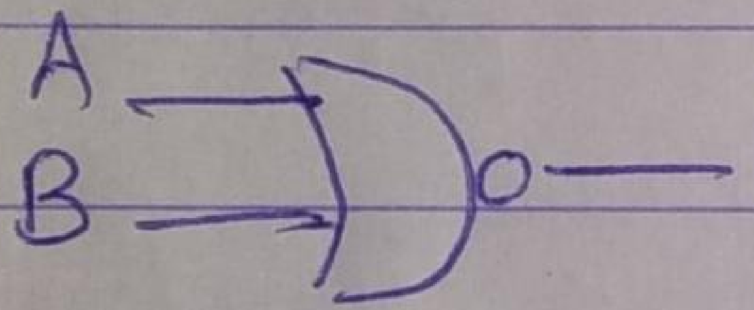
$F = (A \cdot B \cdot C)'$



inverter *

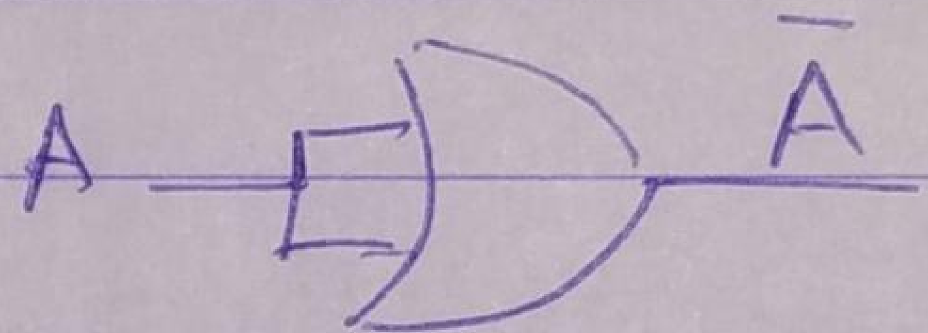
AND مصنوع

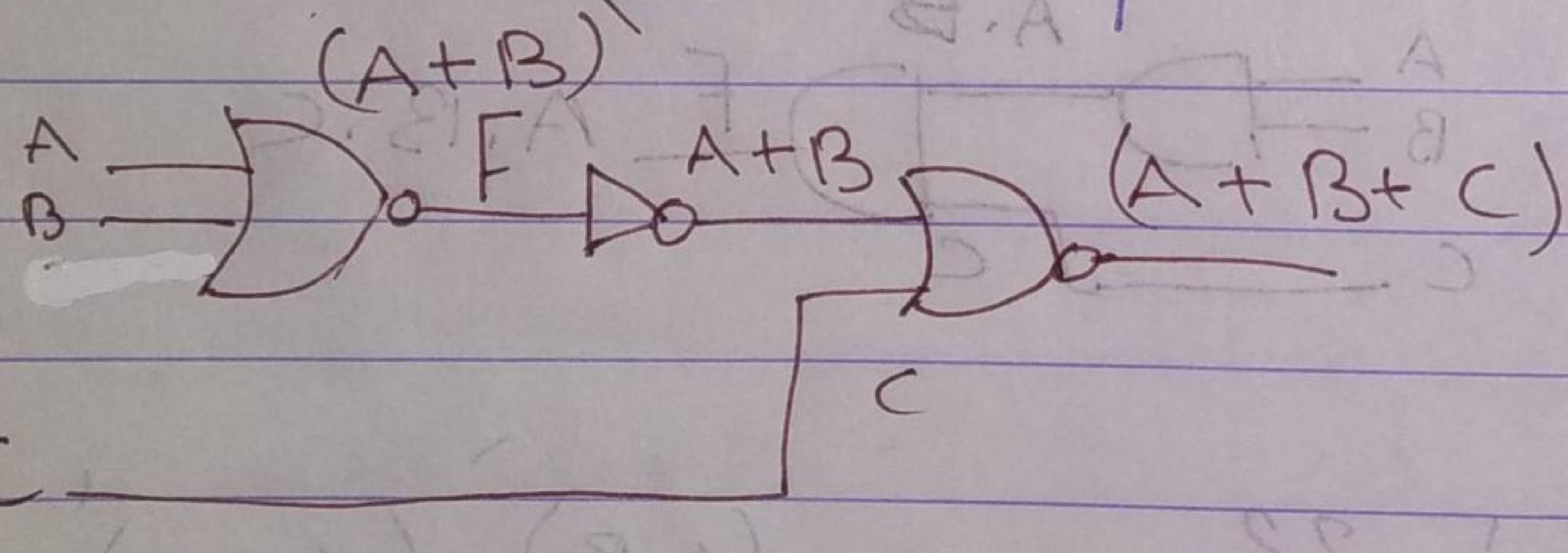
NOR .



$F = (A+B)'$

$= A' \cdot B'$

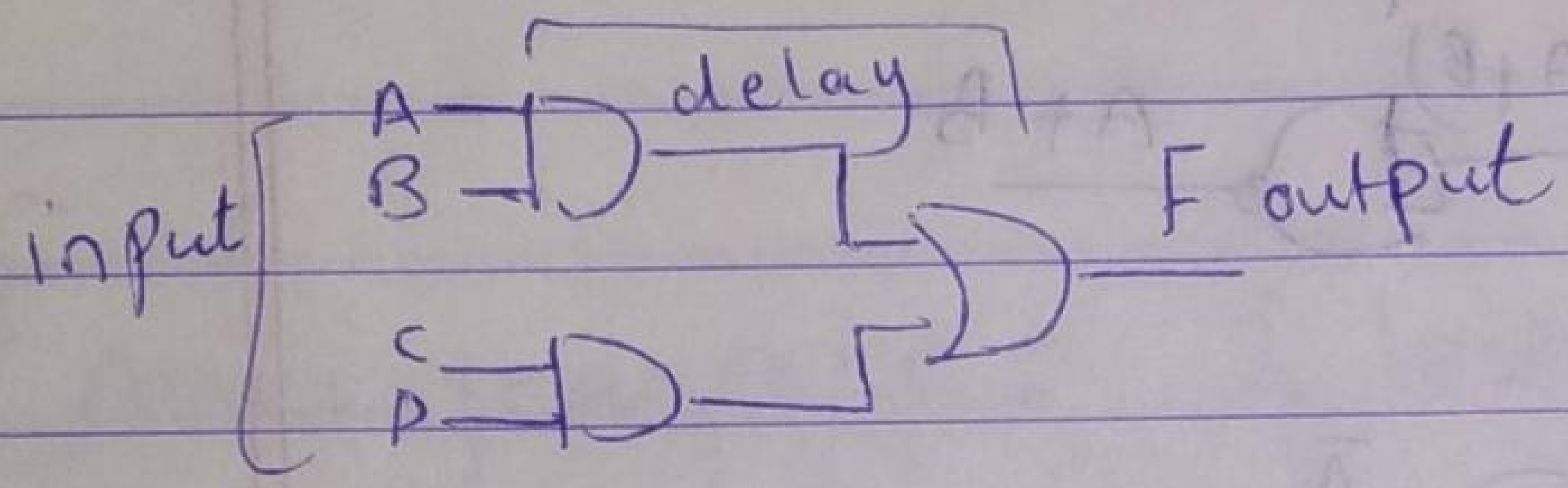


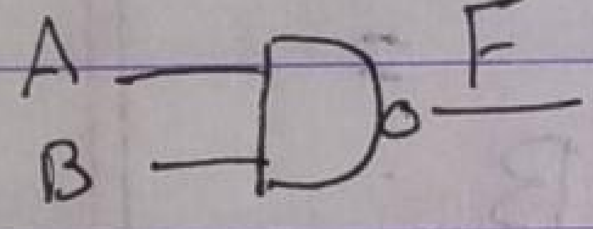


NAND & NOR Implementation:

Digital circuits are more frequently constructed with NAND/NOR than and/OR.

① NAND/NOR use fewer number of transistors.
so ① it's cheaper. ② less delay.
③ NAND/NOR universal gates.

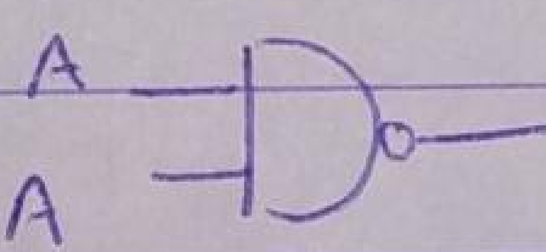




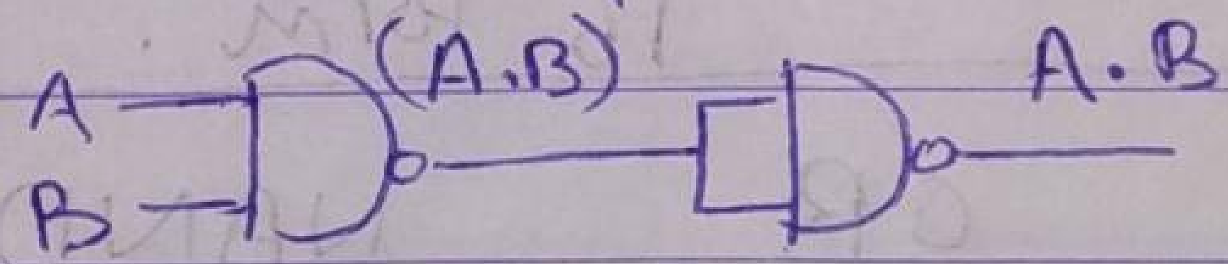
$F = (A \cdot B)' = A' + B'$.

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

invertor → ~~NOT gate~~

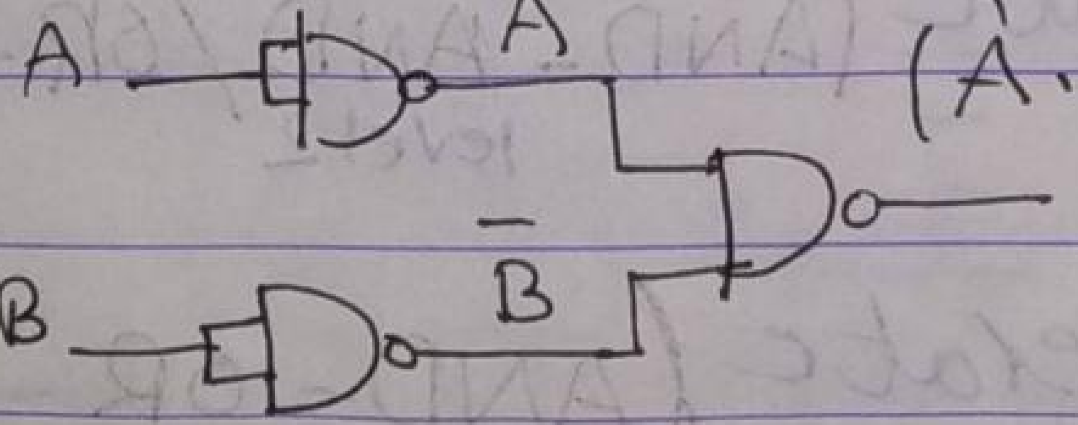


$(A \cdot A)' = \overline{A}$



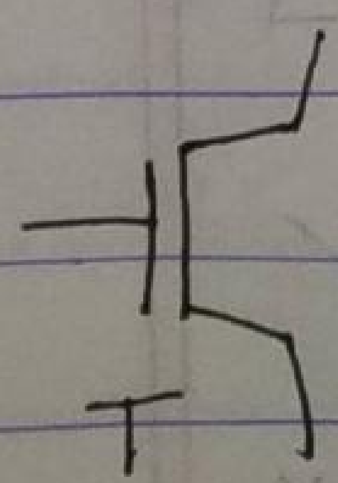
← AND

③ OR.



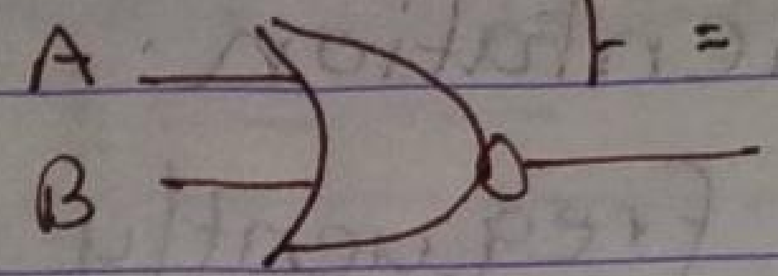
$(A' \cdot B') = A + B$.

Transistor :- is an Electronic Device



اما يمرر التيار الكهربائي .
أو ، open circuit

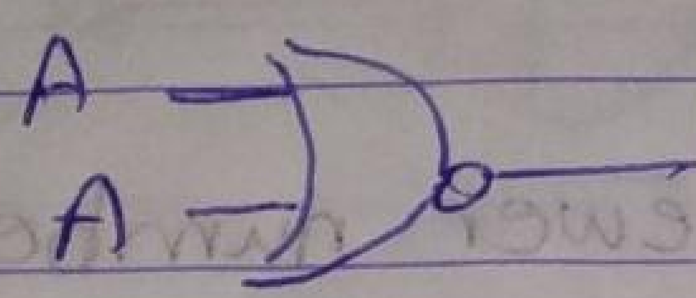
NOR.



$F = (A+B)' = A' \cdot B'$

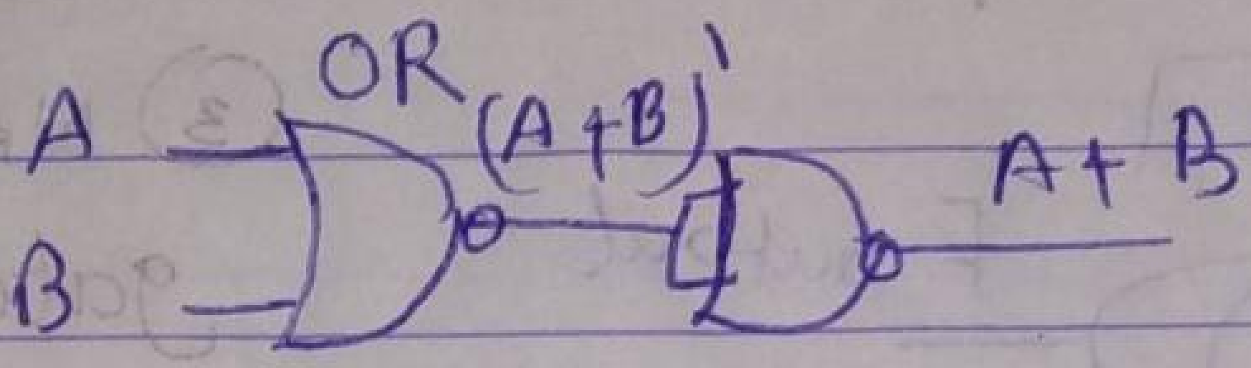
A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

1. Inverter

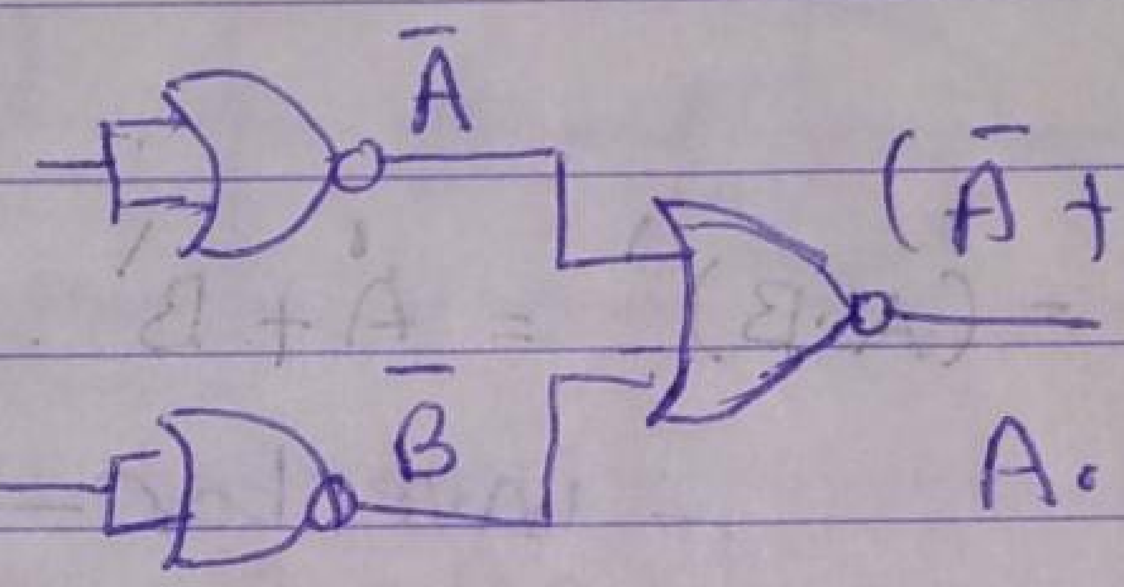


$(A+A)' = \bar{A}$

2.



3. AND



$= A \cdot B$

Basic Gates :-

$\overbrace{\text{AND} \qquad \text{OR} \qquad \text{NAND} \qquad \text{NOR}}^{16 \text{ Form.}}$

8 Form / degenarate (AND - AND / OR - OR) — level 1, level 2

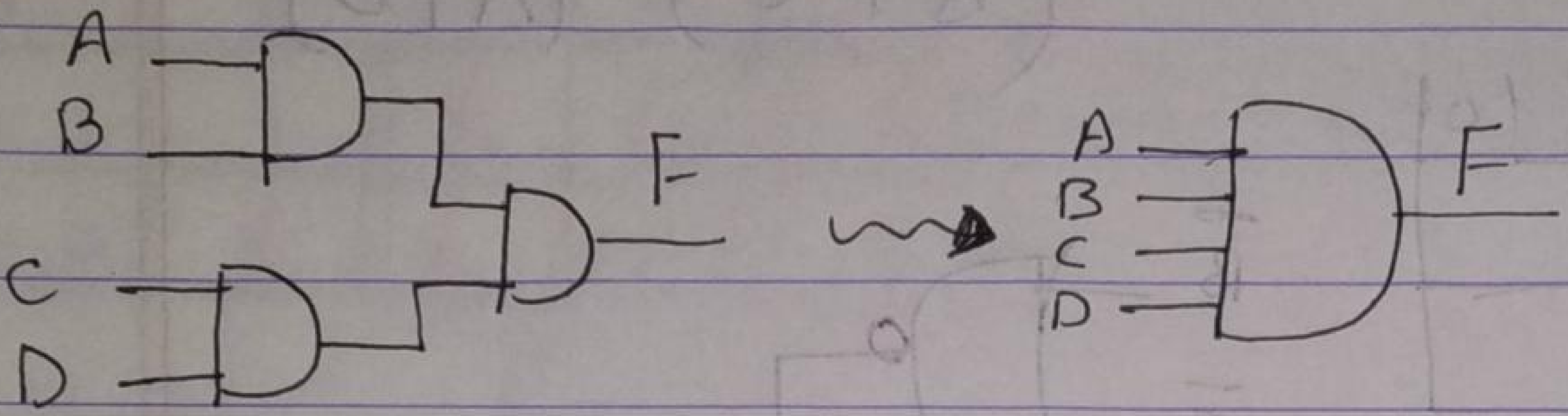
8 Form / non degenerate

minterms ←
- AND - OR] ✓
- NAND - NAND
- NOR - OR] ✗
- OR - NAND

maxterms ←
- OR - AND] ✓
- NOR - NOR
- NAND - AND] ✗
- AND - NOR

Degenerate :- انو بنقدر نحول ال 2 level ل 1 level

مثال :



nondegenerate :- مابنقدر نحولهم ل 1 level

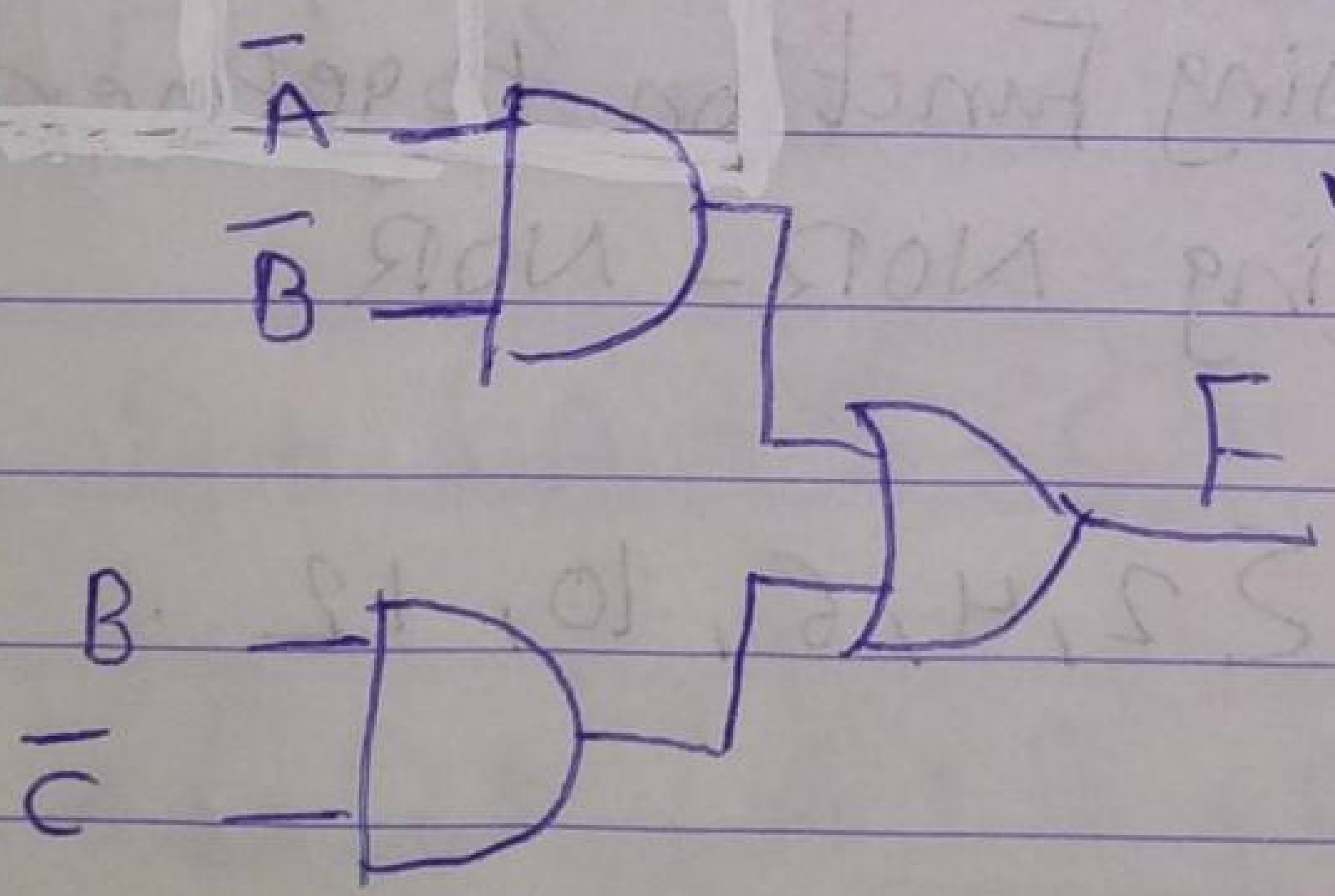
ex $F(A,B,C) = \Sigma\, 0, 1, 2, 6$ ← minterm.

$= \bar{A}\bar{B} + B\bar{C}$

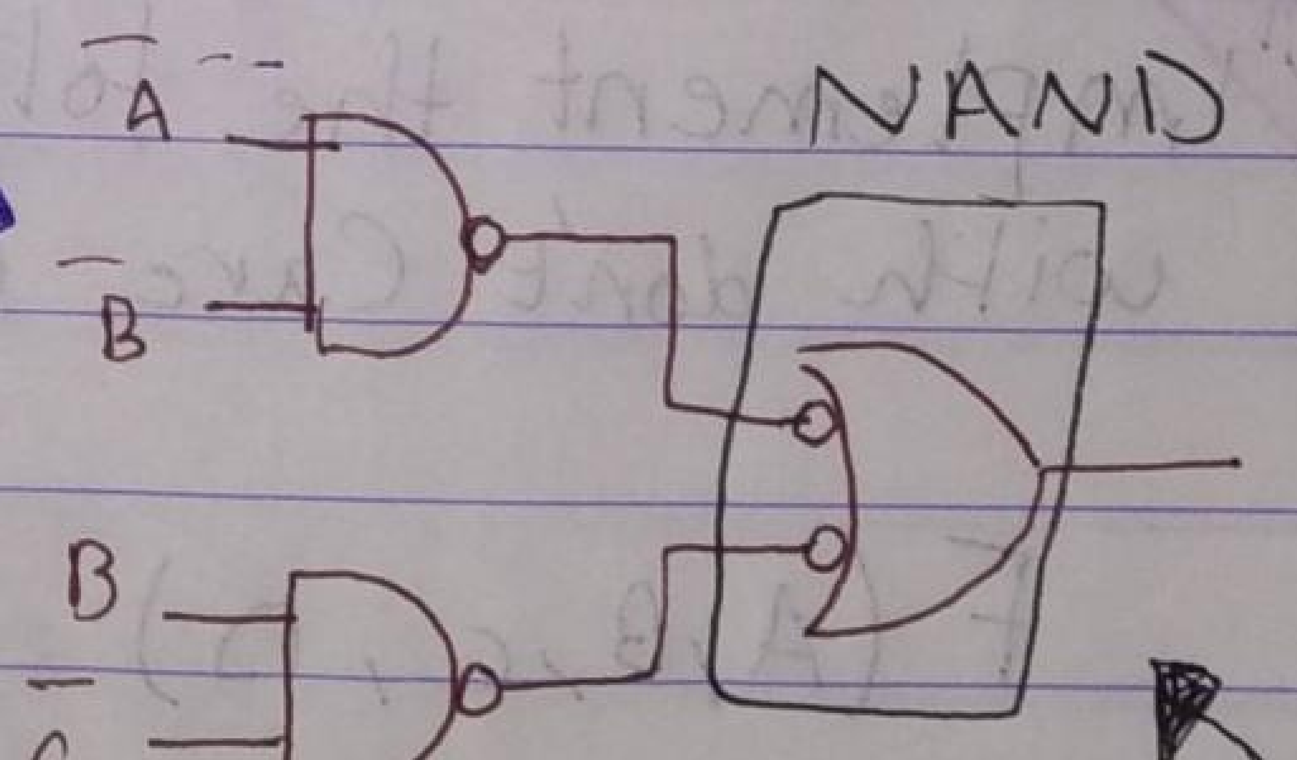
A \ BC	00	01	11	10
0	1	1		1
1				1

→ $\bar{A}\bar{B}$

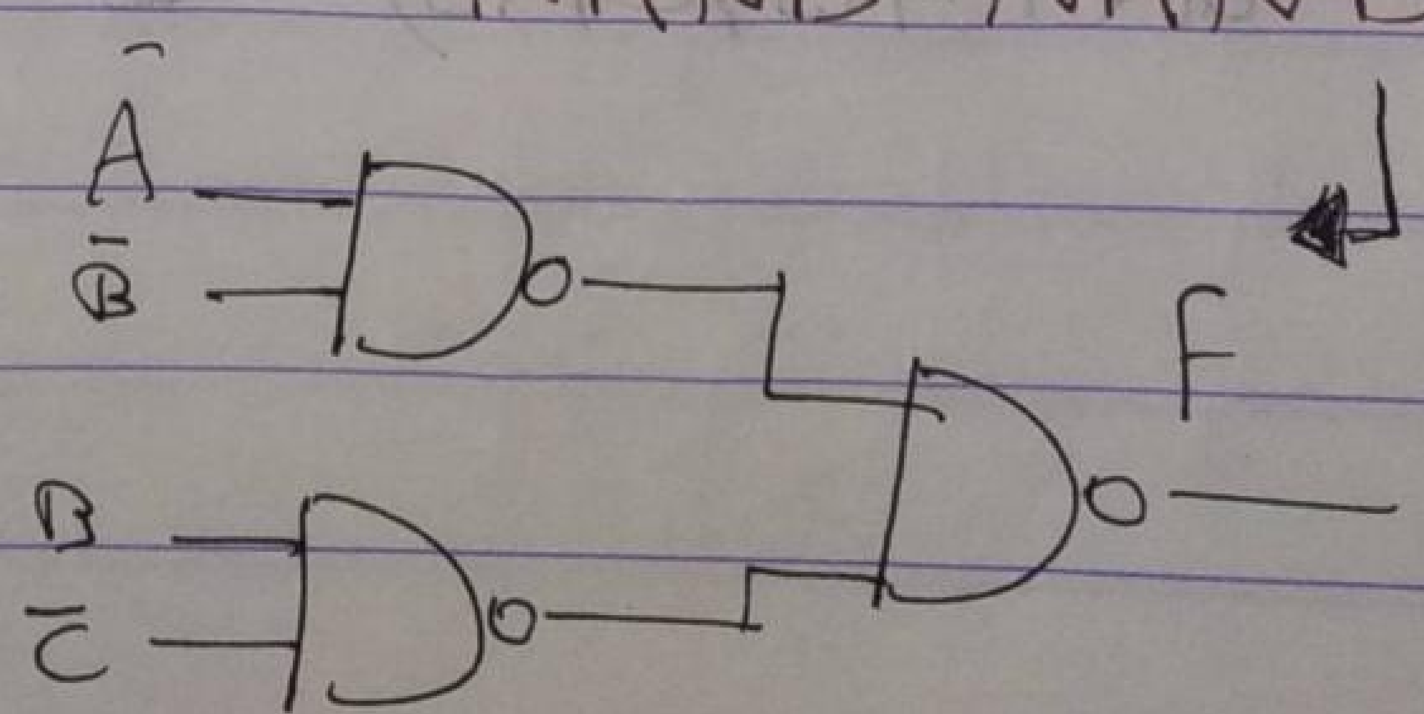
→ $B\bar{C}$



AND — OR.



NAND-NAND



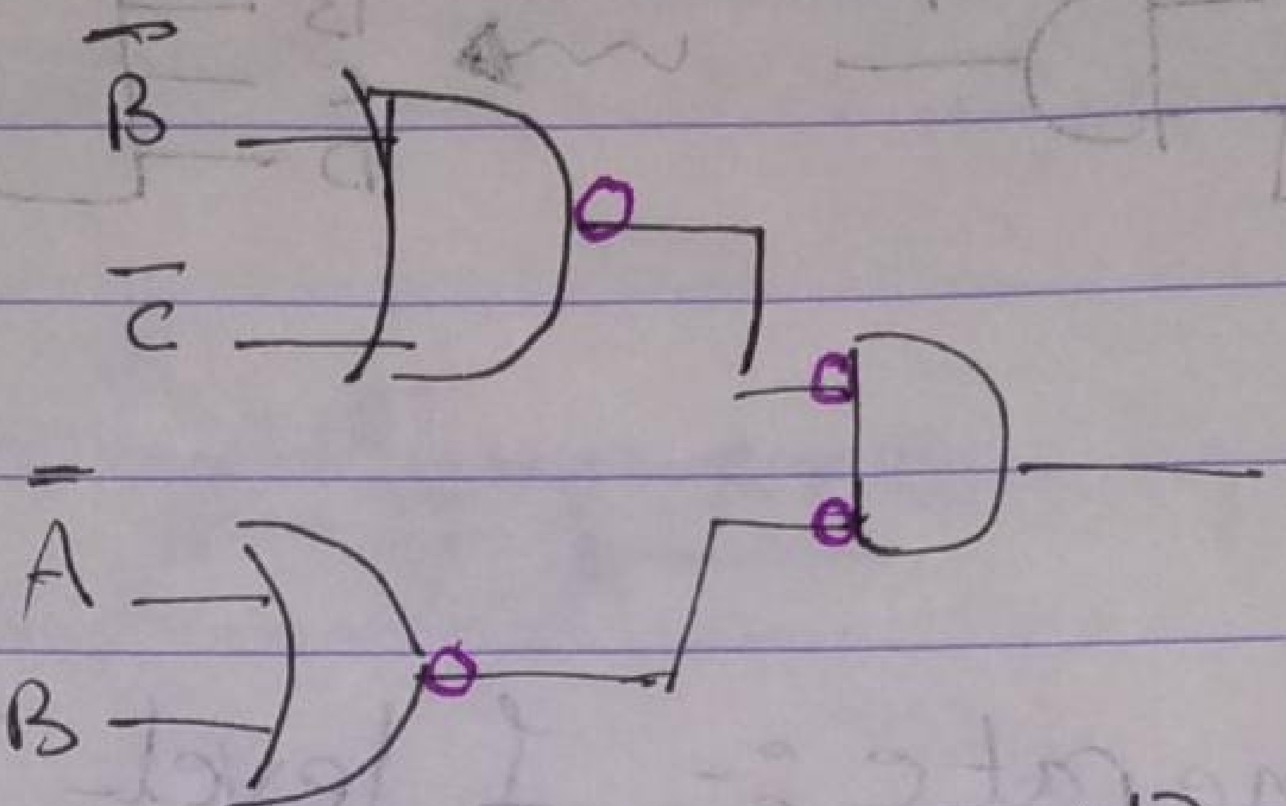
ex $F(A,B,C) = \pi(3,4,5,7)$.

NOR - NOR.

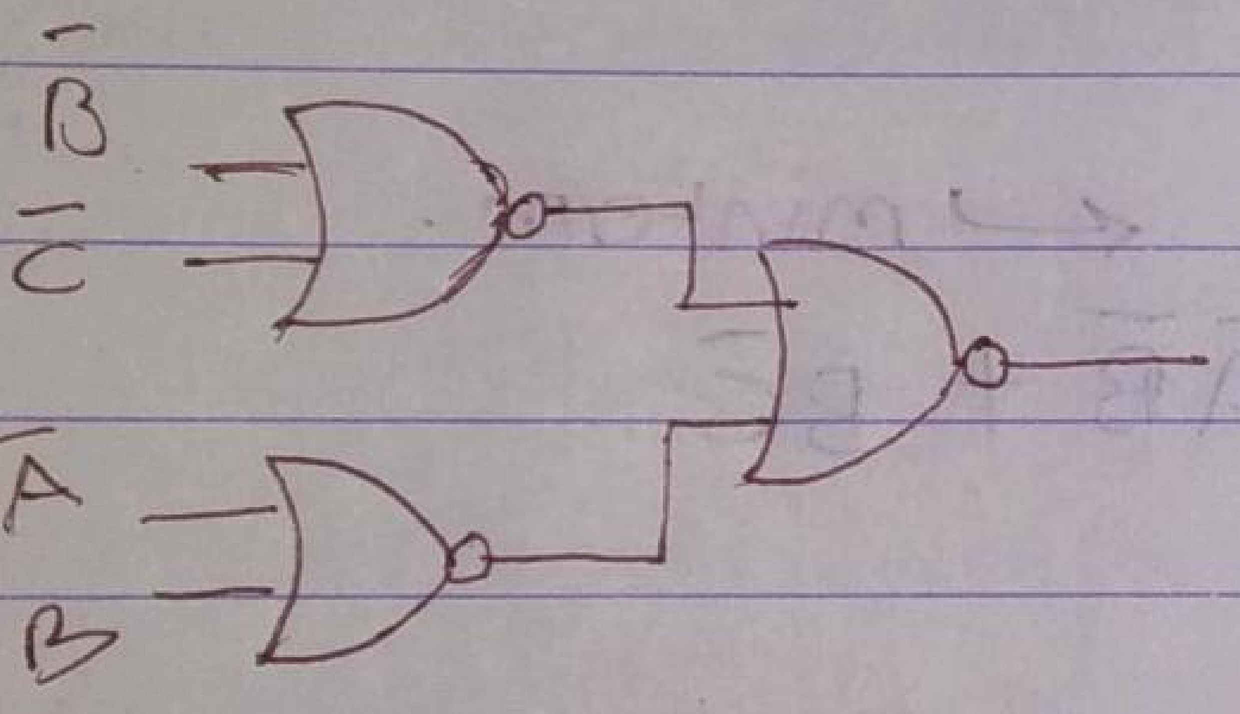
$$= (\bar{B} + \bar{C}) \cdot (\bar{A} + B)$$

A \ BC	00	01	11	10
0			0	
1	0	0	0	

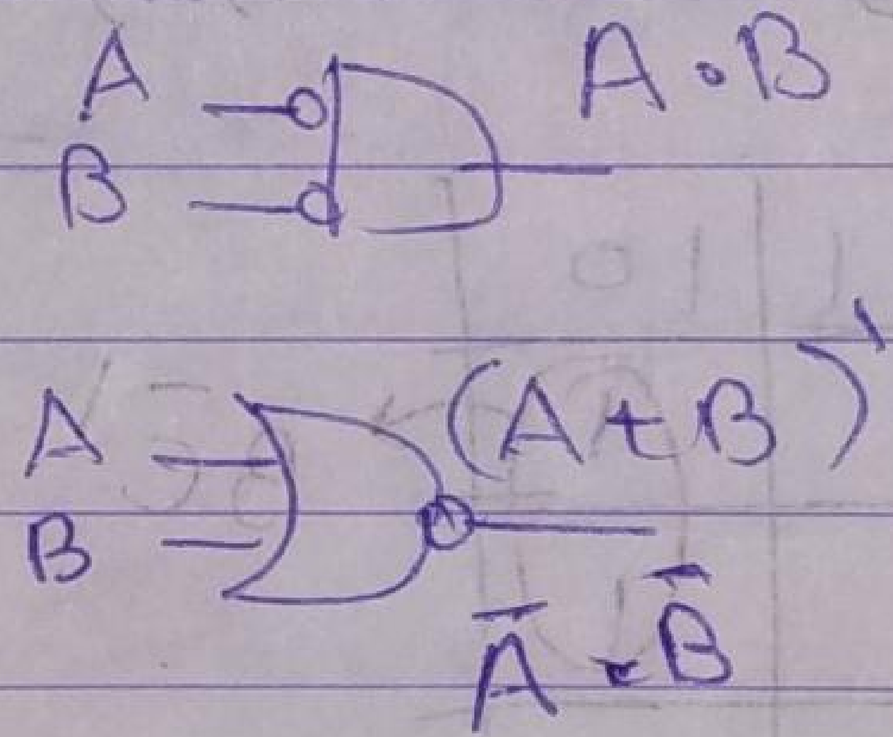
$\bar{A} + B$ $\quad$ $\bar{B} + \bar{C}$



OR - AND



NOR - NOR.



ex Implement the following Function together with don't Care using NOR - NOR.

$$F(A,B,C,D) = \sum 2, 4, 6, 10, 12.$$

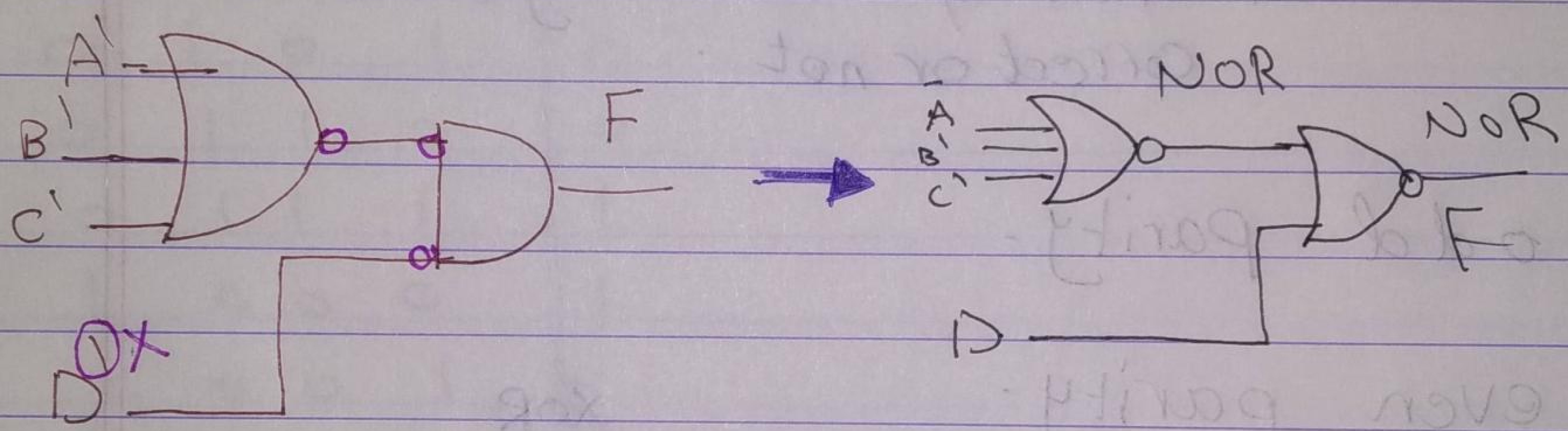
$$d(A,B,C,D) = \sum 0, 8, 9, 13.$$

$\Rightarrow$

AB \ CD	00	01	11	10
00	X	0	0	1
01	1	0	0	1
11	1	X	0	0
10	X	X	0	1

$\rightarrow (\bar{A} + \bar{B} + \bar{C})$

D' $\quad F = D' \cdot (A' + B' + C')$



طريقة ثانية :

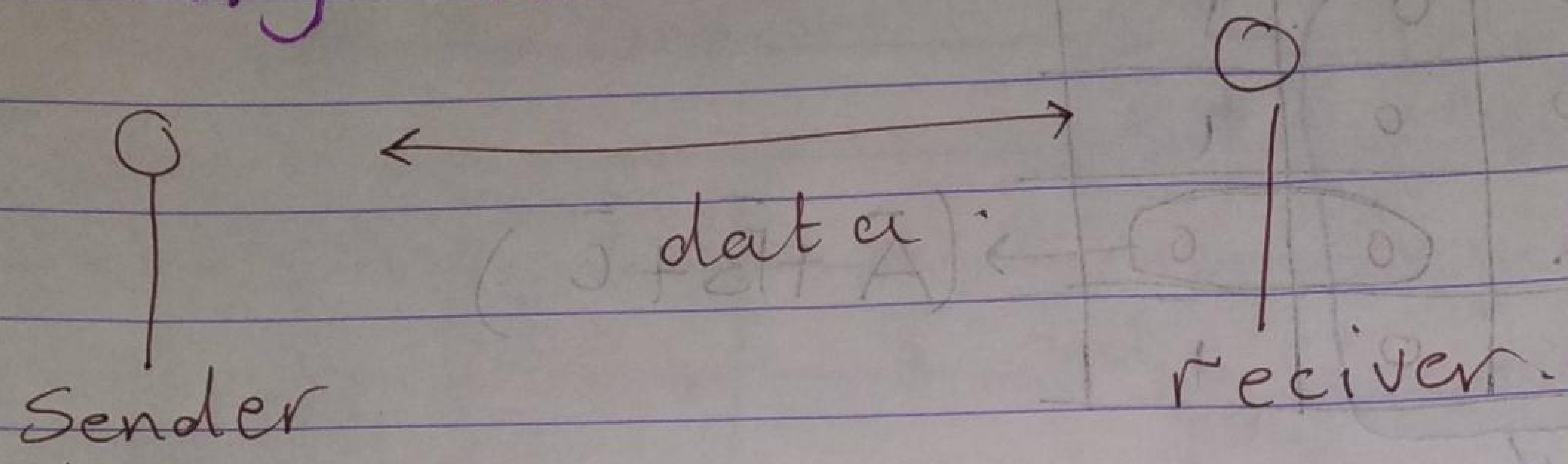
$$F = D' \cdot (A' + B' + C')$$

$$F' = [D' \cdot (A' + B' + C')]'$$

$$F' = D + (A' + B' + C')'$$

$$F'' = F = [D + \underbrace{(A' + B' + C')'}_{NOR}]'$$

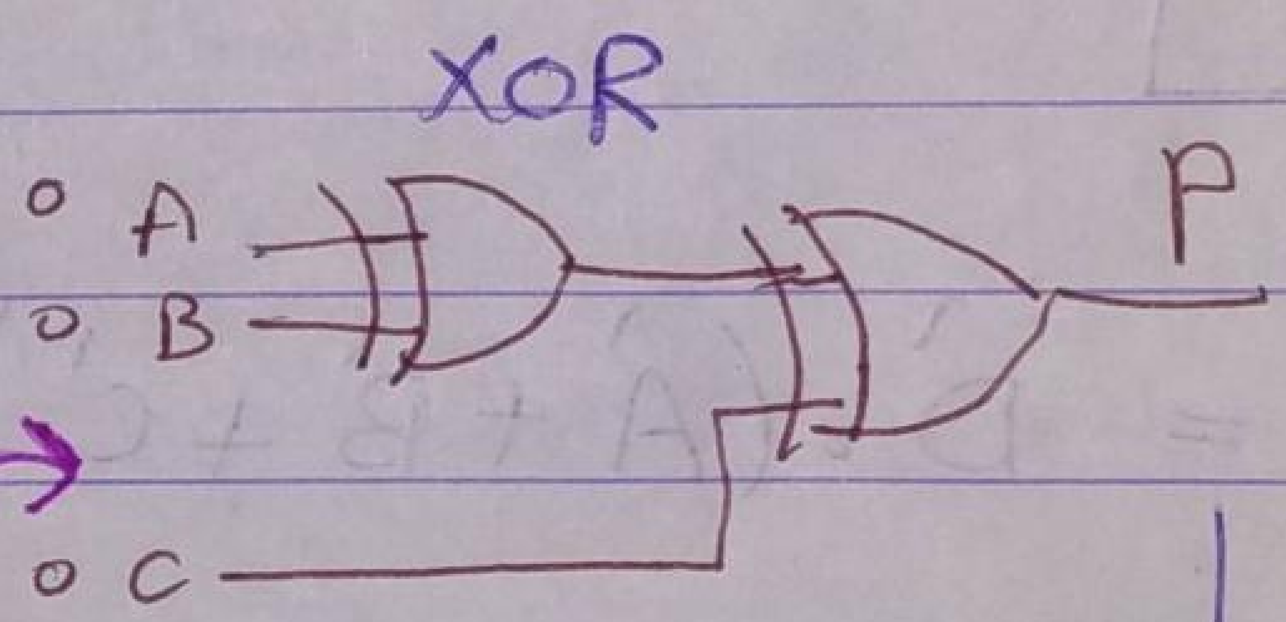
Parity Generator :-



parity :- Extra bit added by sender. To check if the message is correct or not.

① odd parity.

② even parity.



ex sender

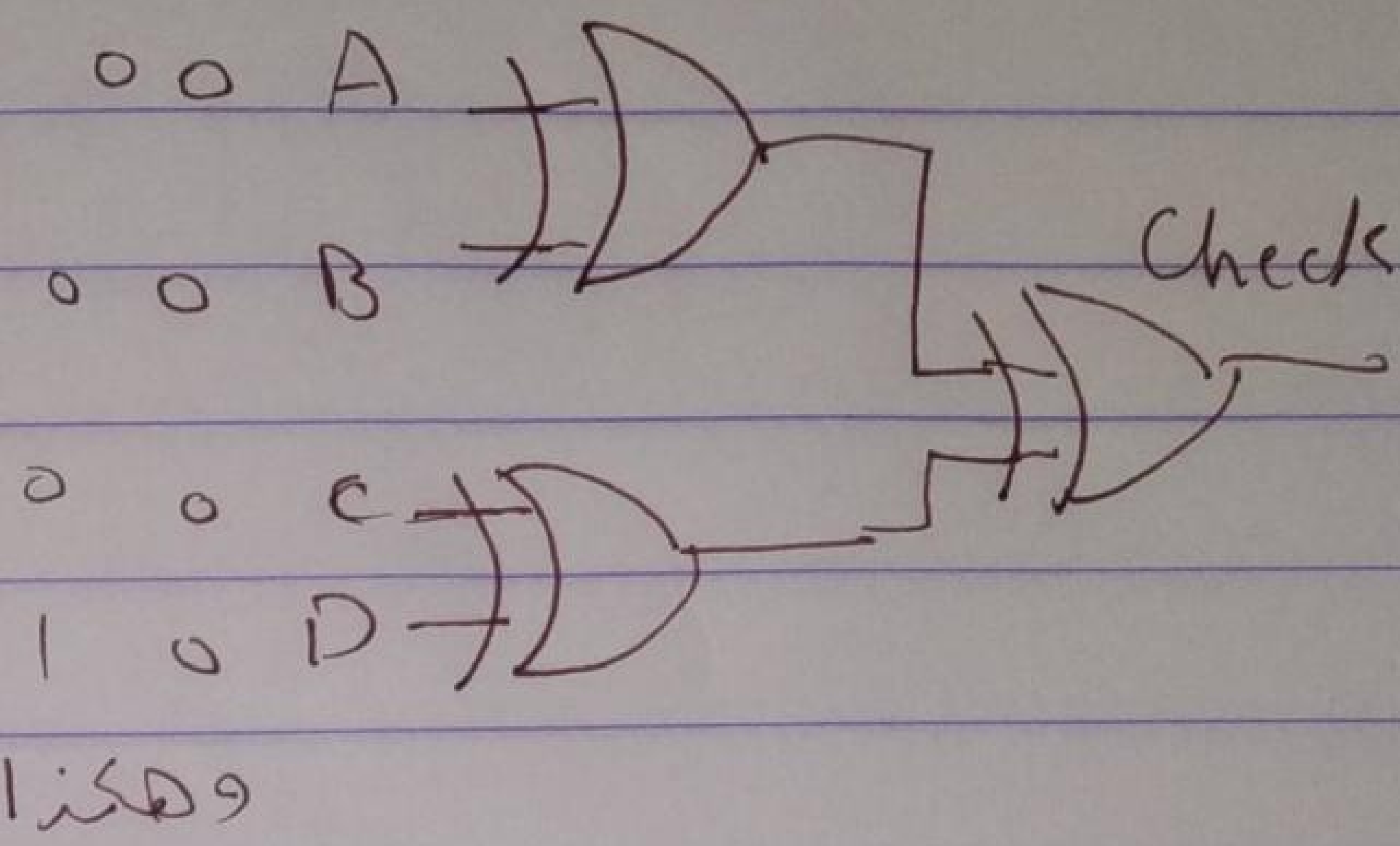
A	B	C	P (even)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

← عشان نخلي
عدد الواحدات زوجي

اذا ال odd
بتكون XNOR

Reciver

A	B	C	D	Check
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
		⋮		⋮



parity (odd).

x	y	z	P
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

x \ yz	00	01	11	10
0	1		1	
1		1		1

$P = x'y'z' + x'yz + xy'z + xyz'$

$P = x'(y'z' + yz) + x(y'z + yz')$

$= x'(y \oplus z)' + x(y \oplus z)$

$= x'w' + xw$

$= (x \oplus w)' = (x \oplus y \oplus z)'$