Ch5

Saturday, April 3, 2021 9:04 AM

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$$1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

① c $\int (1 + \cot^2\theta)\, d\theta$

$$\int \csc^2\theta \, d\theta$$

$$= -\cot\theta + C$$

$(\sin\theta)' = \cos\theta$

$(\cos\theta)' = -\sin\theta$

$(\tan\theta)' = \sec^2\theta$

$\cot\theta = \int (\cot\theta)' = \int -\csc^2\theta = -\int \csc^2\theta$

$(\sec\theta)' = \sec\theta \tan\theta$

$(\csc\theta)' = -\csc\theta \cot\theta$

Q2 ⓒ

$$\left(\int_{\tan x}^{0} \frac{dt}{1+t^2}\right)' = \left(-\int_{0}^{\tan x} \frac{dt}{1+t^2}\right)'$$

$$= -\frac{1}{1+\tan^2 x}(\sec^2 x) - \frac{1(0)}{1+0^2} = -\frac{\sec^2 x}{\sec^2 x} = -1$$

Q3 Find $L(x)$ for $g(x) = 3 + \int_{1}^{x^2} \sec(t-1)\, dt$ at $x_0 = -1$

$(-1)^2$

Q3 Find $L(x)$ for $g(x) = 3 + \int \sec($ ...

$L(x) \approx g(x)$ about $x = -1$

$$g(-1) = 3 + \int_{1}^{(-1)^2} \sec(t-1)\,dt = 3$$

$$L(x) = g(x_0) + g'(x_0)(x - x_0)$$

$$= g(-1) + g'(-1)(x - -1)$$

$$= 3 + (-2)(x+1)$$

$$= 1 - 2x$$

$$g'(x) = 0 + \sec(x^2 - 1)\boxed{2x}$$

$$= 2x \sec(x^2-1)$$

$$g'(-1) = 2(-1)\sec(1-1)$$

$$= -2 \sec 0$$

$$= -2 \frac{1}{\cos 0}$$

$$= -2 (1)$$

$$= -2$$

Estimat $g(-1.1)$

$$g(-1.1) \approx L(-1.1) = 1 - 2(-1.1)$$

$$= 1 + 2.2$$

$$= 3.2$$

Q4 ⓐ

$$\int_{1}^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2}\,ds = \int_{1}^{\sqrt{2}^4} \left(1 + \frac{s^{\frac{1}{2}}}{s^2}\right) ds$$

$$= \int_{1}^{\sqrt{2}^4} \left(1 + s^{\frac{1}{2} - 2}\right) ds = \int_{1}^{\sqrt{2}^4} \left(1 + s^{-\frac{3}{2}}\right) ds = s + \frac{s^{-\frac{1}{2}}}{-\frac{1}{2}} \Big|_{1}^{\sqrt{2}^4}$$

$$= \left(s - 2\frac{1}{\sqrt{s}}\right)\Big|_{1}^{\sqrt{2}^4} = \left(4 - 2\frac{1}{\sqrt{4}}\right) - \left(1 - 2\frac{1}{\sqrt{1}}\right)$$

$$= \left(4 - \cancel{2}\frac{1}{\cancel{2}}\right) - (1 - 2) = (4 - 1) - -1$$

$$= 3 + 1$$

$$= 4$$

Q5 d $\int x^3 \sqrt{x^2+1}\, dx$

$x^3 = x^2\, x$

$= x^2 \sqrt{x^2} \quad , x > 0$

$\int x^2 \sqrt{x^2}\, \sqrt{x^2+1}$

$\int x^2 \sqrt{x^2(x^2+1)}$

$= \int x^2 \sqrt{x^4 + x^2}$

يوجد مشكلة

$u = x^2+1 \Rightarrow x^2 = u-1$

$\int x^3 \sqrt{x^2+1}\, dx = \int x^3 u\, dx$

يوجد مشكلة

$u = x^2+1$

$du = 2x\, dx$

$\int x^3 \sqrt{u}\, \frac{du}{2x}$

$\int x^2 \sqrt{u}\, du$

$\int (u-1)\sqrt{u}\, du$

$\ldots$

$\int x^3 \sqrt{x^2+1}\, dx = \int x^2 \sqrt{x^4+x^2}\, dx$

$= \int x^2 \sqrt{(x^2+x)^2}$

محاولة غبية

$-\frac{1}{2}$

Q5 d $\int x^3 \sqrt{x^2+1}\, dx$

$u = x^2+1 \Rightarrow x^2 = u-1$

$du = 2x\, dx \Rightarrow dx = \frac{du}{2x}$

$\int \cancel{x^3}^{\,2} \sqrt{u}\, \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int x^2 \sqrt{u}\, du = \frac{1}{2}\int (u-1)\, u^{\frac{1}{2}}\, du = \frac{1}{2}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$

$= \frac{1}{2}\left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{1.5}}{1.5}\right) + c \quad = \frac{1}{2}\left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) + c$

$$= \frac{1}{2}\left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{1.5}}{1.5}\right) + c \qquad = \frac{1}{2}\left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) + c$$

$$= \frac{1}{5}\sqrt{u^5} - \frac{1}{3}\sqrt{u^3} + c \quad \checkmark$$

$$= \frac{1}{5}\sqrt{(x^2+1)^5} - \frac{1}{3}\sqrt{(x^2+1)^3} + c \quad \checkmark$$

---

d) $\int x^3 \sqrt{x^2+1}\, dx$

$u = \sqrt{x^2+1}$ ✓

$u^2 = x^2+1 \Rightarrow u^2 - 1 = x^2$

$du = \frac{\cancel{2x}}{\cancel{2}\sqrt{x^2+1}}\, dx$

$du = \frac{x}{u}\, dx$

$\frac{u\, du}{x} = dx$

$\int x^{\cancel{3}\,2}\; u\; \frac{u\,du}{\cancel{x}}$

$\int x^2 u^2\, du$

$\int (u^2-1)\, u^2\, du$

$\int (u^4 - u^2)\, du$

$$\frac{u^5}{5} - \frac{u^3}{3} + C = \frac{1}{5}\left(\sqrt{x^2+1}\right)^5 - \frac{1}{3}\left(\sqrt{x^2+1}\right)^3 + C$$

Q6 (a) Find Area between $y = x^2 - 2x$ and $y = x$

$= \left(x + \frac{-2}{2}\right)^2 - 1$

$= (x-1)^2 - 1$

$y = x^2 - 2x = (x-1)^2 - 1$

$y = x^2 - 2x = \underline{\underline{(x-1)^2 - 1}}$

$= (x-1)^2 - 1$

$= x^2 - 2x + \cancel{1} \cancel{-1}$

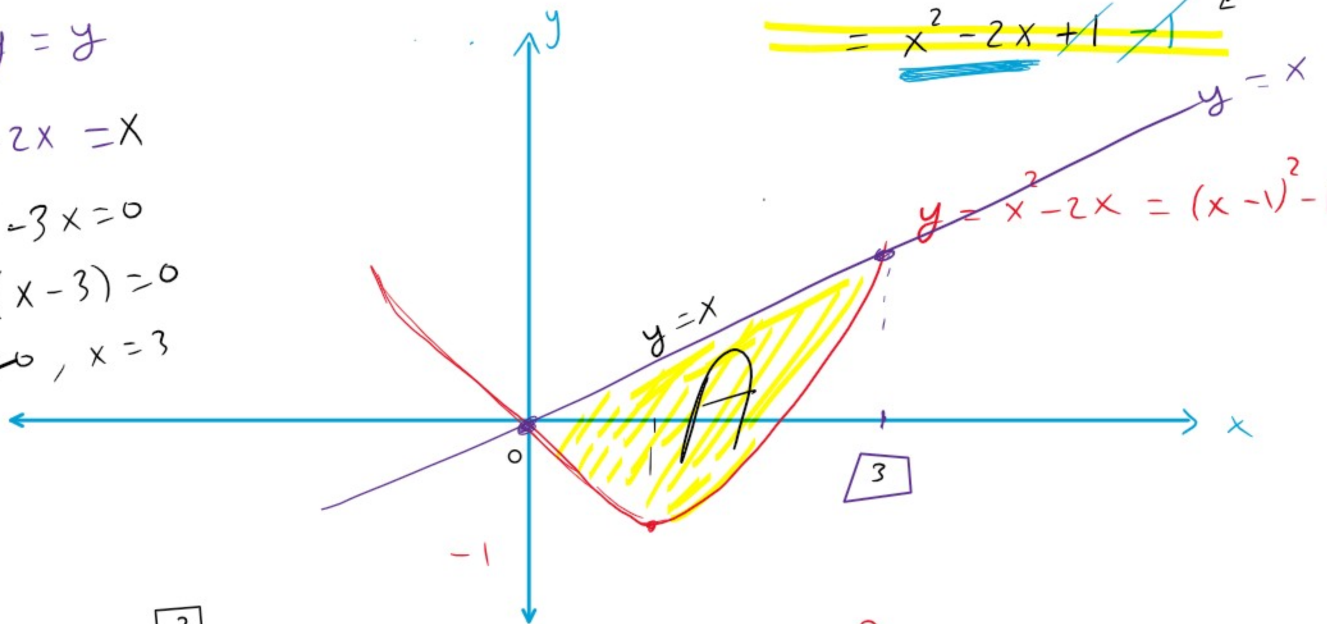
$y = y$

$x^2 - 2x = x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 , \ x = 3$

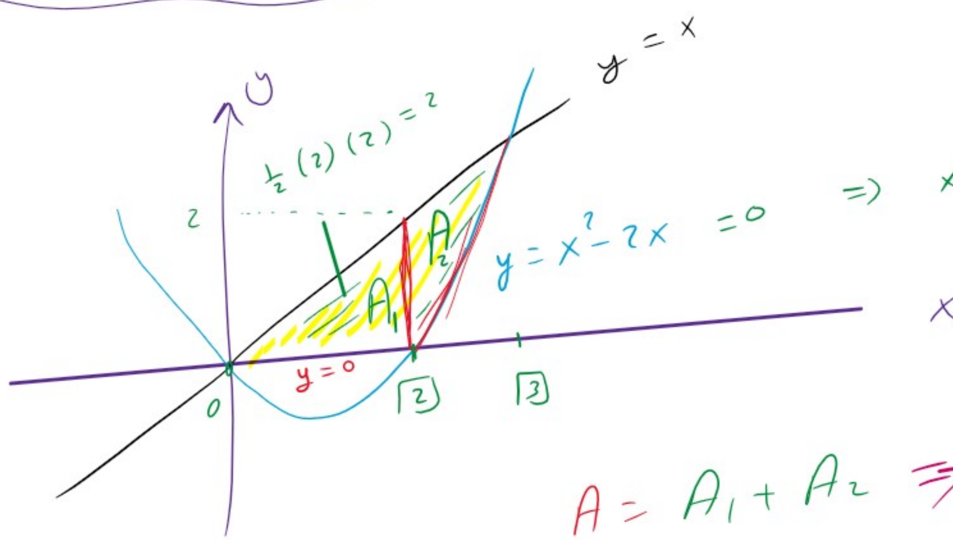


$$A = \int_{\boxed{0}}^{\boxed{3}} \overset{up}{\boxed{x}} - \overset{down}{\boxed{(x^2 - 2x)}} \, dx = \int_0^3 (x - x^2 + 2x) \, dx$$

$$= \int_0^3 (3x - x^2) \, dx = 3\frac{x^2}{2} - \frac{x^3}{3} \Big|_0^3 = \left(\frac{3}{2}(9) - \frac{27}{3}\right) - 0$$

$$= \frac{27}{2} - \frac{27}{3}$$

$$= \frac{81 - 54}{6} = \underline{9}$$



$y = x^2 - 2x = 0 \Rightarrow x(x-2) = 0$

$x = 0 , \ x = 2$

$A = A_1 + A_2$

$A = A_1 + A_2$

$$A_1 = \int_0^2 \left(\boxed{x} - \boxed{0} \right) dx = \frac{x^2}{2}\Big|_0^2 = \frac{4}{2} = 2$$

$$A_2 = \int_2^3 \left(\boxed{x} - \boxed{(x^2 - 2x} \right) dx = \frac{3x^2}{2} - \frac{x^3}{3}\Big|_2^3$$

$$= \left(\frac{3}{2}(9) - \frac{27}{3}\right) - \left(\frac{3}{2}(4) - \frac{8}{3}\right) = \frac{27}{2} - 9 - \left(6 - \frac{8}{3}\right)$$

$$= \frac{27}{2} - 15 + \frac{8}{3}$$

$$A = A_1 + A_2 = 2 + \frac{27}{2} - 15 + \frac{8}{3} = \frac{27}{2} + \frac{8}{3} - 13$$

Q1 d $\int (1 + \cot^2\theta)\, d\theta$

$\sin^2\theta + \cos^2\theta = 1$

$$1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

$$= \int \csc^2\theta \, d\theta$$

$$= -\cot\theta + C$$

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$(\sin\theta)' = \cos\theta$

$(\cos\theta)' = -\sin\theta$

$(\tan\theta)' = \sec^2\theta$

$\cot\theta = \int \boxed{(\cot\theta)' = \int -\csc^2\theta} = -\int \csc^2\theta$

$\cot\theta = \int$ $(\cot\theta)' = \int -\csc^2\theta$ $= -\int \csc\theta$

$(\sec\theta)' = \sec\theta \tan\theta$

$(\csc\theta)' = -\csc\theta \cot\theta$

Q2 (c) $\left(\int_{\tan x}^{0} \frac{dt}{1+t^2}\right)'$ $= \frac{\boxed{0}}{1+0^2} - \frac{\boxed{\sec^2 x}}{1+\tan^2 x} = \frac{-\sec^2 x}{\sec^2 x}$

up, down, (up), (down)', 0

$= -1$

Q3 Find $L(x)$ for $g(x) = 3 + \int_1^{x^2} \sec(t-1)\,dt$ at $x_0 = -1$

up

$L(x) \approx g(x)$ about $x_0 = -1$

line, curve

$g(-1) = 3 + \int_1^{(-1)^2} \sec(t-1)\,dt = 3$

$0 = \int \ldots$

$L(x) = g(x_0) + g'(x_0)(x - x_0)$

$= g(-1) + g'(-1)(x - -1)$

$= 3 + (-2)(x+1)$

$g'(x) = 0 + \sec(x^2-1)\,\boxed{2x} - \cdots \boxed{0}$

$= \sec(x^2-1)\,(2x)$

$g'(-1) = \sec((-1)^2-1)\,(2(-1))$

$= -2\sec 0$

$= -2\,\frac{1}{\cos 0}$

$= (-2)(1)$

$= -2$

$\boxed{L(x) = 1 - 2x}$

$x_0 = -1$

Estimate $g(-1.1)$

$g(-1.1) \approx L(-1.1) = 1 - 2(-1.1)$

$= 1 + 2.2$

$$g(-1.1) \approx L(-1.1) = 1 - 2(-1.1) \quad = -2$$

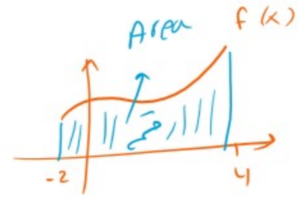
$$= 1 + 2.2$$

$$= 3.2$$

$$\left(\int_{-2}^{4} \frac{1+\cos^3\theta}{\sec^2\theta - \sin^2\theta}\, d\theta\right)' = \left(\text{رقم}\right)' = 0$$

$+$

موجب فقط اذا كان ما داخل التكامل اشاران موجب وبالتالي يمثل مساحة



---

up

$$\left(\int_{1}^{x^2} (2t+1)\, dt\right)' = \left(t^2 + t \Big|_{1}^{x^2}\right)' = \left((x^2)^2 + x^2 - (1^2+1)\right)'$$

رقم

ليست رقم

Function of x

$$= \left(x^4 + x^2 - 2\right)' = 4x^3 + 2x - 0$$

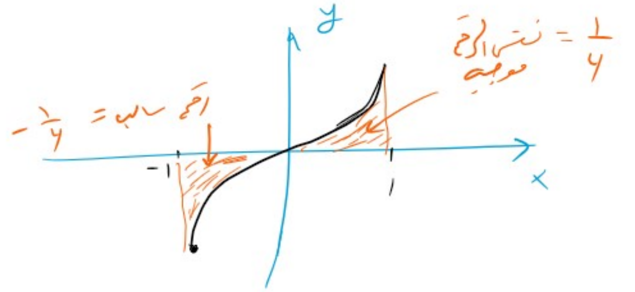
(up) (down)

$$= \left(2x^2+1\right)\boxed{2x} - \cancel{(2(1)+1)}\,\boxed{0}$$

$$= 4x^3 + 2x$$

$$\int_{-1}^{1} x^3\, dx = 0$$

$$\frac{x^4}{4}\Big|_{-1}^{1} = \frac{1}{4} - \frac{1}{4} = 0$$

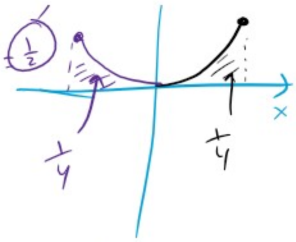


مساحة

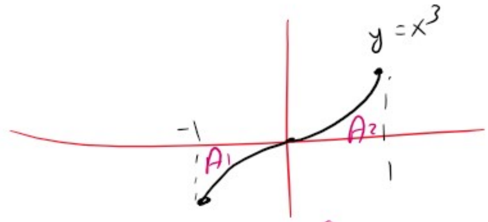
$$\int_{-1}^{1} |x^3|\, dx = \frac{2}{4} = \frac{1}{2}$$

$\frac{x^4}{4}\Big|_{-1}^{1} = \frac{1}{4} - \frac{1}{4}$

$\int_{-1}^{1} |x^3|\,dx = \frac{2}{4} = \frac{1}{2}$



Exp Find area bounded by $y = x^3$ and x-axis on $[-1, 1]$



$$\text{area} = |A_1| + A_2$$
$$= \left|-\frac{1}{4}\right| + \frac{1}{4}$$
$$= \frac{1}{2}$$

$$A_1 = \int_{-1}^{0} x^3\,dx = \frac{x^4}{4} = 0 - \frac{1}{4} = -\frac{1}{4}$$

Q5 d $\int x^3 \sqrt{x^2+1}\,dx$

Anas $+\frac{1}{2}$ $-\frac{1}{2}$ $= 0$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$\int x^{\cancel{3}\,2} \sqrt{u}\,\frac{du}{2\cancel{x}} = \frac{1}{2}\int x^2 \sqrt{u}\,du$$

$$= \frac{1}{2}\int (u-1)\sqrt{u}\,du = \frac{1}{2}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$$

$$= \frac{1}{\cancel{2}}\left[\frac{2}{5}\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{2}{3}\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$$

$$= \frac{1}{5}\sqrt{u^5} - \frac{1}{3}\sqrt{u^3} + C = \frac{1}{5}\sqrt{(x^2+1)^5} - \frac{1}{3}\sqrt{(x^2+1)^3} + C$$

Aya $+\frac{1}{4}$

$u = \sqrt{x^2+1} \checkmark \Rightarrow u^2 = x^2 + 1 \Rightarrow u^2 - 1 = x^2$

$du = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\,dx \Rightarrow du = \frac{x}{u}\,dx \Rightarrow dx = \frac{u\,du}{x}$

$$\int x^3\sqrt{x^2+1}\,dx = \int x^3\,u\,dx = \int x^{\cancel{3}\,2}\,u\,\frac{u\,du}{\cancel{x}}$$

$$\int x^3\sqrt{x^2+1}\,dx = \int x^3\,u\,dx = \int x^3\,u\,\frac{u\,du}{\cancel{x}}$$

$$= \int (u^2-1)\,u^2\,du = \int (u^4-u^2)\,du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \frac{1}{5}\left(\sqrt{x^2+1}\right)^5 - \frac{1}{3}\left(\sqrt{x^2+1}\right)^3 + C$$

ⓐ Find Area bounded by $y = x^2 - 2x$ and $y = x$

$$\cancel{= \left(x + \frac{-2}{2}\right)^2 - 1}$$

$$= (x-1)^2 - 1$$

$$\cancel{= x^2 - 2x + 1 - 1}$$

$$y = x^2 + 6x + 1$$

$$= \left(x + \frac{6}{2}\right)^2 - 8$$

$$= (x+3)^2 - 8$$

$$= x^2 + 6x + 9 - 8$$

$$y = x^2 - 2x = (x-1)^2 - 1$$

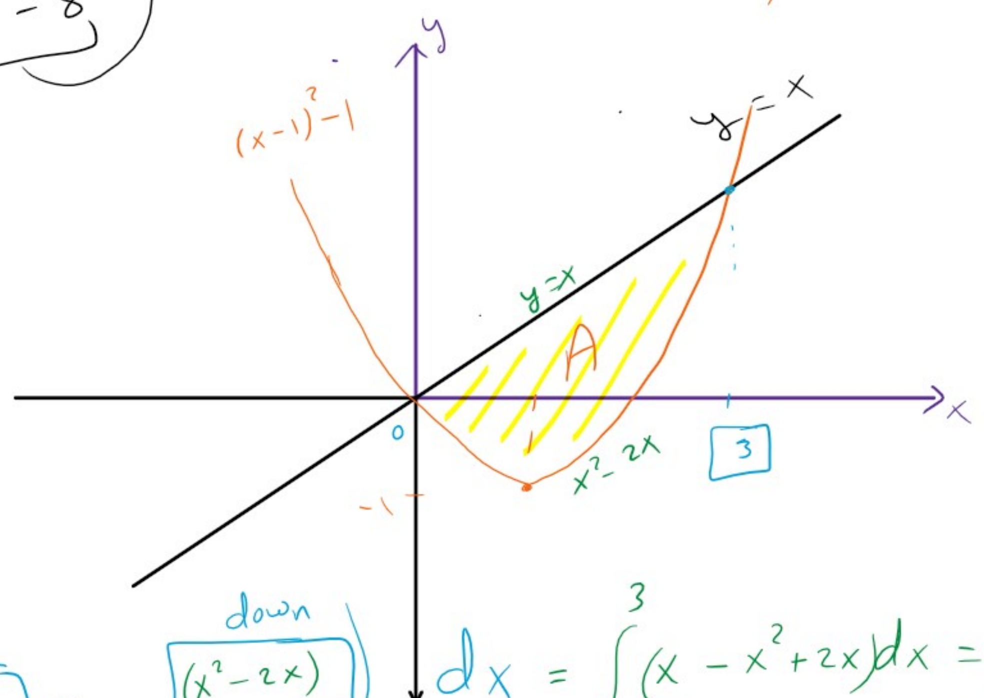
$y = y$

$x = x^2 - 2x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0,\ x = 3$



$$\underset{\uparrow\ \text{Area}}{A} = \int_0^3 \Big(\underbrace{\overset{\text{up}}{\boxed{x}} - \overset{\text{down}}{\boxed{(x^2-2x)}}}_{+}\Big)\,dx = \int_0^3 (x - x^2 + 2x)\,dx = \int_0^3 (3x - x^2)\,dx$$

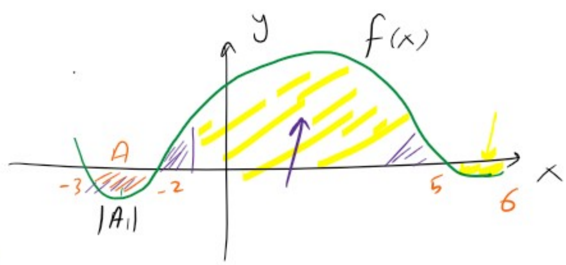
$$= 3\frac{x^2}{2} - \frac{x^3}{3}\Big|_0^3 = \left(3\,\frac{9}{2} - \frac{27}{3}\right) - 0 = \frac{27}{2} - 9$$

$$= \frac{9}{2}$$

> 0 = 1/2

Exp Find Area bound by $y=x^3$ and x-axis on $[-1,1]$

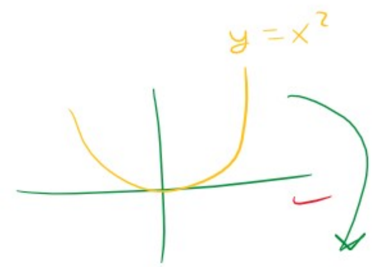
ممنوع $A=0 \Rightarrow |A|=0$ ×



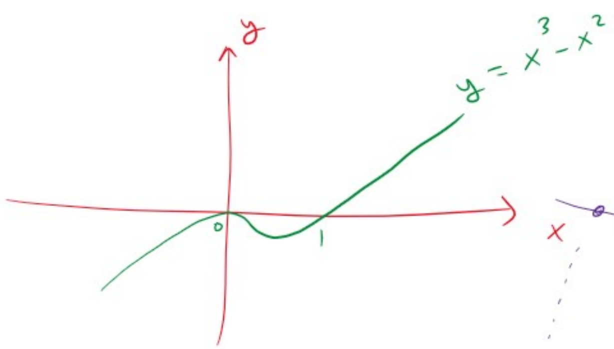
$\int_{-3}^{6} f(x)\,dx \neq$ Area

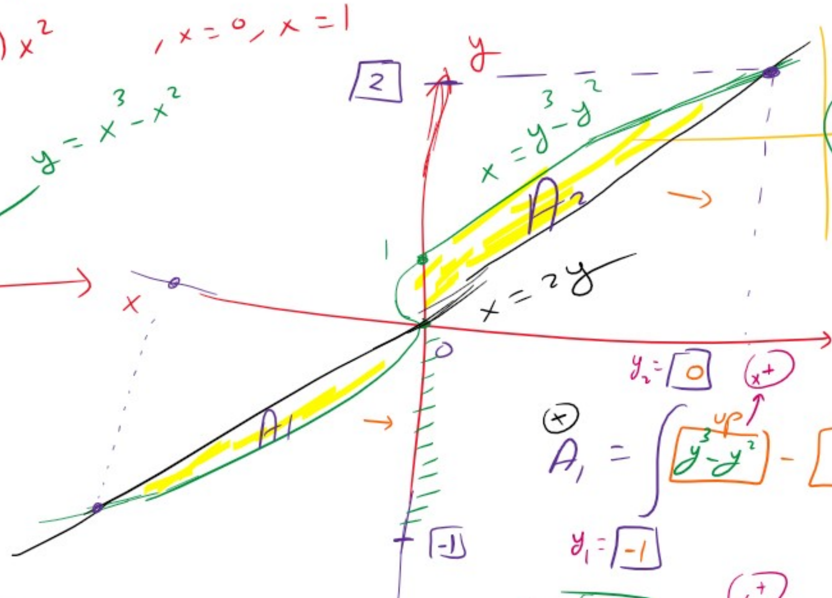
$$\text{Area} = \int_{-3}^{-2} |f(x)|\,dx + \int_{-2}^{5} f(x)\,dx + \int_{5}^{6} |f(x)|\,dx$$

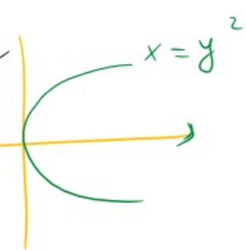
Q6 (d) $x = y^3 - y^2$, $x = 2y$



$y = x^3 - x^2$
$= (x-1)x^2$, $x=0$, $x=1$







$x = x$

$2y = y^3 - y^2$

$y^3 - y^2 - 2y = 0$

$y_2 = \boxed{0}$ up, down

$$A_1 = \int_{y_1=-1}^{y_2=0} \left[\boxed{y^3 - y^2} - \boxed{2y}\right] dy$$

$y_2 = \boxed{2}$ up, down

$A = \int ...$

$$y^3 - y^2 - 2y = 0$$

$$y(y^2 - y - 2) = 0$$

$$y = 0, \quad (y-2)(y+1) = 0$$

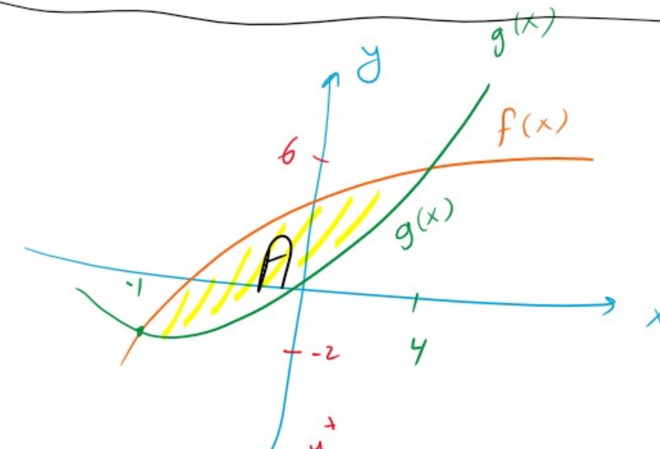
$$y = 2, \quad y = -1$$

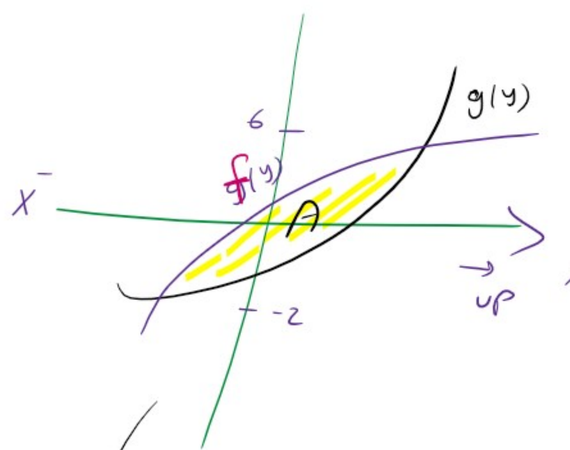
$$A_2 = \int_{y_1 = \boxed{0}}^{y_2 = \boxed{2}} \underbrace{\boxed{2y}}_{up} - \underbrace{\left(\boxed{y^3 - y^2}\right)}_{down}\, dy$$

$$A_1 = \frac{y^4}{4} - \frac{y^3}{3} - y^2 \Big|_{-1}^{0} = 0 - \left(\frac{1}{4} - \frac{-1}{3} - 1\right) = -\frac{1}{4} - \frac{1}{3} + 1 = \frac{5}{12}$$

$$A_2 = y^2 - \frac{y^4}{4} + \frac{y^3}{3} \Big|_{0}^{2} = \cancel{4} - \cancel{\frac{16}{4}} + \frac{8}{3} = \frac{8}{3}$$

$$A = A_1 + A_2 = \frac{5}{12} + \frac{8}{3} = \boxed{\frac{37}{12}}$$





$$A = \int_{-1 = \boxed{x_1}}^{4 = \boxed{x_2}} \underbrace{\boxed{f(x)}}_{up(x)} - \underbrace{\boxed{g(x)}}_{down(x)}\, dx$$

$$= \int_{-2 = \boxed{y_1}}^{6 = \boxed{y_2}} \underbrace{\boxed{g(y)}}_{up(y)} - \underbrace{\boxed{f(y)}}_{down(y)}\, dy$$

نفس الجواب