Problem

$F(F\text{-}1(C)) \subseteq C$

Step-by-step solution

Step 1 of 2

The objective is to prove that,

$$F\left(F^{-1}(C)\right) \subseteq C.$$

The proof can be divided into two parts.

Part-1:

Let F be a function from X to Y and $C \subset Y$.

Let $x \in F\left(F^{-1}(C)\right)$.

By the definition of inverse image,

$$x = F(y) \text{ for some } y \in F^{-1}(C)$$

$$F(y) \in C$$

So, $x \in C$.

Therefore,

$$F\left(F^{-1}(C)\right) \subset C. \quad \text{...... (1)}$$

Step 2 of 2

Part-2:

To show that $F\left(F^{-1}(C)\right) = C.$

Let $x \in C.$

By the definition of function F,

$$x = F(y) \text{ for some } y.$$

Hence, $y \in F^{-1}(C).$

So, $x = F(y) \in F\left(F^{-1}(C)\right).$

This shows that $C \subset F\left(F^{-1}(C)\right).$

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$$F\left(F^{-1}(C)\right) = C. \quad \text{...... (2)}$$

From equations (1) and (2),

$$\boxed{F\left(F^{-1}(C)\right) \subseteq C}$$