11.3 Implicit Differentiation

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الاشتقاق الضمني

Exp Find y' if $y = 2x^3 - 5x + 1$ $\quad y = f(x) \Rightarrow y' = f'(x)$

$$\frac{dy}{dx} = y' = 6x^2 - 5$$

اشتقاق عادي

Ordinary derivative

Exp ① Find $\frac{dy}{dx}$ if $x^2 + y^2 = 26$

$y = f(x)$

$$2x + 2yy' = 0$$

chain Rule

قاعدة السلسلة

$$\frac{\cancel{2}yy'}{\cancel{2}y} = \frac{-\cancel{2}x}{\cancel{2}y}$$

$$\boxed{y' = \frac{dy}{dx} = -\frac{x}{y}}$$

??

② Find $\left.\frac{dy}{dx}\right|_{x=1}$

$x^2 + y^2 = 26$

$1^2 + y^2 = 26$

$\cancel{1} + y^2 = 26$

$\quad -1 \quad -1$

$y^2 = 25$

$x = 1 \Rightarrow (1, 5)$

$\Rightarrow (1, -5)$

$\frac{dy}{dx} \Big|$ = — — 1 slope

$\frac{dy}{dx}\Big|_{(1,5)} = -\frac{1}{5}$ slope

$\frac{dy}{dx}\Big|_{(1,-5)} = -\frac{1}{-5} = \frac{1}{5}$ slope

$\sqrt{y^2} = \sqrt{25}$

$|y| = 5$

$y = \pm 5$

Exp ① Find y' if $6y = x^3 + y^2$

$y = f(x)$

$\frac{dy}{dx} = y' = f'(x)$

$6y' = 3x^2 + 2yy'$

$6y' - 2yy' = 3x^2$

$\frac{\cancel{2}y'\cancel{[3-y]}}{\cancel{2}\cancel{(3-y)}} = \frac{3x^2}{2(3-y)}$

$y' = \frac{dy}{dx} = \frac{3x^2}{2(3-y)}$??

② Find slope at $\boxed{x=0}$

3 .. 2

To find $y \Rightarrow$ $\quad 6y = x^3 + y^2$

$$6y = 0^3 + y^2$$

$$6y = y^2$$

$$y^2 - 6y = 0$$

$$y(y-6) = 0$$

$$\boxed{y=0} \quad \text{or} \quad \boxed{y=6}$$

$$x = 0 \qquad\qquad x = 0$$

$$(0,0) \checkmark \qquad\qquad (0,6) \checkmark$$

slope at $(0,0)$ is

$$\left.\frac{dy}{dx}\right|_{(0,0)} = \left.\frac{3x^2}{2(3-y)}\right|_{(0,0)}$$

$$= \frac{3(0)^2}{2(3-0)} = \frac{0}{6} = 0$$

slope at $(0,6)$ is

$$\left.\frac{dy}{dx}\right|_{(0,6)} = \left.\frac{3x^2}{2(3-y)}\right|_{(0,6)}$$

$$= \frac{3(0)^2}{2(3-6)} = \frac{0}{2(-3)} = 0$$

Exp If $3y^4 + x^5 - 6xy^2 = 0$ Find y'

$$3(4)y^3 y' + 5x^4 - [6x\,(2)y\,y' + y^2(6)] = 0$$

$$12y^3 y' - 12xy\,y' + 5x^4 - 6y^2 = 0$$

$$12yy' - 12xyy' + 5x' - 6y = 0$$

$$\frac{\cancel{12y}\, y' \left[\cancel{y^2 - x}\right]}{\cancel{12y}\ \cancel{(y^2 - x)}} = \frac{6y^2 - 5x^4}{12y\,(y^2 - x)}$$

$$\boxed{y' = \frac{dy}{dx} = \frac{6y^2 - 5x^4}{12y\,(y^2 - x)}}$$

Exp Find tangent line for the curve

$$x\ln y + 2xy = 2 \quad \text{at} \quad (\overset{x_0}{1}, \overset{y_0}{1})$$

$$y - y_0 = m(x - x_0) \qquad m = y'\Big|_{(1,1)} = \frac{dy}{dx}\Big|_{(1,1)} = -\frac{2}{3}$$

$$y - 1 = \boxed{\frac{-2}{3}}(x - 1)$$

$$x\ln y + 2xy = 2 \qquad (\overset{x}{1}, \overset{y}{1})$$

$$\left[x\,\frac{y'}{y} + \ln y\,(1)\right] + \left[2x\,y' + y\,(2)\right] = 0$$

$$\left[x \, \frac{y'}{y} + \ln y \quad (1) \right] + \left[2x\,y' + 2(\ldots) \right]$$

$$(1)\,\frac{y'}{(1)} + \ln(1) + 2(1)\,y' + (1)(2) = 0$$

$$\ln(1) \to 0$$

$$y' + 0 + 2y' + 2 = 0$$

$$3y' = -2$$

$$y' = -\frac{2}{3} = m$$

Tangent line

$$y - 1 = -\frac{2}{3}(x - 1)$$

$$y - \cancel{1} = -\frac{2}{3}x + \frac{2}{3}$$

$$+1 \qquad\qquad +1$$

$$\frac{2}{3} + 1$$

$$\frac{2}{3} + \frac{3}{3}$$

$$\frac{5}{3}$$

$$\boxed{y = -\frac{2}{3}x + \frac{5}{3}}$$

Exp Find $\left.\frac{dq}{dp}\right|_{p=50}$ if $\underbrace{2p^2}_{\text{اول}}\,\underbrace{q}_{\text{ثاني}} + p^2 = 100{,}000$

$q' \leftarrow \frac{dq}{dp}$

(find $q \Rightarrow$)

$p = 50$

$$2p^2 q' + q(4p) + 2p = 0$$

$$2(50)^2 q' + (19.5)(4)(50) + 2(50) = 0$$

$$2(2500) q' + 3900 + 100 = 0$$

$$5000 q' + 4000 = 0$$

$$-4000 \qquad -4000$$

$$\frac{5000}{5000} q' = \frac{-4000}{5000}$$

$$q' = \frac{dq}{dp} = -0.8$$

To find q ⇒

$$2(50)^2 q + (50)^2 = 100,000$$

$$2(2500) q + 2500 = 100,000$$

$$5000 q + 2500 = 100,000$$

$$-2500 \qquad -2500$$

$$\frac{5000 q}{5000} = \frac{97,500}{5000}$$

$$\boxed{q = 19.5}$$