

Exp Find y' if $y = \left(\frac{x^2}{2x+1} \right)^4$
(Power of Quotient)

$$y' = 4 \left(\frac{x^2}{2x+1} \right)^3 \left[\frac{(2x+1)(2x) - (x^2)(2)}{(2x+1)^2} \right]$$

$$= 4 \left(\frac{x^2}{2x+1} \right)^3 \left[\frac{4x^2 + 2x - 2x^2}{(2x+1)^2} \right]$$

$$= 4 \frac{x^{\frac{(2)}{(3)}}}{(2x+1)^3} \left[\frac{2x^2 + 2x}{(2x+1)^2} \right]$$

$$= \frac{4 x^6 (2x^2 + 2x)}{(2x+1)^{3+2}}$$

$$= \frac{4 x^6 2x(x+1)}{(2x+1)^5}$$

$$= \frac{8 x^7 (x+1)}{(2x+1)^5}$$

$$x^2 x^3 = x^5$$

$$y'(1) = \frac{8 \cdot 1^7 \cdot (1+1)}{(2 \cdot 1 + 1)^5} = \frac{8 \cdot (2)}{3^5} = \frac{16}{\underbrace{(3)(3)(3)(3)(3)}} = \frac{16}{243}$$

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Exp (Quotient of two Powers)

Find y' if $y = \frac{(x^4+3)^3}{(x-1)^2}$. Simplify

$$y' = \frac{\frac{d}{dx}(x^4+3)^3 \cdot (x-1)^2 - (x^4+3)^3 \cdot \frac{d}{dx}(x-1)^2}{(x-1)^4}$$

$$= \frac{12x^3 (x-1)^2 (x^4+3)^2 - 2(x-1)(x^4+3)^3}{(x-1)^4}$$

$$= \frac{(x-1) \left[12x^3 (x-1) (x^4+3)^2 - 2(x^4+3)^3 \right]}{(x-1)^4}$$

$$= \frac{2(x^4+3)^2 \left[6x^3 (x-1) - (x^4+3) \right]}{(x-1)^3}$$

$$= \frac{2(x^4+3)^2 \left[\frac{(x-1)}{(x-1)^3} (6x^4 - 6x^3 - x^4 - 3) \right]}{(x-1)^3}$$

$$= \frac{2(x^4+3)^2 (5x^4 - 6x^3 - 3)}{(x-1)^3}$$

Exp (Product with Power)

Find $f'(x)$ if $f(x) = (x^2-1)^2 \sqrt{3-x^2}$

$$= (x^2-1)^2 (3-x^2)^{\frac{1}{2}}$$

$$f'(x) = (x^2-1)^2 \left(\frac{1}{2} \right) (3-x^2)^{\frac{1}{2}-1} (-2x) + (3-x^2)^{\frac{1}{2}} (2x)$$

$$= -x(x^2-1)(3-x^2)^{-\frac{1}{2}} + 2x\sqrt{3-x^2}$$

$$= -x(x^2-1) \frac{1}{(3-x^2)^{\frac{1}{2}}} + 2x\sqrt{3-x^2}$$

$$\begin{aligned}
 & \quad (3-x) \\
 &= \frac{-x(x^2-1)}{\sqrt{(3-x^2)'}} + \frac{2x \sqrt{3-x^2} \sqrt{3-x^2}}{\sqrt{3-x^2}} \\
 &= \frac{-x(x^2-1) + 2x(3-x^2)}{\sqrt{3-x^2}} \\
 &= \frac{-x^3 + x + 6x - 2x^3}{\sqrt{3-x^2}} \\
 &= \frac{-3x^3 + 6x}{\sqrt{3-x^2}} \\
 &= \frac{3x(2-x^2)}{\sqrt{3-x^2}} \quad \checkmark
 \end{aligned}$$

Exp Find $g'(-2)$ if $g(x) = (x^2+3x)^4 \left(5-\frac{x}{2}\right)^3$

Exp Find $g'(-2)$ if $g(x) = \underbrace{(x^2+3x)}_{u_1} \underbrace{\left(5-\frac{x}{2}\right)}_{v_1}$

$$g'(x) = \underbrace{(x^2+3x)}_{u_1}^4 (3) \underbrace{\left(5-\frac{x}{2}\right)^2}_{v_1}^2 \left(-\frac{1}{2}\right) + \underbrace{\left(5-\frac{x}{2}\right)^3}_{v_1}^3 (4) \underbrace{(x^2+3x)^3}_{u_1}^{2x+3}$$

$$\begin{aligned} g'(-2) &= \left(\boxed{-2}^2 + 3\boxed{-2}\right)^4 (3) \left(5 - \frac{\boxed{-2}}{2}\right)^2 \left(-\frac{1}{2}\right) + \left(5 - \frac{\boxed{-2}}{2}\right)^3 (4) \left(\boxed{-2}^2 + 3\boxed{-2}\right)^3 \\ &= (4 - 6)^4 (3) (6)^2 \left(-\frac{1}{2}\right) + (6)^3 (4) (4 - 6)^3 (-4 + 3) \\ &= (-2)^4 (3) (\cancel{36}^{18}) \left(-\frac{1}{2}\right) + (216) (\cancel{4}) (-2)^3 \cancel{(-1)}^3 \\ &= (16) (3) (18) (-1) + (216) (-4) (-8) \\ &= -864 + 6912 \\ &= 6048 \end{aligned}$$