Fundamentals Physics

Tenth Edition

Halliday

Chapter 8

Potential Energy and Conservation of Energy

8-1 Potential Energy (2 of 15)

- **Potential energy** *U* is energy that can be associated with the configuration of a system of objects that exert forces on one another
- A system of objects may be:
  - Earth and a bungee jumper
    - **Gravitational potential energy** accounts for kinetic energy increase during the fall
    - **Elastic potential energy** accounts for deceleration by the bungee cord
- Physics determines how potential energy is calculated, to account for stored energy

8-1 Potential Energy (3 of 15)

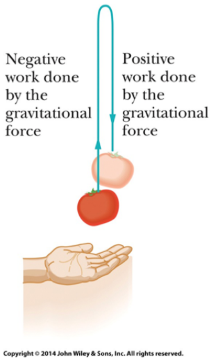
- For an object being raised or lowered:

$$\Delta U = -W. \qquad \textbf{Equation (8-1)}$$

- The change in gravitational potential energy is the negative of the work done
- This also applies to an elastic block-spring system

8-1 Potential Energy (4 of 15)

Against



with

Figure 8-2

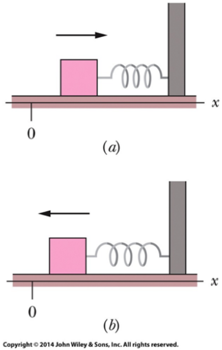


Figure 8-3

8-1 Potential Energy (5 of 15)

- Key points:

1. The system consists of two or more objects
2. A force acts between a particle (tomato/block) and the rest of the system
3. When the configuration changes, the force does work W_1, changing kinetic energy to another form
4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2

- Thus the **kinetic energy** of the tomato/block **becomes potential energy**, **and** then kinetic energy **again**

8-1 Potential Energy (6 of 15)

- **Conservative forces** are forces for which $W_1 = -W_2$ is always true
  - Examples: gravitational force, spring force
  - Otherwise we could not speak of their potential energies
- **Nonconservative forces** are those for which it is false
  - Examples: kinetic friction force, drag force
    - Kinetic energy of a moving particle is transferred to heat by friction
    - Thermal energy cannot be recovered back into kinetic energy of the object via the friction force
    - Therefore the force is not conservative, thermal energy is not a potential energy

8-1 Potential Energy (7 of 15)

- When only conservative forces act on a particle, we find many problems can be simplified:

  The net work done by a conservative force on a particle moving around any closed path is zero.

- A result of this is that:

  The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

8-1 Potential Energy (8 of 15)

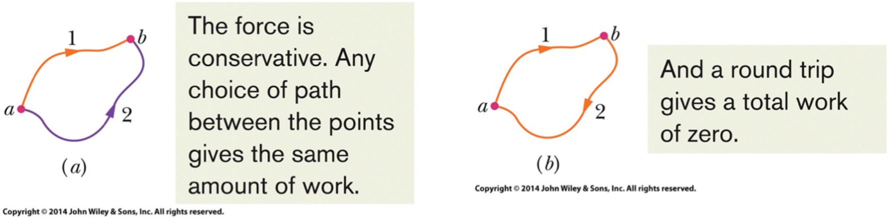


Figure 8-4

8-1 Potential Energy (9 of 15)

- Mathematically:

$$W_{ab,1} = W_{ab,2},$$ **Equation (8-2)**

- This result allows you to substitute a simpler path for a more complex one if only conservative forces are involved

8-1 Potential Energy (10 of 15)

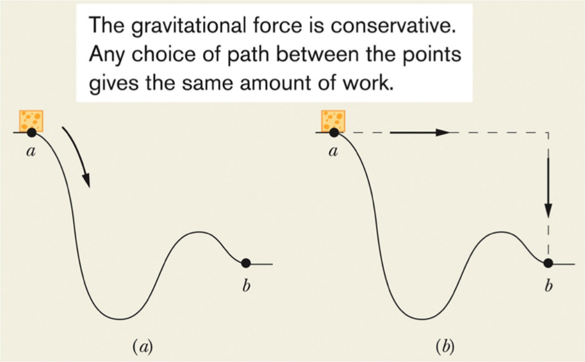
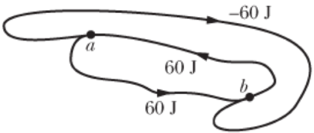


Figure 8-5

8-1 Potential Energy (11 of 15)

Checkpoint 1

The figure shows **three paths** connecting points *a* and *b*. A single force $\vec{F}$ does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force $\vec{F}$ **conservative?**



Answer:

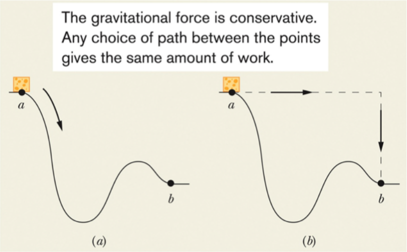
No. The paths from $a \rightarrow b$ have different signs. One pair of paths allows the formation of a zero-work loop. The other does not.

Sample Problem 8.01

Equivalent paths for calculating work, slippery cheese

2.0 kg block of slippery cheese, slides, frictionless, from *a* to *b*. Total distance = 2.0 m, net vertical distance of 0.80 m.

How much work is done on the cheese by the gravitational force during the slide?



Choose the dashed path in Fig.

Two straight segments, Along the horizontal segment, the angle is a constant 90˚:

$W_h = mgd \cos 90^\circ = 0.$

Along the vertical segment, the displacement d is 0.80 m and, with $\vec{F}_g$ and $\vec{d}$ both downward, the angle is a constant 0˚:

$W_v = mgd \cos 0^\circ = (2.0 \text{ kg})(9.8 \text{ m/s}^2)(0.80 \text{ m})(1) = 15.7 \text{ J}.$

The total work = $W = W_h + W_v = 0 + 15.7\text{J} = 16\text{J}$. (this is the work along any track)

8-1 Potential Energy (12 of 15)

- For the general case, we calculate work as:

$$W = \int_{x_i}^{x_f} F(x)\,dx. \qquad \textbf{Equation (8-5)}$$

- So we calculate potential energy as:

$$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx. \qquad \textbf{Equation (8-6)}$$

- Using this to calculate gravitational PE, relative to a **reference configuration** with **reference point** $y_i = 0$:

$$U(y) = mgy \qquad \textbf{Equation (8-9)}$$

8-1 Potential Energy (13 of 15)

The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, **not on the horizontal position**.

8-1 Potential Energy

- Use the same process to calculate spring PE:

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\,dx = \int_{x_i}^{x_f} x\,dx = \frac{1}{2}k\left[x^2\right]_{x_i}^{x_f},$$ **Equation (8-10)**

$$\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2.$$

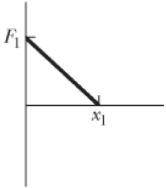
- With reference point $x_i = 0$ for a relaxed spring:

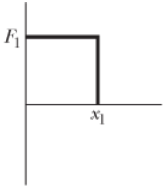
$$U(x) = \frac{1}{2}kx^2$$ **Equation (8-11)**

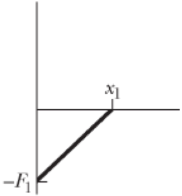
8-1 Potential Energy (15 of 15)

Checkpoint 2

A particle is to move along an x axis from $\boldsymbol{x = 0}$ **to** $\boldsymbol{x_1}$ while a **conservative force**, directed along the x axis, acts on the particle. The figure shows three situations in which the $\boldsymbol{x}$ **component of that force varies with** $\boldsymbol{x}$. The force has the **same maximum magnitude** $\boldsymbol{F_1}$ in all three situations. **Rank** the situations according to the change in the associated **potential energy during** the particle's **motion**, most positive first.

(1) 

(2) 

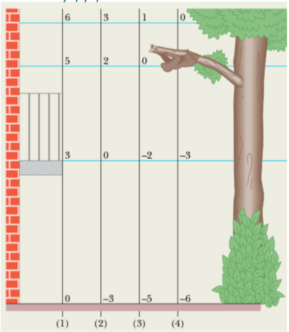
(3) 

Answer:

(3), (1), (2); a positive force does positive work, decreasing the PE; a negative force (e.g., 3) does negative work, increasing the PE

Sample Problem 8.02

Choosing reference level for gravitational potential energy, sloth



A 2.0 kg sloth hangs 5.0 m above the ground:
(a) What is the gravitational potential energy U of the sloth – Earth system if we take the reference point $y = 0$ to be (1) at the ground, (2) at a balcony floor that is 3.0 m above the ground, (3) at the limb, and (4) 1.0 m above the limb? Take the gravitational potential energy to be zero at $y = 0$.

For choice (1) the sloth is at y=5.0m, and $U = mgy$ = (2.0 kg)(9.8 m/s2)(5.0 m) = 98 J
(2) $U=mgy=mg$(2.0m)=39J,
(3) $U=mgy=mg$(0)=0J,
(4) $U = mgy = mg$(–1.0 m) = –19.6 J =–20 J.

8-2 Conservation of Mechanical Energy (2 of 5)

- The mechanical energy of a system is the sum of its potential energy U and kinetic energy K:

$$E_{\text{mec}} = K + U$$ **Equation (8-12)**

- Work done by conservative forces increases K and decreases U by that amount, so:

$$\Delta K = -\Delta U.$$ **Equation (8-15)**

- Using subscripts to refer to different instants of time:

$$K_2 + U_2 = K_1 + U_1$$ **Equation (8-17)**

8-2 Conservation of Mechanical Energy (3 of 5)

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

8-2 Conservation of Mechanical Energy (4 of 5)

- This is the principle of the **conservation of mechanical energy:**

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0. \qquad \textbf{Equation (8-18)}$$

- This is very powerful tool:

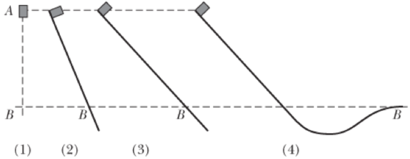
When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant without considering the intermediate motion and without finding the work one by the forces involved.

- One application:
  - Choose the lowest point in the system as $U = 0$
  - Then at the highest point $U = \text{max}$, and $K = \text{min}$

8-2 Conservation of Mechanical Energy (5 of 5)

Checkpoint 3

The figure shows **four situations—one** in which an **initially stationary block is dropped** and **three** in which the **block is allowed to slide** down frictionless ramps. (a) **Rank** the situations according to the **kinetic energy** of the block **at** point ***B***, greatest first. (b) **Rank** them according to the **speed** of the block **at** point ***B***, greatest first.



Answer: all are equal in (a) and (b)

Since there are **no nonconservative forces**, all of the difference in potential energy must go to kinetic energy. Therefore **all are equal in (a)**. Because of this fact, they are **also all equal in (b).** $\Delta E_{mec} = \Delta K + \Delta U = 0.$

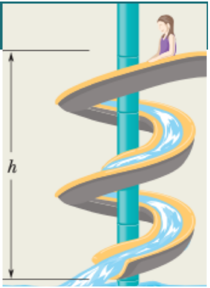
Sample Problem 8.03

Conservation of mechanical energy, water slide

**The advantage of using the conservation of energy instead of Newton's laws of motion is that we do not need to consider all the intermediate motion.

Child, mass *m,* released from rest at the top of a water slide, $h = 8.5$ m, the slide is frictionless because of the water.

Find the child's speed at the bottom of the slide?



The total mechanical energy at the top is equal to the total at the bottom.

$E_{\text{mec},bottom} = E_{\text{mec},top}$

$$\tfrac{1}{2}mv_b^2 + mgy_b = \tfrac{1}{2}mv_t^2 + mgy_t$$

Dividing by *m and rearranging:*

$$v_b^2 = v_t^2 + 2g(y_t - y_b)$$ ($v_t = 0$ and $y_t - y_b = h$)

$$v_b = \sqrt{2gh} = \sqrt{(2)(9.8\text{ m/s}^2)(8.5\text{ m})} = 13\text{ m/s.}$$

Extra example 1:

13. A 2-kg block is thrown upward from a point 20 m above the Earth's surface. At what height above Earth's surface will the gravitational potential energy of the Earth-block system have increased by 500 J?

A) 5 m

B) 25 m

C) 46 m

D) 70 m

E) 270 m

Extra example 2:

A 0.20-kg particle moves along the x axis under the influence of a conservative force. The potential energy is given by $U(x) = (8.0\ \text{J/m}^2)x^2 + (2.0\ \text{J/m}^4)x^4$,

where x is in coordinate of the particle. If the particle has a speed of 5.0 m/s when it is at $x = 1.0$ m, its speed when it is at the origin is:

A) 0 m/s

B) 2.5 m/s

C) 5.7 m/s

D) 7.9 m/s

E) 11 m/s