Chapter 2: Utility and Choices

Utility

Utility is a concept in economics that refers to the satisfaction or happiness that individuals derive from consuming goods and services.

Utility is a subjective concept, as different people derive different levels of satisfaction from the same goods and services.

Understanding utility is important in economics because it helps to explain consumer behavior and choices.

المنفعة هي مفهوم في الاقتصاد يشير إلى الرضا أو الاشباع الذي يحصل عليه الأفراد من استهلاك السلع والخدمات. الاشباع مفهوم شخصي ذاتي، حيث يستمد الأشخاص المختلفون مستويات مختلفة من الرضا او الاشباع من نفس السلع والخدمات. فهم المنفعة مهم في الاقتصاد. لأنه يساعد على تفسير سلوك المستهلك واختياراته.

Utility from Consuming Two Goods

Assume that the person receives utility from consuming two goods X and Y. The utility function is given by:

$$\text{Utility} = U(X, Y ; \text{other things})$$

This notation indicates that the utility an individual receives from consuming X and Y over some period of time depends on the quantities of X and Y consumed and on ‘‘other things.’’

يشير هذه العلاقة الى ان الاشباع الذي يحصل عليه المستهلك عند استهلاكه سلعتين X و Y خلال فترة زمنية معينة تعتمد على كميات المستهلكة من السلعة X و Y وعلى "أشياء أخرى".

These other things might include easily quantifiable items such as the amounts of other kinds of goods consumed, the number of hours worked, or the amount of time spent sleeping. They might also include such unquantifiable items as love, security, and feelings of self-worth.

تعتمد المنفقة التي يحصل عليها المستهلك من استهلاكه سلعتين X و Y خلال فترة زمنية على الكمية التي يستهلكها من السلعةX و الكمية التي يستهلكها من السلعة Y و على أشياء أخرى. تشمل الأشياء الأخرى على عناصر قابلة للقياس بسهولة مثل كميات من أنواع أخرى من السلع التي يستهلكها الشخص، وعدد ساعات العمل، أو مقدار الوقت الذي يقضيه في النوم. وقد تشمل أيضا أشياء لا يمكن تحديدها كمياً كالحب، والأمن، ومشاعر الذاتية.

These other things appear after the semicolon because we assume that they do not change (other things equal) while we look at the individual’s choice between X and Y.

تم وضع الأشياء الأخرى بعد الفاصلة المنقوطة لأننا نفترض أنها لا تتغير (الأشياء الأخرى متساوية) عندما يتم دراسة الاشباع الذي يحصل عليه المستهلك من استهلاكه السلعتين X و.Y

By assuming other things equal, the utility function form become:

$U = U(X, Y)$ → *Utility function*

U = refer to the level of satisfaction you receive from consuming the bundle of goods consisting X and Y.

Measuring Utility قياس المنفعة

A quantification of the satisfaction of wants and needs achieved through the consumption of goods and services. In principle, utility measurement can take one of two forms: (1) cardinal, which is based on numerical values (1, 2, 3, etc.) and (2) ordinal which is based on rankings (first, second, third, etc.). While the

Hypothetical instructional analysis of utility relies on cardinal utility, ordinal utility is a more realistic way to measure satisfaction.

Ordinal versus Cardinal Utility المنفعة الترتيبية والمنفعة العددية

Cardinal utility: The belief that utility can be measured and compared on a unit by unit basis, e.g. Utility measure of 200 is twice as big as a utility measure of 100.

Ordinal utility: Where you rank a bundle of goods but you cannot say how big one bundle is compared to the other.

A utility function that describes by how much one market basket is preferred to another is called cardinal utility. For example, if the consumer can say, the total utility derived from consuming good A is equal to 20, the utility is cardinal.

تقوم المنفعة العددية على التعبير عن عدد وحدت المنفعة المشتقة من سلعة ما عند استهلاكها. فعلى سبيل المثال يمكن أن نقول بأن عدد وحدات المنفعة التي يحصل عليها ماجد عند استهلاكه سلعة معينة تساوي 20 وحدة منفعة.

A utility that generates a ranking of goods or market baskets is called ordinal utility, for example, good A is preferred to good B. or ranking a utility from the same goods by comparing the 1st unit and 2nd unit from utility.

تقوم المنفعة الترتيبية على المقارنة المشتقة من السلع أو مقارنة المنفعة المشتقة من الوحدة الأولى و الوحدة الثانية من نفس السلعة. فعلى سبيل المثال يمكن القول بأن المنفعة التي يحصل عليها ماجد من تفاحة أكبر من المنفعة التي يحصل عليها من برتقالة. وكذلك يمكن القول بأن المنفعة التي يحصل عليها ماجد من كوب الشاي الأول أكبر من المنفعة التي يحصل عليها من كوب الشاي الثاني.

Assumptions about Preferences:

Some Basic Assumptions about Preferences:

Completeness:
Complete preferences: The assumption that an individual is able to state which of any two options is preferred.

Between two consumption bundles, A and B, we might expect a person to be able to state clearly "I prefer A to B" or" I prefer B to A" or "A and B are equally attractive to me"(will be indifferent between the two).

Note that these preferences ignore costs. A consumer might prefer steak to hamburger but buy hamburger because it is cheaper.

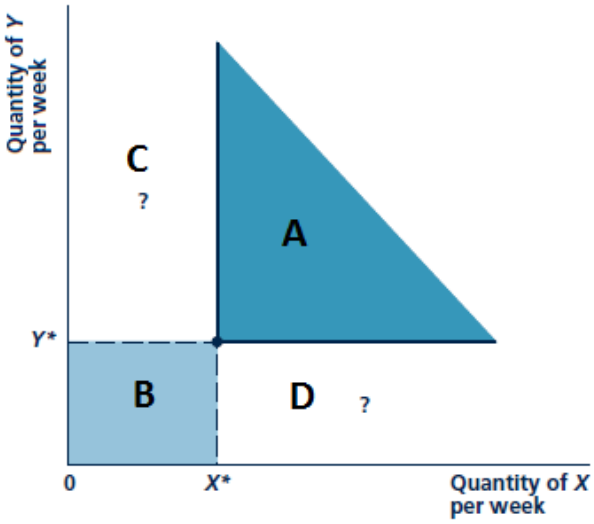
Transitivity:
Transitivity of preferences: The property that if A is preferred to B, and B is preferred to C, then A must be preferred to C.

Example
If a man prefers Tea to Pepsi and prefers Pepsi to Orange Juice, but he is indifferent between Tea and Coffee, transitivity of preferences means that he must

A. prefer orange juice to tea
B. prefer coffee to orange juice
C. prefer Pepsi to coffee
D. be indifferent between Pepsi and coffee

More Is Better: Defining an Economic "Good":

A third assumption we make about preferences is that a person prefers more of a good to less. In Figure 2.1, all points in the darkly shaded area are preferred to the amounts of X^* of good X and Y^* of good Y. Movement from point X^*, Y^* to any point in the shaded area is an unambiguous improvement, since in this area this person gets more of one good without taking less of another. This idea leads us to define an "economic good" as an item that yields positive benefits to people. That is, more of a good is, by definition, better. Combinations of goods in the lightly shaded area of Figure 2.1 are definitely inferior to X^*, Y^* since they offer less of both goods.



إن من إحدى البديهيات حول سلوك المستهلك أنه يفضل الأكثر على الأقل عند استهلاك. بمعنى, أنه لو حصل المستهلك على كمية أكبر من أحدى السلعتين التي يستهلكها دون أن يقلل استهلاكه من السلعة الأخرى, فأن ذلك سيزيد من منفعته. أما إذا حصل المستهلك على كمية أقل من إحدى السلعتين التي يستهلكها دون أن يزيد استهلاكه من السلعة الأخرى، فإن ذلك سوف يقلل من منفعته. في الشكل المجاور، تيفضل المستهلك جميع النقاط في المنطقة A على كميات X* من السلعة X وY* من من السلعة Y. الانتقال من النقطة X*، Y* إلى أي نقطة في المنطقة A يحصل هذا الشخص على كميات أكثر من سلعة دون أن يأخذ أقل من سلعة أخرى. وتقودنا هذه الفكرة إلى تعريف "السلعة الاقتصادية" باعتبارها سلعة تعود بفوائد إيجابية على الشخص. وهذا يعني أن المزيد من السلعة هو، بحكم التعريف، أفضل. من

المؤكد أن مجموعات السـلع الموجودة في المنطقة Bفي الشـكل هي أقل من X* وY* لأنها تقدم كمية أقل من كلا السـلعتين. الانتقال من النقطة *X، Y* إلى أي نقطة في المنطقة C , D يحصـل هذا الشـخص على كميات أكثر من سـلعة بينما يحصـل على كميات اقل من السـلعة الاخرى وبالتالي لا نستطيع تحديد وضع المستهلك.

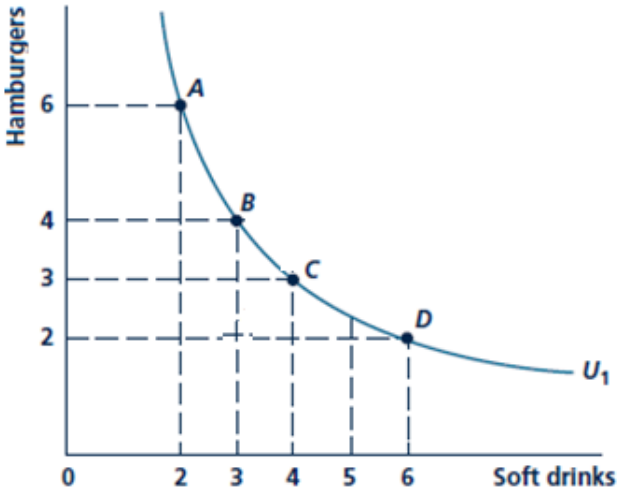
Voluntary Trade and Indifference Curves

Indifference Curve

Indifference curve: A curve that shows all the combinations of goods or services that provide the same level of utility.

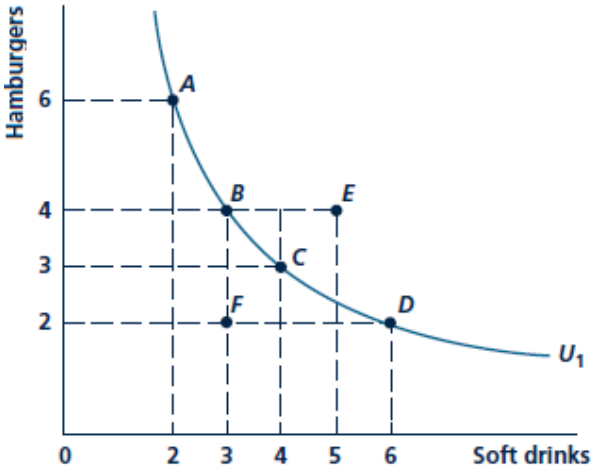
هو ذلك المنحنى الذي يمثل المجموعات المختلفة (من سلعتين) الذي يحصل المستهلك عند استهلاكها على نفس المستوى من المنفعة.

The curve U1 in the Figure includes all combinations of Hamburgers and Soft drink that yield the same level of utility. Point A, with 6 units of Hamburgers and 2 units of Soft drink, has the same utility as point B, 4 Hamburgers and 3 Soft drink. Since **all points on the curve (A, B,C and D) yield the same utility**, the person has no reason to prefer one point over another.



Points above and below an Indifference Curve:

Point *E* (five soft drinks and four hamburgers) is preferred to point *C* because it provides more of both goods. As in Figure, our definition of economic goods assures that combination *E* is preferred to combination *C*. Similarly, the assumption of transitivity assures that combination *E* is also preferred to combinations *A*, *B*, and *D* and to all other combinations on *U*1.



- Point E is preferred to A, B, C, and D

Points above an indifference curve are preferred to points on the curve.

In the Figure, points such as F are below of an indifference curve U_1. Point C is preferred to point F since it contains more of both goods. Because of transitivity, all points on U1 (A, B, C and, D) are preferred to point F.

- Points A, B, C, and D are preferred to point F

Points on an indifference curve are preferred to points below it.

Example

Consider the utility function for goods X and Y as given by U (X, Y) = $\sqrt{X\,Y}$. Which of the following bundles lie on the same indifference curve as the bundle containing 20 units of X and 5 units of Y?

A. The bundle containing 10 units of X and 5 unit of Y
B. The bundle containing 8 unit of X and 12 units of Y
C. The bundle containing 25 units of X and 4 units of Y
D. The bundle containing 20 units of X and 10 units of Y

Example

Suppose the person has preferences for two goods X and Y given by: $U = \sqrt{X\,Y}$

A. If X = 5 and Y = 80, what will utility be?

U = $\sqrt{5 * 80}$ = $\sqrt{400}$ = 20

B. If X = 10, what value for Y will provide that same utility as in part a?

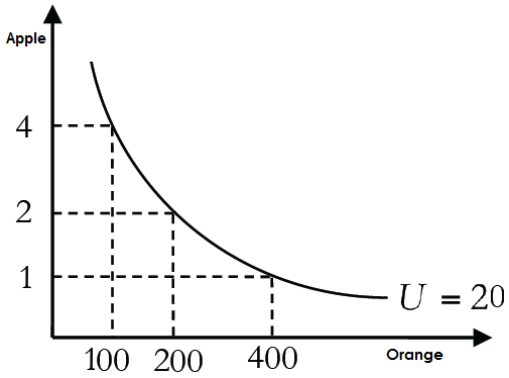
U= $\sqrt{XY}$ → 20 = $\sqrt{10\,Y}$ → 10 Y = 400 → Y = $\frac{400}{10}$ = 40

C. Graph the indifference curve when U= 20.

When U = 20 → 20 = $\sqrt{XY}$

بتربيع الطرفين → 400 = X Y → Y = $\frac{400}{X}$

X	Y
1	400/1 = 400
2	400/2 = 200
4	400/4 = 100



Example

Taleen consumes only two goods X and Y. Her utility function is given by: $U\,(X,Y) = X\,Y^2$.

1. Is the assumption that more is better satisfied for both goods?

   Yes, the "more is better" assumption is satisfied for both goods since as amounts of both good X and Y increases, utility increase.

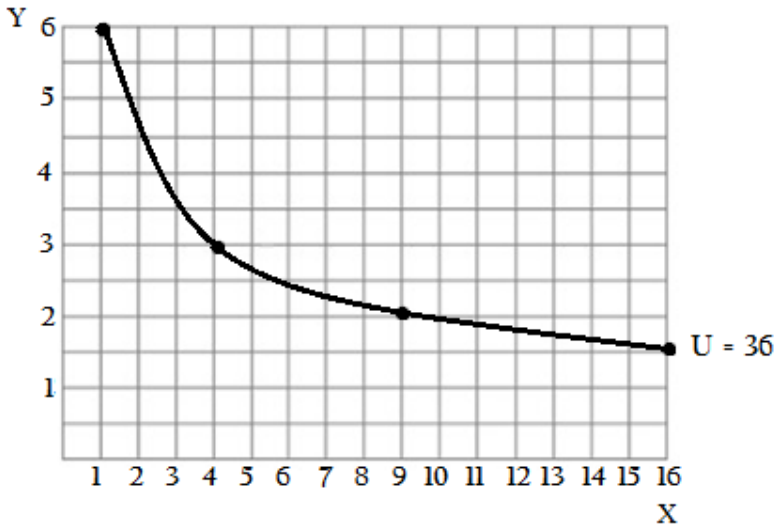
   $X \uparrow \rightarrow U \uparrow \; and \; Y \uparrow \rightarrow U \uparrow$

2. Graph the indifference curve: U = 36

$U = X\,Y^2 \rightarrow 36 = X\,Y^2 \rightarrow Y^2 = \frac{36}{X}$ بأخذ الجذر التربيعي للطرفين

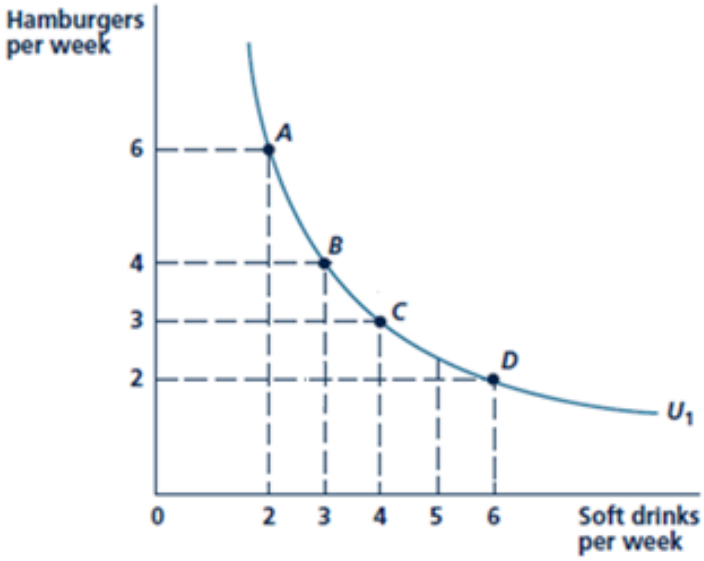
$Y = \frac{6}{\sqrt{X}}$

X	Y
1	$\frac{6}{\sqrt{1}} = 6$
4	$\frac{6}{\sqrt{4}} = 3$
9	$\frac{6}{\sqrt{9}} = 2$
16	$\frac{6}{\sqrt{16}} = 1.5$



Indifference Curve and the Marginal Rate of Substitution (MRS)

What happens when a person moves from point A (six hamburgers and two soft drinks) to point B (four hamburgers and three soft drinks)? This person remains equally well-off because the two commodity bundles lie on the same indifference curve.



This person will voluntarily give up two of the hamburgers that were being consumed at point A in exchange for one additional soft drink. The slope of the curve U1 between A and B is therefore approximately −2. That is, Y (hamburgers) declines two units in response to a one-unit increase in X (soft drinks).

Marginal rate of substitution (MRS) = The rate at which an individual is willing to reduce consumption of one good when he or she gets one more unit of another good.

هو عبارة عن عدد الوحدات التي يتخلى عنها المستهلك من سلعة في مقابل الحصول على وحدة إضافية من سلعة أخرى مع المحافظة على نفس مقدار المنفعة.

Marginal rate of substitution (MRS) = The negative of the slope of an indifference curve.

Marginal rate of substitution (MRS) = the absolute value of slope of indifference curve

The MRS (of soft drinks for hamburgers) between points A and B $= \frac{\Delta H}{\Delta S} = \left|\frac{4-6}{3-2}\right| = 2$

This person is willing to give up 2 hamburgers in order to get one more unit of soft drinks.

المستهلك مستعد التخلي عن وحدتين من هامبورجر مقابل الحصول على وحدة إضافية من المشروبات الغازية

The MRS between points B and C $= \frac{\Delta H}{\Delta S} = \left|\frac{3-4}{4-3}\right| = 1$

This person is willing to give up 1 hamburger's in order to get one more unit of soft drinks.

The MRS between points C and D = $\frac{\Delta H}{\Delta S} = \left|\frac{2-3}{6-4}\right| = ½$

This person is willing to give up ½ hamburger's in order to get one more unit of soft drinks.

Diminishing Marginal Rate of Substitution

The MRS diminishes (decreases) along an indifference curve moving from point A to point D.

The MRS between points A and B is equal 2
The MRS between points B and C is equal 1
The MRS between points C and D is equal ½

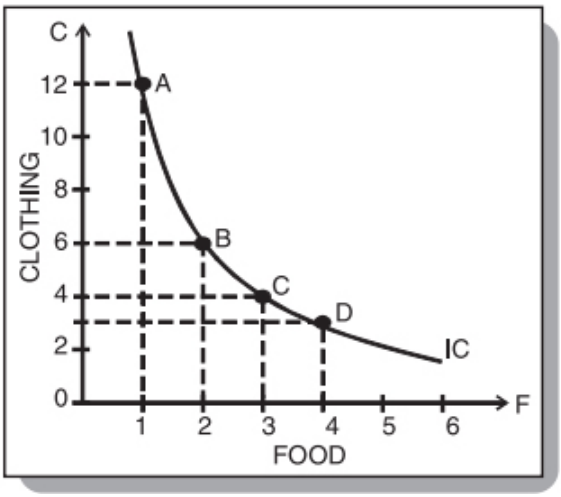
- As we move from point A to D MRS diminishing.

Example
The price of gum is 20 cents and the price of candy is 10 cents. A student is willing to give up 2 pieces of gum (vertical axis) for 1 piece of candy (horizontal axis). Money income is $2. What is the marginal rate of substitution?

MRS: The rate at which an individual is willing to reduce consumption of one good (vertical axis) when he or she gets one more unit of another good (horizontal axis).
MRS = $\frac{2}{1} = 2$

Example

The diagram shows an Indifference curve for two goods clothing's and food.



1. Calculate the MRS when the consumer increase amount of food from 1 to 2 units and interpret it in words.

   The MRS = $\frac{\Delta \text{clothing}}{\Delta \text{food}} = \left|\frac{6-12}{2-1}\right| = 6$

   The consumer is willing to give up 6 clothing's in order to get one more unit of food.

   المستهلك مستعد ان يتخلى عن 6 وحدات من الملابس مقابل الحصول على وحدة اضافية من الطعام

2. Is MRS diminishing, constant, or increasing as the consumer substitute's food for clothing along an indifference curve?

   The MRS between points A and B = $\frac{\Delta \text{clothing}}{\Delta \text{food}} = \left|\frac{6-12}{2-1}\right| = 6$

   The MRS between points B and C = $\frac{\Delta \text{clothing}}{\Delta \text{food}} = \left|\frac{4-6}{3-2}\right| = 2$

   The MRS between points C and D = $\frac{\Delta \text{clothing}}{\Delta \text{food}} = \left|\frac{3-4}{4-3}\right| = 1$

   → As we move from point A to D MRS diminishing (decreasing).

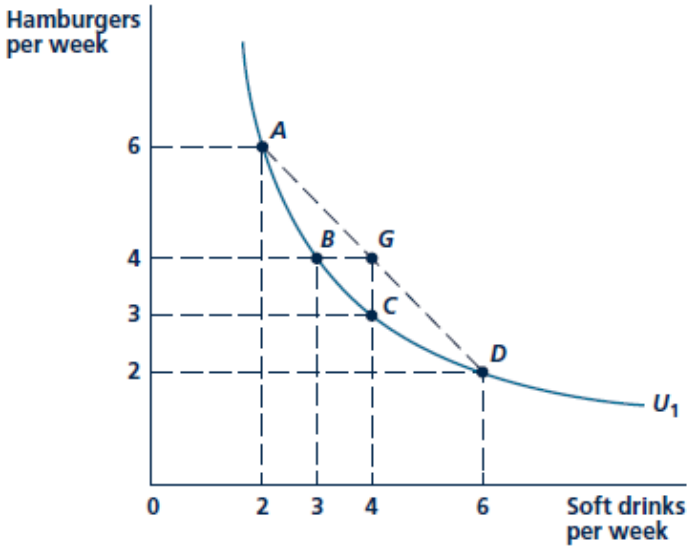
The Shape of Indifference Curves

Indifference curves are downward sloping. In our example of hamburgers and soft drinks, when the amount of soft drinks increases along an indifference curve, the amount of hamburger decreases. The fact *that indifference curves slope downward follows directly from our assumption that more of a good is better to less*. If an indifference curve sloped upward, a consumer would be indifferent between two market baskets even though one of them had more of both hamburger and soft drink.

Balance in Consumption

The conclusion of a diminishing MRS is based on the idea that people prefer balanced consumption bundles to unbalanced ones. This assumption is illustrated precisely in Figure below, where the indifference curve U1 is redrawn. Our discussion here concerns the two extreme consumption options A and D. In consuming A, this person gets six hamburgers and two soft drinks; the same satisfaction could be received by consuming D (two hamburgers and six soft drinks).

Now consider a bundle of commodities (say, G) "between" these extremes. With G (four hamburgers and four soft drinks), this person obtains a higher level of satisfaction (point G is northeast of the indifference curve U1) than with either of the extreme bundles A or D. The reason for this increased satisfaction should be geometrically obvious. All of the points on the straight-line joining A and D lie above U1. Point G is one of these points (as the figure shows, there are many others).



As long as the indifference curve obeys the assumption of a diminishing MRS, it will have the type of convex shape shown in Figure below. Any consumption bundle that represents an "average" between two equally attractive extremes will be preferred to those extremes. The assumption of a diminishing MRS (or convex indifference curves) reflects the notion that people prefer variety in their consumption choices.

Indifference Curve Maps

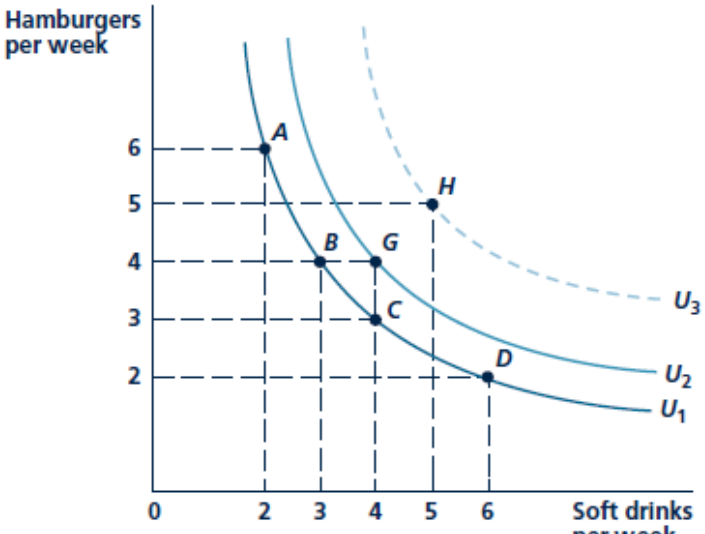
To describe a person's preferences for all combinations of hamburger and soft drink, we can graph a set of indifference curves called an indifference map.
Indifference map: Graph containing a set of indifference curves showing the market basket among which a consumer is indifferent.

خريطة منحنيات السواء هي مجموعة منحنيات السواء للمستهلك والتي تظهر تفضيله أو ذوقه تجاه السلع التي يستهلكها.

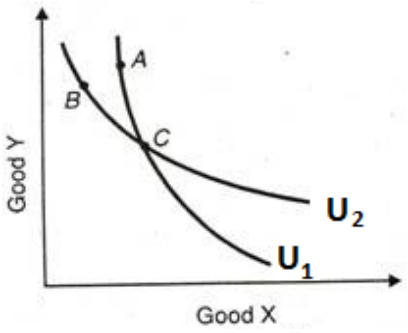
Any market basket on indifference curve U_3, such as basket H, is preferred to any basket on curve U_2 (e.g., basket G), which in turn is preferred to any basket on U_1, such as C.

$$U3 > U2 > U1$$

- *Point H is preferred to point G, and point G is preferred to point C*
- *All points A, B, C, and D give the consumer the same level of satisfaction.*



Indifference Curves Cannot Intersect



Points A and C give the consumer the same level of utility (both in U1)

Points B and C give the consumer the same level of utility (both in U2)

A = C and B = C this yield A =B, but A and B not in the same indifference curve (*conflict)*

If indifference curves U_1 and U_2 intersect, one of the assumptions of consumer theory is violated. According to this diagram, the consumer should be indifferent among market baskets A, B, and C. Yet A should be preferred to B because A has more of both goods. Thus, *intersecting indifference curves contradict our assumption that more is preferred to less*.

Marginal Rate of Substitution (MRS) and Marginal Utility:

Marginal utility (MU): The additional satisfaction obtained from consuming one additional unit of a good.

هو عبارة عن الاشباع الاضافي الذي يحصل عليه المستهلك من استهلاكة وحدة اضافية من سلعة

The marginal utility of good X (MUX): The extra utility obtained by consuming one more unit of good X.

هو عبارة عن الاشباع الاضافي الذي يحصل عليه المستهلك من استهلاكة وحدة اضافية من سلعة X

$$MUX = \frac{dU(X,Y)}{dx}$$

The marginal utility of good Y (MUY): The extra utility obtained by consuming one more unit of good Y.

هو عبارة عن الاشباع الاضافي الذي يحصل عليه المستهلك من استهلاكة وحدة اضافية من سلعة Y

$$MUY = \frac{dU(X,Y)}{dY}$$

$$Utility = U(X,Y)$$

$$MRS = \frac{MUX}{MUY}$$

Example

Consider the utility function $U(X,Y) = X^2\,Y$

1. Calculate the marginal utility of X.

   $\text{MUX} = \frac{dU(X,Y)}{dX} = 2YX$

2. Calculate the $\text{MRS}_{x,y}$ at the bundle (X = 2, Y= 3) and interpret it in words

   $\text{MRS} = \frac{\text{MUX}}{\text{MUY}} = \frac{2\text{XY}}{\text{X}^2} = \frac{2\text{Y}}{\text{X}} = \frac{2(3)}{2} = 3$

The consumer is willing to give up 3 clothing's in order to get one more unit of food.

المستهلك مستعد ان يتخلى عن 3 وحدات من السلعة Y مقابل الحصول على وحدة اضافية من السلعة X

3. Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve?

$$MRS = \frac{2Y}{X}$$

As X increases (and Y decreases) (البسط يقل والمقام بزيد) MRS is diminishing.

Example

Consider the utility function $U(X,Y) = 3X + Y$

1. Calculate the marginal utility of X. Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more X? Explain.

$$MUX = \frac{dU(X,Y)}{dX} = 3$$

So as you consume more X, the MUX constant

4. Calculate the $MRS_{x,y}$. Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y?

$$MUX = \frac{dU(X,Y)}{dX} = 3$$

$$MUY = \frac{dU(X,Y)}{dY} = 1$$

$$MRS = \frac{MUX}{MUY} = \frac{3}{1} = 3$$

As X increases MRS is constant and equal 3.

Example

Maher has \$80 to spend on eggs (E) and meat (M). Eggs are \$4 each and a strip of meat is \$2. His utility function is given by the following: U (E, M) = E*M. What is the Maher's MRS between meat and eggs at the bundle (15 eggs and 3 meats)?

MRS between meat and eggs = $\frac{MUE}{MUM}$

$$MUE = \frac{dU(E,M)}{dE} = M$$

$$MUM = \frac{dU(E,M)}{dM} = E$$

$$MRS = \frac{M}{E} = \frac{3}{15} = \frac{1}{5}$$

This person is willing to give up 1/5 meat in order to get one more eggs). Or this person is willing to give up 1 meat in order to get 5 more eggs).

Example

A consumer spends his income to purchase two goods, X and Y. His preferences are represented by the following utility function $U(X,Y) = ½X - Y^2$. What is the MRS for consumer when X = 4, Y=2?

MRS between meat and eggs = $\frac{\text{MUX}}{\text{MUY}}$

MUX = $\frac{dU(X,Y)}{dX} = ½$

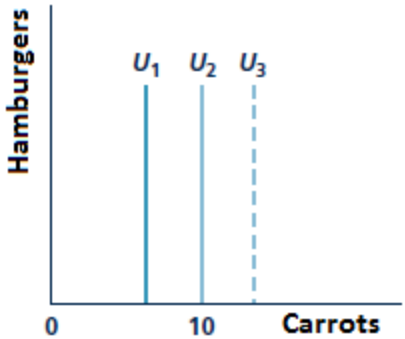
MUY = $\frac{dU(X,Y)}{dY} = -2Y$

MRS = $\frac{MUX}{MUY} = \frac{½}{-2Y} = \frac{-1}{4Y} = \frac{-1}{4(2)} = \frac{-1}{8}$

Illustrating Particular Preferences

- **A Useless Good**

  Figure shows an individual's indifference curve map for vegetarians who eat carrots (on the horizontal axis) and hamburger (on the vertical axis). Because hamburger is completely useless (Vegetarians do not eat meat.), increasing purchases of them does not increase utility. Only by getting more carrots does this vegetarian person enjoy a higher level of utility. The vertical indifference curve $U2$, for example, shows that utility will be $U2$ as long as this person has 10 units of carrot no matter how many hamburgers he or she has.



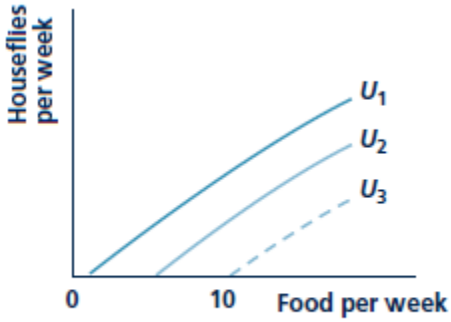
A useless good

يوضح الشكل في الاعلى خريطة منحنى السواء للفرد للنباتيين اللذين يتناولون الجزر (على المحور الأفقي) والهمبرغر (على المحور الرأسي). نظرًا لأن الهامبرغر عديم الفائدة تمامًا (النباتيون لا يأكلون اللحوم، للنباتيين الذين لا يحبون اللحوم ولا يكرهونها)، فإن زيادة مشترياتهم لا تزيد من فائدتهم. فقط من خلال الحصول على المزيد من الجزر، يتمتع الشخص النباتي بمستوى أعلى من المنفعة. على سبيل المثال، يوضح منحنى االسواء الرأسي U2 أن المنفعة ستكون U2 طالما أن هذا الشخص لديه 10 وحدات من الجزر بغض النظر عن عدد الهامبرغر الذي لديه.

- **An Economic Bad**

  Useless goods cause no harm—having more useless smoke grinders causes no problem since one can always throw them away. In some cases, however, such free disposal is not possible, and additional units of a good can cause actual harm. For example, Figure (b) shows an indifference curve map for food and houseflies. Holding food consumption constant at 10, utility declines as the number of houseflies increases. Because additional houseflies reduce utility, an individual might even be willing to give up some food (and buy flypaper instead, for example) in exchange for fewer houseflies.

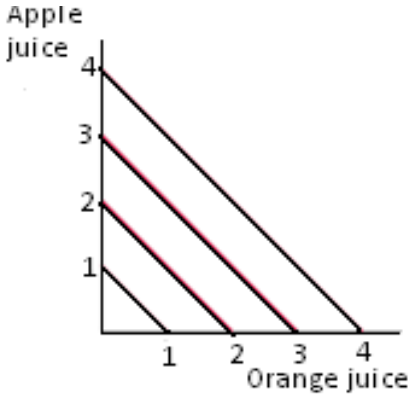
السلع عديمة الفائدة لا تسبب أي ضرر - فوجود المزيد من السلع عديمة الفائدة لا يسبب أي مشكلة حيث يمكن للمرء دائمًا التخلص منها. ومع ذلك، في بعض الحالات، لا يكون هذا التصرف المجاني ممكنًا، ويمكن أن تسبب وحدات إضافية من السلعة ضررًا فعليًا. على سبيل المثال، يوضح الشكل (ب) خريطة منحنى السواء للطعام والذباب المنزلي. ومع بقاء استهلاك الغذاء ثابتًا عند 10، تنخفض المنفعة مع زيادة عدد الذباب المنزلي. نظرًا لأن الذباب المنزلي الإضافي يقلل من فائدته، فقد يكون الفرد على استعداد للتخلي عن بعض الطعام (وشراء ورق صائدة بدلاً من ذلك، على سبيل المثال) مقابل عدد أقل من الذباب المنزلي.



(b) An economic bad

- **Perfect Substitutes**

  The consumer views orange juice and apple juice as perfect substitutes: He is always indifferent between a glass of one and a glass of the other.



Perfect Substitutes

The utility function for these goods: $U\ (X, Y)\ =\ a\ X\ +\ b\ Y$

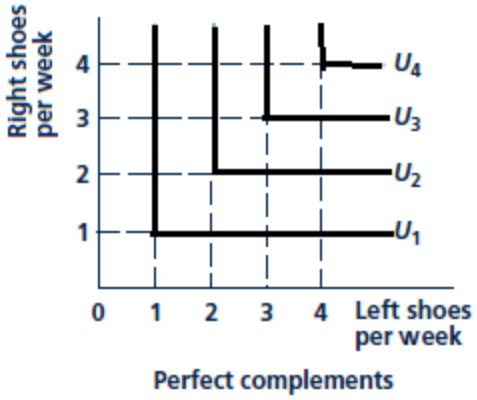
For perfect substitutes marginal rate of substitution of one for the other is a constant.

Perfect Complement

Two goods for which the MRS is infinite; the indifference curves are shaped as right angles.

Jane views left shoes and right shoes as perfect complements: An additional left shoe gives her no extra satisfaction unless she also obtains the matching right shoe.

The utility function for these goods: $U\ (X, Y)\ =\ min\ \{\ a\ X\ , b\ Y\}$

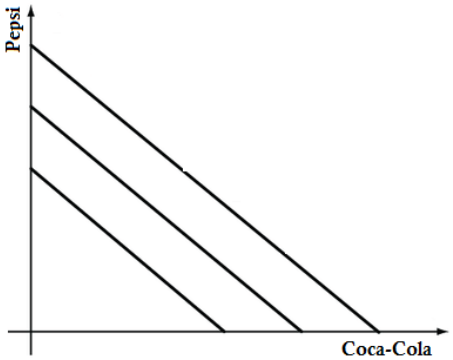


Example

Draw indifference curves that reflects the following preferences.

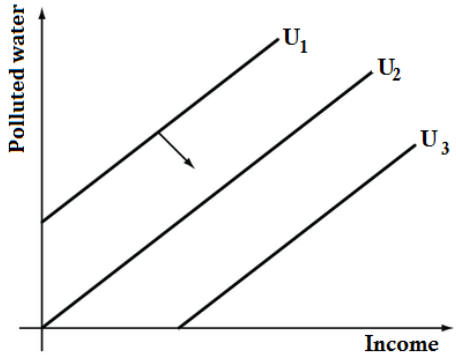
A. Coca-Cola and Pepsi

Coca-Cola and Pepsi are substitutes goods



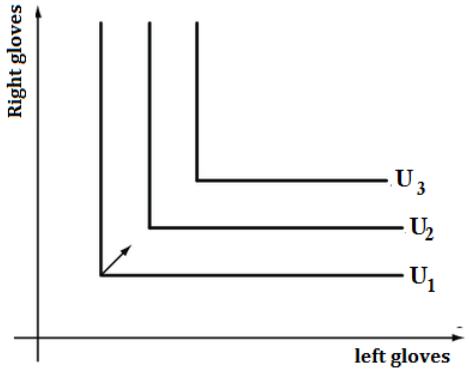
B. Income (الدخل) and polluted water (تلوث الماء)

Polluted water is a bad good



C. Right gloves and left gloves

Right gloves and left gloves are perfect complements



Budget Constraints

Constraints that consumers face as a result of limited incomes.

The budget line: all combinations of goods for which the total amount of money spent is equal to income.
خط الميزانية هو خط مستقيم يظهر مجموعات مختلفة من سلعتين ينفق عليهما المستهلك كامل دخله.

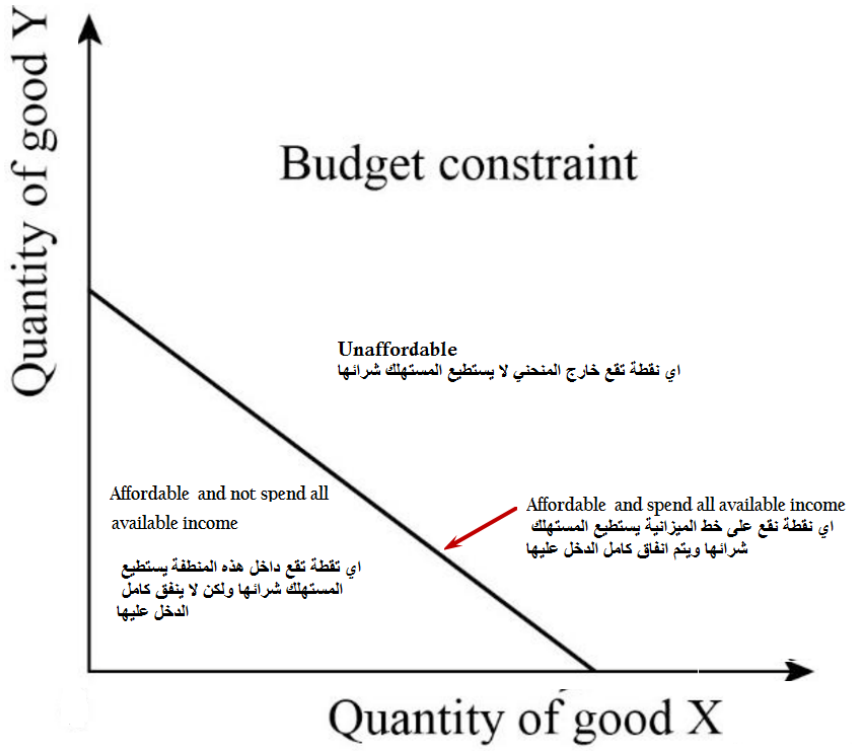
To see how a budget constraint limits a consumer's choices, let's consider a situation in which a woman has a fixed amount of income, *I*, that can be spent on good X and good Y. Let X be the amount of good x purchases and Y be the amount of goody. We will denote the prices of the two goods P_x and P_y.

If the consumer spent all enter income on the two goods (X and Y), the combinations of good X and good Y that she can by will all lie on this line.

$Px\ X\ +\ Py\ Y\ =\ I$

$Px\ X$: Is the amount of money spend on good X

$Py\ Y$: Is the amount of money spend on good Y



- All points on the budget line are affordable, and the consumer spent all available income.
  جميع النقاط التي تقع على خط الميزانية يقدر المستهلك على شرائها، وعند شرائهل يتم انفاق كامل الدخل

- Points in the area below the budget line are affordable, but these leave some portion of income unspent.
  النقاط التي تقع تحت خط الميزانية يستطيع المستهلك شرائها، وعند شرائها لا يتم انفاق كامل الدخل

- Points in the area above the budget line are not affordable.
  النقاط التي تقع خارج خط الميزانية لا يستطيع المستهلك شراؤها (الدخل غير كافي).

- The downward slope of the budget line shows that the individual can afford more good X only if good Y purchases are cut back.
  خط الميزانية منحدر للاسفل بسبب أن المستهلك اذا زاد استهلاكة من السلعة X فإن الكمية المستهلكة من السلعة Y تقل .

The slope of the budget line:

Using the budget line equation: $P_X\,X\ +\ P_Y\,Y\ =\ I$, we can see how much of Y must be give up to consume more of X.

$Y\ =\ \frac{I}{P_Y}\ -\ \frac{P_X}{P_Y}\ X$: This is the equation of the budget line; its slope of $(-\frac{P_X}{P_Y}\ )$, and intercept of $\frac{I}{P_Y}$

The slope of the budget line is the negative of the ratio of the prices of the two goods.

Example:
Suppose a person has $80 to spend only on Apples (X) and Oranges (Y). Apples cost $4 each and oranges cost $1 each.

a. Write the budget line equation.

   Budget line: $P_X\,X\ +\ P_Y\,Y\ =\ I$

   $4\,X\ +\ Y\ =\ 80\qquad\rightarrow\quad$ *Budget line equation*

b. If this person buys only apples, how many can be bought?

   If this person buys only apples → the amount of oranges purchases equal zero

   $4\,X\ =\ 80\ \rightarrow X\ =\ 80/4\ =\ 20$ units.

c. If this person buys only oranges, how many can be bought?

   $0\ +\ Y\ =\ 80\quad\rightarrow\quad Y\ =\ 80$ units.

d. If the person were to buy 10 apples, how many oranges could be bought with the funds left over?

$4X + Y = 80 \rightarrow 4 * 10 + Y = 80 \rightarrow 40 + Y = 80 \rightarrow Y = 80 - 40 = 40$ units.

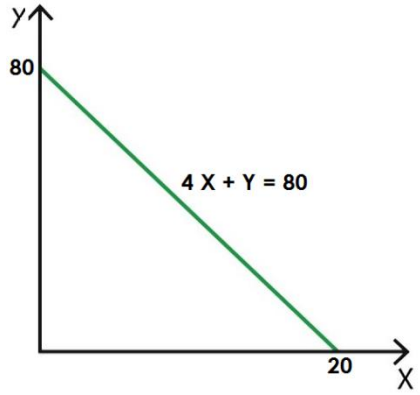
e. Find the slope of the budget line, and graph it.

Slope $= -\frac{P_X}{P_Y} = -\frac{-4}{1} = -4$

Vertical intercept $= \frac{I}{P_Y} = \frac{80}{1} = 80$

Horizontal intercept $= \frac{I}{P_X} = \frac{80}{4} = 20$

Graph the budget line: $4X + Y = 80$

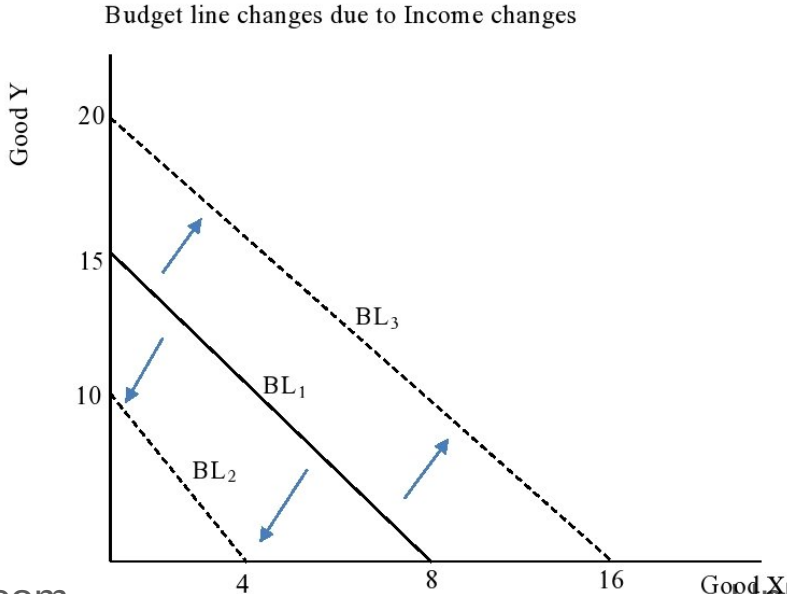


The Effects of Changes in Income and Prices

Income changes:

From the budget line equations, we can see that a change in income alters the vertical intercept of the budget line but does not change the slope.

- If income increase, the budget line shifts outward (to the right), from budget line BL_1 to budget line BL_3.
- If income decreases, the budget line shifts inward (to the left), from budget line BL_1 to budget line BL_2.

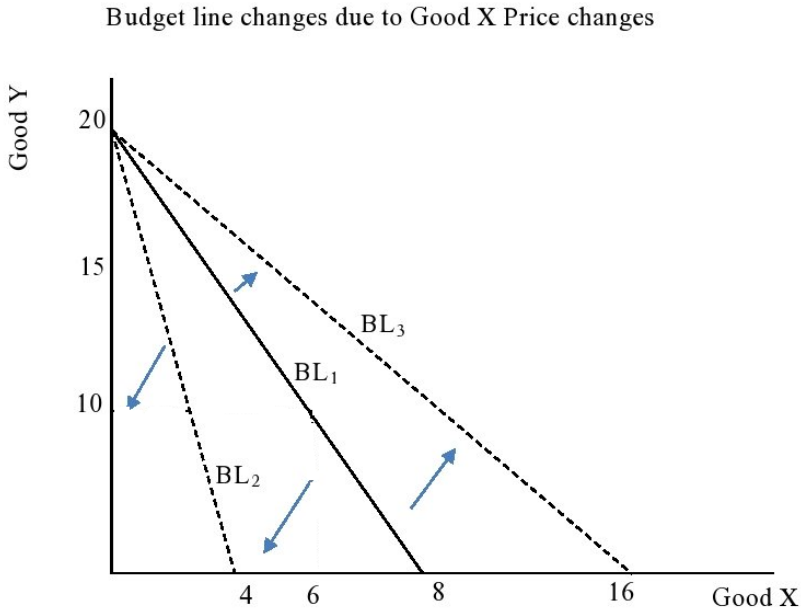


Price Change:

A change in the price of one good (with income and price of another good unchanged) causes the budget line to rotate about the intercept.

- When the price of food falls, the budget line rotates outward to line BL_3.

- When the price of food increases, the budget line rotates inward to line BL_2.



Utility Maximization (Consumer Choice)

Given preferences and budget constraint, we can determine how much consumers buy in order to maximum utility.

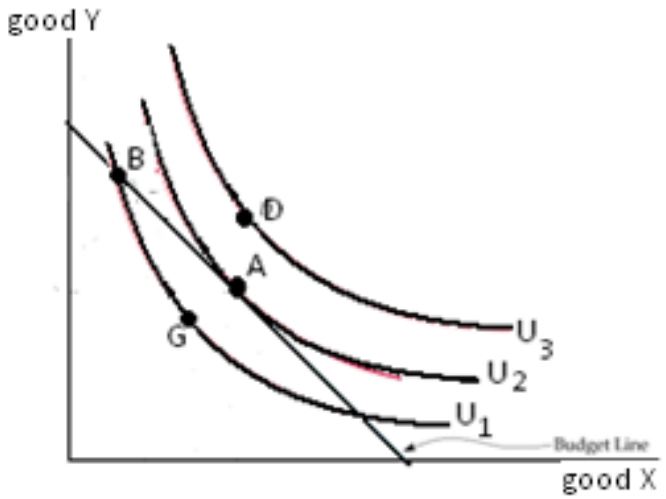
The maximizing market basket must satisfy two conditions:

1. It must be located on the budget line.
2. It must give the consumer the most preferred combination of goods and services.

- Point G is affordable but not all of the consumer's income would be spent.
- Point B is affordable but is not on the highest indifference curve that can be reached by the consumer.

- Point D is on a higher indifference curve than A, but is not affordable given the budget constraint.

- Point A is the point that is affordable that lies on the highest indifference curve, so it represents utility maximization.



The basket which maximizes satisfaction must lie on the highest indifference curve that touches the budget line. Point A is the point of tangency between indifference curve and the budget line.

At point A, the slope of the budget line is exactly equal to the slope of the indifference curve.

The satisfaction (utility) is maximized at the point where: MRS = $\frac{P_X}{P_Y}$

Satisfaction is maximized when the marginal rate of substitution (of good X for good Y) is equal to the ratio of the price (of X for Y).

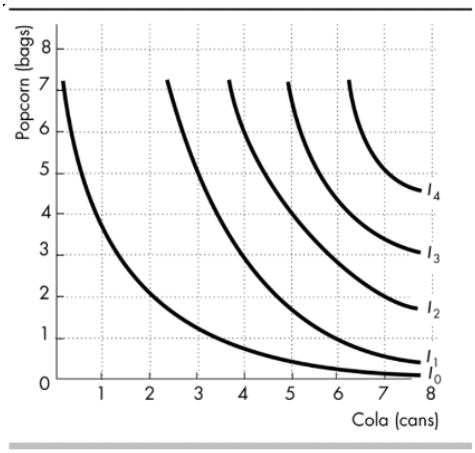
Because the MRS is also equal to the ratio of the marginal utilities of consuming good X and good Y, it follows that:

$\frac{MUX}{MUY} = \frac{P_X}{P_Y}$ or $\frac{MUX}{P_X} = \frac{MUY}{P_Y}$

This equation tells us that utility maximization is achieved when the budget is allocated so that the *marginal utility per dollar* of expenditure is the same for each good.

Example:

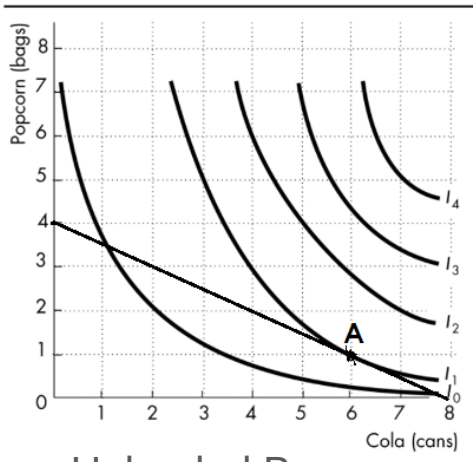
Sara's income is \$12 a week. The price of popcorn is \$3 a bag, and the price of cola is \$1.5 a can. Figure below shows Sara's preference map for popcorn and cola.



What quantities of popcorn and cola does Sara buy in order to maximize utility? What is Sara's marginal rate of substitution at the point which she consumes?

Budget line equation: $P_X X + P_Y Y = I \rightarrow 1.5X + 3Y = 12$ (X: cola; Y: popcorn)

Graph the budget line on the same graph: Vertical intercept = $\frac{I}{P_Y} = \frac{12}{3} = 4$,horizontal intercept = $\frac{I}{P_X} = \frac{12}{1.5} = 8$



Utility maximization point is the point of tangency between indifference curve and the budget line, at point A

Popcorn = 1 bag

Cola = 6 cans

MRS = $\frac{Price\ _{(Cola)}}{Price\ _{(Popcorn)}} = \frac{1.5}{3} = 0.5$

Example

Assume the individual can choose between hamburgers (Y) and soft drinks (X) whose prices are PY = \$1and PX=\$0.5. The individual has \$10to spend.

The individual gets measurable utility from X and Y as follows: $U(X,Y) = \sqrt{X\,Y}$

How much hamburgers (Y) and soft drinks (X) should he buy to maximize her utility?

Budget line: $Px\ X + Py\ Y = I \rightarrow 0.5\,X + Y = 10$ ... ... (1)

To maximize utility: $\frac{MUX}{MUY} = \frac{P_X}{P_Y}$

$MUX = \frac{dU(X,Y)}{dx} = \frac{Y}{2\sqrt{X\,Y}}$

$MUY = \frac{dU(X,Y)}{dY} = \frac{X}{2\sqrt{X\,Y}}$

$\frac{\frac{Y}{2\sqrt{X\,Y}}}{\frac{X}{2\sqrt{X\,Y}}} = \frac{0.5}{1} \rightarrow \frac{Y}{X} = \frac{1}{2} \rightarrow X = 2Y$ -------- (2)

Solve equations (1) and (2):

$0.5(2Y) + Y = 10 \rightarrow Y + Y = 10 \rightarrow 2Y = 10 \rightarrow Y = 5\ units$

From equation (2): $X = 2Y \rightarrow X = 2 * 5 = 10\ units$

Another solution:

Budget line equation: $0.5\,X + Y = 10$

Y	X	U(X,Y) = $\sqrt{XY}$
0	20	$\sqrt{0 * 20} = 0$
1	18	$\sqrt{1 * 18} = \sqrt{18}$
2	16	$\sqrt{2 * 16} = \sqrt{32}$
3	14	$\sqrt{3 * 14} = \sqrt{42}$
4	12	$\sqrt{4 * 12} = \sqrt{48}$
5	***10***	$\sqrt{\mathbf{5 * 10}} = \sqrt{\mathbf{50}}$ ***→ max***
6	8	$\sqrt{6 * 8} = \sqrt{48}$
7	6	$\sqrt{7 * 6} = \sqrt{42}$

8	4	$\sqrt{8*4} = \sqrt{32}$
9	2	$\sqrt{9*2} = \sqrt{18}$
10	0	$\sqrt{10*0} = 0$

To max utility the consumer should consume 5 units of Y and 10 units of X.

Example

Jammel receives utility from days spent traveling on vacation domestically (D), and days spend travelling on vacation in a foreign country (F), as given by the utility function $U(D,F) = 10DF$. In addition, the price of a day spend traveling on vacation domestically is \$100, the price of a day spent travelling on vacation in a foreign country is \$400, and Jammel's annual travel budget is \$4000. Find Jammel's utility maximizing choice of days spent traveling domestically and days spent in a foreign country.

Budget line: $P_D\ D + P_F\ F = I \quad \rightarrow \quad 100\ D + 400\ F = 4000$ ..... (1)

To maximize utility: $\frac{MUD}{MUF} = \frac{P_D}{P_F}$

$MUD = \frac{dU(D,F)}{dD} = 10F$

$MUF = \frac{dU(D,F)}{dF} = 10D$

$\frac{10F}{10D} = \frac{100}{400} \quad \rightarrow \quad \frac{F}{D} = \frac{1}{4} \quad \rightarrow$ D = 4F -------- (2)

Solve equations (1) and (2):

$100(4F) + 400F = 4000 \quad \rightarrow \quad 400F + 400F = 4000 \quad \rightarrow 800F = 4000 \quad \rightarrow F = 5\ days$

From equation (2): $D = 4F \quad \rightarrow \quad D = 4 * 5 = 20\ days$

Example

A consumer has the following utility function $U(X,Y) = XY + 10Y$. If the price of good X is \$2 and the price of good Y is \$5, and his income is \$80.How much goods X and Y should the consumer purchase to maximize utility?

Budget line: $Px\ X + Py\ Y = I \quad \rightarrow \quad 2\ X + 5\ Y = 80$ ..... (1)

To maximize utility: $\frac{MUX}{MUY} = \frac{P_X}{P_Y}$

$MUX = \frac{dU(X,Y)}{dx} = Y$

$MUY = \frac{dU(X,Y)}{dY} = X + 10$

$\frac{MUX}{MUY} = \frac{P_X}{P_Y} \quad \rightarrow \quad \frac{Y}{X+10} = \frac{2}{5} \quad \rightarrow$ ضرب تبادلي ينتج $5Y = 2(X+10) \rightarrow 5Y = 2X + 20$ -------- (2)

بتعويض المعادل 2 في المعادلة 1 ينتج

$2X + 2X + 20 = 80 \quad \rightarrow 4X = 60 \rightarrow X = = \frac{60}{4} = 15$

$5Y = 2X + 20 \rightarrow Y = \frac{2X+20}{5} = \frac{2(15)+20}{5} = \frac{50}{5} = 10$

X = 15
Y = 1

Example

Hani consumes goods X and Y according to the utility function $U(X,Y) = X^2 Y^3$. If the price of good X is $4 and the price of good Y is $2, and his income is $90. How much goods X and Y should Hani purchase to maximize utility?

Budget line: $Px\ X + Py\ Y = I \quad \rightarrow \quad 4X + 2Y = 90 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$

To maximize utility: $\frac{MUX}{MUY} = \frac{P_X}{P_Y}$

$MUX = \frac{dU(X,Y)}{dx} = 2XY^3$

$MUY = \frac{dU(X,Y)}{dY} = 3X^2 Y^2$

$\frac{MUX}{MUY} = \frac{P_X}{P_Y} \rightarrow \quad \frac{2XY^3}{3X^2 Y^2} = \frac{4}{2} \quad \rightarrow \frac{2Y}{3X} = 2 \quad \rightarrow 2Y = 6X$ ..... *(2)*

بتعويض المعادلة 2 في المعادلة 1 ينتج

$4X + 6X = 90 \rightarrow 10X = 90 \rightarrow X = \frac{90}{10} = 9$

$Y = \frac{6X}{2} = 3X = 3(9) = 27$

When utility maximization conditions not satisfy

- If the equation (MU per dollar of good X = MU per dollar of good Y) is not fulfilled ((غير متحقق, then some reallocation of the consumer's expenditures between good X and good Y will increase the consumer's total utility.

  If $\frac{MUX}{MUY} > \frac{P_X}{P_Y}$ *or* $\frac{MUX}{P_X} > \frac{MUY}{P_Y}$

  The last dollar spent on good X provide more utility than the last dollar spent on good Y. so *the consumer can increase total utility by purchasing (consuming) more of good X and less of good Y*.

  If $\frac{MUX}{MUY} < \frac{P_X}{P_Y}$ *or* $\frac{MUX}{P_X} < \frac{MUY}{P_Y}$

- The last dollar spent on good Y provide more utility than the last dollar spent on good X. so *the consumer can increase total utility by purchasing (consuming) more of good Y and less of good X*.

Example

Muhanad divides her consumption between orange juice (O) and bagels (B). Orange juice costs $1 per glass and bagels cost $3each. Muhanad consumes positive amounts of both goods, and has chosen consumption quantities where his marginal utility of orange juice is 10 utile per glass, his marginal utility of bagels is 25 utils per bagel, and he spends all her income. Is Muhanad maximizing utility? If no, what he do to increase his utility?

To max utility: $\frac{MUO}{MUB} = \frac{P_O}{P_B} \rightarrow \frac{10}{25} \neq \frac{1}{3} \rightarrow$ Muhanad does not maximizing utility

But $\frac{MUO}{MUB} > \frac{P_O}{P_B}$ Since, $\frac{10}{25} > \frac{1}{3}$

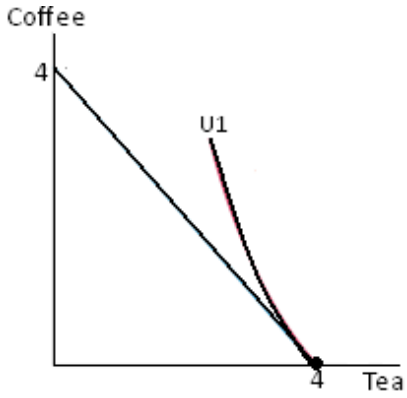
The last dollar spent on orange juice provides more utility than the last dollar spent on bagels. So, the consumer can increase total utility *by consuming more of orange juice and less of bagels.*

Utility maximization for perfect substitute's goods case:

لمعرفة اذا كانت الحالة تمثل حالة السلع البديلة من خلال معادلة منحنى المنفعة $(U(X,Y) = aX + bY)$

Example

Ms. Caffeine enjoy coffee (C) and tea (T) according to the function $U(C,T) = 3C + 4T$. If coffee and tea cost $3 each and Ms. Caffeine has $12 to spend on these products, how much coffee (C) and tea (T) should she buy to maximize her utility?



Budget line: $Pc\ C + P_T\ T = I \Rightarrow 3C + 3T = 12$ ..... (1)

To maximum utility: $\frac{MUC}{MUT} = \frac{P_C}{P_T}$

MUC = $\frac{dU(C,T)}{dC} = 3$

$MUT = \frac{dU(C,T)}{dT} = 4$

$\frac{MUC}{MUT} = \frac{P_C}{P_T} \rightarrow \frac{3}{4} \neq \frac{3}{3} \Rightarrow$ Condition fail

By using another solution:

C	T	U(C,T) = 3C + 4T
0	**4**	**3*0 + 4 * 4 = 16 ⇨ max**
4	0	3*4 + 4*0 = 12

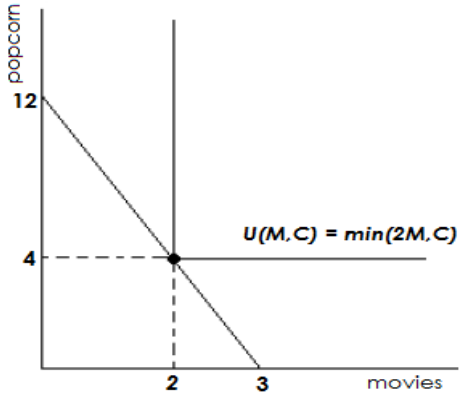
Ms. Caffeine should consume 4 units of tea and gets 16 utile

Utility maximization for perfect complement goods case:

لمعرفة اذا كانت الحالة تمثل حالة السلع المكملة من خلال معادلة منحنى المنفعة ($U(X,Y) = min\{X,Y\}$)

Example

A consumer consumes two goods movies (M) and popcorn (C) according to the utility function :
$U(M,C) = min\ (2M, C)$, the price of movies ticket is \$10 and the price of popcorn is \$2.5 and consumer budget is \$30. how much movies and popcorn should she buy to maximize her utility?



Where *"Min"* means that utility is given by the smaller of the two terms in parentheses. If, for example, this person attends a movie but buys no popcorn, utility is zero. If he or she attends a movie and buys three bags of popcorn, utility is 2—the extra bag of popcorn does not raise utility.

Budget line: $PM * M + PC * C = I \quad \Rightarrow \quad 10\,M + 2.5C = 30 \quad ------(1)$

$U(M,C) = min\ (2M, C)$: this person should only consume bundles for which $C = 2M$ (that is, two bags of popcorn for each movie).
$C = 2M \quad ----------(2)$

Solve the 2 equations:

$10\,M + 2.5C = 30$

$C = 2M$

بتعويض المعادلة 2 في المعادلة 1 ينتج:

$$10\text{M} + 2.5(2\text{M}) = 30 \quad \rightarrow \quad 10\text{M} + 5\text{M} = 30 \quad \rightarrow 15\text{M} = 30 \quad \rightarrow \text{M} = \frac{30}{15} = 2$$

$$\text{C} = 2\text{M} = 2 * 2 = 4$$

$U(M,C) = min\ (2M, C) = min\ (2 * 2, 4) == min\ (4, 4) = 4$

Example

Hani consumes goods X and Y according to the utility function $U(X,Y) = min\ (X, ½Y)$. If the price of good X is \$2 and the price of good Y is \$1, and his income is \$100. How many units of goods X and Y does Hani consume in order to maximize utility?

Budget line: $Px\ X + P_Y\ Y = I \quad \rightarrow \quad 2\,X + Y = 100 \quad \ldots\ldots\ldots\ldots (1)$

$UU(X,Y) = min\ (X, ½Y)$: this person should only consume bundles for which $X = ½Y$ (that is, 1 unit of X equal ½ unit of Y)
$X = ½Y \quad \ldots\ldots\ldots\ldots (2)$

$2\ (½Y) + Y = 100 \quad \rightarrow Y + Y = 100 \rightarrow 2Y = 100 \rightarrow Y = 50$

$X = ½Y \quad = ½(50) = 2$