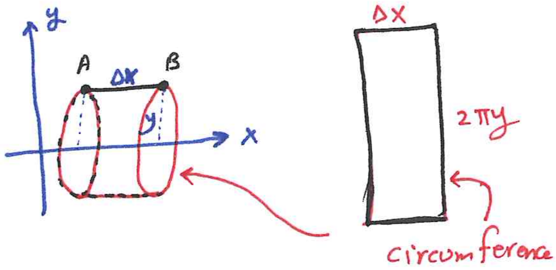
6.4 Surfaces Areas of Revolution

* The surface area generated by rotating the horizontal line AB of length Δx about x-axis is $2\pi y\,\Delta x$



Def: If $y = f(x) \geq 0$ is continuously differentiable on $[a, b]$, then the area of surface generating by revolving the graph of $y = f(x)$ about the x-axis is

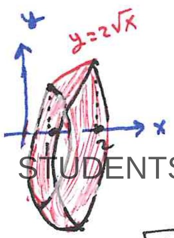
$$S = \int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$$

Def: If $x = g(y) \geq 0$ is 〃 〃 〃 $[c, d]$, 〃
〃 〃 〃 〃 〃 〃 〃 〃 〃 〃 $x = g(y)$
〃 〃 y-axis is

$$S = \int_c^d 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy = \int_c^d 2\pi g(y)\sqrt{1+[g'(y)]^2}\,dy$$

Example: Find the area of the surface generated by revolving the curve

1) $y = 2\sqrt{x}$, $1 \leq x \leq 2$ about x-axis



$$S = 2\int_1^2 2\pi\sqrt{x}\sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}\,dx = 4\pi\int_1^2\sqrt{x}\sqrt{\frac{x+1}{x}}\,dx = 4\pi\int_1^2\sqrt{x+1}\,dx$$

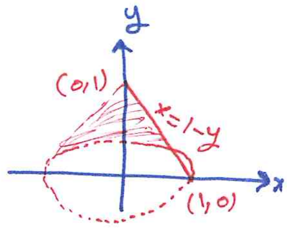
$u = x+1$
$du = dx$

$$= 4\pi\int_2^3 u^{\frac{1}{2}}\,du = \frac{8\pi}{3}u^{\frac{3}{2}}\Big|_2^3 = \frac{8\pi}{3}(3\sqrt{3}-2\sqrt{2}).$$

2) $x = 1-y$, $0 \leq y \leq 1$ about y-axis.

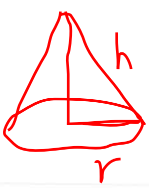
$$S = 2\pi\int_0^1 (1-y)\sqrt{1+(-1)^2}\,dy = 2\pi\int_0^1 \sqrt{2}\,(1-y)\,dy$$

$$= 2\sqrt{2}\pi\left[y-\frac{y^2}{2}\right]_0^1 = 2\pi\sqrt{2}\left(1-\frac{1}{2}\right) = \pi\sqrt{2}$$



Cone
The base not included
"Only the Lateral surface area".

$S = \pi r h$



[redacted]

$\boxed{3}$ $x = \sqrt{2y-1}$, $\frac{5}{8} \le y \le 1$, y-axis

$$S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$\frac{dx}{dy} = \frac{1}{2}(2y-1)^{-\frac{1}{2}}(2)$$

$$= \frac{1}{\sqrt{2y-1}}$$

$$(x')^2 = \frac{1}{2y-1}$$

$$1 + (x')^2 = \frac{2y-1+1}{2y-1} = \frac{2y}{2y-1}$$

$$= \int_{\frac{5}{8}}^{1} 2\pi \sqrt{2y-1} \sqrt{\frac{2y}{2y-1}}\, dy$$

$$= \int_{\frac{5}{8}}^{1} 2\pi \sqrt{2y}\, dy$$

$$= 2\pi \sqrt{2}\, y^{\frac{3}{2}} \frac{2}{3} \Big|_{\frac{5}{8}}^{1} = \frac{4\sqrt{2}\pi}{3}\left[1 - \sqrt{\left(\frac{5}{8}\right)^3}\right]$$