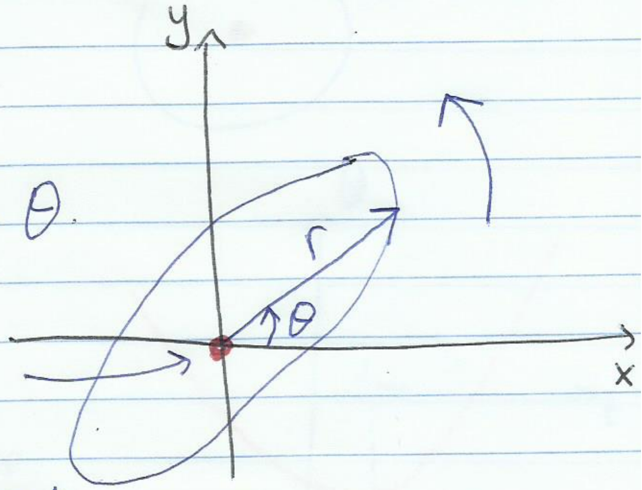
Chapter 10 – Rotation

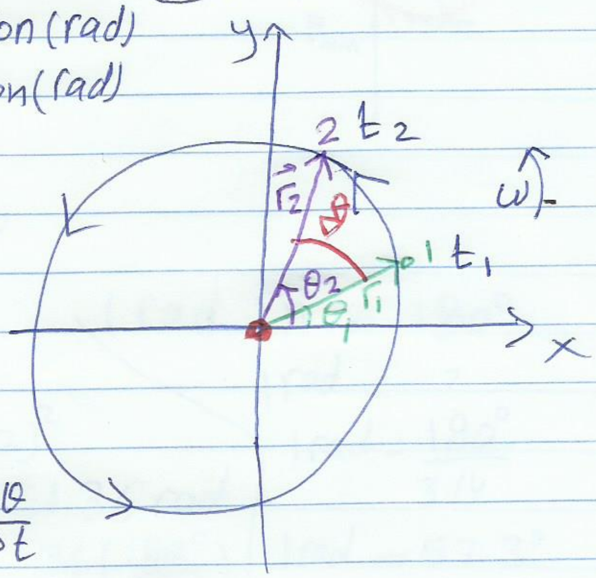
Rotational Variables:

Motion
- → translation, in which an object moves along a straight or curved line.
- → rotation, in which an object turn about an axis



Rotational variables:
The body has rotated Counterclockwise by angle θ.
This is the positive direction

rotation axis



θ_1 = is the initial angular position (rad)
θ_2 = is the final angular position (rad)
$\Delta\theta$ = Angular Displacement

Average Angular Velocity $= \dfrac{\Delta\theta}{\Delta t}$

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} \text{ rad/s}$$

instantanous Angular velocity $= \lim\limits_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t}$

$$\omega_{inst} = \frac{d\theta}{dt} \text{ rad/s}$$

Angular Average Acceleration $= \dfrac{\Delta\omega}{\Delta t}$ rad/s²

$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t} \text{ rad/s}^2$$

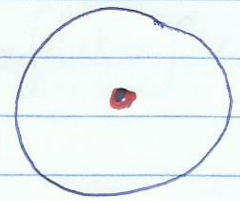
Instantaneous Angular Acceleration $= \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$

$$\alpha_{inst} = \frac{d\omega}{dt} \text{ rad/s}^2$$

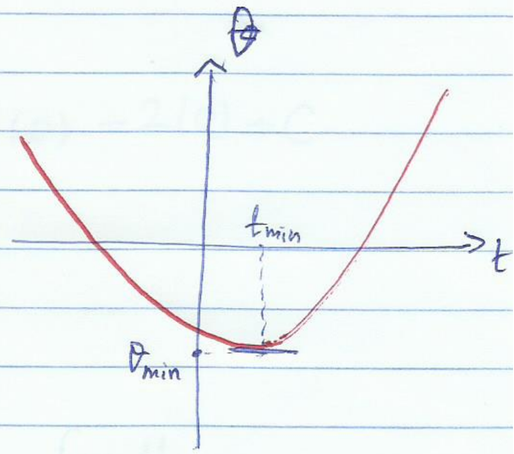
$(\theta, \Delta\theta, \omega, \alpha)$ are positive if they are Counterclockwise
$(\theta, \Delta\theta, \omega, \alpha)$ are negative if they are Clockwise.

Sample Problem 10.01

The Disk is rotating about its central axis according to the equation



$$\theta = -1 - 0.6t + 0.25t^2 \text{ rad.}$$



a) Draw θ vs. t
b) find t_{min} ? for θ to reach minimum value

θ is minimum, when $\frac{d\theta}{dt} = 0$

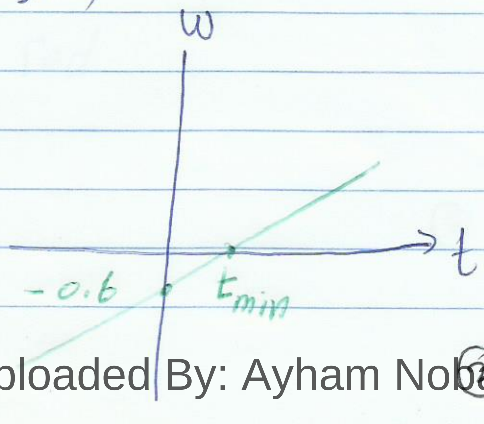
$$\frac{d\theta}{dt} = -0.6 + 0.5t = 0 \Rightarrow$$

$$t_{min} = \frac{0.6}{0.5} = 1.2 \text{ sec.}$$

$$\theta_{min} = -1 - 0.6(1.2) + 0.25(1.2)^2$$
$$= -1 - 0.72 + 0.36 = -1.36 \text{ rad}$$
$$= -1.36\left(\frac{180^\circ}{3.14}\right)$$
$$= -77.9^\circ$$

$(\text{rad})\pi = 180^\circ$
$1\text{rad} = ?$
$1\text{rad} = \frac{180^\circ}{3.14}$
$1\text{rad} = 57.3^\circ$

c) Graph ω vs. t



$$\omega = \frac{d\theta}{dt} = -0.6 + 0.5t$$

$$\alpha = \frac{d\omega}{dt} = 0.5 \text{ rad/s}^2 \text{ is constant}$$

Sample Problem 10.02

A child's top is spun with angular acceleration

$\alpha = 5t^3 - 4t$ rad/s²

At $t=0$ $\begin{cases} \omega_0 = 5 \text{ rad/s} \\ \theta_0 = 2 \text{ rad} \end{cases}$

a) find $\omega(t) = ?$

$\alpha = 5t^3 - 4t$

$\frac{d\omega}{dt} = \alpha \Rightarrow d\omega = \alpha\, dt \Rightarrow \int d\omega = \int \alpha\, dt$

$\omega = \int (5t^3 - 4t)\, dt$

$\omega = \frac{5t^4}{4} - \frac{4t^2}{2} + C$

At $t=0 \longrightarrow \omega_0 = 5$ $\quad 5 = \frac{5}{4}(0) - 2(0) + C$

$C = 5$

$\omega = \frac{5}{4}t^4 - 2t^2 + 5$ rad/s

b) find $\theta(t)$?

$\omega = \frac{d\theta}{dt} \Rightarrow d\theta = \omega\, dt \Rightarrow \int d\theta = \int \omega\, dt$

$\theta = \int (\frac{5}{4}t^4 - 2t^2 + 5)\, dt$

$= \frac{5}{4}(\frac{t^5}{5}) - 2(\frac{t^3}{3}) + 5t + C'$

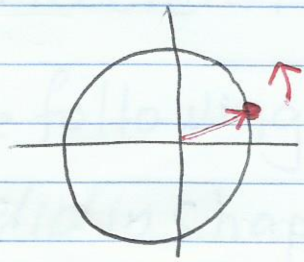
$\theta = \frac{1}{4}t^5 - \frac{2}{3}t^3 + 5t + C'$, at $t=0$, $\theta_0 = 2$

$2 = C'$

$\theta = \frac{1}{4}t^5 - \frac{2}{3}t^3 + 5t + 2$ rad

(10-4) Problem:

$\theta = 2 + 4t^2 + 2t^3$ rad.



a) At $t=0$, find the point's angular position?

$\theta_0 = 2$ rad.

b) At $t=0$, find the point's angular velocity?

$$W = \frac{d\theta}{dt} = 8t + 6t^2 \qquad W = 8t + 6t^2 \text{ rad/s}$$

$W_0 = 0$

c) At $t = 4s$, find its angular velocity

$W_4 = 8(4) + 6(4)^2 = 32 + 96$

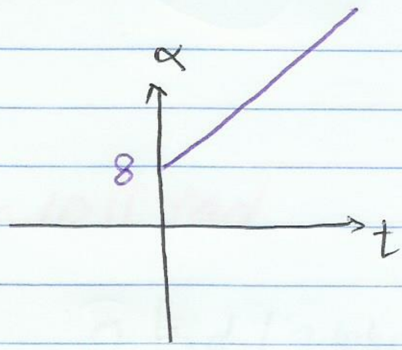
$W_4 = 128$ rad/s

d) At $t = 2s$, find its angular acceleration?

$$\alpha = \frac{dw}{dt} = 8 + 12t \qquad \alpha = 8 + 12t \text{ rad/s}^2$$

$\alpha_2 = 8 + 12(2) = 32$ rad/s^2

e) α is variable



Remember

1 rev = 360° = 2π rad

1 rad = 57.3° = 0.159 rev.

Rotation with constant Angular Acceleration

α Constant, you can use the following equation, as we did in chapter 2

$$\omega = \omega_0 + \alpha t$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \left(\frac{\omega_0 + \omega}{2}\right)\Delta t = \left(\frac{\omega_0+\omega}{2}\right)t$$

Sample Problem 10.03:

$\alpha = 0.35\ rad/s^2$

at $t=0$ $\to$ $\omega_0 = -4.6\ rad/s$, $\theta_0 = 0$

ⓐ find t? when $\theta = 5.0\ rev$
$= (5.0)(2\pi) = 10\pi\ rad$
$= 31.4\ rad$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$31.4 = -4.6t + \frac{1}{2}(0.35)t^2$$

$$0.175t^2 - 4.6t - 31.4 = 0$$

$$t^2 - 26.29t - 179.43 = 0$$

$$t = \frac{-26.29 \pm \sqrt{(26.29)^2 - 4(1)(-179.43}}{2}$$

$$= \frac{26.29 \pm 37.54}{2} = 32\ sec.$$

c) find t? when $\omega = 0$

$$\omega = \omega_0 + \alpha t$$

$$0 = -4.6 + 0.35t$$

$$t = \frac{4.6}{0.35}$$

$$= 13\ s.$$

Sample Problem 10.04 :-

$$\omega_0 = 3.40\ rad/s \xrightarrow[\alpha ?\ t ?]{\Delta\theta = 20 rev} \omega = 2.00\ rad/s$$

a) $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$, $\Delta\theta = 20 rev = 20(2\pi)\ rad$

$= 125.66\ rad$

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\Delta\theta}$$

$$= \frac{(2)^2 - (3.4)^2}{2(125.66)} = \frac{4 - 11.56}{251}$$

$$\alpha = -0.03\ rad/s^2$$

b) find t?

$$\omega = \omega_0 + \alpha t$$

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{2 - 3.4}{-0.031} = 46.5\ sec.$$

Problem 10.09

$$\omega_0 = 12.60\ rad/s \xrightarrow[\text{at constant rate of } 4.20\ rad/s^2]{\text{slows down}} \omega = 0$$

t? $\Delta\theta$?

slows means $\alpha = -4.2\ rad/s^2$

a) $\omega = \omega_0 + \alpha t$, $t = \frac{\omega - \omega_0}{\alpha}$

$$t = \frac{0 - 12.6}{-4.2} = 3 sec.$$

b) $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

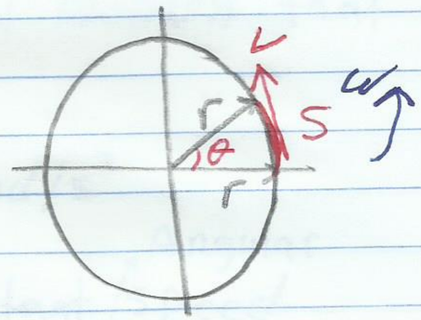
$$= (12.6)(3) + \frac{1}{2}(-4.2)(3)^2 = 18.9\ rad$$

or

$$\Delta\theta = \left(\frac{\omega_0 + \omega}{2}\right)t = \left(\frac{12.6 + 0}{2}\right)(3) = 18.9\ rad$$

Relating the Linear and Angular variables:

$S = r\theta$ S in m
r in m
$\theta \Rightarrow$ in radian



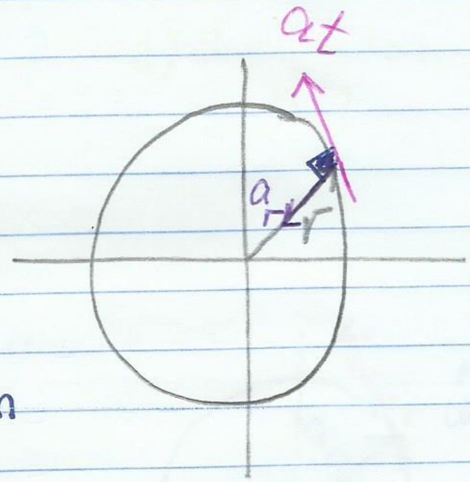
$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$V = \omega r$, V in m/s ,,, r in m
$\omega \Rightarrow$ rad/s

$$V = \frac{2\pi r}{T} = (2\pi r)f \quad m/s$$

$$\omega = \frac{2\pi}{T} \text{ rad/s} , \omega = 2\pi f \text{ rad/s}$$

$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

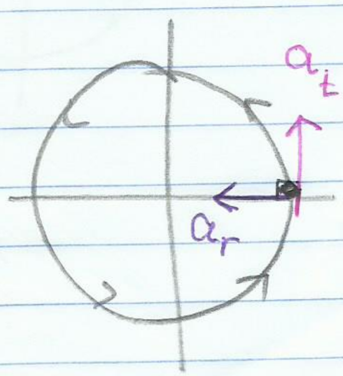


$a_t = \alpha r$ — m — rad/s^2 — m/s^2

a_t = tangential acceleration

$a_t = \alpha r$

$a_r = \frac{V^2}{r} = \omega^2 r$ radial acceleration



Sample Problem 10:05

The radius = 33.1m for rotational motion that rotates according to

$$\theta = ct^3, \quad c = 6.39\times10^{-2}\ \text{rad}/s^3$$

from $t_1 = 0 \longrightarrow t_2 = 2.30s$

After t_2 it will move at constant Angular speed

At $t = 2.20s$ find:

1) Angular speed ω?
2) Linear speed V?
3) Angular Acceleration α?
4) tangential acceleration a_t?
5) radial acceleration a_r?
6) acceleration a?

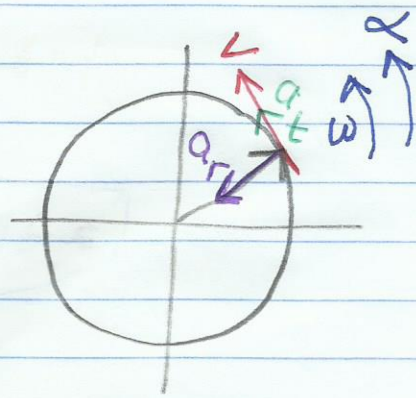
1) $\omega = \frac{d\theta}{dt} = 3ct^2$

$\omega = 3(6.39\times10^{-2})(2.20)^2$

$\omega = 0.928$ rad/s

2) $V = \omega r = (0.928)(33.1)$

$V = 30.7$ m/s



3) $\alpha = \frac{d\omega}{dt} = 6ct$

$\alpha = 6(6.39\times10^{-2})(2.20)$

$\alpha = 0.843$ rad/s^2

4) $a_t = \alpha r = (0.843)(33.1)$

$a_t = 27.9$ m/s^2

5) $a_r = \frac{V^2}{r} = \frac{(30.7)^2}{33.1} \Rightarrow a_r = 28.5$ m/s^2

6) $a = \sqrt{a_r^2 + a_t^2} = \sqrt{(28.5)^2 + (27.9)^2}$

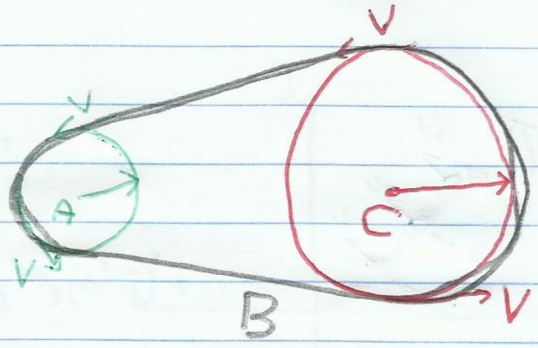
$a = 39.9$ m/s^2

(10 - 28)

Wheel A $\rightarrow r_A = 10cm$

Belt B

Wheel C $\longrightarrow r_C = 25cm$



$\omega_{0A} = 0 \quad \alpha_A = 1.6\ rad/s^2$

Find the time needed for wheel C to reach $\omega_C = 100 rev/min$?

$V_C = (\omega r)_C = \left(\frac{100(2\pi)}{60}\right)(0.25)$, $\omega_C = 100\ rev/min$

$= (10.5)(0.25)$ $\quad = \frac{100(2\pi)\ rad}{60\ s}$

$= 2.6\ m/s$ $\quad = 10.5\ rad/s$

$V_C = V_A = 2.6 m/s$

$V_A = \omega_A r_A \Rightarrow \omega_A = \frac{V_A}{r_A} = \frac{2.6}{0.1} = 26\ rad/s$

$\omega_A = 26\ rad/s$,,, $V_A = 2.6\ m/s$

$\omega_A = \omega_{0A} + \alpha_A t$

$26 = 0 + 1.6t \Rightarrow t = \frac{26}{1.6} = 16\ s$

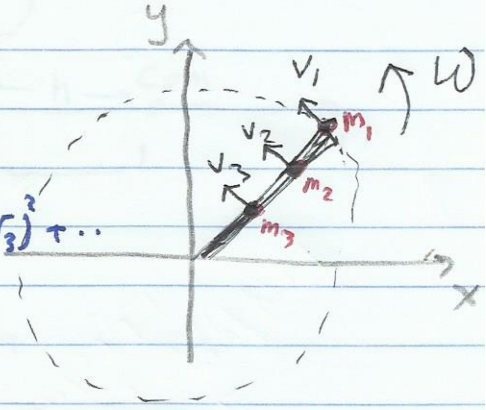
Kinetic Energy of Rotation:-

$$K_{rot} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \dots$$

$$= \frac{1}{2}m_1(\omega r_1)^2 + \frac{1}{2}m_2(\omega r_2)^2 + \frac{1}{2}m_3(\omega r_3)^2 + \dots$$

$$= \frac{1}{2}\left[m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \dots\right]\omega^2$$

$$= \frac{1}{2}\left(\sum m_ir_i^2\right)\omega^2$$



Note: that $\boxed{\omega}$ is th same for all particles

$$K_{rot} = \frac{1}{2}I\omega^2$$, where $I = \sum m_ir_i^2$ (Rotational Inertia) (Moment of Inertia)

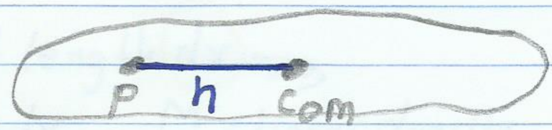
Calculating the Rotational Inertia:

for continous body $I = \int r^2\,dm$ kg.m²

Page 274 contains some rotational Inertias of different Bodies.

Parall-Axis Theorem

$$I_p = I_{com} + Mh^2 \quad kg.m^2$$

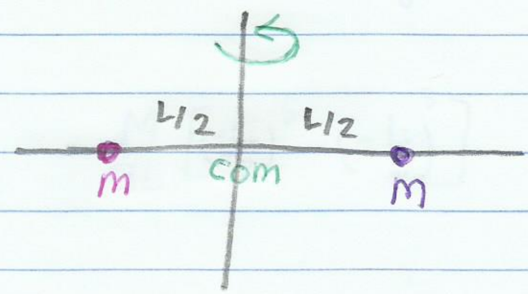


Sample Problem 10.8:

a) $m_1 = m_2 = m$

connected by massless rod of Length L

find I_{com} ?

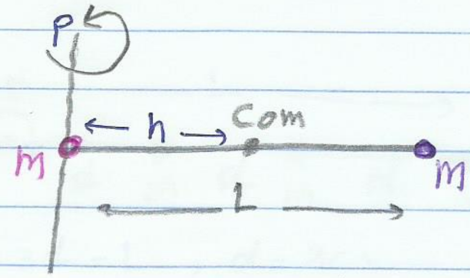


$$I = m_1r_1^2 + m_2r_2^2$$

$$I_{com} = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{2}mL^2 \quad kg.m^2$$

b) find I of the system about an axis through the left end of the rod Parallel to the first axis



$$I_p = \sum m_i r_i^2$$

$$= m(0)^2 + mL^2$$

$$I_p = mL^2$$

or → from Parallel axis theorem

$$I_p = I_{com} + Mh^2$$

$$= \frac{1}{2}ML^2 + 2M\left(\frac{L}{2}\right)^2$$

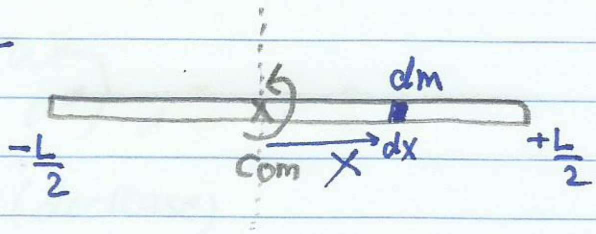
$$= \frac{1}{2}mL^2 + 2m\left(\frac{L}{2}\right)^2 = \frac{1}{2}mL^2 + \frac{1}{2}mL^2$$

$$= mL^2$$

Sample Problem 10.7: Rotational Inertia of a Uniform rod.

A thin Uniform rod → mass = M, → Length = L

a) What is the Rotational Inertia (I) of the rod about the Perpendicular rotation axis through the center?



$$I_{Com} = \int_{-L/2}^{+L/2} r^2\,dm \quad , r = x \quad ,, \text{ dm is of length dx} \Rightarrow$$

$$dm = \left(\frac{M}{L}\right)dx \quad , \quad \frac{M}{L} = \text{linear density kg/m}$$

$$= \int_{-L/2}^{+L/2} x^2\left(\frac{M}{L}\,dx\right)$$

$$= \frac{M}{L}\int_{-L/2}^{+L/2} x^2\,dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{+L/2} = \frac{M}{3L}\left[\left(+\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right]$$

$$I_{com} = \frac{1}{12}ML^2$$

b) What is the rod's I about a new rotation axis that is Perpendicular(عمودي) to the rod and through the left end?

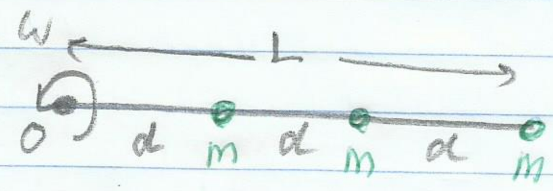
$$I_{left} = I_{com} + Mh^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$$

$$I_{end} = \frac{1}{3}ML^2$$

Problem 10.38

$m = 0.0100\,kg$

rod → $L = 6.00\,cm$ negligible mas



$I_i = \sum m_i r_i^2$

$= md^2 + m(2d)^2 + m(3d)^2$, $3d = L$, $d = 2cm$

$I_i = 14md^2 = 5.6 \times 10^{-5}\ kgm^2$

a) Find percentage of decreasing I, when the innermost mass was removed?

$I_1 = \sum m_i r_i^2$

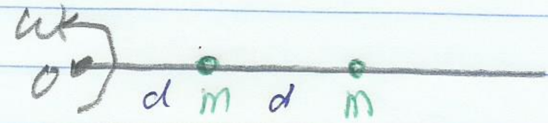
$= m(2d)^2 + m(3d)^2$

$= 13md^2 = 5.2 \times 10^{-5}\ kgm^2$

$\Delta I = 14md^2 - 13md^2 = md^2$ (decreasing)

$\frac{\Delta I}{I_i} 100\% = \frac{md^2}{14md^2} 100\% = 7.14\%$

b) find $\frac{\Delta I}{I_2}$? when the outermost m was removed?



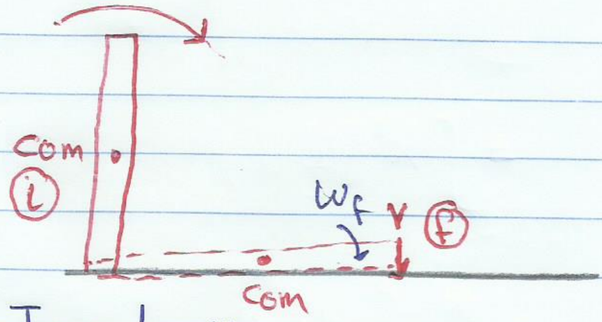
$I_2 = md^2 + m(2d)^2$

$= 5md^2$

$\Delta I = 14md^2 - 5md^2 = 9md^2$ (decrease)

$\frac{\Delta I}{I_i} = \frac{9md^2}{14md^2} = 0.6429 = 64.29\%$

(Problem 10.63) $L = 1m$



E is Conserved

$(K+U)_i = (K+U)_f$

$0 + mg\frac{L}{2} = \frac{1}{2} I \omega_f^2 + 0$, $I_{one\ end} = \frac{1}{3} mL^2$

$mg\frac{L}{2} = \frac{1}{2}\left(\frac{1}{3} ML^2\right)\omega_f^2$

$\omega_f = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3(9.8)}{1}} = 5.42\ rad/s$

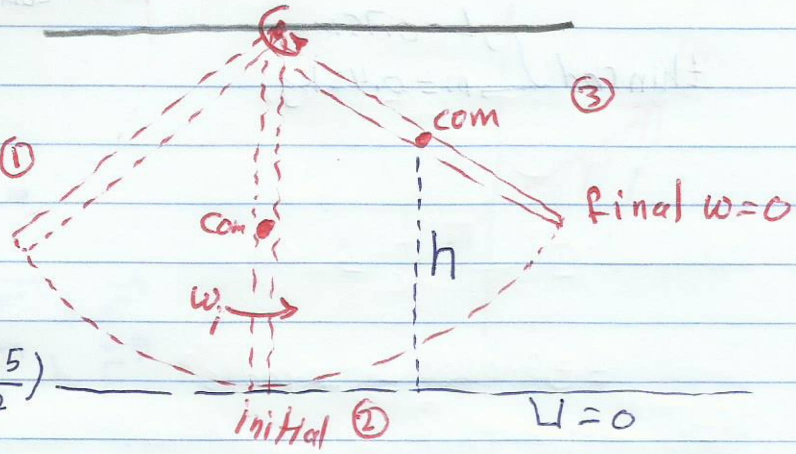
$V = \omega r$, $V_{right\ end} = \omega(1) = 5.42\ m/s$

$V_{com} = \omega(0.5) = 2.71\ m/s$

$V_{left\ end} = \omega(0) = 0$

Problem 10.60

$$(K+U)_i = (K+U)_f$$



$$\frac{1}{2}I\omega_i^2 + mg\frac{L}{2} = 0 + mgh$$

$$E_i = K_i + U_i$$

$$\frac{1}{2}(0.07875)(4)^2 + 0.42(9.8)(\frac{0.75}{2})$$

$$E_i = (0.63\,J) + (1.5435\,J)$$

rod
- $L = 0.75\,cm$
- $m = 0.42\,kg$
- $I = \frac{1}{3}Ml^2 = 0.07875\,kg\,m^2$

$\omega_i = 4\,rad/s$

$$E_f = mgh = 4.116h$$

$$E_i = E_f$$

$$2.1735 = 4.116h$$

$$h = 0.528\,m$$

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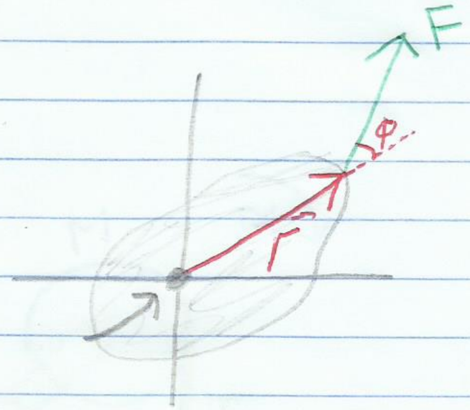
Torque:-

$\overrightarrow{Torque} = \vec{r} \times \vec{F}$



$\vec{\tau} = \vec{r} \times \vec{F}$ N.m

$\vec{\tau} = rF\sin\phi$

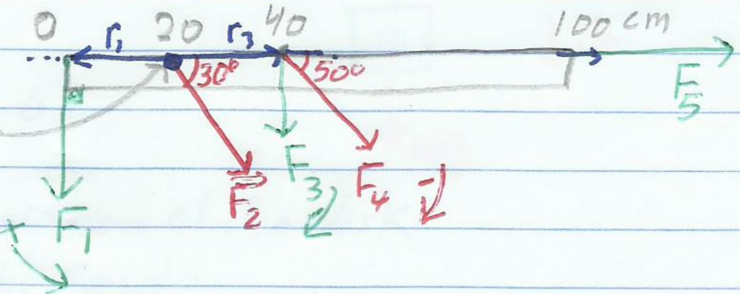
$\vec{\tau} \perp \vec{r}$

$\vec{\tau} \perp \vec{F}$

example (checkpoint 6)

$F_1 = F_2 = F_3 = F_4 = F_5 = 15N$

Pivot Point at mark 20cm



$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau_1 \overset{+}{=} (20 \times 10^{-2})(15)\sin 90 \quad = +3\ N.m$ Counter clockwise

$\tau_2 = (0)(15) \quad = 0$

$\tau_3 = (0.2)(15)\sin 90 \quad = -3\ N.m$ clockwise

$\tau_4 = (0.2)(15)\sin 50 \quad = -2.3\ N.m$ clockwise

$\tau_5 = (0.8)(15)\sin 0$

$= 0$

$\tau_1 = \tau_3 > \tau_4 > \tau_2 = \tau_5$

$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \vec{\tau}_4 + \vec{\tau}_5$

$= +3 + 0 + -3 + -2.3 + 0$

$= -2.3\ N.m$ clockwise

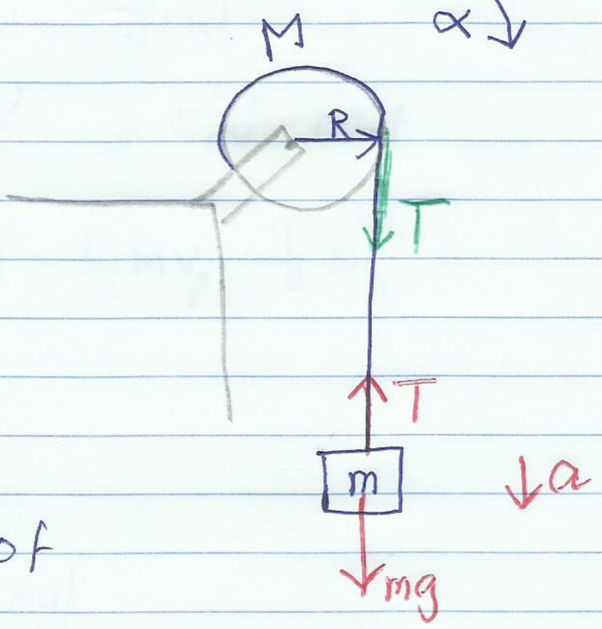
$\vec{\tau}_{net} \Longrightarrow$ Angular Acceleration

$\vec{\tau}_{net} \Longrightarrow \alpha$

Newton's Second Law for Rotation:

$$\vec{\tau}_{net} = I\vec{\alpha}$$

Sample Problem 10.10 :-



Uniform Disk → $M = 2.5 kg$
→ $R = 20 cm$
→ $I = \frac{1}{2} M R^2 = 0.05 kg.m^2$

$m = 1.2 kg$

find 1) Find the acceleration of Falling block (m) ?

2) Find the Angular Acceleration of the disk

3) Find the tension in the cord ?

1) For the Falling block (m) $\sum \vec{F} = m\vec{a}$

$T - mg = -ma$ ①

For the Rotating disk $\vec{\tau}_{net} = I\alpha$

$-RT = I(-\alpha)$

$RT = (\frac{1}{2}MR^2)\alpha$, $a = R\alpha$

$RT = (\frac{1}{2}MR^2)(\frac{a}{R})$

$T = \frac{1}{2}Ma$ ② → in ① get

$\frac{1}{2}Ma - mg = -ma \Rightarrow (\frac{1}{2}M + m)a = mg$

$a = \frac{mg}{(M/2) + m} = \frac{2mg}{M + 2m} = \frac{2(1.2)(9.8)}{2.5 + 2(1.2)} = 4.8 m/s^2$

2) $a = R\alpha$

$\alpha = \frac{4.8}{0.2} = 24 rad/s^2$

3) from ②

$T = \frac{1}{2}Ma$

$= \frac{1}{2}(2.5)(4.8)$

$= 6N$

Work and Rotational kinetic Energy:-

Remember: in translational motion

$$W = \int_{x_i}^{x_f} F\,dx \qquad \text{Joul}$$

$$\text{Power} = \frac{dW}{dt} = \vec{F}\cdot\vec{v} \quad \text{Watt}$$

$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{net} = \Delta K$$

In Rotational Motion:

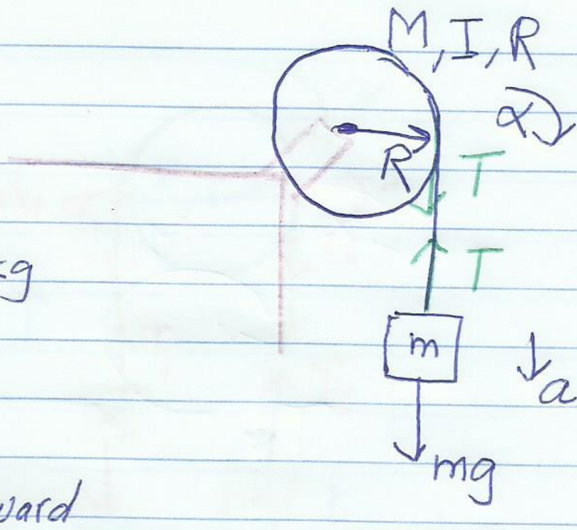
$$W = \int_{\theta_i}^{\theta_f} \tau\,d\theta \qquad \text{Joul}$$

$$\text{Power} = \frac{dW}{dt} = \tau\omega \qquad \text{Watt}$$

$$\Delta K = K_f - K_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 \quad \text{Joul}$$

$$W_{net} = \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 \qquad \text{Joul}$$

Sample Problem 10.11 :-



Uniform Disk → $M = 2.5\,kg$
→ $R = 20\,cm$
→ $I = \frac{1}{2}MR^2 = 0.05\,kg$

Falling block $m = 1.2\,kg$

From sample Problem 10.10
We found $a = -4.8\,m/s^2$ downward
$\alpha = -24\,rad/s^2$ clockwise
$T = 6N$

1) find Rotational kinetic Energy at $t = 2.5s$?

$\omega = \omega_0 + \alpha t$
$= 0 + 24(2.5)$
$= -60\,rad/s$ clockwise

$K_R = \frac{1}{2}I\omega^2$
$= \frac{1}{2}(0.05)(-60)^2$

$K_R = 90\,J$

2)* Find translational Kinetic Energy at $t = 2.5s$?

$v = v_0 + at$
$= 0 + -4.8(2.5)$
$= (-)12\,m/s$ downward

$K_t = \frac{1}{2}mv^2 = \frac{1}{2}(1.2)(12)^2 = 86.4\,J$

3)* Find the distance (Δy) moved by m ? during $t = 2.5s$

$\Delta y = V_{0y}t + \frac{1}{2}a_y t^2$
$= 0 + \frac{1}{2}(-4.8)(2.5)^2$

$\Delta y = -15\,m$ (Displacement)

distance $= 15\,m$

Problem 10.51:

$m_1 = 460g = 0.46 kg$

$m_2 = 500g = 0.5 kg$

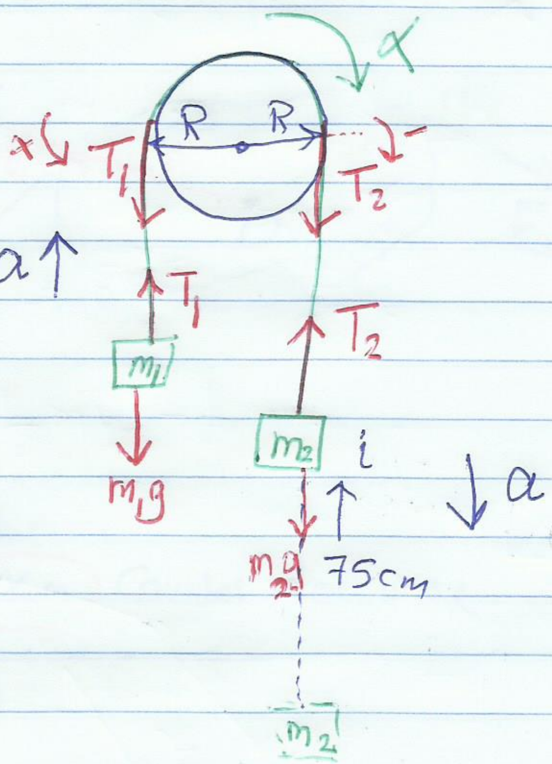
Pulley → $R = 5cm$

m_2 falls 75cm in $t = 5s$

a) Find a?

b) T_2 ? c) T_1 ? d) α ? of the pulley

e) I ? of the pulley



a) $\Delta y = v_{oy} t + \frac{1}{2} a_y t^2$ for m_2

$-0.75 = 0 + \frac{1}{2}(a_y)(5)^2$

$a_y = \frac{-0.75}{12.5} = -0.06 m/s^2$ downward.

$\boxed{a = 0.06 m/s^2}$ for each block as shown

Apply Newton's 2nd Law for each block $\sum \vec{F} = m\vec{a}$

b) for m_1

$m_1 a = T_1 - m_1 g \Rightarrow T_1 = m_1 a + m_1 g$

$= 0.46(0.06 + 9.8)$

$\boxed{T_1 = 4.536 N}$

For m_2

c) $m_2(-a) = T_2 - m_2 g \Rightarrow T_2 = m_2(g - a)$

$= 0.5(9.8 - 0.06)$

$\boxed{T_2 = 4.87 N}$

d) $a = R\alpha \Rightarrow \alpha = \frac{a}{R} = \frac{0.06}{0.75} = \boxed{0.08 \, rad/s^2}$ clockwise

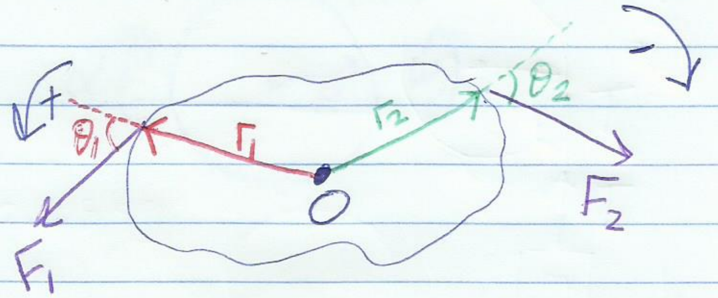
e) $\vec{\tau}_{net} = I\vec{\alpha}$

$+RT_1 + \bar{R}T_2 = I(-\alpha) \Rightarrow I = \frac{R(T_1 - T_2)}{-\alpha} = \frac{0.05(4.536 - 4.87)}{-0.08}$

$\boxed{I = 0.209 \, kgm^2}$

Problem 10.45

$\vec{F}_1 = 4.2N$, $\theta_1 = 75°$, $r_1 = 1.3m$
$F_2 = 4.9N$, $\theta_2 = 60$, $r_2 = 2.15m$



Find Net torque about the pivot O ?

$\vec{\tau}_{net}$? $\quad \vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau}_1 \overset{+}{=} r_1 F_1 \sin\theta_1$ counterclockwise

$\overset{+}{=} (1.3)(4.2)\sin 75 = 5.274\ N.m$ Counter clockwise

$\vec{\tau}_2 \overset{-}{=} r_2 F_2 \sin\theta_2$

$\overset{-}{=} (2.15)(4.9)\sin 60$ clockwise

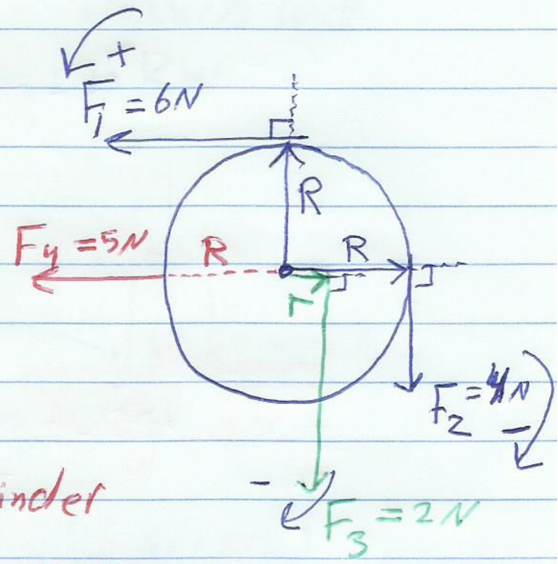
$\vec{\tau}_2 = -9.124\ N.m$

$\vec{\tau}_{net} = 5.274 + {}^{-}9.124$

$\vec{\tau}_{net} = -3.85 N.m$ (clockwise)

Problem 10.52:

Cylindr
- → mass = 2.0kg
- → R = 12cm
- → Rotate about the pivot through the center
- → $I = \frac{1}{2}MR^2 = \frac{1}{2}(2)(0.12)^2$
  $= 1.44 \times 10^{-2}$ kg.m
  $= 0.0144$ kg.m^2



4 Forces are Acting as shown , r = 5cm

Find $\vec{\alpha}$? of the rotating cylinder

$\vec{\tau} = \vec{r} \times \vec{F}$

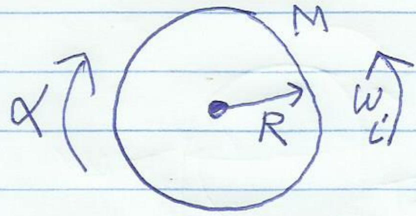
$\vec{\tau}_1 \overset{+}{=} RF_1 \sin 90 = +0.72\ N.m$
$\vec{\tau}_2 \overset{-}{=} RF_2 \sin 90 = -0.48\ N.m$
$\vec{\tau}_3 = {}^{-}rF_3 \sin 90 = -0.1\ N.m$
$\vec{\tau}_4 = RF_4 \sin 0 = 0$

$\vec{\tau}_{net} = {}^{+}0.14 N.m$

$\vec{\tau}_{net} = I\vec{\alpha}$

$\vec{\alpha} = \frac{+0.14}{0.0144} = +9.72\ rad/s^2$

Problem 10.61:-



Wheel, essentially a thin hoop

$I_{hoop} = MR^2 = (32)(1.2)^2$

$m = 32kg$

$R = 1.20m$

$I_{hoop} = MR^2 = (32)(1.2)^2$
$= 46.08 \ kg \cdot m^2$

$\omega_i = 280 \ rev/min$
$= 280 \frac{(2\pi \ rad)}{60s}$

$\omega_i = 29.32 \ rad/s \xrightarrow{t=15s} \omega_f = 0$

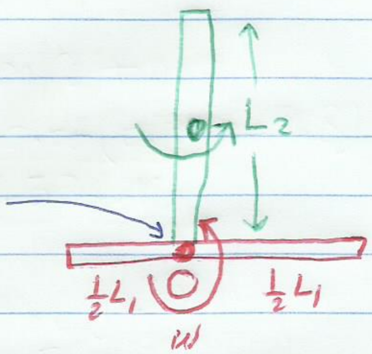
a) Find Work must be done to stop it?

$$W = \Delta K = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$$

$$W = 0 - \frac{1}{2}(46.08)(29.32)^2$$
$$= (-) \ 1.98 \times 10^4 \ J$$

b) $P_{avarage} = \frac{Work}{time} = \frac{-1.98 \times 10^4}{15} = -1.32 \times 10^3 \ Watt$
$= -1.32 \ KW$

Problem (10.100)



$m_1 = m_2 = 0.2kg$

$L_1 = 0.4m$

$L_2 = 0.5m$

a) Find I_O?

$$I_O = (I_1)_{cm} + (I_2)_{lower\ end}$$
$$= \frac{1}{12} m_1 L_1^2 + \frac{1}{3} m_2 L_2^2 = (2.667 \times 10^{-3}) + (16.667 \times 10^{-3})$$
$$= 19.3 \times 10^{-3} \ kg \cdot m^2$$

b) Find I_C? $I_C = (I_1)_C + (I_2)_C = m_1 \left(\frac{L_2}{2}\right)^2 + \frac{1}{3} m_2 L_2^2$
$= (0.2)\left(\frac{0.5}{2}\right)^2 + \frac{1}{3}(0.2)(0.5)^2$
$= 1.25 \times 10^{-2} + 1.67 \times 10^{-2}$

$I_C = 2.92 \times 10^{-2} \ kg \cdot m^2$