ENEE3309

Communication Notes

FM & PM Modulation

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Analog Communication System

$c(t) = A_c \cos(\theta(t))$ — A_c: ثابتة ; $\theta(t) = \omega t + \phi$ (ωt: freq, ϕ: phase)

- Amplitude Mod. ← magnitude proportional with message signal
  - Normal Am "DSB-FC"
  - DSB-SC
  - SSB-SC
    - USB-SC
    - LSB-C
- Angle Mod
  - Freq. Mod (FM)
  - phase mod. (PM)

Note:

$$\omega = \frac{\partial \theta(t)}{\partial t} \quad \text{at AM} \Rightarrow \omega_c = \frac{\partial \theta(t)}{\partial t} = 2\pi f_c$$

f_c: ثابت

But for FM

$f_i(t) = f_c + K_f m(t)$; K_f: frequency sensitivity [Hz/volt]

$$\omega_i = \frac{\partial \theta_i(t)}{\partial t} \Rightarrow \theta_i(t) = \int_0^t \omega_i(\tau)\, \partial\tau$$

$$= \int_0^t [2\pi f_i(\tau)\, \partial\tau] = \int_0^t 2\pi(f_c + K_f m(\tau))\partial\tau$$

$$= 2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, \partial\tau$$

Note: $f_i(t) - f_c = K_f m(t)$

$\Delta f = f_i(t) - f_c = K_f m(t) =$ freq deviation

if $m(t) = A_m \cos(2\pi f_m t)$

$\Delta f_{max} = K_f A_m$: max. freq. deviation
peak freq.

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

if we assume $m(t) = A_m \cos(2\pi f_m t)$

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m \tau)\,d\tau\right)$$

$$= A_c \cos\left(2\pi f_c t + \cancel{2\pi} k_f \cdot \frac{A_m}{\cancel{2\pi} f_m} \sin(2\pi f_m t)\right)$$

$$= A_c \cos\left(2\pi f_c t + \frac{\Delta f_{max}}{f_m} \sin(2\pi f_m t)\right)$$; let $\beta = \frac{\Delta f_{max}}{f_m}$: Modulation index

$$= A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

$$= A_c \cos(2\pi f_c t) \underbrace{\cos(\beta \sin(2\pi f_m t))}_{1} - A_c \sin(2\pi f_c t) \underbrace{\sin(\beta \sin(2\pi f_m t))}_{\beta \sin(2\pi f_m t)}$$

$\sin(0) = 0$ *

if $\beta << 1$ very small [narrow band]

إذاً sin أي زاوية = الزاوية نفسها

(في هذه الحالة) و $1 = \cos(0)$

$$\Rightarrow S_{FM}(t) = A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t) \cdot \beta \sin(2\pi f_m t)$$

Remember: $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

$\Rightarrow \sin(\alpha)\sin(\beta) = \frac{1}{2}\cos(\alpha - \beta) - \frac{1}{2}\cos(\alpha + \beta)$

$$S_{FM}(t) = A_c \cos(2\pi f_c t) - \frac{A_c \beta}{2}\left[\cos(2\pi(f_c - f_m)t) - \cos(2\pi(f_c + f_m)t)\right]$$

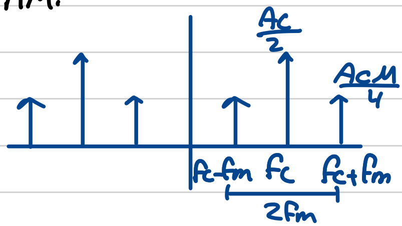
$$= A_c \cos(2\pi f_c t) - \frac{A_c \beta}{2} \cos(2\pi(f_c - f_m)t) + \frac{A_c \beta}{2} \cos(2\pi(f_c + f_m)t)$$

Remember : $S_{AM}(t) = A_c(1 + k_a m(t))\cos(2\pi f_c t)$; $m(t) = A_m \cos(2\pi f_m t)$

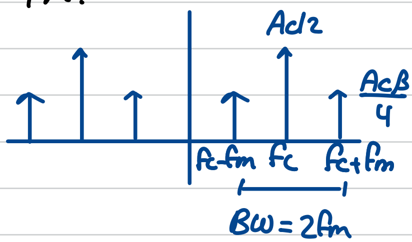
$$= A_c(1 + \mu \cos(2\pi f_m t))\cos(2\pi f_c t)$$

$$= A_c \cos(2\pi f_c t) + \frac{A_c \mu}{2} \cos(2\pi(f_c + f_m)t) + \frac{A_c \mu}{2} \cos(2\pi(f_c - f_m)t)$$

AM:



FM:



$$M_{\%} = \frac{\left(\frac{A_c B}{4}\right)^2 \cdot 4}{\left(\frac{A_c}{2}\right)^2 \cdot 2 + \left(\frac{A_c B}{4}\right)^2 \cdot 4} * 100\%$$

But $B \gg 1$ [wide band]

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau) d\tau\right) \quad \text{since} \quad m(t) = A_m \cos(2\pi f_m t)$$

$$\Rightarrow S_{FM}(t) = A_c \cos\left(2\pi f_c t + B \sin(2\pi f_m t)\right)$$

Periodic signal with fund. period T_m

$$= \operatorname{Re}\left\{A_c e^{j(2\pi f_c t + B\sin(2\pi f_m t))}\right\} = \operatorname{Re}\left\{A_c e^{j 2\pi f_c t} \cdot e^{jB\sin(2\pi f_m t)}\right\}$$

Periodic fund. period $T_m = \frac{1}{f_m}$

$$= \operatorname{Re}\left\{A_c e^{j2\pi f_c t} \cdot \sum_{n=-\infty}^{\infty} x_n e^{j2\pi n f_m t}\right\}$$

$$x_n = \frac{1}{T_m} \int_0^{T_m} x(t) e^{-j2\pi n f_m t} dt = \frac{1}{T_m} \int_0^{T_m} e^{jB\sin(2\pi f_m t)} e^{-j2\pi n f_m t} dt$$

$$= J_n(B)$$ ← Bessel function

$$J_n(x) = (-1)^n J_{-n}(x) \qquad J_0(t) \cong 1 \quad J_1(x) \cong \frac{x}{2}$$

$$J_n(x) = (-1)^n J_n(-x)$$

for small values of x_n, $J_n(x) \cong \frac{x^n}{2^n n!}$

$$\sum_{-\infty}^{\infty} (J_n(x))^2 = 1, \text{ for all } x$$

$$S_{FM}(t) = Re\left\{A_c e^{j2\pi f_c t} \cdot \sum_{n=-\infty}^{\infty} x_n e^{j2\pi n f_m t}\right\}$$

$$= Re\left\{A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi f_c t} \cdot e^{-j2\pi n f_m t}\right\}$$

$$= \boxed{A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi (f_c + n f_m) t)}$$

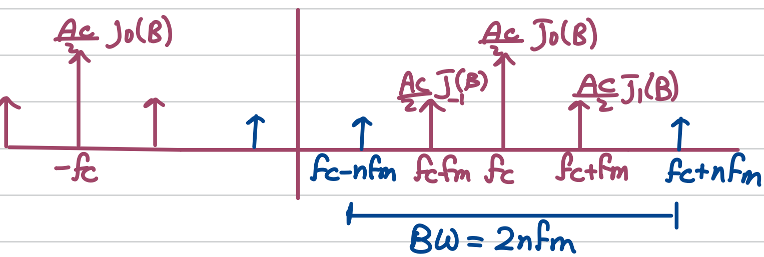
Remember: $S_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

Total avg power: $P_{avg} = \dfrac{A_c^2 \cancelto{1}{\sum_{-\infty}^{\infty} (J_n(\beta))^2}}{2} = \dfrac{A_c^2}{2}$

but $P_c = \dfrac{A_c^2}{2} J_0^2(\beta)$

To evaluate the BW:

$$S_{FM}(t) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - (f_c - n f_m)) + \delta(f + (f_c + n f_m))\right]$$



To evaluate Power

$$S_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi (f_c + n f_m) t)$$

$$P_{tot} = \frac{(A_c J_0(\beta))^2}{2} + \frac{(A_c J_1(\beta))^2}{2} + \frac{(A_c J_{-1}(\beta))^2}{2} + \frac{(A_c J_n(\beta))^2}{2} + \frac{(A_c J_{-n}(\beta))^2}{2}$$

$$= \frac{A_c^2}{2} \underbrace{\left[J_0^2(\beta) + 2J_1^2(\beta) + 2J_2^2(\beta) + \dots + 2J_n^2(\beta) \right]}_{0.99}$$

$$\boxed{\begin{array}{l} S_{FM}(t) = A_c \sum_{n=\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + nf_m)t) \\ \text{Total power} \quad P_{avg} = \frac{A_c^2}{2} \doteq \frac{A_c}{2} \left[J_0^2(\beta) + 2J_1^2(\beta) + 2J_2^2(\beta) + \dots + 2J^2(\beta) \right] \approx 0.99 \\ P_c = \frac{A_c^2 J_0^2(\beta)}{2} \qquad BW = 2nf_m \end{array}}$$

Ex: Find the 99% power bandwith at an FM modulated signal when $B=1$ and $\beta=0.2$ at

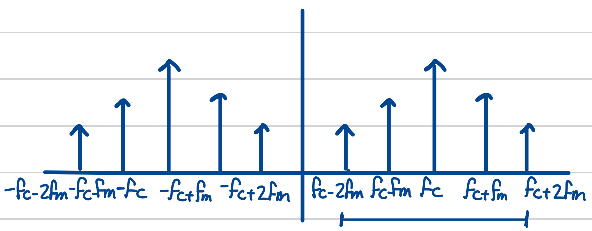
at $\beta = 1$

$$J_0^2(1) \approx (0.7652)^2$$

$$J_1^2(1) = (0.4401)^2$$

$$J_1^2(1) = (0.1149)^2$$

$$\underbrace{J_0^2(1) + 2J_1^2(1) + 2J_2^2(1) + \dots}_{0.99}$$



$BW = 2.2f_m = 4f_m$

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Freq. Mod

$C(t) = A_c\cos(\underbrace{2\pi f_c t + \theta}_{\phi(t)})$

A_c ↑ Amp $\phi(t)$ ↑ Angle

In FM

$S_{FM}(t) = A_c\cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau)$ if $m(t) = A_m\cos(2\pi f_m t)$

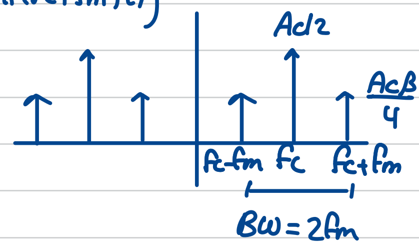
$S_{FM}(t) = A_c\cos(2\pi f_c t + \beta\sin(2\pi f_m t))$; β: modulation index $\beta = \frac{\Delta f_{max}}{f_m}$; Δf_{max}: peak freq. deviation

k_f: freq. sensitivity

if $\beta <<$ very small [narrow band FM]
(NBFM)

$S_{AM}(t) = A_c\cos(2\pi f_c t) - A_c\beta\sin(2\pi f_m t)\sin(2\pi f_c t)$

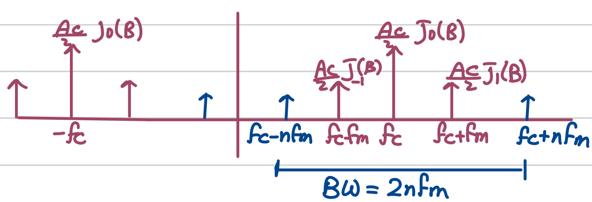
$= A_c\cos(2\pi f_c t) - \frac{A_c\beta}{2}\left[\cos(2\pi(f_c - f_m)t) - \cos(2\pi(f_c + f_m)t)\right]$



$BW_{FM} = 2f_{max}$

if $\beta >>$ [wideband FM (WBFM)]

$S_{FM}(t) = A_c\sum_{n=-\infty}^{\infty} J_n(\beta)\cos(2\pi(f_c + nf_m)t)$



To Evaluate BW

@ General ⇒ 99% Powerbandwith

$P_{avg} = \frac{A_c^2}{2}\left[J_0^2(\beta) + 2J_1^2(\beta) + \dots\right]$

let $\beta = 0.2$

β	$J_0(\beta)$	$J_1(\beta)$	$J_2(\beta)$
0.2	0.99	0.0995	

99% Power Bandwith

$J_0^2(0.2) = (0.99)^2$
$2J_1^2(0.2) = (0.0995)^2$ } 0.99

$BB = 2(1)f_m$

Note:

$$J_n(\beta) = \frac{1}{T_m}\int_0^{T_m} e^{j\beta\sin(2\pi f_m t)}\, e^{-j2\pi n f_m t}\, dt$$

② Carson's Rule (98% Power bandwith)

$$BW = 2f_m(\beta+1) = 2(\Delta f_{max} + f_m)$$

$$= 2BW_{message}(\beta+1) = 2(\Delta f_{max} + BW_{message})$$

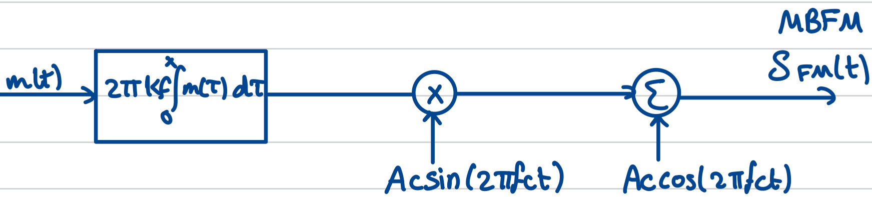
Generation FM

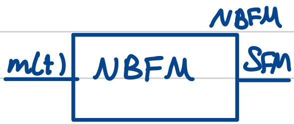
① Narrowband FM

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

if $m(t) = A_m \cos(2\pi f_m t)$

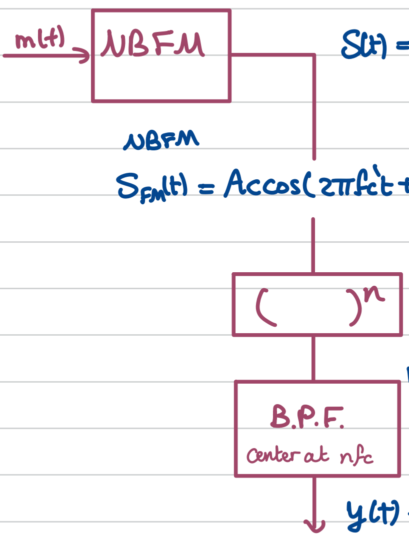
$$S_{FM}(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_m t)\sin(2\pi f_c t)$$





To generate WBFM

① In direct method



$$S(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

if $m(t) = A_m \cos(2\pi f_m t)$

NBFM

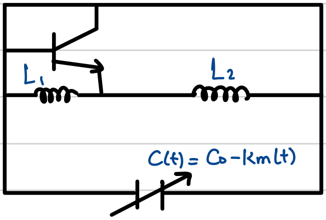
$$S_{FM}(t) = A_c \cos(2\pi f_c' t + \beta' \sin(2\pi f_m t))$$

← freq Multiplier

$$r(t) = A_c^2\left(\tfrac{1}{2} + \tfrac{1}{2}\cos(2\pi(2f_c')t) + 2\beta' \sin(2\pi f_m t)\right)$$

$$y(t) = \frac{A_c^2}{2}\cos(\quad)$$

Direct Method



$$f_c(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C(t)}} = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_0 - Km(t))}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}} \cdot \frac{1}{\sqrt{\left(1-\frac{Km(t)}{C_0}\right)}}$$

$$= f_c \cdot \left(1 - \frac{Km(t)}{C_0}\right)^{-\frac{1}{2}}, \quad \frac{Km(t)}{C_0} <<$$

$$\approx f_c \left(1 + \frac{1}{2} \cdot \frac{Km(t)}{C_0}\right)$$

$$\approx f_c + f_c \cdot \frac{K}{2C_0} m(t) \approx f_c + K_f m(t)$$

Demodulation FM

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right)$$

$$\Big\downarrow \frac{d(\ )}{dt}$$

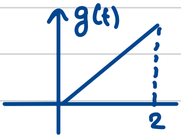
$$= A_c\left(2\pi f_c + 2\pi K_f m(t)\right) \sin\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right)$$

$$\Big\downarrow \text{Envelope detector}$$

$$A_c\left(2\pi f_c + 2\pi K_f m(t)\right)$$

$$A_c 2\pi K_f m(t)$$

Ex1: Consider the message signal $g(t) = \begin{cases} t & 0 \le t < 2 \\ 0 & o.w \end{cases}$ is applied to an FM modulator with sensitivity $K_f = 2$ Hz/volt to produce an FM signal $s(t)$. The unmodulated carrier is 100Hz



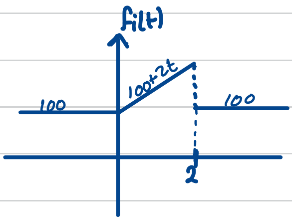
ⓐ Find and plot the instantaneous frequency $s(t)$ versus time.

ⓑ Find the peak frequency deviation of $s(t)$

ⓒ Suggest a method via which $g(t)$ can be recovered from $s(t)$

ⓓ Find the time-domain expression for $s(t)$ for all t.

① $f_i(t) = f_c + K_f m(t) = 100 + 2m(t)$



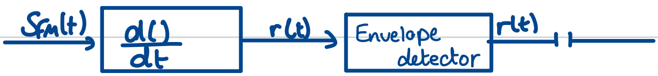
② $f_i(t) = f_c + K_f m(t)$

$f_i(t) - f_c = K_f m(t)$

$\Delta f = K_f m(t)$

$\Delta f_{max} = |K_f m(t)|_{max} = 4$

③



$S_{FM}(t) = A_c \cos(2\pi f_c t + 2\pi K_f) \int_0^t m(\tau) d\tau)$

$r(t) = A_c (2\pi f_c + 2\pi K_f m(t)) \sin(2\pi f_c t + 2\pi K_f) \int_0^t m(\tau) d\tau)$

$v(t) = A_c (2\pi f_c + 2\pi K_f m(t))$

$y(t) = A_c 2\pi K_f m(t)$

ⓓ $S_{FM}(t) = A_c \cos(2\pi f_c t + 2\pi K_f \int_0^t m(\tau) d\tau)$

$= A_c \cos(2\pi f_c t + 2\pi K_f \int_0^t g(\tau) d\tau)$

$= \begin{cases} A_c \cos(2\pi f_c t) & t < 0 \\ A_c \cos(2\pi f_c t + \pi K_f t^2) & 0 \le t < 2 \\ A_c \cos(2\pi f_c t + 2\pi K_f [\int_0^2 \quad + \int_2^\infty \quad] & t > 2 \end{cases}$

Ex: The audio signal $m(t) = A_m \cos(2\pi(100)t)$ frequency modulates the carrier $c(t) = \cos(2\pi(1000)t)$ when $A_m = 1.8$, $S(t)$ shows a peak frequency deviation of 320 Hz.

① Find the FM modulation index

② Use Carson's rule to estimate the bandwith of $S(t)$

③ write the modulated signal in time domain.

④ Find K_f

⑤ If A_m changes to 3.2 find the new frequency modulation index.

① $\beta = \dfrac{\Delta f_{max}}{f_m} = \dfrac{K_f A_m}{f_m}$

$= \dfrac{320}{100} = 3.2$

② $BW = 2f_m(\beta+1) = 2(100)(3.2+1)$

③ $S_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

④ $K_f = \dfrac{\beta f_m}{A_m} = \dfrac{3.2 \times 100}{1.8} = \dfrac{320}{1.8}$

⑤ $\Delta f_{new} = K_f A_{m_{new}} = \dfrac{320}{1.8} \cdot 3.2$ $\quad \beta = \dfrac{\Delta f_{max_{new}}}{f_m}$

Ex: Consider the FM signal $s(t) = 10\cos(2\pi(10000)t + 1.6\sin(2\pi(100)t)]$. The FM modulator sensitivity $K_f = 10 Hz/volt$ The modulated signal $s(t)$ is passed through an ideal bandpass filter with bandwidth 500 Hz centered at the carrier frequency $f_c = 10000$ Hz to produce the signal $g(t)$.

① Find the instantaneous freq. of $s(t)$

② Find the message $m(t)$

③ Find the peak freq. deviation of $s(t)$

④ Find the filter output $g(t)$

⑤ Find the fraction of the power contained in $g(t)$ to that in $s(t)$.

① $f_i(t) = \frac{1}{2\pi} \frac{d\phi_i(t)}{dt} = f_c + K_f m(t)$

$= \frac{1}{2\pi}\left(2\pi(10000) + 1.6(2\pi)(100)\cos(2\pi(100)t)\right]$

② $f_i(t) = f_c + K_f m(t)$

$f_i(t) - f_c = K_f m(t)$

$\frac{(1.6)(100)\cos(2\pi(100)t)}{K_f} = \frac{K_f}{K_f} m(t)$

③ $\Delta f_{max} = \beta f_m = (1.6)(100)$

④ Since $\beta >>$ [wideband]

$S_{FM}(t) = A_c \sum_{n=\infty}^{\infty} J_n(\beta)\cos(2\pi(f_c + n f_m)t)$

Angle Modulation → Freq. Mod. / phase Mod. ; $c(t) = A_c\cos(\underbrace{2\pi f_c t+\theta}_{\phi(t)})$

In freq Mod.

$$f_i(t) = f_c + k_f m(t)$$
$$f_i(t) - f_c = \Delta f = k_f m(t)$$
$$\Delta f_{max} = |k_f m(t)|_{max}$$
$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + k_f m(t)$$
$$\theta_c(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau$$
$$S_{FM}(t) = A_c\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right)$$

If $m(t) = A_m\cos(2\pi f_m t)$

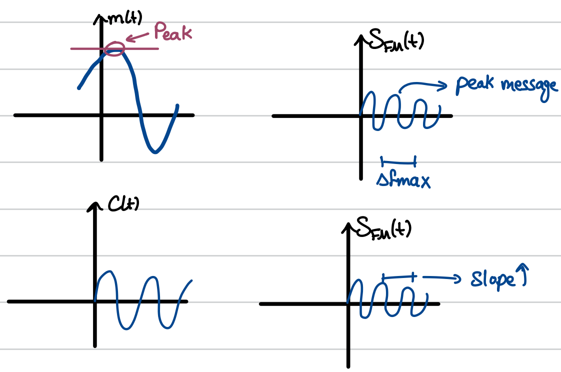
$$S_{FM}(t) = A_c\cos(2\pi f_c t + \beta\sin(2\pi f_m t))$$

where $\beta = \frac{\Delta f_{max}}{f_m} = \frac{k_f A_m}{f_m}$

In PM

$\theta = k_p m(t)$:

$$\Rightarrow S_{PM}(t) = A_c\cos(\underbrace{2\pi f_c t + k_f m(t)}_{\theta_i(t)})$$
$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = \frac{1}{2\pi}\left(2\pi f_c + k_f\frac{dm(t)}{dt}\right)$$
$$f_i(t) = f_c + \frac{k_f}{2\pi}\frac{dm(t)}{dt}$$
$$\Delta f_{max} = \left|\frac{k_f}{2\pi}\frac{dm(t)}{dt}\right|_{max}$$



To Evaluate BW, we can use Carson's Rule:

$$BW = 2f_m(\beta+1)$$

Bandwidth of message Signal

Ex: Consider the following angle modulated signal:

$$S(t) = 10\cos(\overbrace{2\pi*10^6 t + 2\sin(2\pi*10^4 t) + 4\sin(4\pi*10^4 t)}^{\phi_i(t)})$$

$f_c = 1\times10^6$ $\quad$ $f_1 = 10KHz$ $\quad$ $f_2 = 20KHz$

@ Evaluate the Bandwidth (FM & PM), using Carson's Rule:

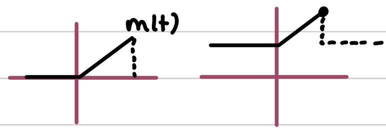
$$BW = 2\,BW_{message}(B+1) = 2f_{max}(B+1) = 2(\Delta f_{max} + \underbrace{f_{max}}_{\Leftarrow BW\ message})$$

Remember:

$$f_i(t) = \frac{1}{2\pi}\frac{d\phi_i(t)}{dt} = \frac{1}{2\pi}\left(2\pi*10^6 + (2)(2\pi*10^4)\cos(2\pi*10^4 t) + (4)(2\pi*10^4)\cos(4\pi*10^4)t\right)$$

$$= \underbrace{10^6}_{f_c} + 2*10^4\cos(2\pi*10^4 t) + 8*10^4\cos(4\pi*10^4)$$

$$\underbrace{f_i(t) - f_c}_{\Delta f} = 2*10^4\cancelto{1}{\cos(2\pi*10^4 t)} + 8\times10^4\cancelto{1}{\cos(4\pi*10^4)}$$



$\Delta f_{max} = 100KHz$

$$\Rightarrow BW = 2(\Delta f_{max} + \overset{BW\,m(t)}{f_{max}})$$
$$= 2(100+20)KHz$$

ⓑ Evaluate $m(t) \begin{cases} \to FM \\ \to PM \end{cases}$

In FM:

$$S_{FM}(t) = A_c\cos\left(2\pi f_c t + 2\pi k_f\int_0^t m(\tau)d\tau\right) = 10\cos(\overbrace{2\pi*10^6 t + 2\sin(2\pi*10^4 t) + 4\sin(4\pi\times10^4 t)}^{\phi_i(t)})$$

To Evaluate $m(t)$ in FM:

① $f_i(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} = k_f m(t) + f_c$

$$= \frac{1}{2\pi}(2\pi f_c + 2\pi k_f m(t))$$

$$\frac{d\theta_i(t)}{dt} = \cancel{2\pi}*\cancel{10^6} + \overset{2}{\cancel{4\pi}}*10^4\cos(2\pi*10^4t) + \overset{8\pi}{\cancel{16\pi}}\times10^4\cos(4\pi*10^4t)$$
$$= \cancel{2\pi} f_i(t) = \cancel{f_c} + K_f m(t)$$

but in PM:

$$S_{PM}(t) = A_c\cos(2\pi f_c t + K_p m(t)) = 10\cos(2\pi*10^6t + 2\sin(2\pi*10^4t) + 4\sin(4\pi*10^4t))$$
$$m(t) = \frac{2\sin(2\pi*10^4t) + 4\sin(4\pi*10^4t)}{K_p}$$

Ⓒ Evaluate the modulation index, B

$B = \frac{\Delta f_{max}}{f_m} \rightarrow$ Single tone

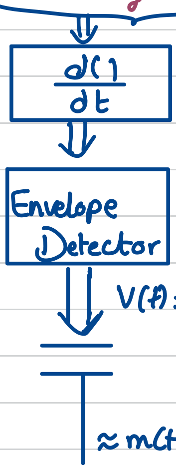
But for multi tone $\Rightarrow B = \frac{\Delta f_{max}}{f_{max}} = \frac{100}{20}$

$\Rightarrow BW_{\text{message Signal}}$

ⓓ Suggest demodulator to recover message:

ⓐ FM:

$$S_{FM}(t) = A_c\cos(2\pi f_c t + \underbrace{2\pi k_f\int_0^t m(\tau)d\tau}$$



$$r(t) = -A_c(2\pi f_c + 2\pi K_f m(t))\sin(2\pi f_c t + 2\pi k_f\int_0^t m(\tau)d\tau)$$

$$V(t) = A_c(2\pi f_c + 2\pi K_f m(t))$$

$$\approx m(t) = A_c 2\pi K_f m(t)$$

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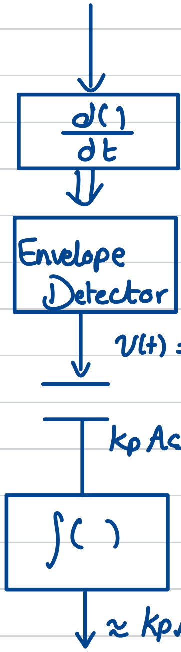
But in PM:

$$S_{PM}(t) = A_c \cos(2\pi f_c t + k_p m(t))$$



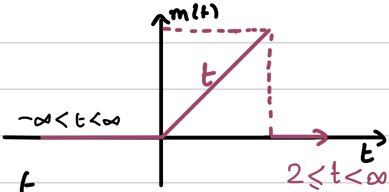
$$r(t) = A_c\left(2\pi f_c + k_f \frac{dm(t)}{dt}\right)\sin(2\pi f_c t + k_f m(t))$$

$$v(t) = A_c\left(2\pi f_c + k_f \frac{dm(t)}{dt}\right)$$

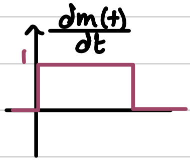
$$k_p A_c \frac{dm(t)}{dt}$$

$$\approx k_p A_c m(t)$$

Ex: Consider the following Signal:



$\Rightarrow$



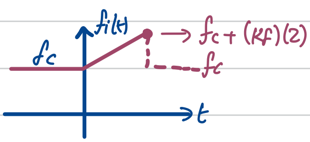
$\int_{-\infty}^{0} f(0) + \int_{0}^{t}$

If m(t) is applied to an angle modulator to produce angle modulated signal S(t)

ⓐ Find and Plot the instaneous freq of S(t) → FM, PM

In FM:

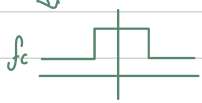
$f_i(t) = f_c + K_f m(t)$



In PM

$S_{PM}(t) = A_c \cos(\underbrace{2\pi f_c t + K_p m(t)}_{\Theta_i(t)})$

$f_i(t) = \frac{1}{2\pi}\frac{d\Theta_i(t)}{dt} = \frac{1}{2\pi}\left(2\pi f_c + K_p \frac{dm(t)}{dt}\right) = \boxed{f_c + \frac{K_f}{2\pi}\frac{dm(t)}{dt}}$



ⓑ Evaluate the peak freq. deviation

① FM

$\Delta f = f_i(t) - f_c = K_f m(t)$

$\Delta f_{max} = \left| K_f m(t) \right|_{max}$

② PM

$\Delta f = f_i(t) - f_c = \frac{K_f}{2\pi}\frac{dm(t)}{dt}$

$\Delta f_{max} = \left| \frac{K_f}{2\pi}\frac{dm(t)}{dt} \right|$

ⓒ Suggest a method via which m(t) can be recovered from S(t) → FM, PM

ⓓ Find the time-domain expression for $S(t)$ for all t

In FM: $S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right)$

For $-\infty < t < 0 \Rightarrow m(t) = 0$

$$S_{FM}(t) = A_c \cos(2\pi f_c t)$$

For $0 \leq t < 2$

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{0} (0)\, dt + 2\pi k_f \int_0^t \tau\, d\tau\right)$$

For $2 \leq t < \infty$

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{0} (0)\, d\tau + 2\pi k_f \int_0^2 (\tau)\, d\tau + 2\pi k_f \int_2^t (0)\, dt\right)$$

In PM:

$$S_{PM}(t) = A_c \cos\left(2\pi f_c t + k_f m(t)\right) = \begin{cases} A_c \cos(2\pi f_c t + k_f (0)) & -\infty < t < 0 \\ A_c \cos(2\pi f_c t + k_f t) & 0 \leq t < 2 \\ \vdots & \end{cases}$$