5.4 Binomial Probability distribution

* Discrete probability distribution
* It deal with experiment called binomial experiment

Ex:- Tossing a coin 10 times $\boxed{n=10}$

Define X : # of Heads

$X = 0, 1, 2, \ldots — 10$

Notice that :-

[1] This experiment consist of 10 identical trails $\boxed{n=10}$

[2] The trails are ~~is~~ independent

[3] Each trail has two outcomes

Head : (success)

Tail : (Failure)

[4] In each trail :-

The probability of success $= P(\text{success})$ is Fixed
$= P$

& the probability of failure $= P(\text{Failure})$ is fixed
$= 1 - P$

This experiment is called $\boxed{\text{a binomial experiment}}$

In general: A binomial experiment satisfies

$\boxed{1}$ The experiment consist of n identical trails

n : Sample size $\boxed{n \geq 2}$

$\boxed{2}$ The trials are independent

$\boxed{3}$ Each trail has two outcomes

success : The one we study (depends on X)

failure : The complement (no success)

$\boxed{4}$ In each trial:-

The probability of success = P(success) is fixed, denoted by P.

& The probability of failure = P(failure) is fixed (1-P)

Ex: 65% of BZU employees own private cars

We took a random sample 25 employees.

Define X :- number of employees who own private cars.

Find $n = ??$ $\quad P =$ $\quad 1-P =$

$X =$

How to find $P(X)$? X: number of success

We use the formula :-

$$\boxed{f(x) = P(X) = C_x^n \, P^x (1-P)^{n-x}}$$

n: # of trials (Sample size)

P: probability of success (if any trial)

Note:

1. $P(0) + P(1) + \text{———} P(n) = 1$

2. $E(x) = \boxed{\mu = nP}$

3. $Var(X) = \boxed{\sigma^2 = nP(1-P)}$

4. Standard deviation of $x = \sigma = \sqrt{nP(1-P)}$

Ex:- The percentag of students who like early classes is 40%. In a random sample of 8 students.

1. What is the probability that 3 student like early classes?
2. What is the probability that ALL of them like early classes
3. What is the expected number of students who like early classes
4. What is the = = = = = dislike early classes
5. What is the variance = = = like early classes
6. What is the prob. that at most 2 student like early classes
7. What is the prob. that at least 2 student like early classes

Exp:- let $X = B(6, \underline{0.4})$ $P = 0.4$

$1-P = 0.6$

$\boxed{1}$ Write the prob. function

$\boxed{2}$ Find $E(x)$ $Var(X)$, $\sigma(x)$

$\boxed{4}$ Find the prob. of exactly 4 successes

$\boxed{5}$ Find the prob of no more than (at most) 2 failures.

Solution:-

$\boxed{1}$ $f(x) = C^6_x (0.4)^x (0.6)^{6-x}$ $x = 0, 1, \ldots 6\}$

$\boxed{2}$ $E(x) = n \cdot p = 6 * 0.4 = 2.4$

$\boxed{3}$ $Var(x) = n\,p\cdot(1-p) = 6 * 0.4 * 0.6 = 1.44$

$\boxed{4}$ $f(4) = C^6_4 (0.4)^4 (0.6)^2 = 0.1382$.

Calculater [6 $\boxed{nCr}$ 4 $\boxed{x}$ 0.4 $\boxed{\wedge}$ 4 $\boxed{x}$ 0.6 $\boxed{\wedge}$ 2 =]

$\boxed{5}$ At most 2 failures $\xrightarrow{\text{Mean}}$ At least 4 Success.

* Define a new Binomial Variable with 6 trial

Prob. of success (0.6) $X = B(6, 0.6)$

$P(\text{At most 2 failures}) = f(2) + f(1) + f(0)$

$= C^6_2 (0.6)^2 (0.4)^4 + C^6_1 (0.6)^1 (0.4)^5 + C^6_0 (0.6)^0 (0.4)^6$

A survey founded that one out of five Palestinians say he or she visited a doctor last month

If 10 people are ~~selected~~ selected at random, what is the probability that none of them visited a doctor last ~~month~~ month

Solution:- X:- # of palestinian visited a doctor last month

$n = 10$

$1 - p = \frac{4}{5}$

$p = \frac{1}{5}$

$X = 0, 1, 2, 3, 4, \ldots\ldots 10$

$P(0) = C^{10}_{0} \left(\frac{1}{5}\right)^{0} \left(\frac{4}{5}\right)^{10}$

$= 0.1074$