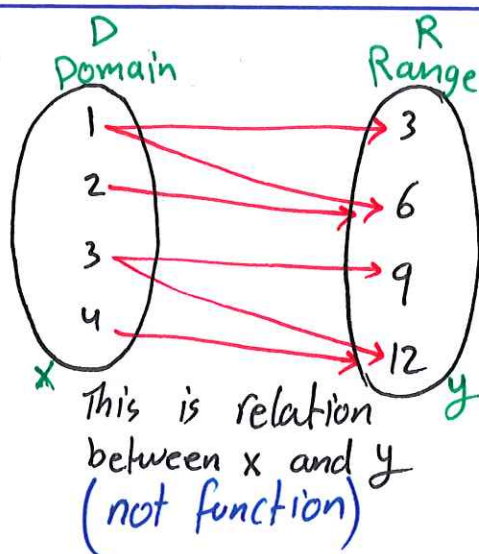


- The set of ordered pairs $\{(1,3), (1,6), (2,6), (3,9), (3,12), (4,12)\}$ represents relation between the 1st component $x \in \{1,2,3,4\}$ and the 2nd component $y \in \{3,6,9,12\}$.
- The set of first components is called the domain (input)
- The set of second components is called the range (output)

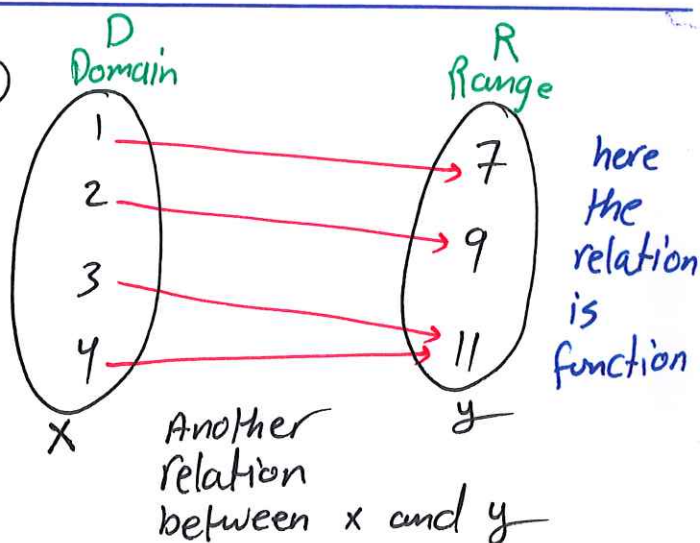
Def. A relation is set of ordered pairs or is a rule that determines how the ordered pairs are found

- A relation is also defined by a table or graph or equation or inequality

Exp
①



②

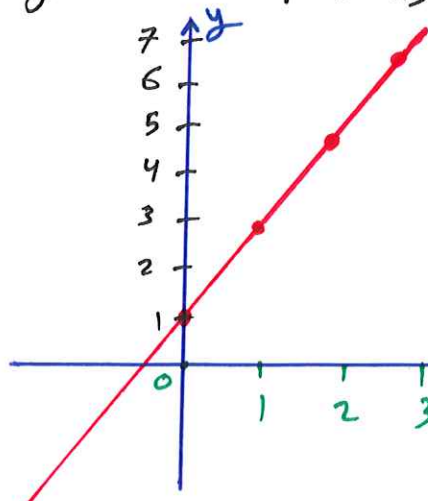


③ The equation $y = 2x + 1$ represents relation between x and y

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x	y
0	1
1	3
2	5
3	7
$\vdots$	$\vdots$

Table



x : independent variable
 y : dependent variable

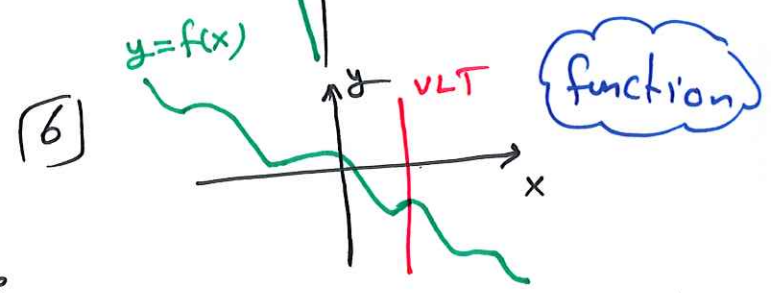
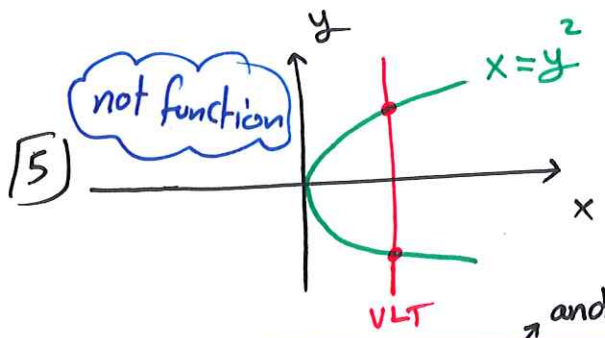
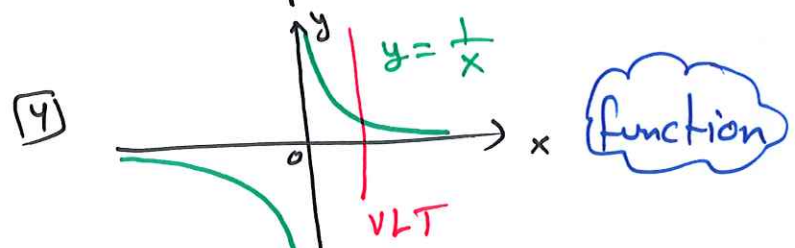
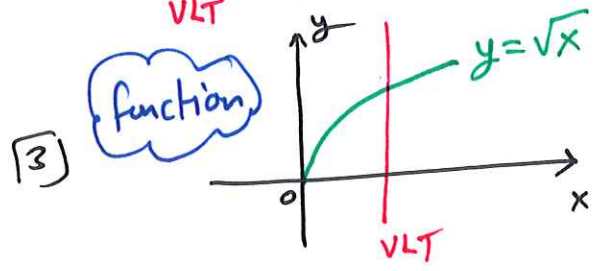
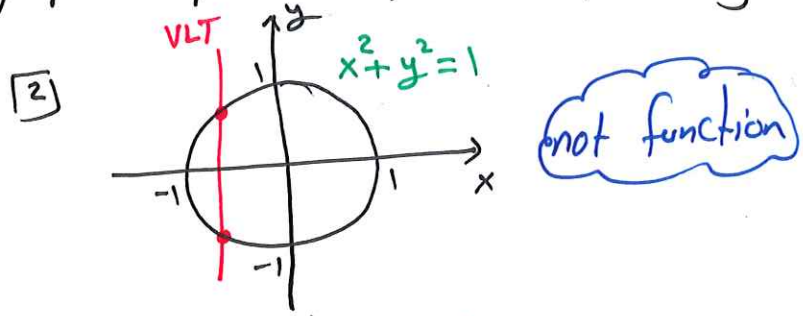
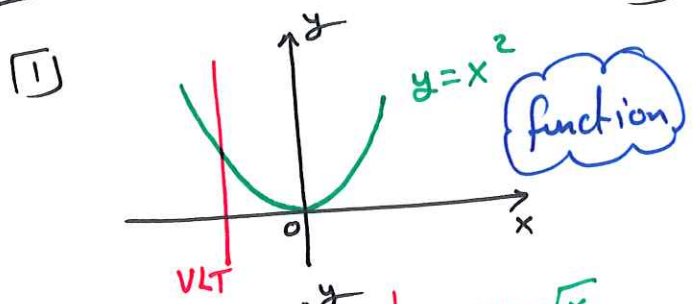
Each value of x is substituted to give only one value of y

so this relation represents function

Def A function is relation between two sets such that each element in domain is assigned to a unique element in range.

Remark: If any vertical line intersects the graph at most once, then this graph represents function.
VLTest Otherwise, the graph is not function

Exp* which of the following graphs represent function? Justify



Exp Find the domain and Range of functions in Exp*

- [1] Domain (values of x) is the set of all real numbers $\mathbb{R}$
- [3] Domain (values of x) is $[0, \infty)$
- [4] Domain (values of x) = $\mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$
- [6] Domain = $\mathbb{R} = (-\infty, \infty)$

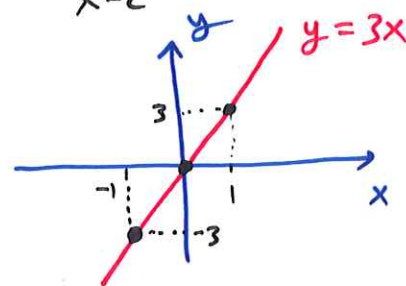
- Range (values of y) $\Rightarrow$
- [1] Range = $[0, \infty)$
 - [3] Range = $[0, \infty)$
 - [4] Range = $\mathbb{R} \setminus \{0\}$
 - [6] Range = $\mathbb{R}$

Exp Find domain and Range of the functions

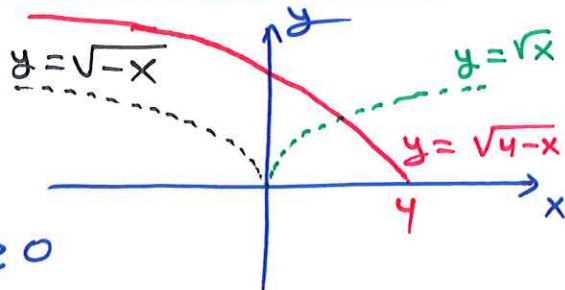
47

1) $y = 3x$ 2) $y = \sqrt{4-x}$ 3) $y = 1 + \frac{1}{x-2}$

1) $y = 3x \Rightarrow D = \mathbb{R} = (-\infty, \infty)$
 $R = \mathbb{R} = (-\infty, \infty)$



2) $y = \sqrt{4-x}$



Domain $\Rightarrow 4-x \geq 0$

$4 \geq x \Rightarrow \text{Domain} = (-\infty, 4]$

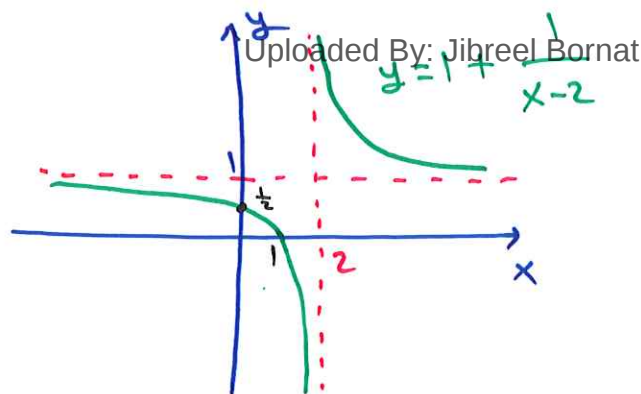
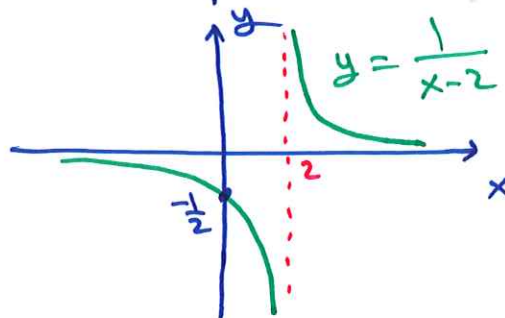
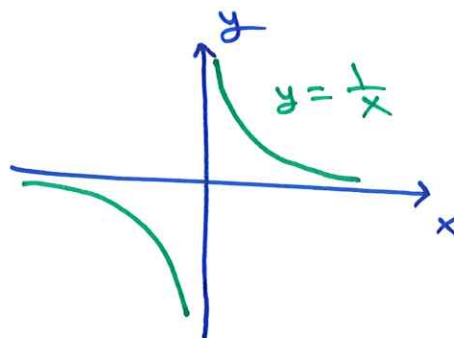
Range = $[0, \infty)$ "values of y"

3) $y = 1 + \frac{1}{x-2}$

Domain $\Rightarrow x-2 \neq 0$
 $x \neq 2$

$\Rightarrow \text{Domain} = \mathbb{R} \setminus \{2\}$

Range = $\mathbb{R} \setminus \{1\}$



Exp Suppose $g(x) = 8x - 10$ Find

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$$\textcircled{1} g(0) = 8(0) - 10 = 0 - 10 = -10$$

$$\textcircled{2} g(1) = 8(1) - 10 = 8 - 10 = -2$$

$$\textcircled{3} g(-\frac{3}{4}) = \cancel{8}^2(-\frac{3}{4}) - 10 = -6 - 10 = -16$$

$$\textcircled{4} g(3) = 8(3) - 10 = 24 - 10 = 14$$

$$\textcircled{5} \frac{g(x+h) - g(x)}{h} = \frac{8(x+h) - 10 - (8x - 10)}{h}$$

$$= \frac{\cancel{8x} + 8h - \cancel{10} - \cancel{8x} + \cancel{10}}{h} = \frac{8h}{h} = 8, h \neq 0$$

Operations with Functions

Let f and g be functions of x

$$\text{Sum : } (f + g)(x) = f(x) + g(x)$$

$$\text{Difference : } (f - g)(x) = f(x) - g(x)$$

$$\text{Product : } (f \cdot g)(x) = f(x) \cdot g(x)$$

$$\text{Quotient : } \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$$

Exp Let $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$. Find

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$$\textcircled{1} (f + g)(4) = f(4) + g(4) = \sqrt{4} + 4^2 - 1 = 2 + 16 - 1 = 17$$

$$\textcircled{2} (f \cdot g)(1) = f(1) \cdot g(1) = \sqrt{1} \cdot (1^2 - 1) = 1 \cdot 0 = 0$$

$$\textcircled{3} \left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{\sqrt{4}}{4^2 - 1} = \frac{2}{16 - 1} = \frac{2}{15}$$

$$\textcircled{4} (f \cdot f)(4) = f(4) f(4) = \sqrt{4} \cdot \sqrt{4} = 2 \cdot 2 = 4 = [f(4)]^2$$

Def • Let f and g be any functions.

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The composite function g of f is defined by

$$(g \circ f)(x) = g(f(x))$$

• Similarly we define the composite function f of g :

$$(f \circ g)(x) = f(g(x))$$

Exp Let $f(x) = 1 - 2x$ and $g(x) = 3x^2$. Find

$$① (g - f)(x) = g(x) - f(x) = 3x^2 - (1 - 2x) = 3x^2 + 2x - 1$$

$$② (f \circ g)(1) = f(g(1)) = [1 - 2(1)][3(1)^2] = (1 - 2)(3) = -3$$

$$③ (f \circ g)(0) = f(g(0)) = f(3(0)^2) = f(0) = 1 - 2(0) = 1 - 0 = 1$$

$$④ (g \circ f)(x) = g(f(x)) = g(1 - 2x) = 3(1 - 2x)^2$$

$$⑤ (f \circ f)(x) = f(f(x)) = f(1 - 2x) = 1 - 2(1 - 2x) = 1 - 2 + 4x = 4x - 1$$

$$⑥ (f \circ g)(x) = f(g(x)) = f(3x^2) = 1 - 2(3x^2) = 1 - 6x^2$$

Note $(f \circ g)(x)$ not necessarily equals to $(g \circ f)(x)$

see ④ and ⑥ in E_{xp}^{**}