

Th (Fundamental Th of Calculus)

• $f(x)$ cont. on $[a, b]$

• $F(x)$ is any antiderivative of $f(x)$

Then ① $\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$

$$f(x) = 2x$$

$$F_1(x) = x^2 + 1$$

$$F_2(x) = x^2 - \sqrt{5}$$

⋮

② If $F(x) = \int_a^x f(t) dt$ cont. on $[a, b]$ and
diff on (a, b)

then $F'(x) = f(x) \quad 1 = f(x)$

Exp Find the following derivatives

① $\frac{d}{dx} \int_0^x \cot t dt = \cot x \cdot 1 = \cot x$

② $\frac{d}{dx} \int_{\frac{\pi}{2}}^{\pi} \left(\frac{\csc \theta}{1 - \cos \theta} \right) d\theta = 0$

تکس یکدرد
جواب رفته
= 0

③ $\left(\int_{\tan x}^{\frac{\pi}{4}} \frac{dt}{1+t^2} \right)' = \left(- \int_{\frac{\pi}{4}}^{\tan x} \frac{dt}{1+t^2} \right)'$

$\tan x$

$$= - \frac{1}{1 + (\tan x)^2} \cdot \sec^2 x$$

$$= - \frac{\sec^2 x}{1 + \tan^2 x} = - \frac{\sec^2 x}{\sec^2 x} = -1$$

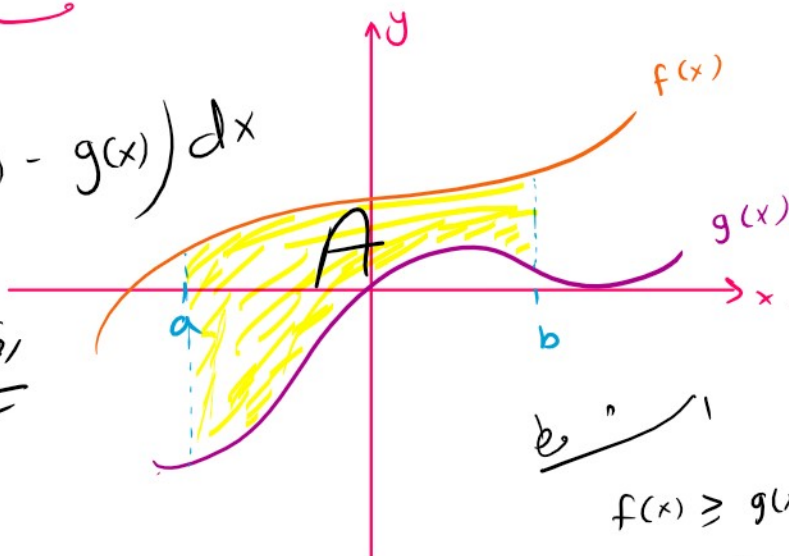
$$F(x) = \int_a^{u(x)} f(t) dt$$

$$F'(x) = f(u(x)) u'(x) \quad \checkmark$$

Area المساحة

$$A = \int_a^b (f(x) - g(x)) dx$$

المساحة



b

$f(x) \geq g(x)$ on $[a, b]$

f, g integrable

b

محدود

$$(f(x) - g(x)) dx$$

$$d \uparrow y = f(x) \downarrow$$

(b) $A = \int_a^b (f(x) - g(x)) dx$ (عمودي)

(d) $A = \int_c^d (g(y) - f(y)) dy$ (افقي)

Exp $y = 5 - x^2$, $y = 1$
Find area enclosed by these functions

$y = 5 - x^2$
 $5 - x^2 = 1$
 $x^2 = 4$
 $x = \pm 2$

$A = \int_{-2}^2 (5 - x^2 - 1) dx$

$= \int_{-2}^2 (4 - x^2) dx = 4x - \frac{x^3}{3} \Big|_{-2}^2 = 8 - \frac{8}{3} = \frac{16}{3}$

$A = \int_1^5 (\sqrt{5-y} - 0) dy = \frac{16}{3}$

$2A = 2 \left(\frac{16}{3} \right) = \frac{32}{3}$ ✓

$= \int_{-2}^2 (4 - x^2 - 1) dx$

$$= \int_{-2}^2 (5 - x^2 - 1) dx$$

Exp Find area enclosed by $f(x) = 3x\sqrt{x^2+1}$ and x-axis on $[0,1]$

موجبة، متزايدة $[0,1]$

$$A = \int_0^1 3x\sqrt{x^2+1} dx$$

$$= \int_1^2 3\sqrt{u} \frac{du}{2}$$

$$= \frac{3}{2} \int_1^2 u^{\frac{1}{2}} du$$

$$= \frac{3}{2} \left[\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_1^2$$

$$= \frac{3}{2} \left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^2$$

$$= \sqrt{u^3} \Big|_1^2$$

$$= \sqrt{8} - \sqrt{1}$$

$$= 2\sqrt{2} - 1$$

$$u = x^2 + 1$$

$$du = 2x dx$$

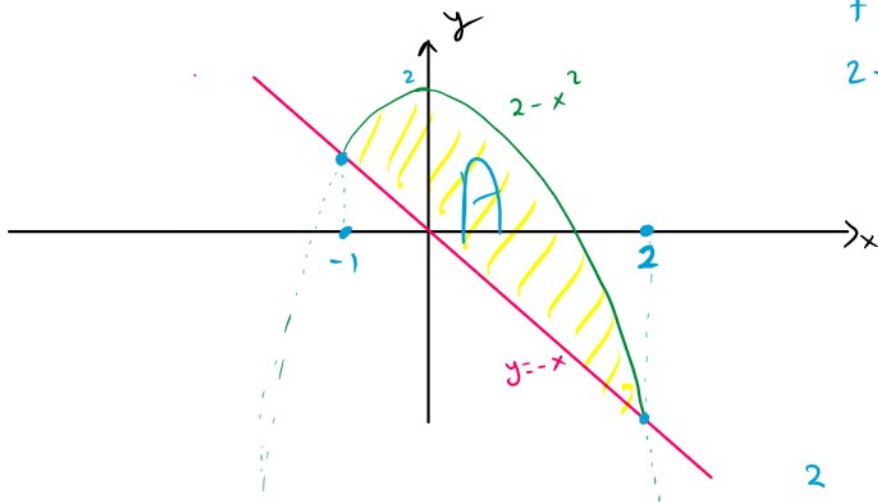
$$\frac{du}{2} = x dx$$

$$x=0 \Rightarrow u = 0^2 + 1 = 1$$

$$x=1 \Rightarrow u = 1^2 + 1 = 2$$

$$= 2\sqrt{2} - 1$$

Exp Find area enclosed by $f(x) = 2 - x^2$ and $y = -x$



$$f(x) = y$$

$$2 - x^2 = -x$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2, x = -1$$

$$A = \int_{-1}^2 (2 - x^2 - (-x)) dx = \int_{-1}^2 (2 - x^2 + x) dx$$

$$= 2x - \frac{x^3}{3} + \frac{x^2}{2} \Big|_{-1}^2$$

$$= \left(4 - \frac{8}{3} + \frac{4}{2}\right) - \left(-2 - \frac{1}{3} + \frac{1}{2}\right)$$

$$= 4 - \frac{8}{3} + 2 + 2 - \frac{1}{3} - \frac{1}{2}$$

$$= 8 - \frac{8}{3} - \frac{1}{3} - \frac{1}{2} = \boxed{\frac{9}{2}}$$

Exp $y = \sqrt{x}$, x -axis, $y = x - 2$
Find area enclosed by these curves and lines

Q18

Find area enclosed by these curves and lines

(1) with respect to x-axis

(2) " " " " y-axis

$$\sqrt{x} = x - 2$$

$$x = (x-2)^2$$

$$x = x^2 - 4x + 4$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

$$x=4, x=1$$

$$A = \int_0^2 (y+2 - y^2) dy$$

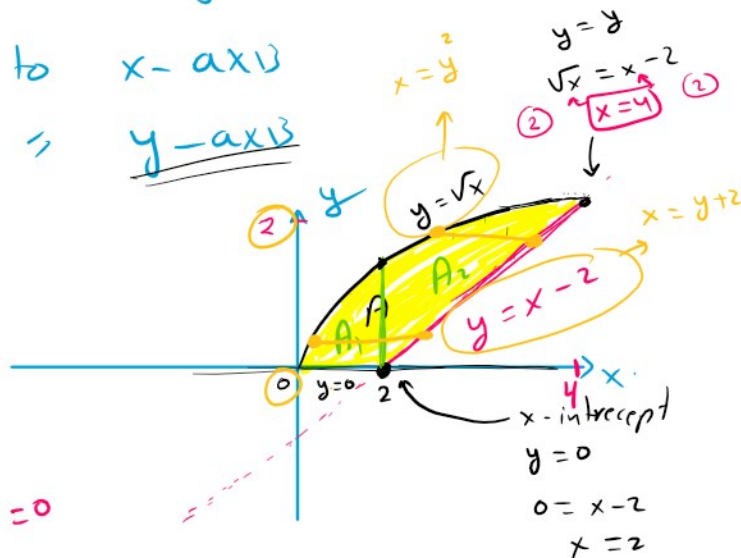
$$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_0^2$$

$$= 2 + 4 - \frac{8}{3}$$

$$= 6 - \frac{8}{3}$$

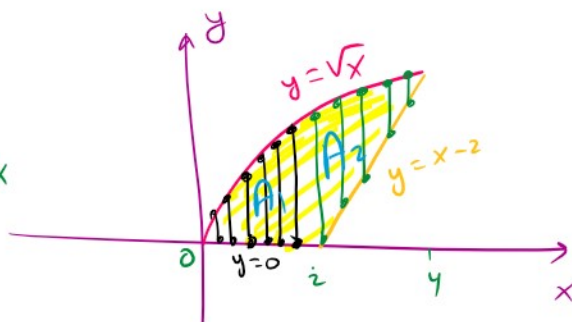
$$= \frac{10}{3}$$

$$A = \int (\text{top} - \text{bottom}) dx$$



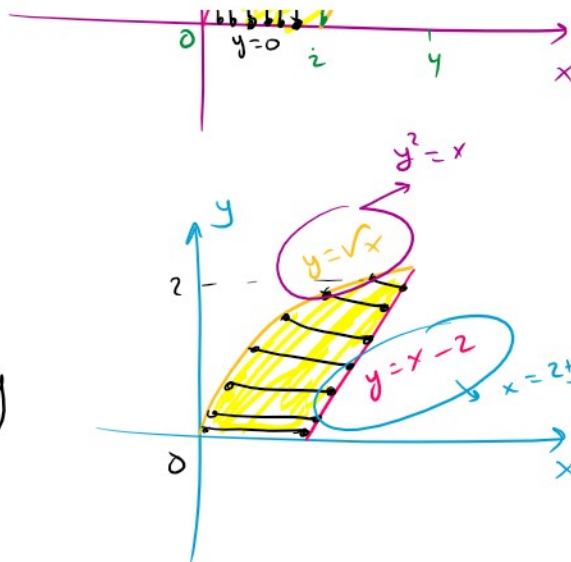
(1) x-axis

$$\begin{aligned} A &= A_1 + A_2 \\ &= \int_0^2 (\sqrt{x} - 0) dx + \int_2^4 (\sqrt{x} - (x-2)) dx \\ &= \int_0^2 x^{\frac{1}{2}} dx + \int_2^4 (x^{\frac{1}{2}} - x + 2) dx \\ &= \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^2 + \left[\frac{2}{3} x^{\frac{3}{2}} - \frac{x^2}{2} + 2x \right]_2^4 \\ &= \dots = \frac{10}{3} \end{aligned}$$



$$A = A_1 + A_2$$

$$A = \int_0^2 \left(\sqrt{2+y} - \sqrt{y} \right) dy$$



Exp $\int \sqrt{\frac{x^4}{x^3-1}} dx$

$$\int \frac{\sqrt{x^4}}{\sqrt{x^3-1}} dx$$

$$dx$$

$$\left(\frac{x^4}{x^3-1} \right)^{\frac{1}{2}}$$

$$\sqrt{\frac{x^4}{x^3(1-\frac{1}{x^3})}}$$

$$u = x^3 - 1$$

$$\int \frac{|x|^2}{\sqrt{x^3-1}} dx$$

$$u = x^3 - 1$$

$$du = 3x^2 dx$$

$$\int \frac{x^2}{\sqrt{x^3-1}} dx$$

$$\frac{du}{3} = x^2 dx$$

$$\int \frac{\frac{du}{3}}{\sqrt{u}} = \frac{1}{3} \int u^{-\frac{1}{2}} du$$

$$\sqrt{u}$$

$$= \frac{1}{3} \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{1}{3} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{1}{3} \cdot \frac{2}{1} \sqrt{u} + C$$

$$= \frac{2}{3} \sqrt{x^3 - 1} + C$$

نتيجة

$$\Downarrow$$

$$\sqrt{\frac{x^4}{x^3-1}}$$