Statics of particles

CHAPTER 2

Objectives

- **Describe** force as vector quantity
- **Examine** vector operation useful for the analysis of forces
- **Determine** the resultant of multiple forces acting on a particle
- **Resolve** forces into components
- **Add** forces that have been resolved into rectangular components.
- **Introduce** the concept of F.B.D
- **Use** F.B.D to assist in the analysis of planer and spatial particle equilibrium problems.

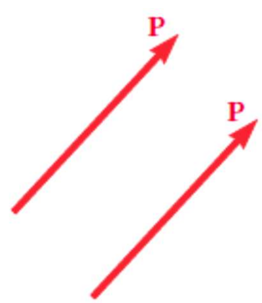
Objectives

- **Recall:** The focus on particles does not imply a restriction to very small bodies. Rather, the study is restricted to analyses in which the size and shape of the bodies is not significant so that all forces may be assumed to be applied at a single point.

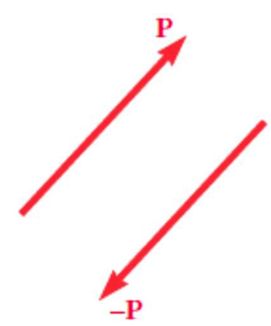
- **Application:** The tension in the cable supporting this person can be found using the concepts in this chapter

2.1B & C Vectors and vectors addition

- ***Vectors are defined as mathematical expressions possessing magnitude and direction***, which add according to the parallelogram law. Examples: Forces, displacements, velocities, accelerations, momenta …Etc.
- Vectors are represented by arrows in diagrams and or as boldface type (**P**) or as $\left(\vec{P}\right)$ in longhand writing.
- ***Vectors Types:*** A vector used to represent a force can be
  - *fixed, or bound, vector*
  - *free vectors*
  - *sliding vectors.*
- *Equal and negative vector*



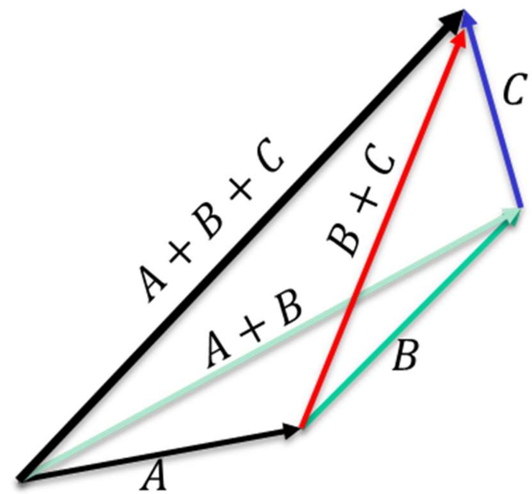
Equal vector



negative vector

Addition of Vectors

- Vectors add according to the parallelogram law, graphically and analytically.
- Vector addition has the following properties

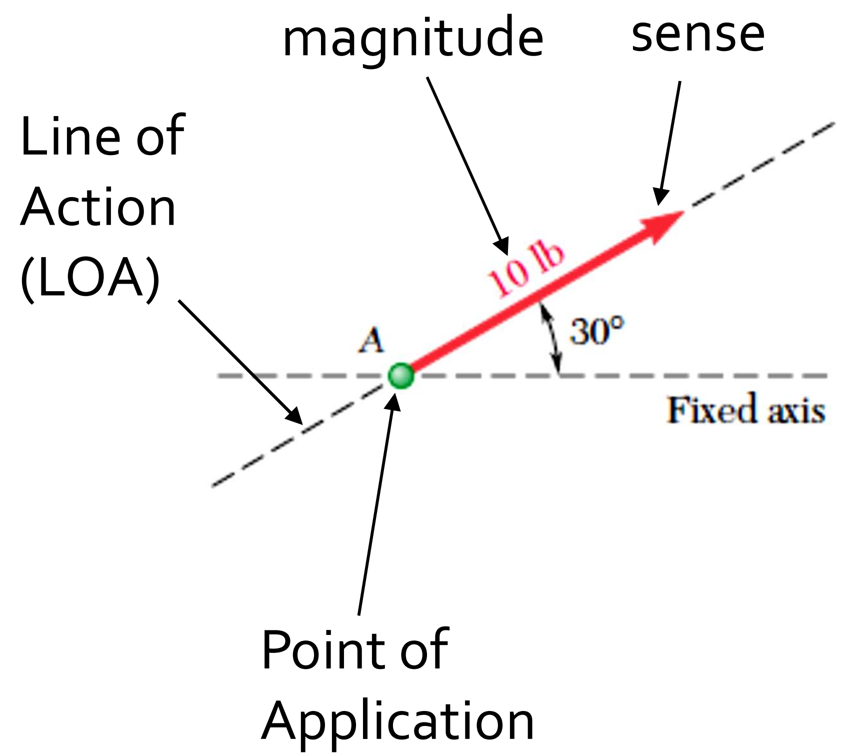


$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \qquad \text{commutative property,}$$
$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C}) \qquad \text{associative property,}$$

- Multiplication of a vector $\vec{A}$ and a scalar S produces a new vector $\vec{R}$ where $\vec{R} = S\vec{A} = \vec{A}S$. The magnitude of $\vec{R}$ is equal to the magnitude of $\vec{A}$ multiplied by $|S|$.

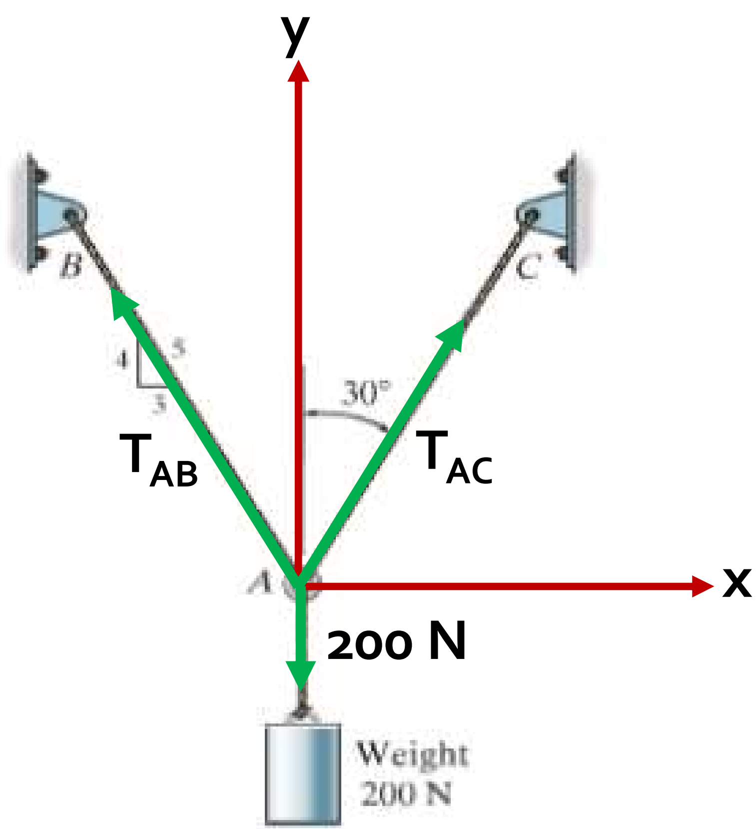
RECALL – Force Characteristics

- A force is **vector** characterized by its **point of application**, its **magnitude**, and its **direction**.
- The direction of a force is define by its **line of action** and the **sense** of the force.

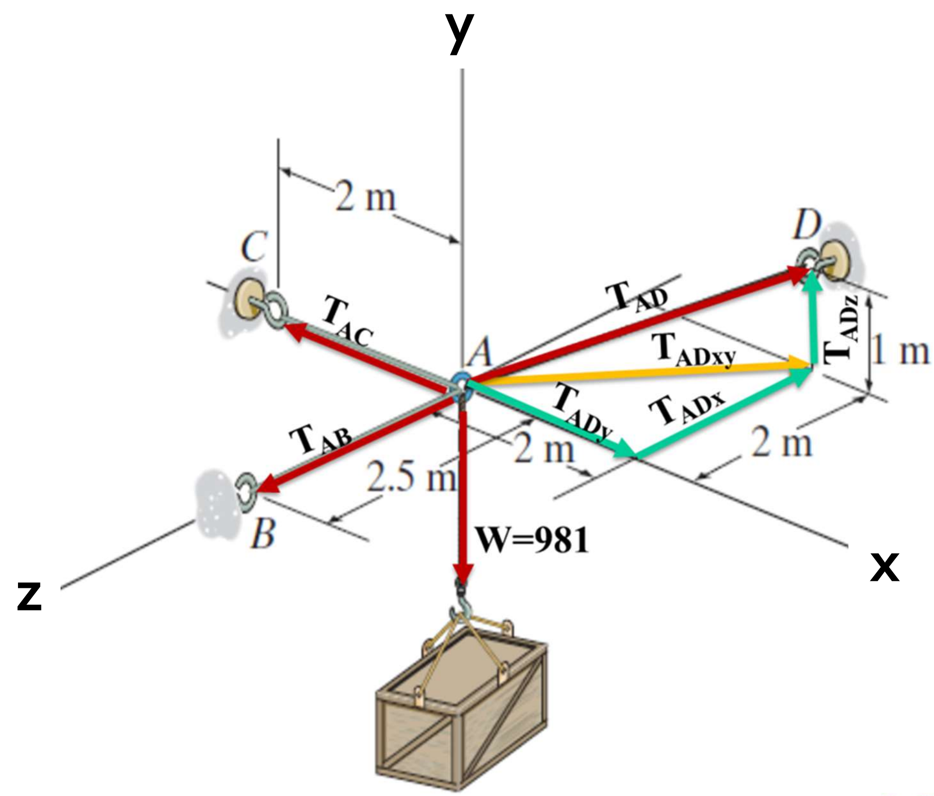


- The line of action is the infinite straight line along which the force acts; it is characterized by the angle it forms with some fixed axis

Force in a plane - Sections 2.1 to 2.3



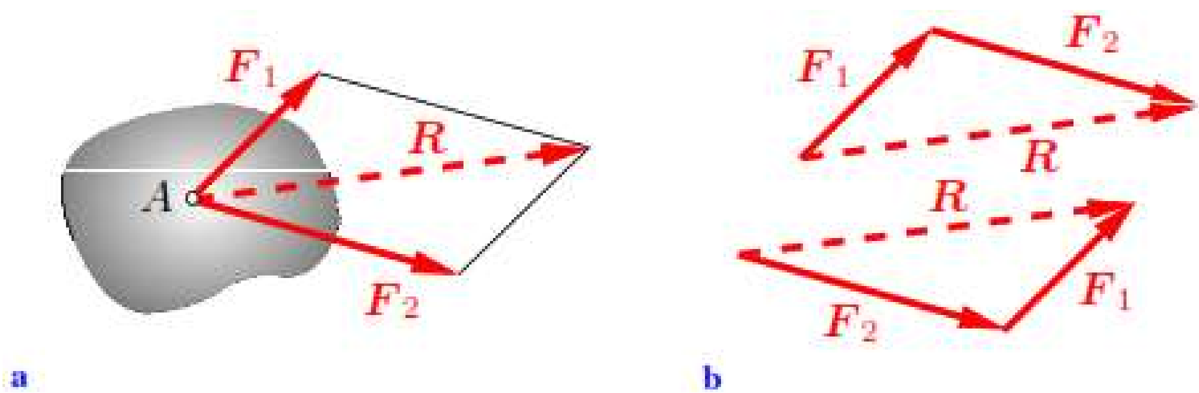
Forces in a plane



Forces in space

Resultant of Concurrent Forces

- Forces acting in a body is usually added to obtain the resultant (R).
- **Resultant (R) of two forces**: The effect of two nonparallel forces F1 and F2 acting at a point A of a body is the same as the effect of the single force R acting at the same point and obtained as the diagonal of the parallelogram formed by F1 and F2.

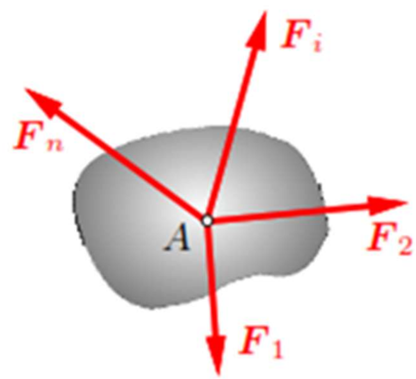


$$R = F_1 + F_2$$

Resultant of Concurrent Forces

- **Resultant of a system of n forces** that all lie in a plane and whose lines of action intersect at single point A. The resultant can be obtained by adding the forces head-to-tail or through successive application of the parallelogram law of forces.

$$R = F_1 + F_2 + \ldots + F_n = \sum F_i \,.$$



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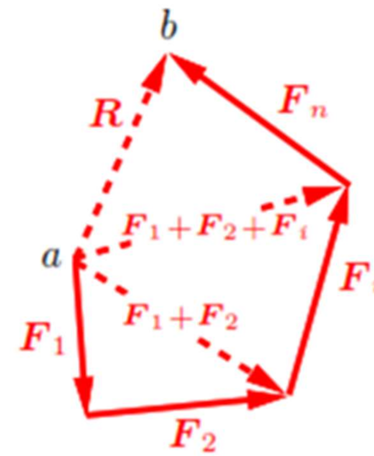


b

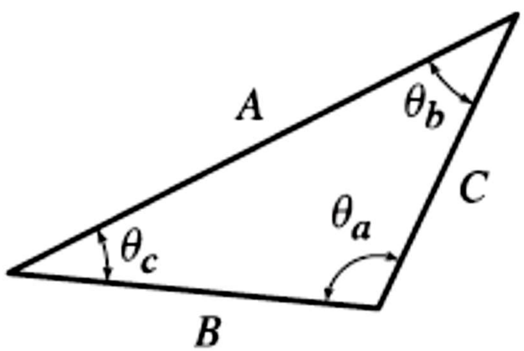
Notes:

- The sequence of the addition is arbitrary; in particular, it is immaterial which vector is chosen to be the first one
- Such a system is called a coplanar system of concurrent forces.

2.1 Addition of planar forces

Forces can be added:

1. As vectors using the parallelogram law of forces
   - Graphically
   - Analytically



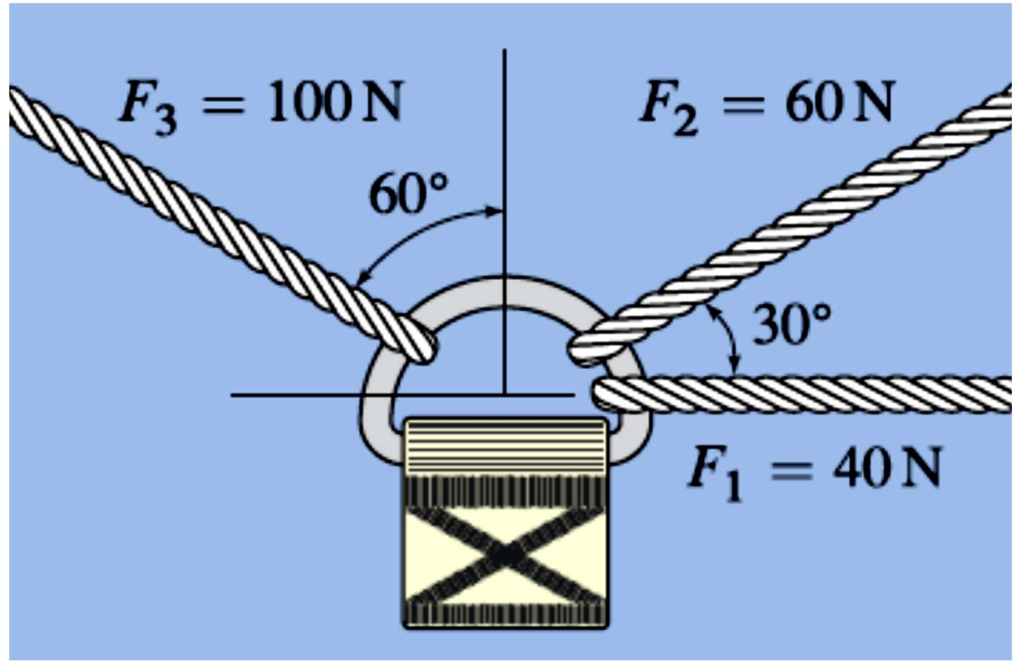
$$\frac{\sin\theta_a}{A} = \frac{\sin\theta_b}{B} = \frac{\sin\theta_c}{C} \qquad \text{law of sines,}$$

$$\begin{aligned} A &= \sqrt{B^2 + C^2 - 2BC\cos\theta_a} \\ B &= \sqrt{A^2 + C^2 - 2AC\cos\theta_b} \\ C &= \sqrt{A^2 + B^2 - 2AB\cos\theta_c} \end{aligned} \qquad \text{law of cosines.}$$

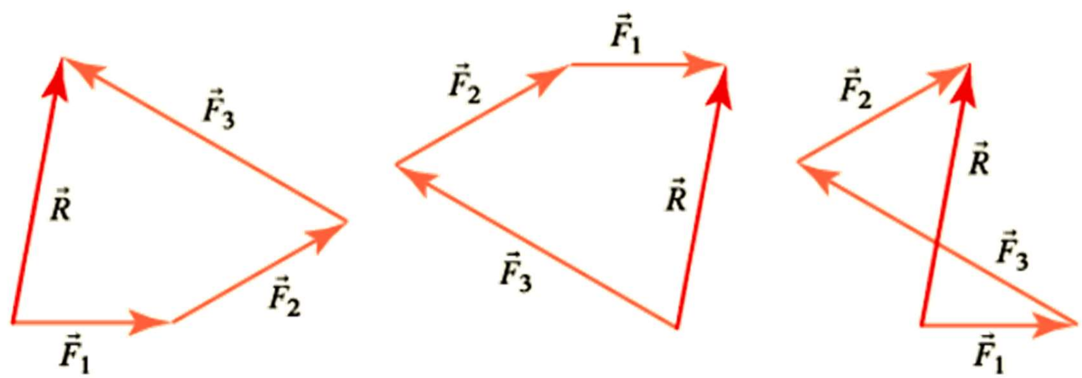
2. Decomposition of Forces and summation of the components

Example 1

The D ring shown has three cords tied to it and the cords support the forces shown, determine the resultant force applied to the D ring by the cords, expressing the result as a vector.



Note: you can construct several force polygons as shown

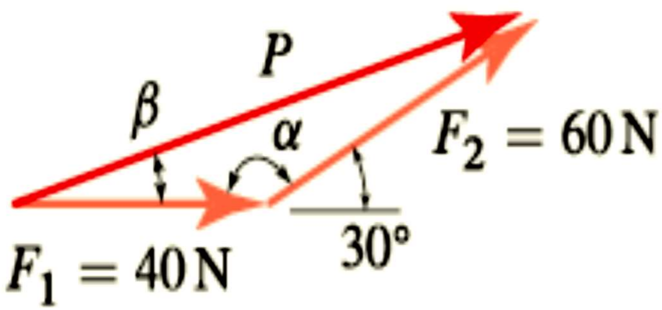


Solution

1. Find F1 + F2

$$P = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos\alpha}$$

$$= \sqrt{(40\,\text{N})^2 + (60\,\text{N})^2 - 2(40\,\text{N})(60\,\text{N})\cos(150^\circ)} = 96.73\,\text{N},$$



- use the law of sines, to find angle β

$$\frac{P}{\sin\alpha} = \frac{F_2}{\sin\beta},$$

$$\sin\beta = \frac{F_2}{P}\sin(150^\circ) = \frac{60\,\text{N}}{96.73\,\text{N}}\sin(150^\circ) = 0.3101.$$

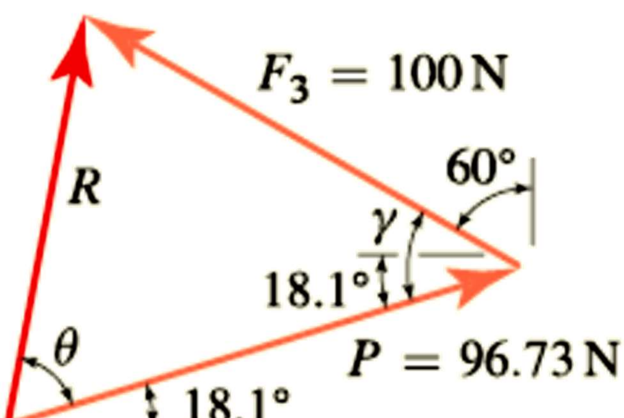
$$\beta = \sin^{-1}(0.3101) = 18.1^\circ.$$

2. Find P + F3

$$R = \sqrt{P^2 + F_3^2 - 2PF_3\cos\gamma}$$

$$= \sqrt{(96.73\,\text{N})^2 + (100\,\text{N})^2 - 2(96.73\,\text{N})(100\,\text{N})\cos(48.1^\circ)}$$

$$= 80.23\,\text{N}.$$



- use the law of sines, to find angle θ

$$\frac{F_3}{\sin\theta} = \frac{R}{\sin\gamma}.$$

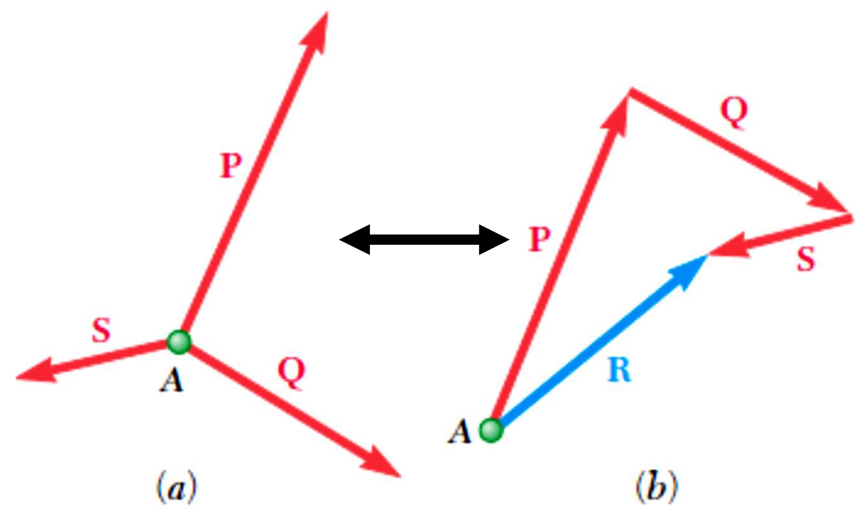
$$\sin\theta = \frac{F_3}{R}\sin\gamma = \frac{100\,\text{N}}{80.2\,\text{N}}\sin 48.1^\circ = 0.928,$$

$$\theta = \sin^{-1}(0.928) = 68.1^\circ.$$

2.1E Resolution of a Force into Components

- If the forces (or vectors) **P**, **Q** and **S** acting on particle A may be replaced by a single force **R** that has the same effect on the particle. Then conversely, a single force **R** acting on a particle may be replaced by two or more forces that, together, have the same effect on the particle. These forces are called components of the original force **R.**

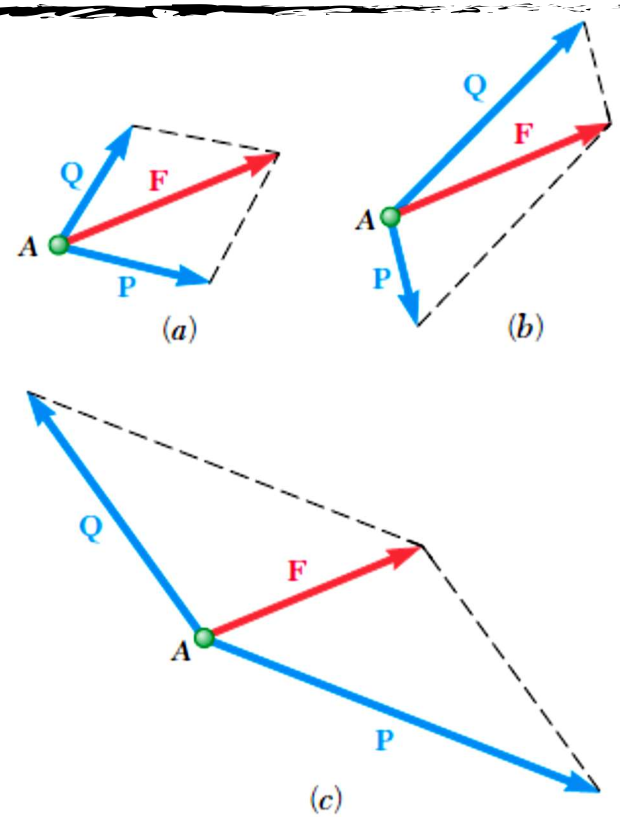
- Clearly, each force F can be resolved into an infinite number of possible sets of components.



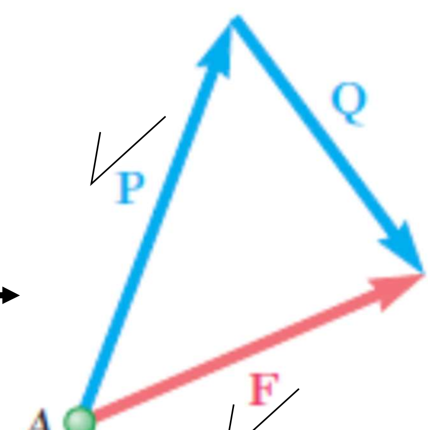
- However, sets of two components are the most important as far as practical applications are concerned.

2.1E Resolution of a Force into Components

- Even when a force is to be resolved to two components, the number of ways in which a given force F may be resolved is unlimited (see figure).

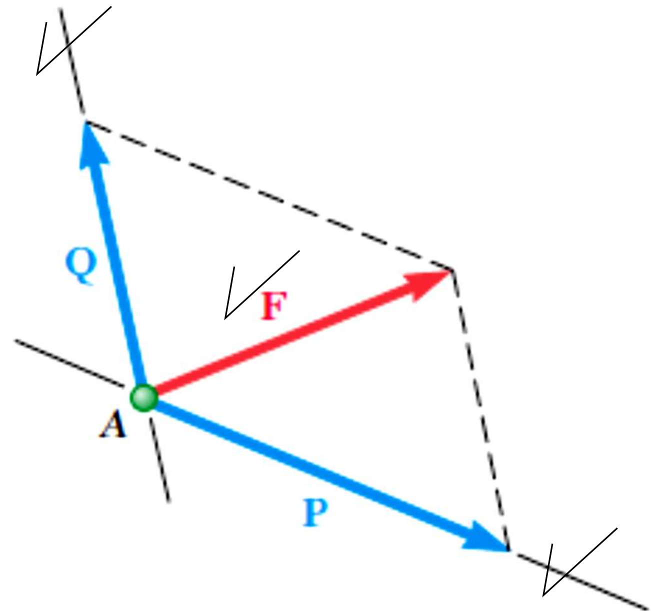


- The cases are of particular interest are:
- One of the Two Components, P, Is Known. We obtain the second component, Q, by applying the triangle rule and joining the tip of P to the tip of F.

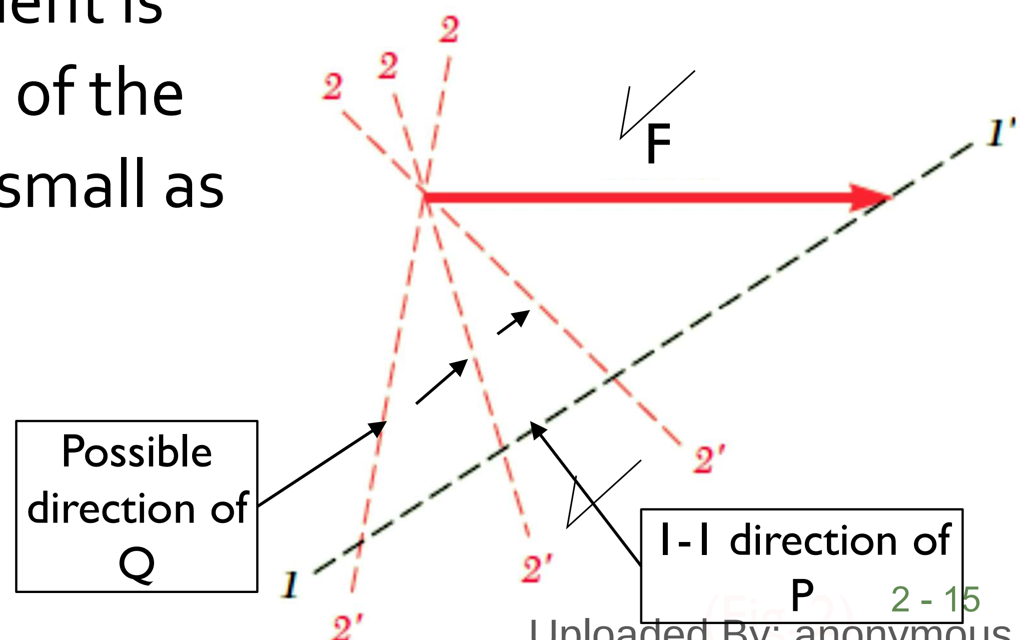


2.1E Resolution of a Force into Components

2. The Line of Action of Each Component Is Known. We obtain the magnitude and sense of the components by applying the parallelogram law.



3. The direction of one component is known, while the magnitude of the other component is to be as small as possible.

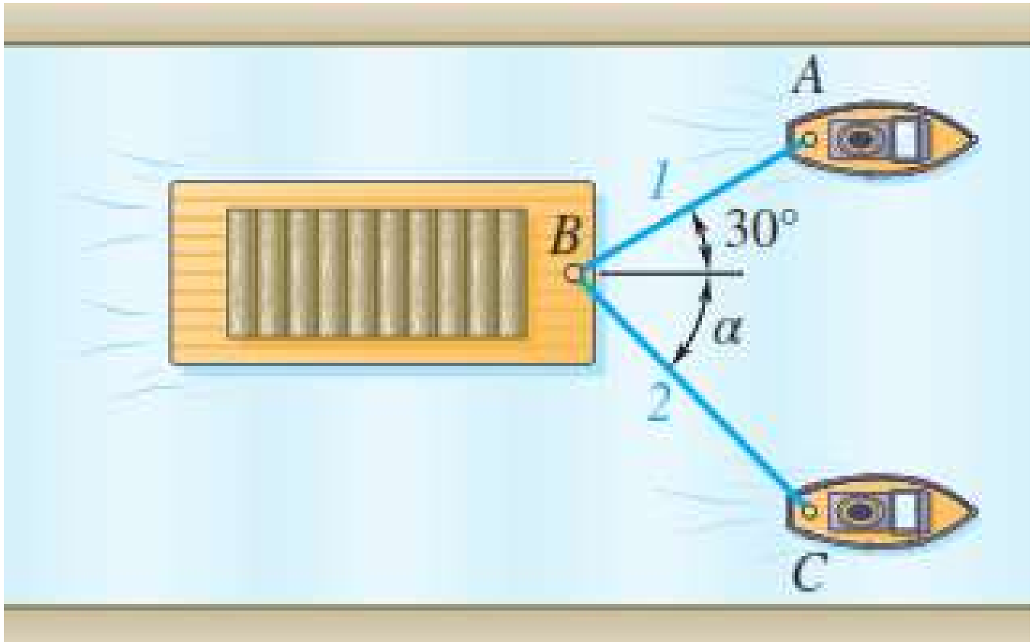


Sample Problem 2.2

Two tugboats are pulling a barge. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine

(a) the tension in each of the ropes, given that $\alpha = 45^\circ$

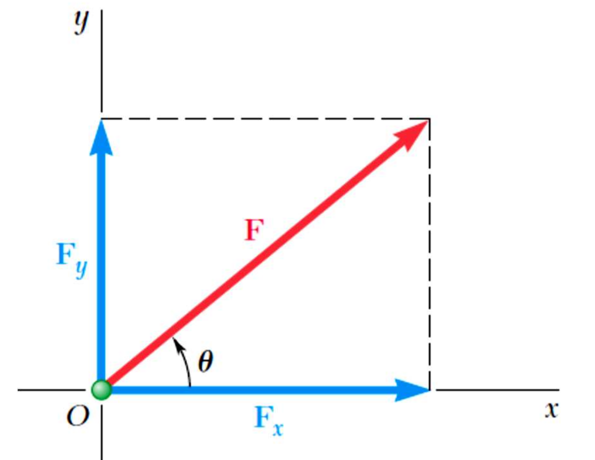
(b) the value of α for which the tension in rope 2 is a minimum.



2.2 Adding forces by components

2.2A Rectangular Components of a Force

- Rectangular Components of the force are the two components (Fx and Fy) that are perpendicular to each other.

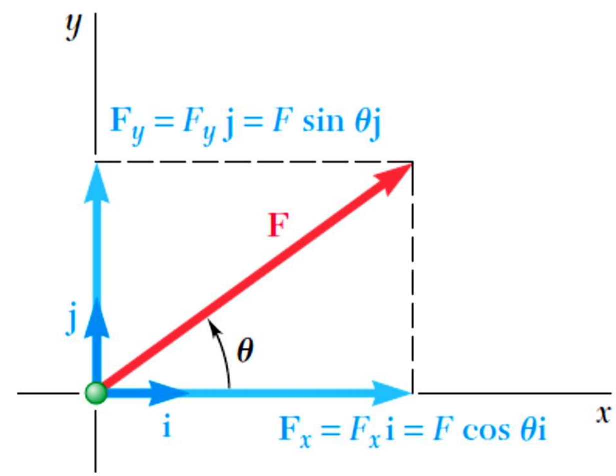


Rectangular components of a force F.

- Those components can be written using the unit vectors I and j as

$$\mathbf{F}_x = F_x\mathbf{i} \qquad \mathbf{F}_y = F_y\mathbf{j}$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

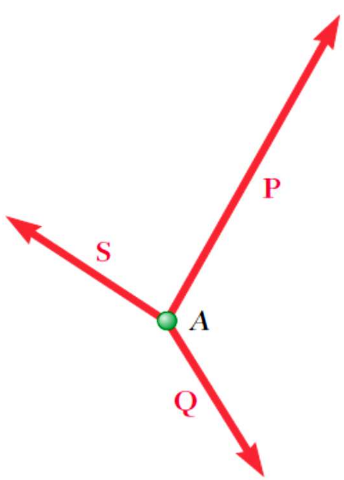


- Direction of a Force: $\tan\theta = \frac{F_y}{F_x}$
- Magnitude F of the force: $F = \sqrt{F_x^2 + F_y^2}$

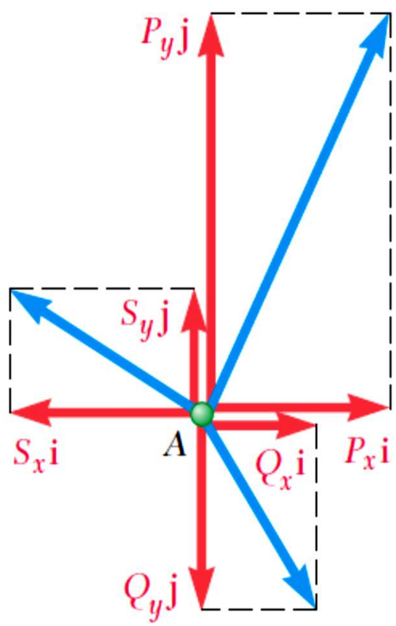
2.2 Adding forces by components

- When several forces are acting on a particle, we obtain the scalar components R_x and R_y of the resultant **R** by adding algebraically the corresponding scalar components of the given forces.

Forces



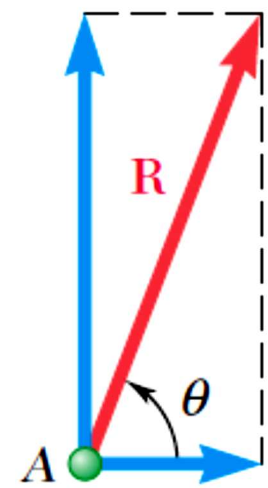
Components



$$R_x = \sum F_x$$

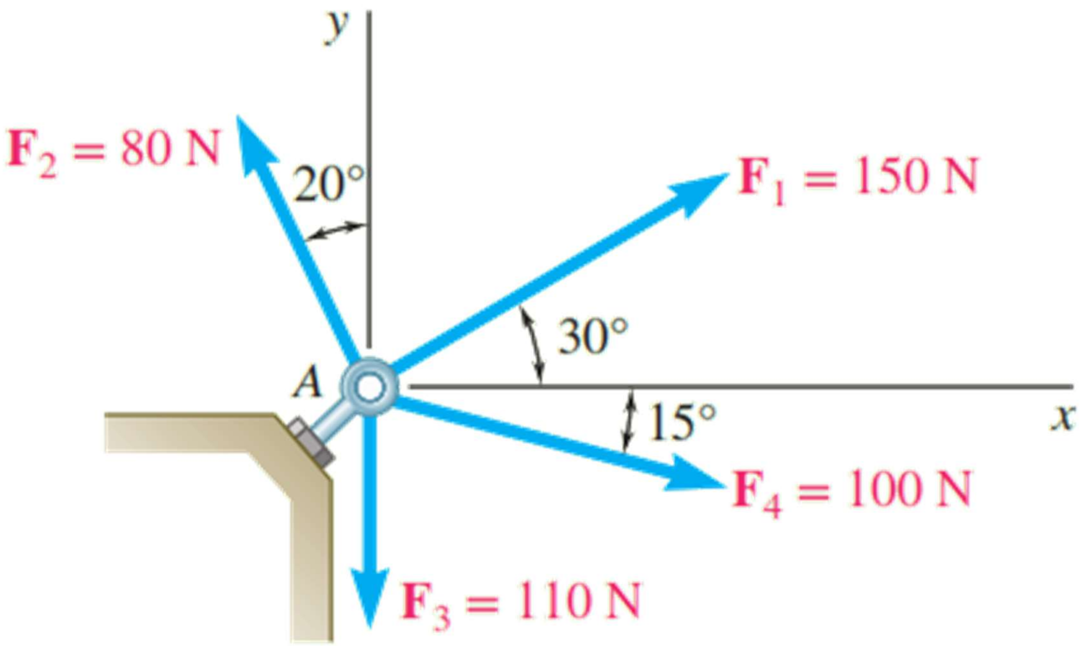
$$R_y = \sum F_y$$

$$R = R_x i + R_y j$$



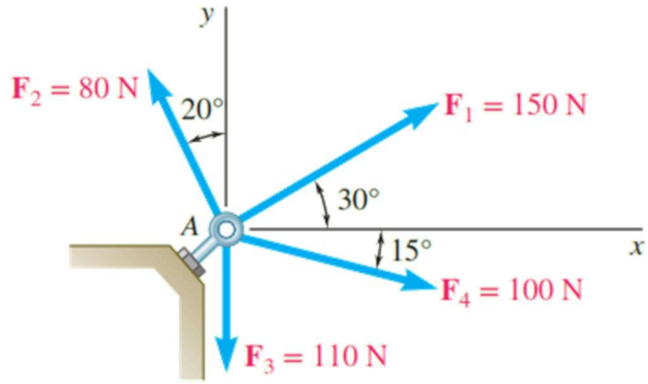
Sample Problem 2.3

Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.



Sample Problem 2.3

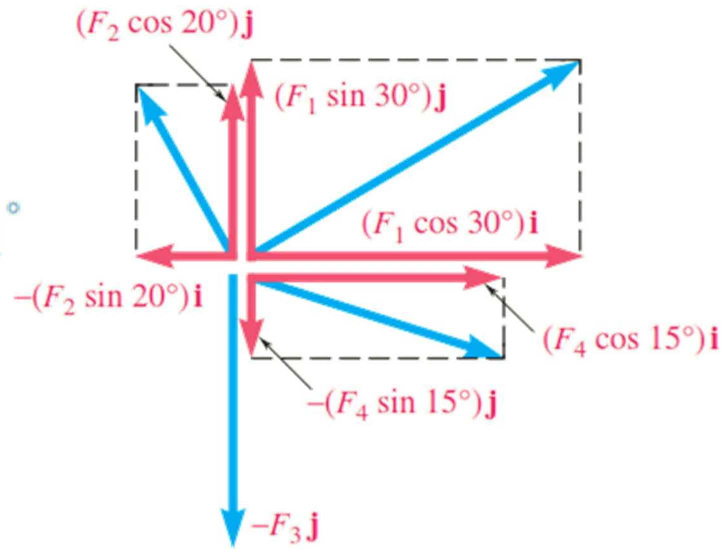
Force	Magnitude, N	x Component, N	y Component, N
$\mathbf{F}_1$	150	+129.9	+75.0
$\mathbf{F}_2$	80	−27.4	+75.2
$\mathbf{F}_3$	110	0	−110.0
$\mathbf{F}_4$	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$

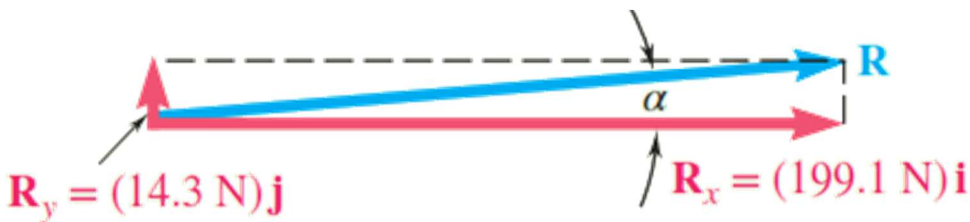


$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} \qquad \mathbf{R} = (199.1\text{ N})\mathbf{i} + (14.3\text{ N})\mathbf{j}$$

$$\tan\alpha = \frac{R_y}{R_x} = \frac{14.3\text{ N}}{199.1\text{ N}} \qquad \alpha = 4.1^\circ$$

$$R = \frac{14.3\text{ N}}{\sin\alpha} = 199.6\text{ N} \qquad \mathbf{R} = 199.6\text{ N} \measuredangle 4.1^\circ$$

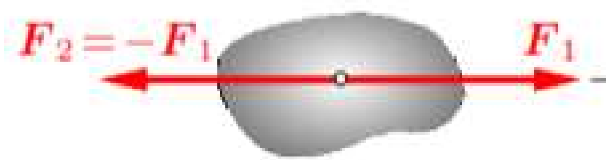




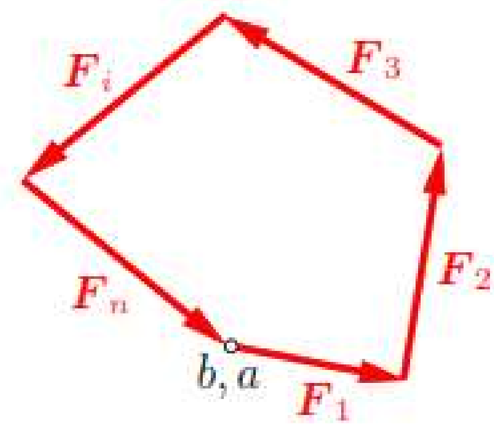
2.3 Forces and equilibrium in a plane

- According to Newton's First Law If the resultant force on a particle is zero, the particle will remain at rest if it was at rest. In this case the particle is said to be in *equilibrium*.

- For example a body subjected to two equal and opposite force is in equilibrium and.



- For example a body subjected to several forces that sums up to a close polygon is in equilibrium or

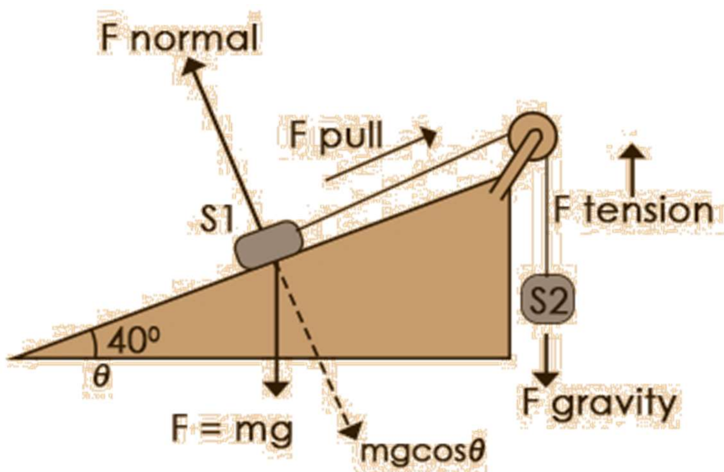


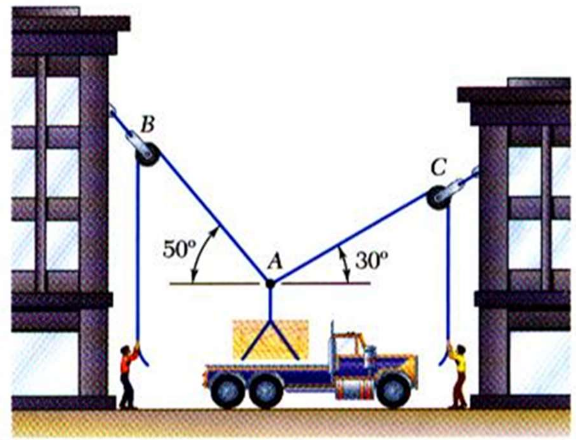
- In the case of equilibrium, the following equations shall be satisfied

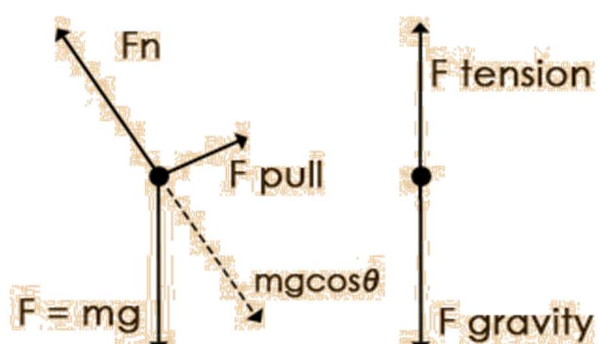
$$R = \sum F = 0, \qquad \sum F_x = 0 \quad and \sum F_y = 0$$

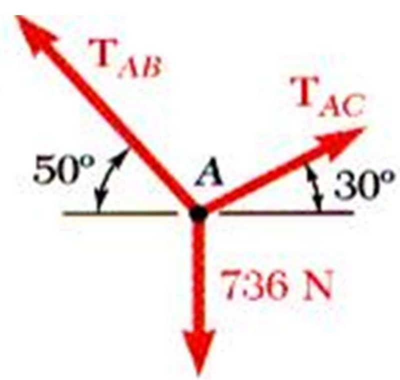
Free-Body Diagrams and Problem Solving

- A free body diagram (FBD) is a sketch of a body or a portion of a body that is separated or made free from its environment and/or other parts of the structural system, that show all forces acting on the body.

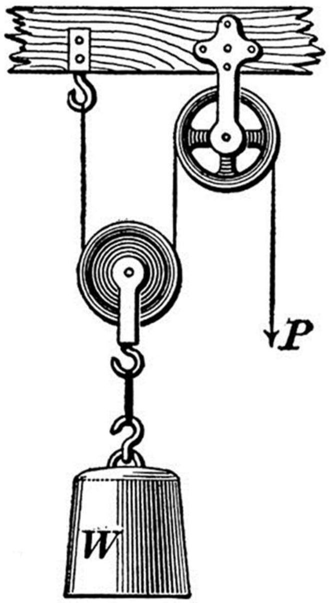




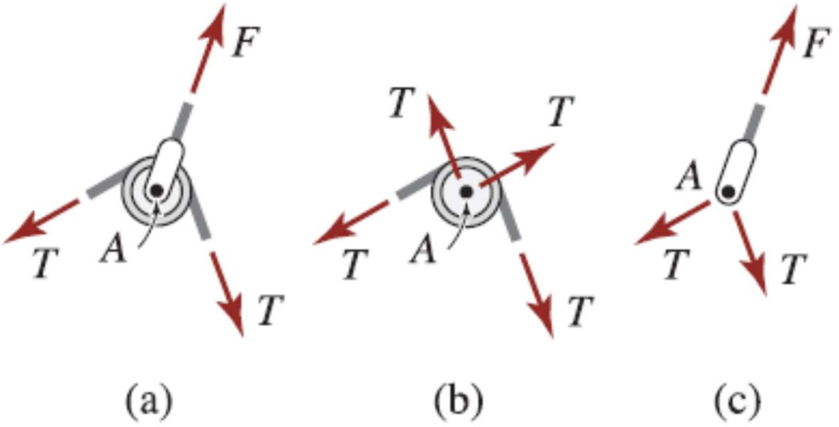




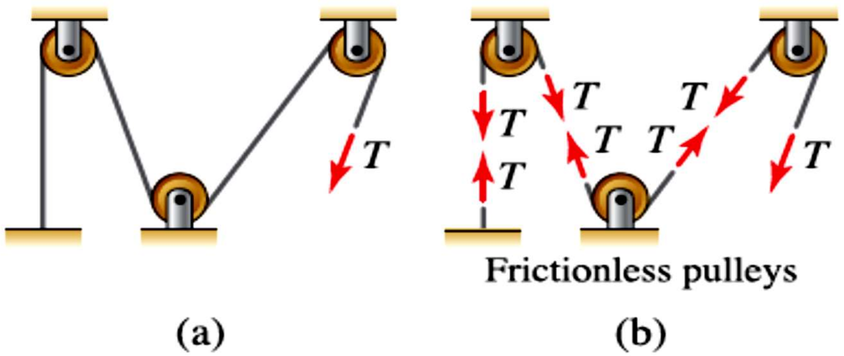
Pulleys as particles in equilibrium



- Pulleys can be modeled as particles as shown below



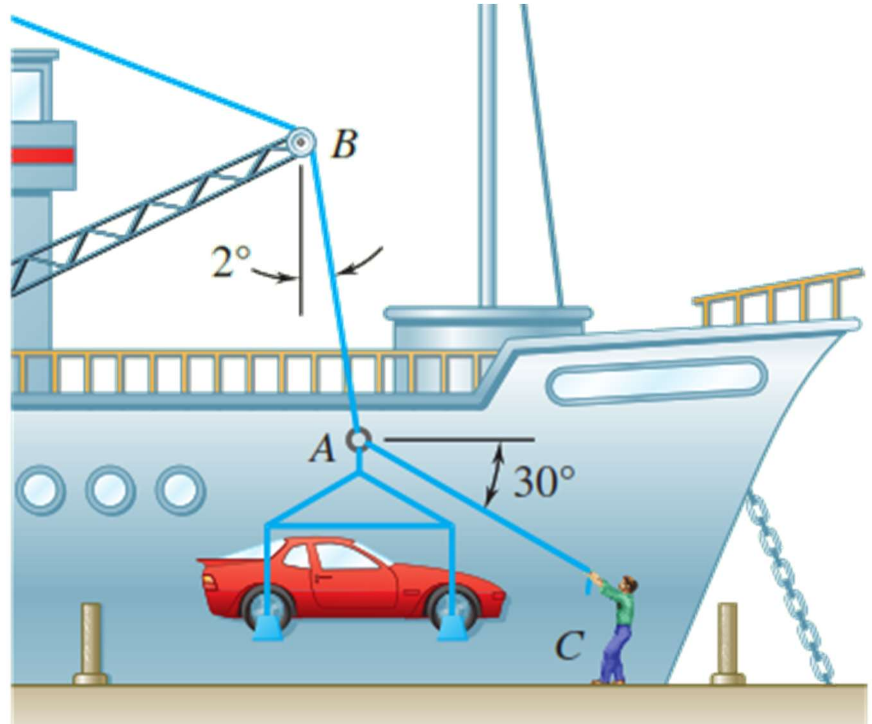
- Forces in ropes or cables is always tension (Pull).
- In friction less pulleys, the force in continuous rope or cable is constant.



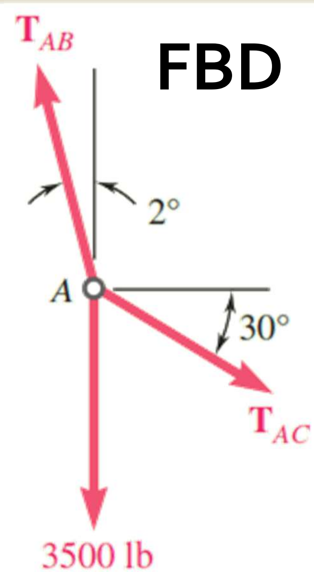
Sample Problem 2.4

In a ship-unloading operation, a 3500-lb automobile is supported by a cable. A worker ties a rope to the cable at A and pulls on it in order to center the automobile over its intended position on the dock. At the moment illustrated, the automobile is stationary, the angle between the cable and the vertical is 2°, and the angle between the rope and the horizontal is 30°

What are the tensions in the rope and cable?

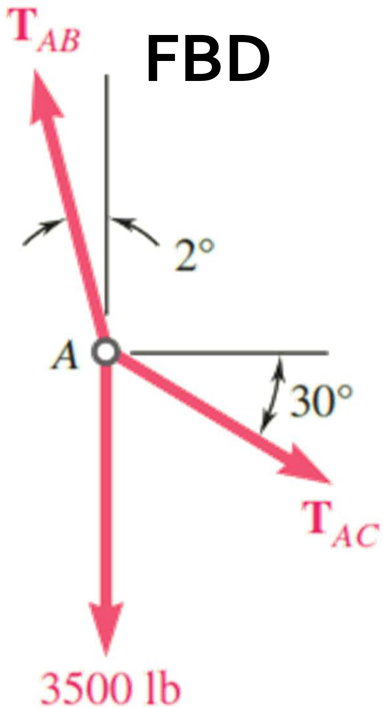


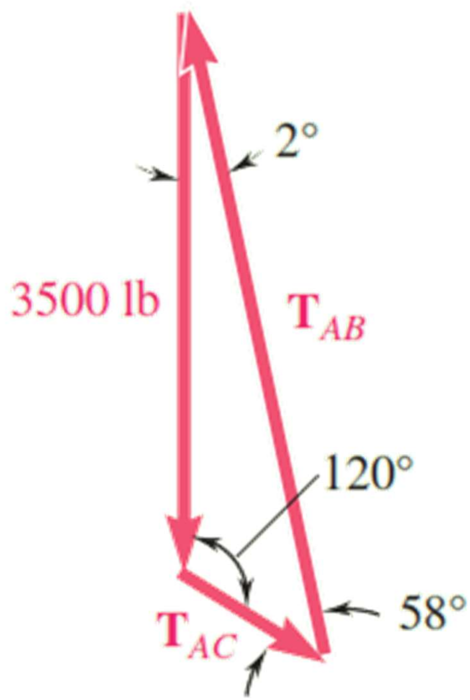
FBD



Sample Problem 2.4

FBD



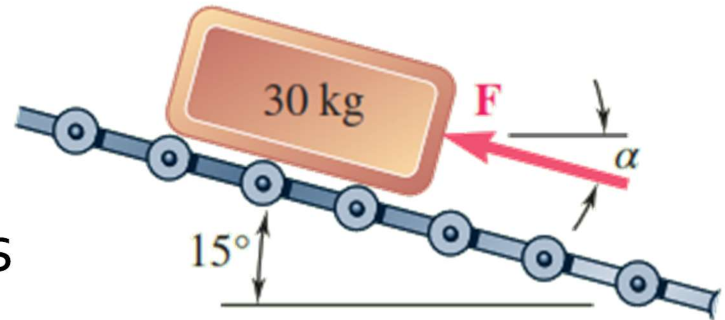


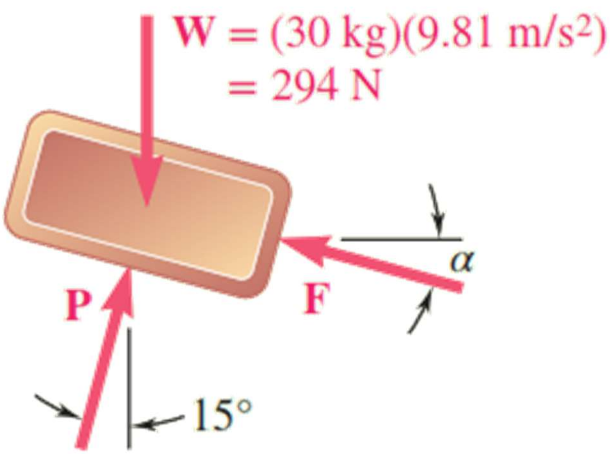
$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{3500\text{ lb}}{\sin 58°}$$

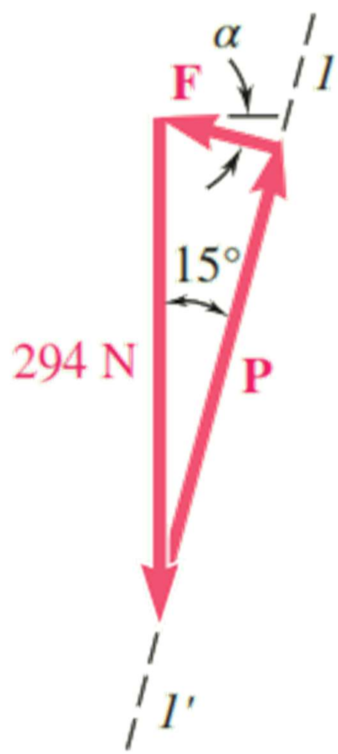
$T_{AB} = 3570$ lb $\qquad T_{AC} = 144$ lb

Sample Problem 2.4

Determine the magnitude and direction of the smallest force F that maintains the 30-kg package shown in equilibrium. Note that the force exerted by the rollers on the package is perpendicular to the incline.





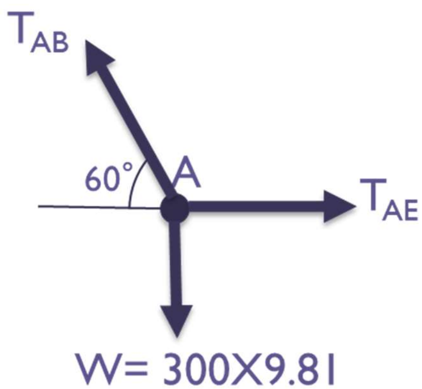


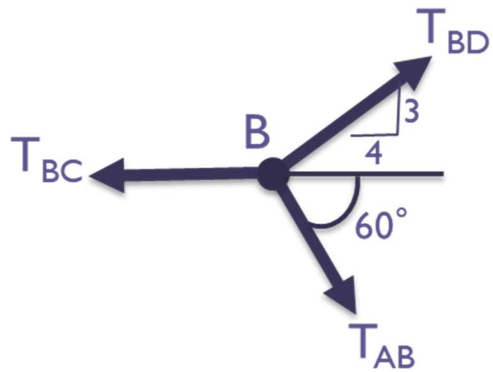
$$F = (294\text{ N})\sin 15^\circ \quad = 76.1\text{ N}$$

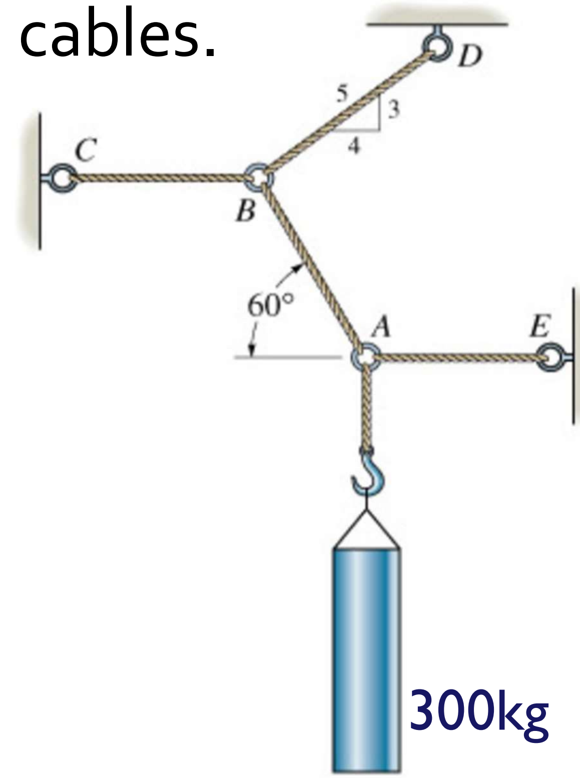
$$\alpha = 15^\circ$$

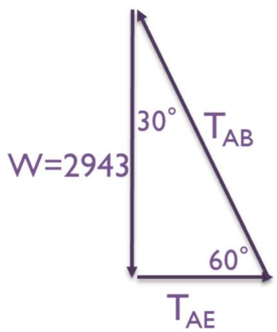
Sample Problem

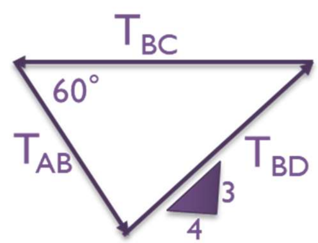
In the figure shown, determine the loads in all cables.











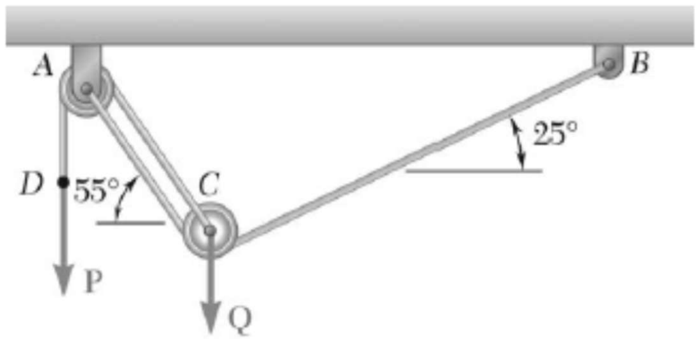
Particle A

$$\frac{2943}{\sin 60} = \frac{T_{AE}}{\sin 30} = \frac{T_{AB}}{\sin 90} \rightarrow T_{AE} = 1699\ N, and\ \ T_{AB} = 3398.3\ N$$

Particle B

$$\frac{3398.3}{.6} = \frac{T_{BD}}{\sin 60} = \frac{T_{BC}}{\sin 83.13} \rightarrow T_{BD} = 4905N\ and\ T_{BC} = 5623.2N$$

Sample Problem

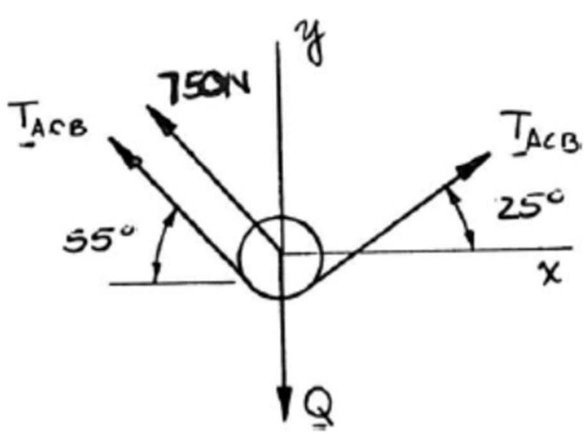


PROBLEM 2.69

A load **Q** is applied to the pulley *C*, which can roll on the cable *ACB*. The pulley is held in the position shown by a second cable *CAD*, which passes over the pulley *A* and supports a load **P**. Knowing that $P = 750$ N, determine (*a*) the tension in cable *ACB*, (*b*) the magnitude of load **Q**.

SOLUTION

Free-Body Diagram: Pulley *C*



(*a*) $\xrightarrow{+} \Sigma F_x = 0$: $T_{ACB}(\cos 25° - \cos 55°) - (750 \text{ N})\cos 55° = 0$

Hence: $T_{ACB} = 1292.88$ N

$T_{ACB} = 1293$ N ◀

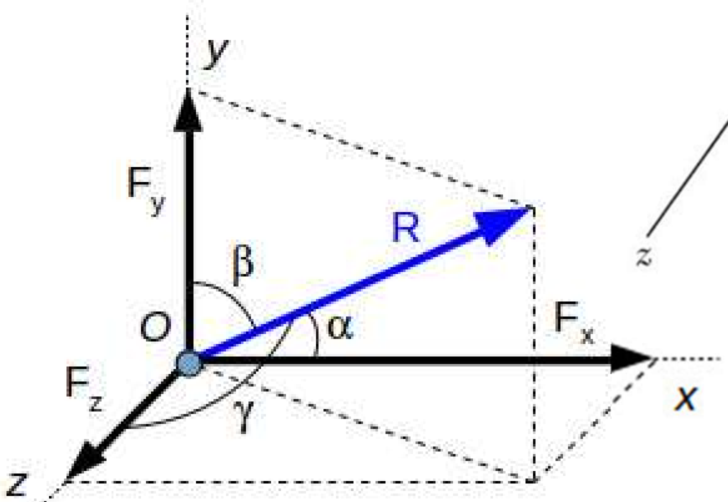
(*b*) $+\uparrow \Sigma F_y = 0$: $T_{ACB}(\sin 25° + \sin 55°) + (750 \text{ N})\sin 55° - Q = 0$

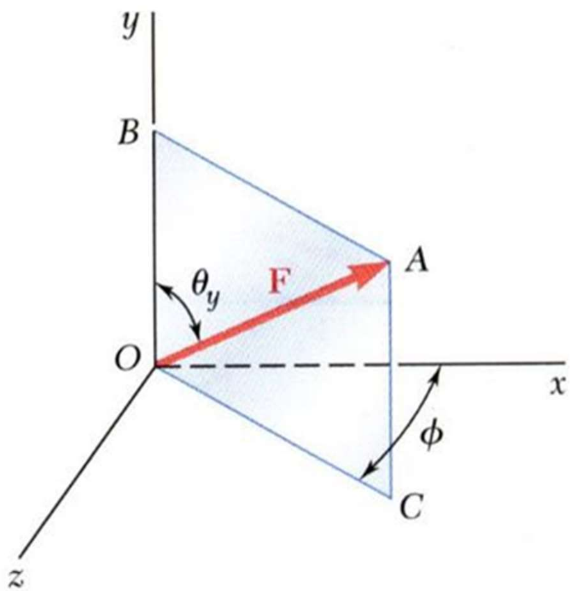
$(1292.88 \text{ N})(\sin 25° + \sin 55°) + (750 \text{ N})\sin 55° - Q = 0$

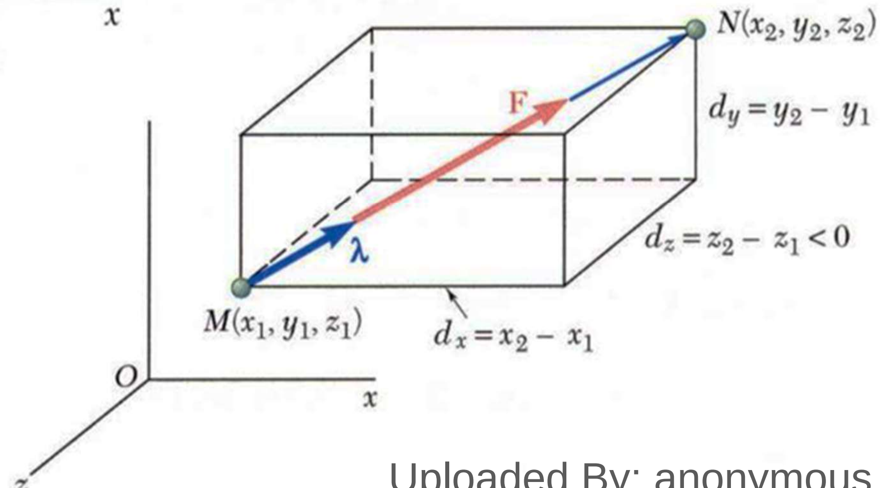
or $Q = 2219.8$ N $Q = 2220$ N ◀

2.4 Forces in space

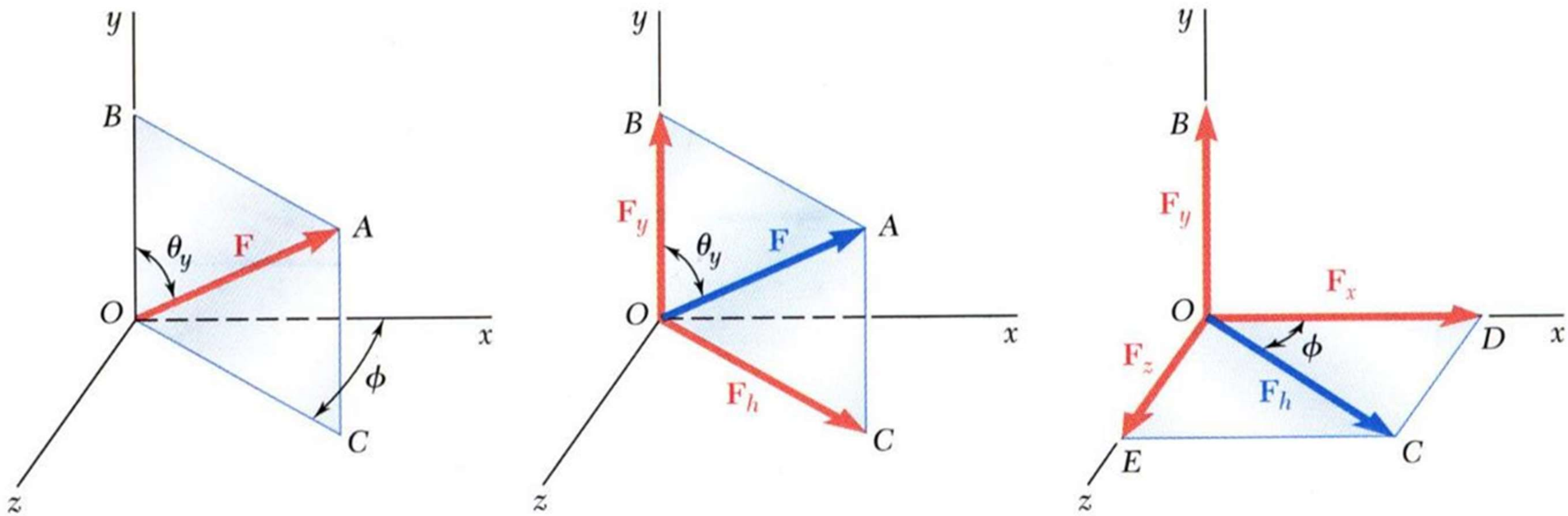
- Forces in space can be defined by
  - Two angles, one with one coordinate axis and a projection angle on the plan of the other axis.
  - Three angles, one with each axis
  - The coordinates of two points in the force line of action







2.4A Rectangular Components of a Force in Space



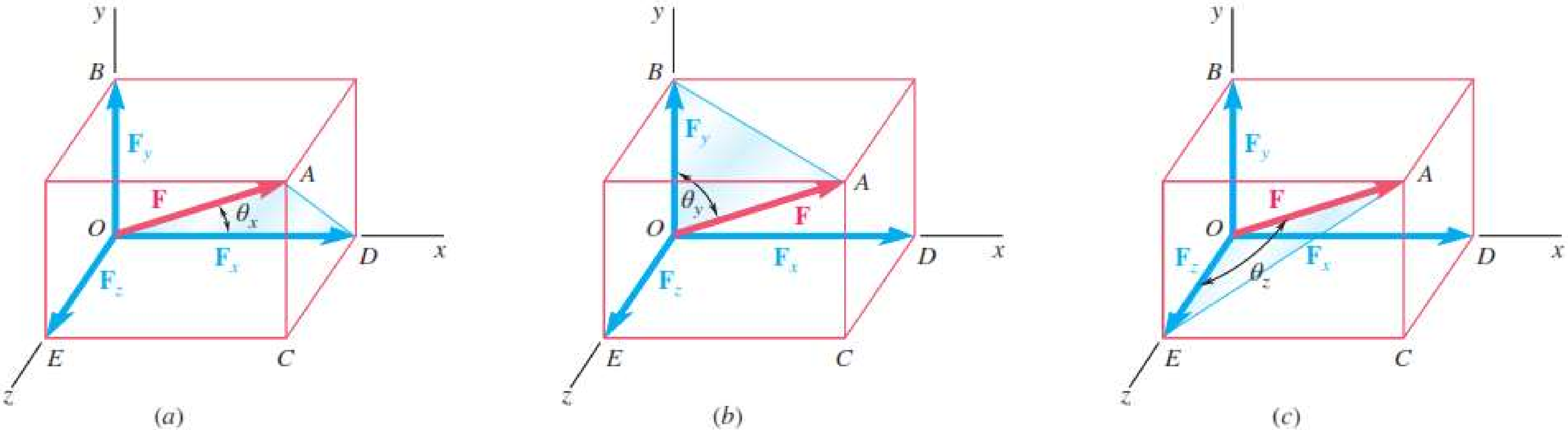
$$F_y = F\cos\theta_y \qquad F_x = F_h\cos\varphi = F\sin\theta_y\cos\varphi$$
$$F_h = F\sin\theta_y \qquad F_z = F_h\sin\varphi = F\sin\theta_y\sin\varphi$$

$$F^2 = F_y^2 + F_h^2, and\ \ F_h^2 = F_x^2 + F_z^2 \quad \rightarrow F^2 = F_x^2 + F_y^2 + F_z^2$$

- Magnitude of a force in space

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Rectangular Components of a Force in Space



- If the force angles with the coordinate axis are known as shown above then as the shaded triangle are right triangles, the Scalar components of a force F are:

$$F_x = F \cos\theta_x \quad F_y = F \cos\theta_y; \quad F_z = F \cos\theta_z$$

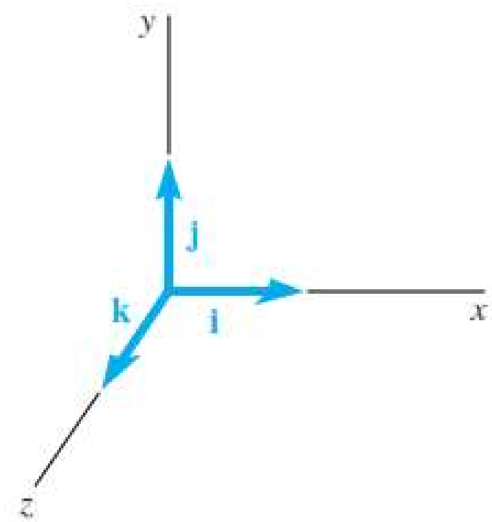
$$\cos\theta_x = \frac{F_x}{F} \quad \cos\theta_y = \frac{F_y}{F} \quad \cos\theta_z = \frac{F_z}{F}$$

Rectangular Components of a Force in Space

- *But considering the unit victors I, j, k*

$$\vec{F} = F_x i + F_y j + F_z k$$

$$\rightarrow \vec{F} = F(\cos\theta_x\, i + \cos\theta_y\, j + \cos\theta_z\, k)$$



- The previous equation shows that the force F can be expressed as the product of the scalar F and the vector $\lambda = \cos\theta_x\, i + \cos\theta_y\, j + \cos\theta_z\, k$

$$\vec{F} = F\vec{\lambda}$$

- The Magnitude of λ shall be 1 so that the magnitude of F to remain constant. So λ is a unit vector which means

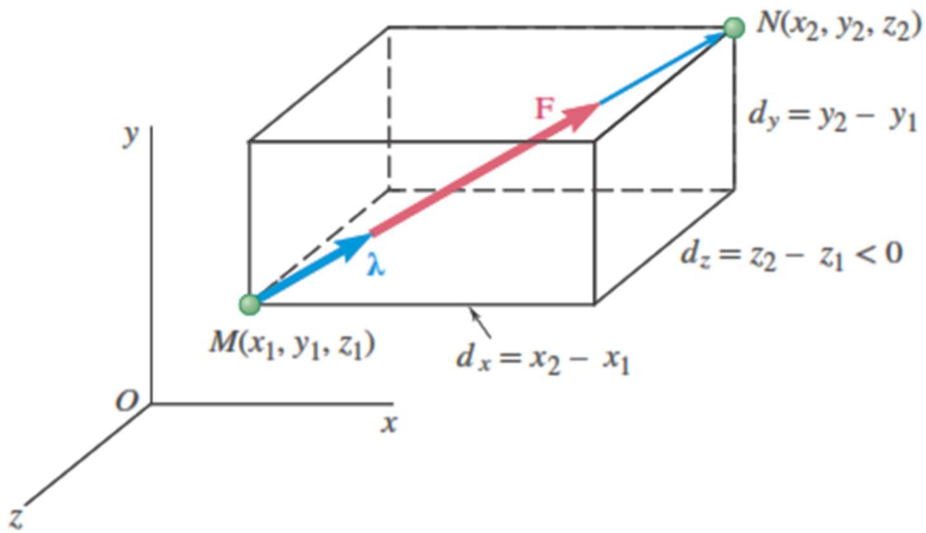
$$1 = \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z$$

Force Defined by its Magnitude and Two Points on its Line of Action

- Consider the vector MN joining M and N and of the same sense as a force F. $\rightarrow \overrightarrow{MN} = d_x i + d_y j + d_z k$
- We can obtain a unit vector $\boldsymbol{\lambda}$ along the line of action of **F** (i.e., along the line *MN*) by dividing the vector $\overrightarrow{MN}$ by its magnitude *MN*

$$\lambda = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}(d_x i + d_y j + d_z k)$$

$$\rightarrow \vec{F} = F\lambda = \frac{F}{d}(d_x i + d_y j + d_z k)$$



- Scalar components of force F

$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

- Direction cosines of force F

$$\cos\theta_x = \frac{d_x}{d} \qquad \cos\theta_y = \frac{d_y}{d} \qquad \cos\theta_z = \frac{d_z}{d}$$

Addition of Concurrent Forces in Space

- Graphical or trigonometric methods are generally not practical in the case of forces in space. However, we can determine the resultant **R** of two or more forces in space by summing their rectangular components.

$$\vec{R} = \sum F$$

Or

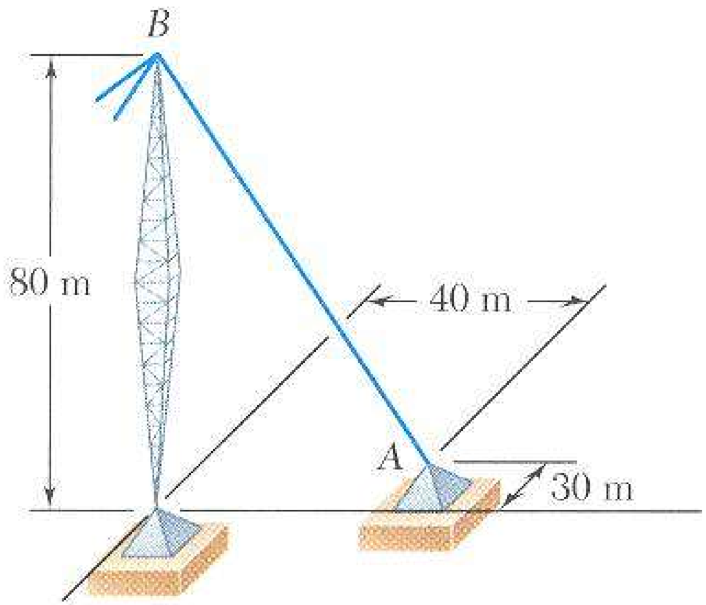
$$R_x i + R_y j + R_z k = \left(\sum F_x\right) i + \left(\sum F_y\right) j + \left(\sum F_z\right) k$$

- Magnitude and direction of the Resultant of concurrent forces in space

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$\cos\theta_x = \frac{R_x}{R} \qquad \cos\theta_y = \frac{R_y}{R} \qquad \cos\theta_z = \frac{R_z}{R}$$

Sample Problem 2.7



The tension in the guy wire is 2500 N. Determine:

a) components F_x, F_y, F_z of the force acting on the bolt at *A*,

b) the angles q_x, q_y, q_z defining the direction of the force (the direction cosines)

SOLUTION:

- Determine the unit vector pointing from *A* towards *B*.

$$\overrightarrow{AB} = (-40\,\text{m})\vec{i} + (80\,\text{m})\vec{j} + (30\,\text{m})\vec{k}$$

$$AB = \sqrt{(-40\,\text{m})^2 + (80\,\text{m})^2 + (30\,\text{m})^2}$$

$$= 94.3\ \text{m}$$

$$\vec{\lambda} = \left(\frac{-40}{94.3}\right)\vec{i} + \left(\frac{80}{94.3}\right)\vec{j} + \left(\frac{30}{94.3}\right)\vec{k}$$

$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

- Determine the components of the force.

$$\vec{F} = F\vec{\lambda}$$

$$= (2500\ \text{N})\left(-0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}\right)$$

$$= (-1060\text{N})\vec{i} + (2120\ \text{N})\vec{j} + (795\ \text{N})\vec{k}$$

- direction cosines

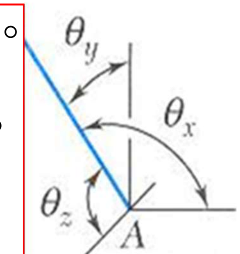
$$\hat{\boldsymbol{\lambda}} = cos\theta_x\hat{\imath} + cos\theta_y\hat{\jmath} + cos\theta_z\hat{k}$$

$$= -0.424\hat{\imath} + 0.848\hat{\jmath} + 0.318\hat{k}$$

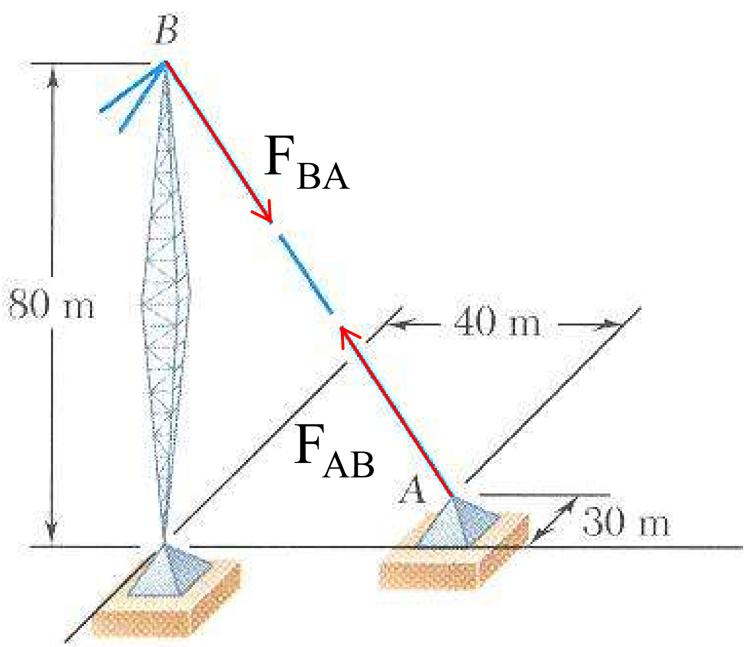
$$\theta_x = 115.1^\circ$$

$$\theta_y = 32.0^\circ$$

$$\theta_z = 71.5^\circ$$



What if…?



SOLUTION:

- Since the force in the guy wire must be the same throughout its length, the force at B (and acting toward A) must be the same magnitude but opposite in direction to the force at A.

$$\vec{F}_{BA} = -\vec{F}_{AB}$$
$$= (1060\text{N})\vec{i} + (-2120\ \text{N})\vec{j} + (-795\ \text{N})\vec{k}$$

What are the components of the force in the wire at point B? Can you find it without doing any calculations?

Give this some thought and discuss this with a neighbor.

2.5 FORCES AND EQUILIBRIUM IN SPACE

The condition of equilibrium of a particle in three dimensions are

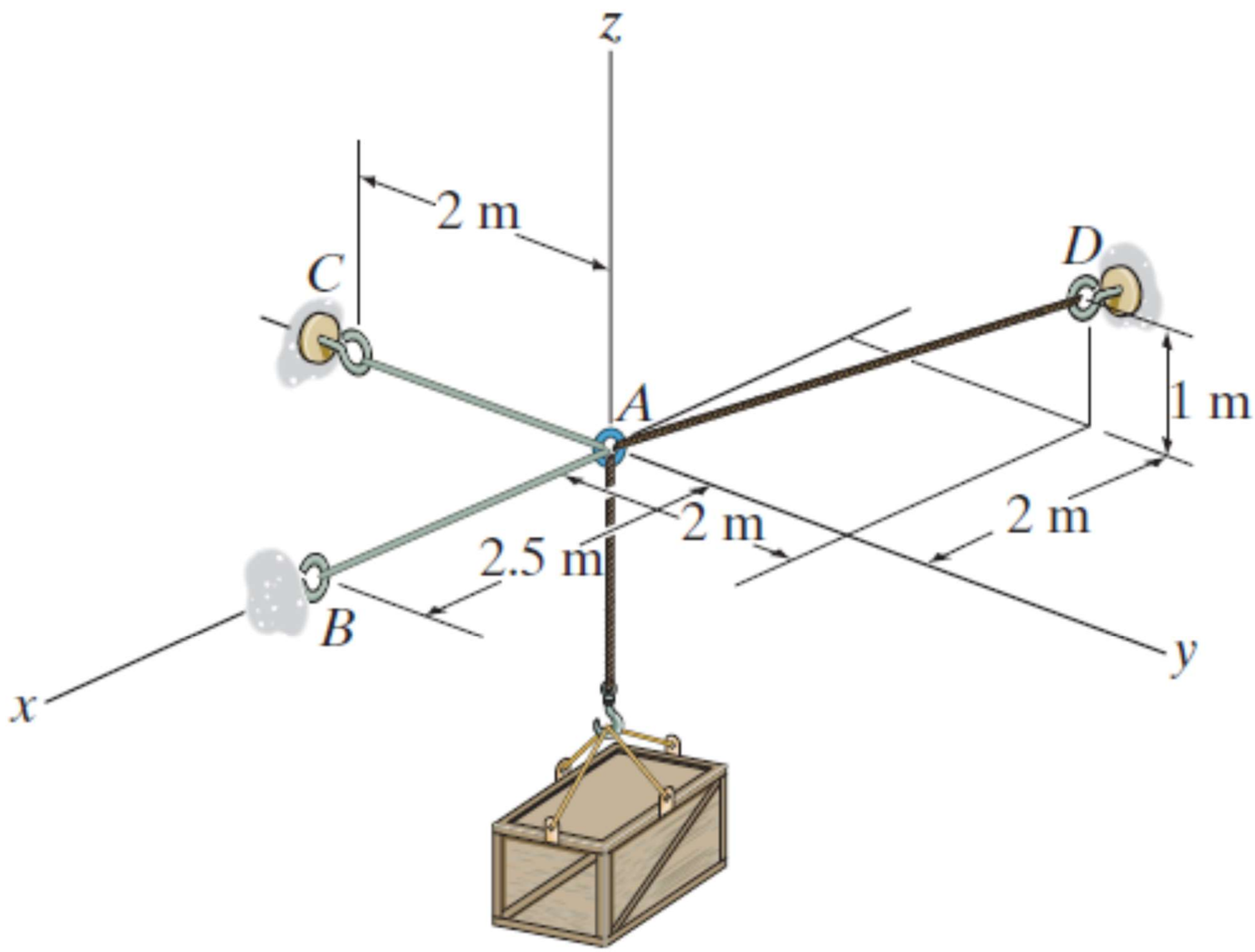
$$\sum \vec{F} = \vec{0},$$

$$\text{or} \quad \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath} + \left(\sum F_z\right)\hat{k} = \vec{0},$$

$$\text{or} \quad \sum F_x = 0 \ \text{ and } \ \sum F_y = 0 \ \text{ and } \ \sum F_z = 0.$$

Sample Problem

Determine the tension in the cables in order to support the 100-kg crate in the equilibrium position shown.



Sample Problem - solution

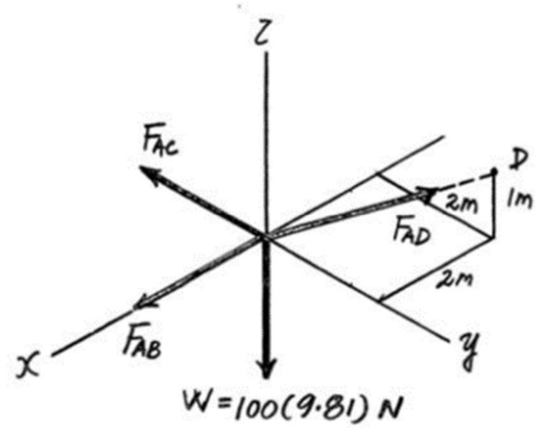
Force Vectors: We can express each of the forces on the free - body diagram shown in Fig. (*a*)

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{i}$$

$$\mathbf{F}_{AC} = -F_{AC}\,\mathbf{j}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(-2-0)\mathbf{i}+(2-0)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-2-0)^2+(2-0)^2+(1-0)^2}}\right] = -\frac{2}{3}F_{AD}\mathbf{i}+\frac{2}{3}F_{AD}\mathbf{j}+\frac{1}{3}F_{AD}\mathbf{k}$$

$$\mathbf{W} = [-100(9.81)\mathbf{k}]\,\mathrm{N} = [-981\,\mathbf{k}]\,\mathrm{N}$$



(a)

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$F_{AB}\,\mathbf{i} + (-F_{AC}\,\mathbf{j}) + \left(-\frac{2}{3}F_{AD}\mathbf{i} + \frac{2}{3}F_{AD}\,\mathbf{j} + \frac{1}{3}F_{AD}\,\mathbf{k}\right) + (-981\mathbf{k}) = \mathbf{0}$$

$$\left(F_{AB} - \frac{2}{3}F_{AD}\right)\mathbf{i} + \left(-F_{AC} + \frac{2}{3}F_{AD}\right)\mathbf{j} + \left(\frac{1}{3}F_{AD} - 981\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$F_{AB} - \frac{2}{3}F_{AD} = 0 \qquad (1)$$

$$-F_{AC} + \frac{2}{3}F_{AD} = 0 \qquad (2)$$

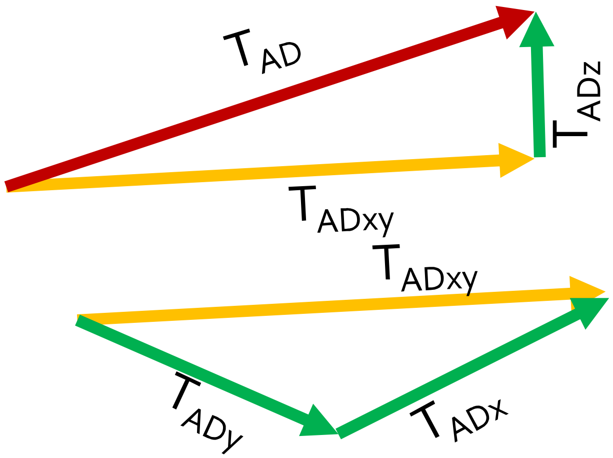
$$\frac{1}{3}F_{AD} - 981 = 0 \qquad (3)$$

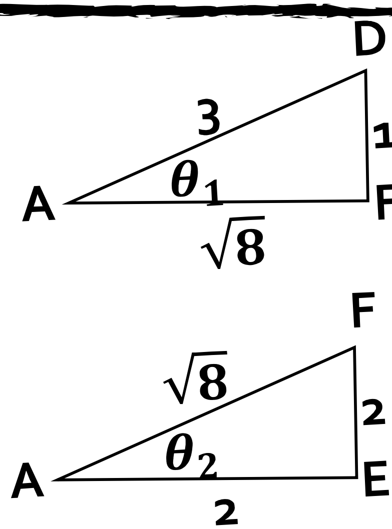
Solving Eqs. (1) through (3) yields

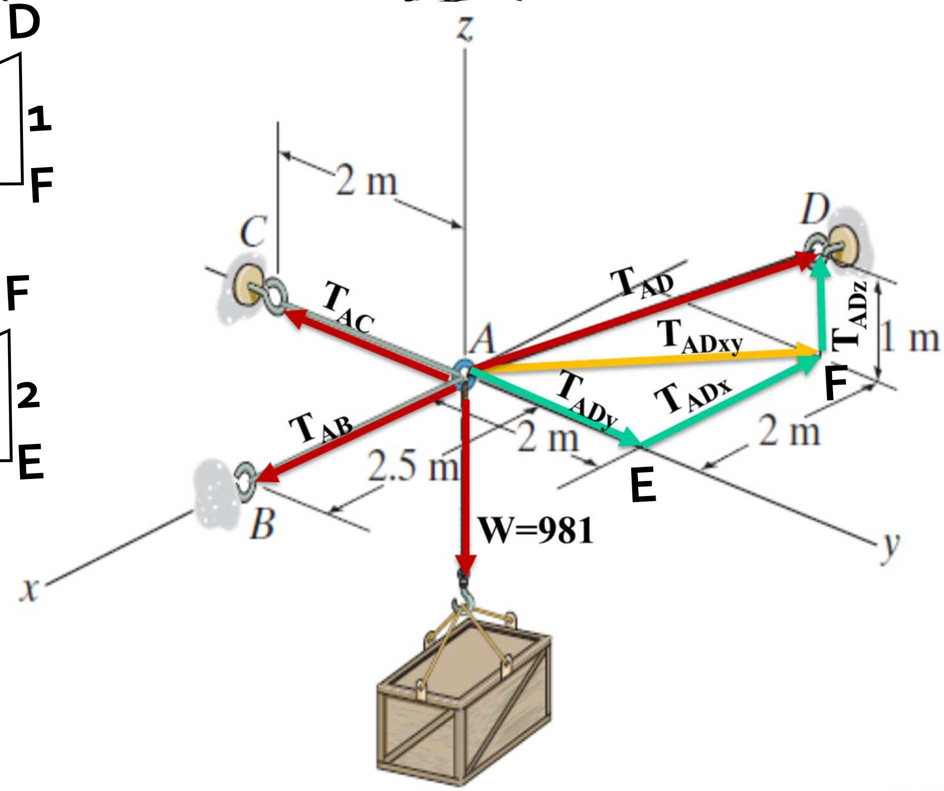
$F_{AD} = 2943\ \mathrm{N} = 2.94\ \mathrm{kN}$ **Ans.**

$F_{AB} = F_{AC} = 1962\ \mathrm{N} = 1.96\ \mathrm{kN}$ **Ans.**

Sample Problem – solution 2







$$\sum F_z = 0 \qquad T_{ADZ} = T_{AD}\sin\theta_1 = \frac{T_{AD}}{3} = W \to T_{AD} = 2943N$$

$$\sum F_y = 0 \qquad T_{AC} = T_{ADY} = T_{ADxy}\cos\theta_2 = \frac{T_{ADxy}}{\sqrt{2}} = \frac{T_{AD}\cos\theta_1}{\sqrt{2}} = \frac{T_{AD}\left(\sqrt{8}/_3\right)}{\sqrt{2}} = 1962\ N$$

$$\sum F_x = 0 \qquad T_{AB} = T_{ADx} = T_{ADxy}\sin\theta_2 = \frac{T_{ADxy}}{\sqrt{2}} = \frac{T_{AD}\cos\theta_1}{\sqrt{2}} = \frac{T_{AD}\left(\sqrt{8}/_3\right)}{\sqrt{2}} = 1962\ N$$