Mathematics Department

Math 1321 - Worksheet #5

"10.6 - 10.7"

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Name.......

Q1 Which of the series converge absolutely, which converge conditionally, and which diverge

1. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{4^n}{3^{n+1}}$

2. $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^2}$

3. $\frac{1}{4} - \frac{1}{6} + \frac{1}{8} - \frac{1}{10} + \frac{1}{12} \cdots$

Q2. (a) Find the radius and interval of Convergence

(b) What values of x does the series converge conditionally

1. $\sum_{n=0}^{\infty} \left(\frac{x^2+1}{3}\right)^n$

2. $\sum_{n=1}^{\infty} \frac{(-1)^n (2x-3)^n}{2n-1}$

Short Answers

Q. $\boxed{1}$ The series of Abs Value $\frac{1}{3}\sum_{n=1}^{\infty}\left(\frac{4}{3}\right)^n$

Div. G.S. because $|R|>1$

*** $\frac{1}{3}\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{4}{3}\right)^n$ Diverge by nth term test

$\boxed{2}$ The series of Abs Value $\sum_{n=1}^{\infty}\frac{\sin n}{n^2}$

By D.C.T the series of Abs value Conv.

$\Rightarrow \sum_{n=1}^{\infty}(-1)^n\frac{\sin n}{n^2}$ Conv. Abs.

$\boxed{3}$ $\sum_{n=1}^{\infty}(-1)^n\frac{1}{2n+2}$

The series of Abs value $\sum_{n=1}^{\infty}\frac{1}{2n+2}$

By D.C.T the series of Abs value Div.

** $\sum_{n=1}^{\infty}(-1)^n\frac{1}{2n+2}$

① U_n's positive $\forall n\geq 1$

② U_n's dec since $\frac{1}{2n+2} > \frac{1}{2n+4}$

③ $\lim_{n\to\infty} U_n = 0$

The $\sum_{n=1}^{\infty}(-1)^n\frac{1}{2n+2}$ Conv. Conditionally.

Q_2 $\boxed{1}$ $\sum_{n=0}^{\infty} \left(\frac{x^2+1}{3}\right)^n$ G.S

Converge geometric series if $\left|\frac{x^2+1}{3}\right| < 1$

$\underbrace{|x^2+1|}_{\text{positive}} < 3$

$x^2+1 < 3$

$x^2 < 2$

$|x| < \sqrt{2}$

$-\sqrt{2} < x < \sqrt{2}$

The series converge on the interval $(-\sqrt{2}, \sqrt{2})$

Its sum $= \dfrac{1}{1-\frac{x^2+1}{3}} = \dfrac{3}{2-x^2}$

$\boxed{2}$ $\sum_{n=1}^{\infty} \frac{(-1)^n (2x-3)^n}{2n-1}$

* The interval of convergence $(1, 2]$
* The Radius of convergence $R = \frac{1}{2}$
* At $x = 2$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$ Conv. Conditionally

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