جامعة بيرزيت

BIRZEIT UNIVERSITY

ANSWER BOOKLET

Student: Digital	Number 4
Course: Department:	Number:
Division:	Instructor:
Date: Day	Month Year

Dr. Abdellatif Abu Issa

For Instructor's Use

Question	Grade
1	
2	
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8	
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10	
11	
12	
Total	

The Tabulation Method

Determination of Prime Implicants

Ex: $F(w, x, y, z) = \Sigma(0, 1, 2, 8, 10, 11, 14, 15)$

Step 1

(a) $wx\ yz$

0 0000 ✓
1 0001 ✓
2 0010 ✓
8 1000 ✓
10 1010 ✓
11 1011 ✓
14 1110 ✓
15 1111 ✓

(b)

(0,1) 000 –
(0,2) 00 – 0 ✓
(0,8) – 000 ✓
(2,10) – 010 ✓
(8,10) 10 – 0 ✓
(10,11) 101 – ✓
(10,14) 1 – 10 ✓
(11,15) 1 – 11 ✓
(14,15) 111 – ✓

(c)

(0,2,8,10) – 0 – 0
(0,8,2,10) – 0 – 0
(10,11,14,15) 1 – 1 –
(10,14,11,15) 1 – 1 –

⇒ Prime implicants are

(0,1) : 000 – → $w'x'y'$
(0,2,8,10) : – 0 – 0 → $x'z'$
(10,11,14,15) 1 – 1 – → wy

* using decimal notations only for determination of prime implicants

(a)

0 ✓	0,1	(1)
—	(0,2)	(2) ✓
1 ✓	(0,8)	(8) ✓
2 ✓	—	
8 ✓	2,10	(8) ✓
—	8,10	(2) ✓
10 ✓	—	
—	(10,11)	(1) –
11 ✓	10,14	(4) –
14 ✓	—	
—	11,15	(4) –
15 ✓	14,15	(1) –

(b)

0,2,8,10	(2,8)
0,2,8,10	(2,8)
—	
10,11,14,15	(1,4)
10,11,14,15	(1,4)

c.

Prime implicants are

Decimal	Binary (w x y z)	Term
0,1 (1)	0 0 0 –	$w'x'z'$
0,2,8,10	– 0 – 0	$x'z'$
10,11,14,15	1 – 1 –	wy

* Selection of prime implicants that minimize the function

Term	Decimal	0	1	2	8	10	11	14	15
~~w + y'~~									
~~x + z'~~									
$w'x'y'$	0, 1 (1)	X	(X)						
~~0, 2, 8, 10~~									
$x'z'$	0, 2, 8, 10 (2, 8)	X		(X)	X	X			
wy	10, 11, 14, 15 (1, 4)					X	X	X	(X)
		✓	✓	✓	✓	✓	✓	✓	✓

all of them are essential prime implicants ⇒

$$F = w'x'y' + x'z' + wy$$

Ex. $F(w,x,y,z) = \Sigma(1,4,6,7,8,9,10,11,15)$

	(a)	(b)	(c)
0001	1 ✓	1,9 (8)	8,9,10,11 (1,2)
0100	4 ✓	4,6 (2)	8,9,10,11 (1,2)
1000	8 ✓	~~8,9~~	
		8,9 (1) ✓	
0110	6 ✓	8,10 (2) ✓	
1001	9 ✓		
1010	10 ✓	6,7 (1)	
		9,11 (2) ✓	
0111	~~7~~ ✓	10,11 (1) ✓	
1011	11 ✓		
		7,15 (8)	
1111	15 ✓	11,15 (4)	

⇒ Prime implicants

~~1,9 (8)~~

Decimal	Binary w	x	y	z	Term
1,9 (8)	-	0	0	1	$x'y'z$
4,6 (2)	0	1	-	0	$w'xz'$
6,7 (1)	0	1	1	-	$w'xy$
7,15 (8)	-	1	1	1	xyz
11,15 (4)	1	-	1	1	wyz
8,9,10,11 (1,2)	1	0	-	-	wx'

Selection of prime implicants

		1	4	6	7	8	9	10	11	15
✓ $x'y'z$	1,9	⊗					X			
✓ $w'xz'$	4,6		⊗	X						
$w'xy$	6,7			X	X					
xyz	7,15				X					X
wyz	11,15								X	X
✓ wx'	8,9,10,11					⊗	X	X	X	
		✓	✓	✓		✓	✓	✓	✓	

Essential prime implicants

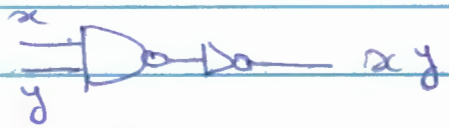
$x'y'z$, $w'xz'$, wx'

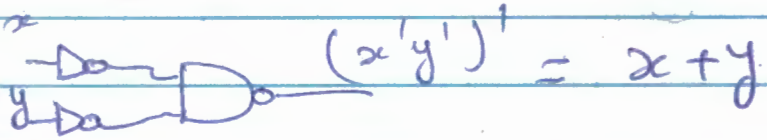
take xyz from the prime implicants for the simplest form

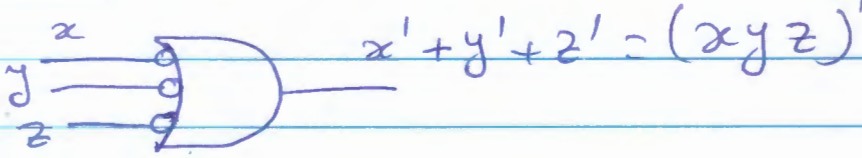
$\Rightarrow F = x'y'z + w'xz' + wx' + xyz$

⊛ NAND AND NOR Implementations

⊛ 2. level implementation

- AND 

- OR 



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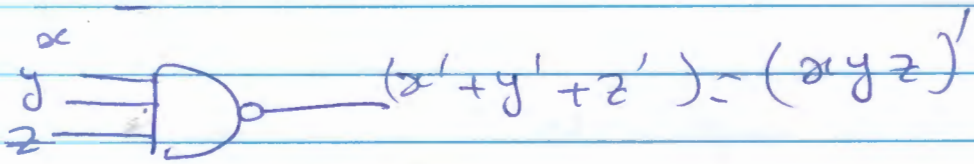
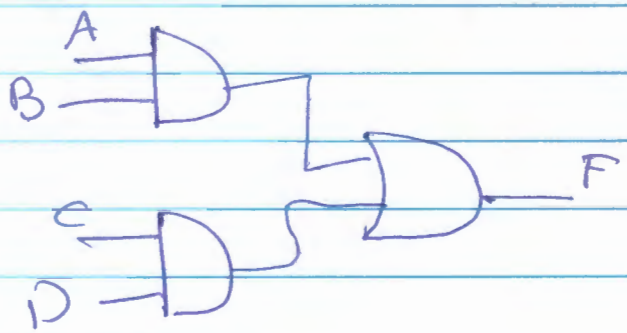
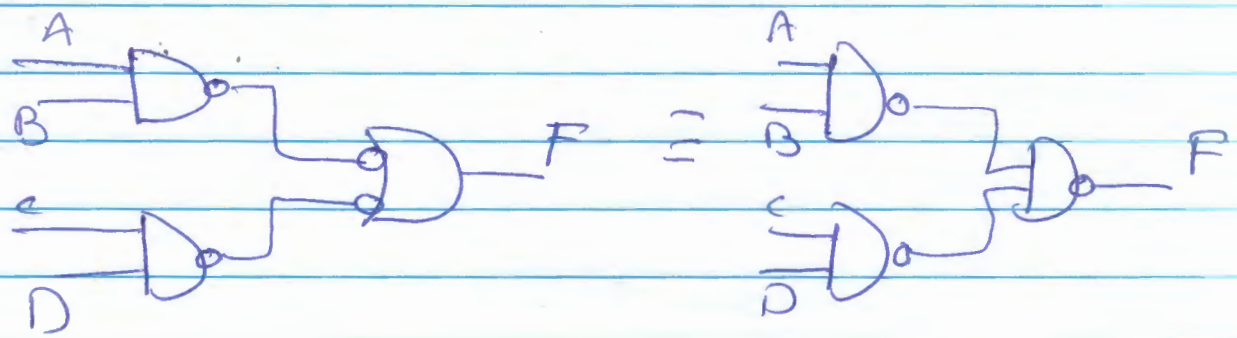


⊛ Two level Implementation

Ex: $F = AB + CD$



(a)

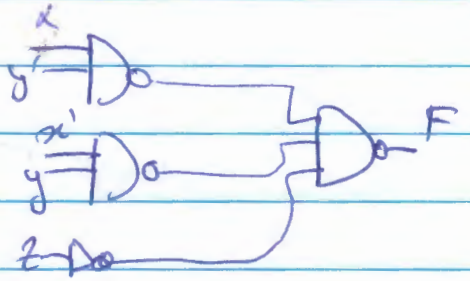


Ex:

$F(x,y,z) = \Sigma(1,2,3,4,5,7)$

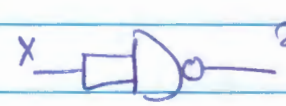
$\Rightarrow$ using k-map $\Rightarrow$ $F = xy' + x'y + z$

$\Rightarrow$

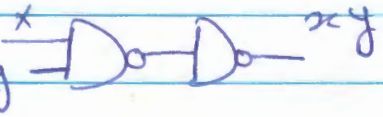


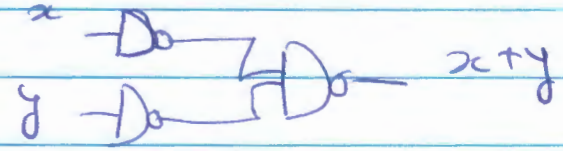
⑥ How NAND and NOR are functional Complete

① NOT



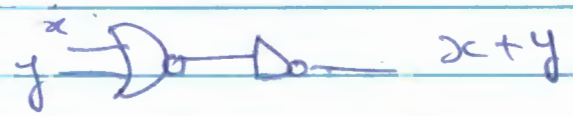
② AND

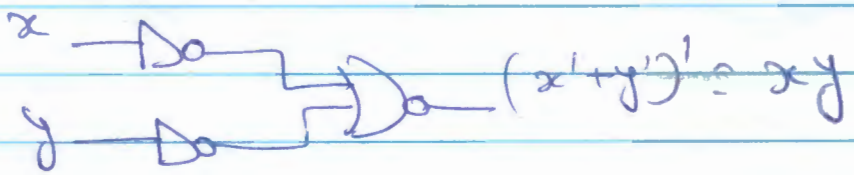




$(x'y')' = x+y$

NOR

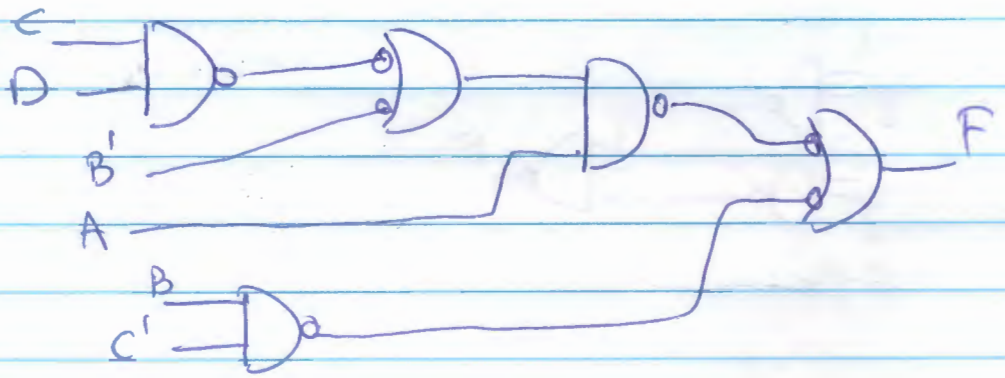




Multilevel NAND circuit

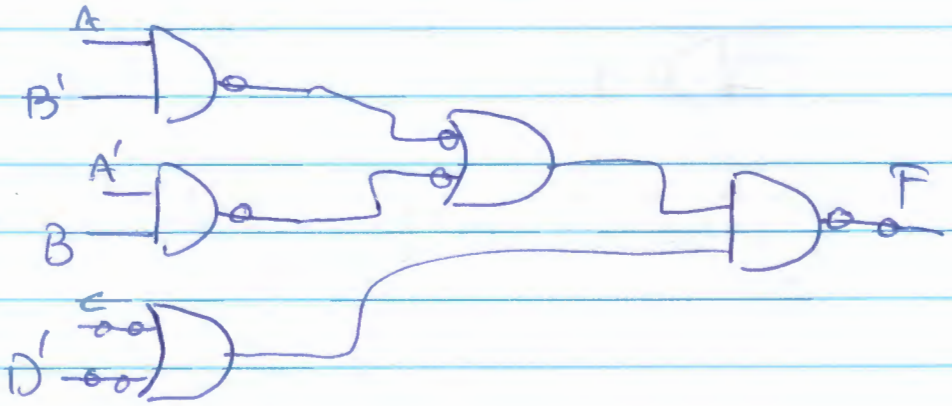
Ex.

$$F = A(CD + B) + BC'$$



Ex.

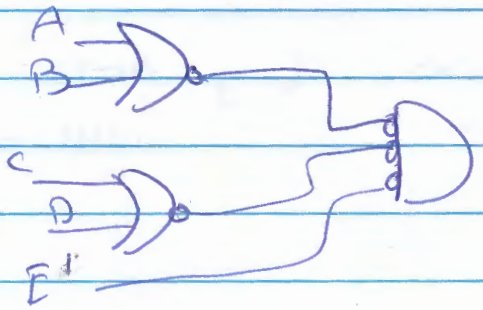
$$F = (AB' + A'B)(C + D')$$



NOR

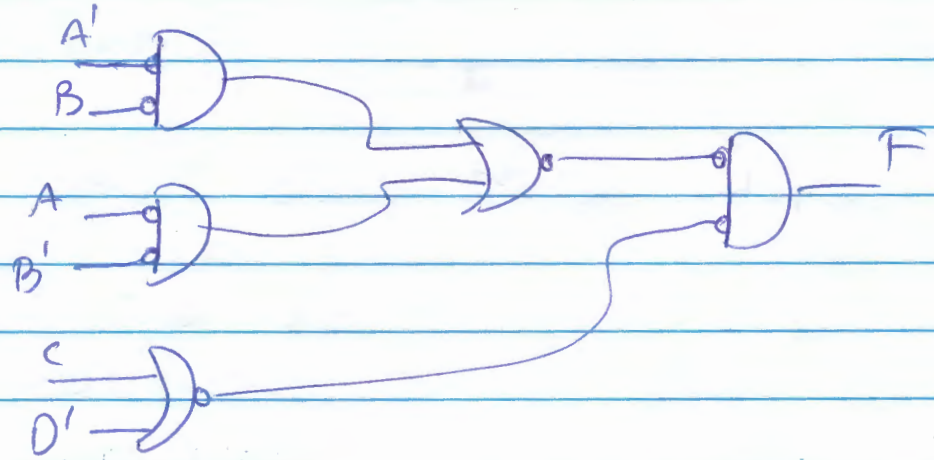
Ex:

$F = (A+B)(C+D)E$



Ex:

$F = (AB' + A'B)(C + D')$



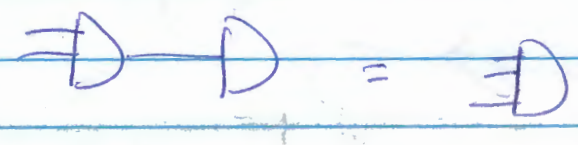
⊛ Wired Logic

⊛ Nondegenerate form

AND
OR
NAND
NOR

} 2 level → 16 forms

- 8 degenerate form (eg AND - AND
NAND - NOR

AND - AND 

NAND - NOR 

≡ 

≡ AND - AND

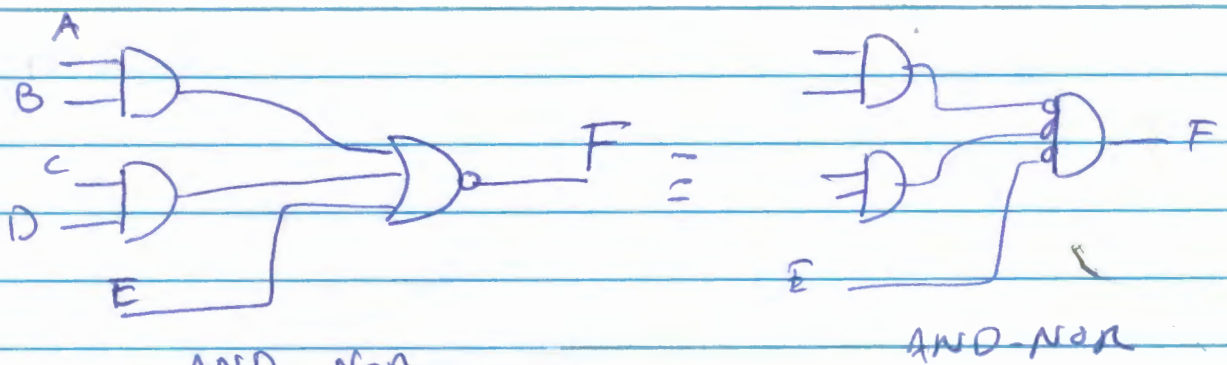
- Nondegenerate form (8 forms)

AND - OR	OR - AND
NAND - NAND	NOR - NOR
NOR - OR	NAND - AND
OR - NAND	AND - NOR

dual for each other

Ex: $F = (AB + CD + E)'$

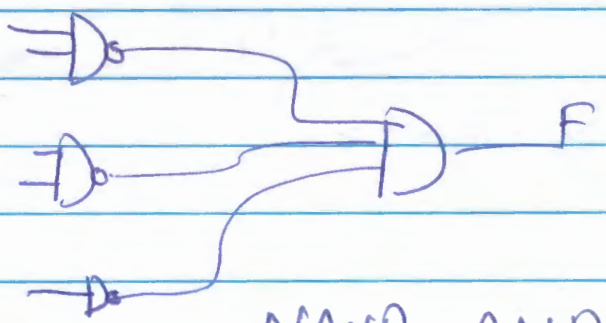
AND – OR – invert



AND – NOR

AND-NOR

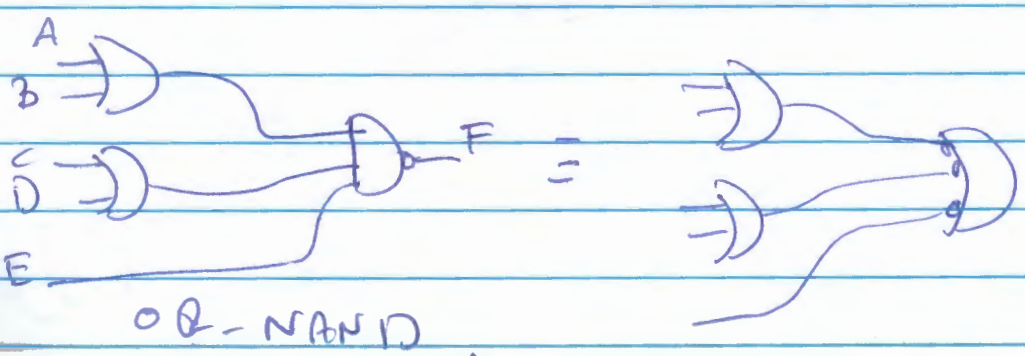
=



NAND – AND

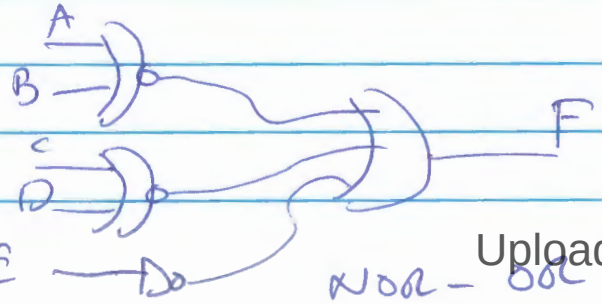
Ex: $F = [(A+B)(C+D)E]'$

OR – AND – invert



OR – NAND

=



NOR – OR

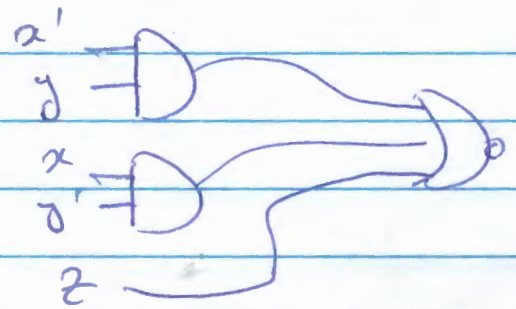
④ In the last 4 cases, we look to simplify f', then use inverter to get f.

$F(x,y,z) \Sigma (0, 6)$ → Page 93

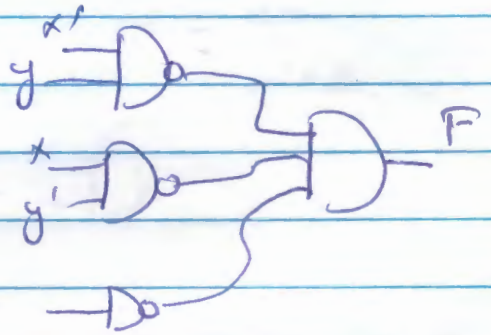
ex: $f' = x'y + xy' + z$

$F = (x'y + xy' + z)'$

① AND - NOR



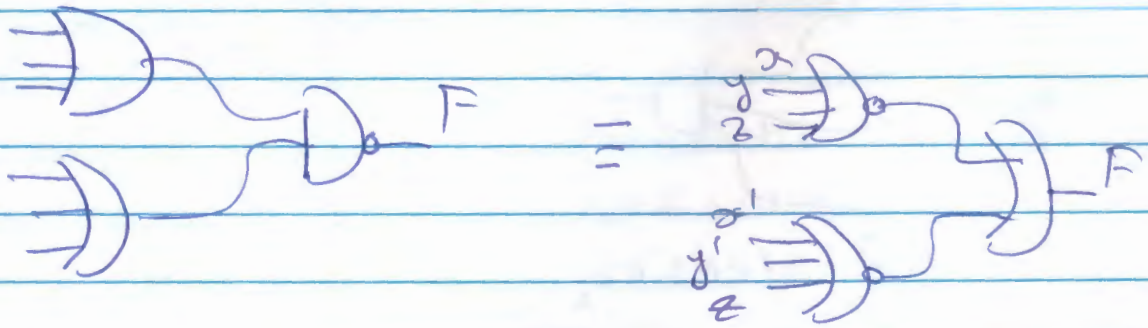
② NAND - AND



③ OR - NAND $(F = x'y'z' + xyz')$

$\rightarrow F' = (x+y+z)(x'+y'+z)$

$\rightarrow F = \{(x+y+z)(x'+y'+z)\}'$



Exclusive-OR function:-

x	y	f
0	0	0
0	1	1
1	0	1
1	1	0

$F = x \oplus y$

x \ y	0	1
0		⊕
1	⊕	

$F = xy' + x'y$

$(x \oplus y)' = xy + x'y'$

⊕ The following are some properties of XOR

$x \oplus 0 = x$

$x \oplus 1 = x'$

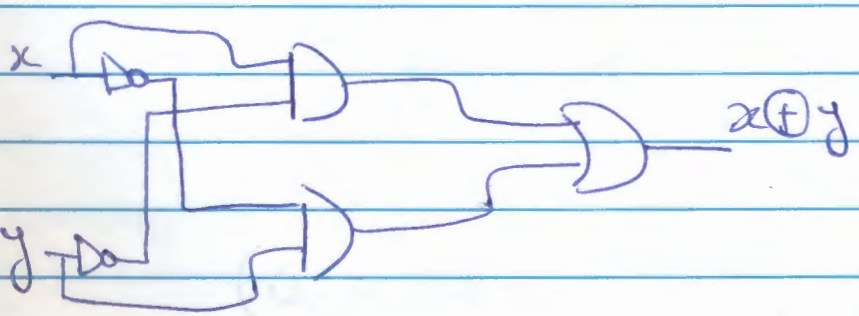
$x \oplus x = 0$

$x \oplus x' = 1$

$x \oplus y = y \oplus x$

$x \oplus y' = x' \oplus y = (x \oplus y)'$

$(x \oplus y) \oplus z = x \oplus (y \oplus z) = x \oplus y \oplus z$



(using nand-nand).

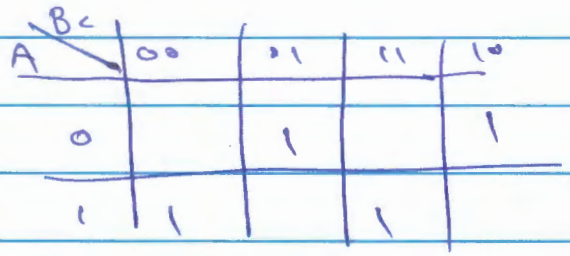
* XOR is odd function

e.g. $A \oplus B \oplus C = (A \oplus B) \oplus C$

$= (AB' + A'B)C' + (AB + A'B')C$

$= AB'C' + A'BC' + ABC + A'B'C$

$= \Sigma(1, 2, 4, 7)$



$\Rightarrow F = (A \oplus B \oplus C)'$ is an even function

i.e	A	B	C	F
	0	0	0	0
	0	0	1	1
	0	1	0	1
	0	1	1	0
	1	0	0	1
	1	0	1	0
	1	1	0	0
	1	1	1	1

* The same thing for $f(A, B, C, D)$

$f(A, B, C, D) = A \oplus B \oplus C \oplus D$

$= (AB' + A'B) \oplus (CD' + C'D)$

$= \Sigma(1, 2, 4, 7, 8, 11, 13, 14)$

⊛ Parity Generation and checking

- A parity bit is an extra bit included with a binary message to make the number of 1's either odd or even.

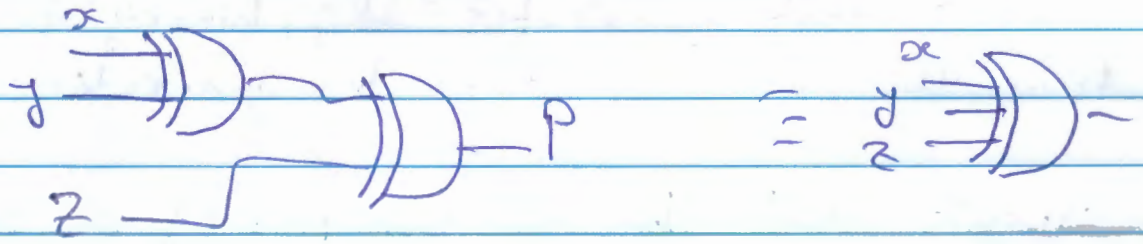
- the circuit that generates the binary bit in the transmitter is called a parity generator.

- The circuit that checks the parity in the receiver is called a parity checker.

Ex: 3 bits x, y, z to be transmitted with an even parity ⇒

Three-Bit Message			Parity bit
x	y	z	P
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$P = x \oplus y \oplus z$



- These 3 bits together with the parity bit are transmitted to their destination

- if the number of 1's in the receiver is odd ⟶ error occured during transmission.