Digital Systems

Chapter 1: Digital systems and Binary Numbers

System: an integrated whole used for a certain task

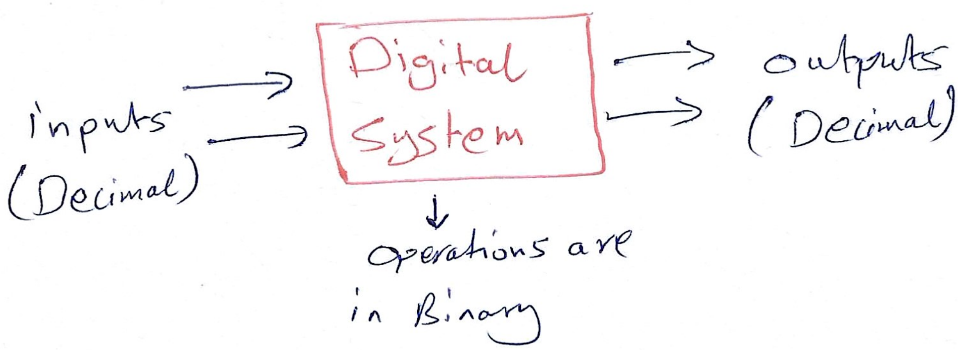
- is a group of interacting elements (sub systems) having an internal structure which links them into a unified whole.

Digital: - Discrete of time and magnitude.

- A collection of components, such as hardware, networks, and software, used to store, process and communicate digital informations, typically in the form of binary data. Digital systems operate using binary numbers, 0 an 1.

* Numbering System:-

1) Binary	2) Decimal	3) octal	4) Hexa Decimal
2-digits	10-digits	8-digits	16-digits
0, 1	0, 1, 2, 3, ... 9	0, 1, 2, ... 7	0, 1, 2, ... 9, A, B, ... F
$(100110)_2$	$(753)_{10}$	$(375)_8$	$(3F)_{16}$
B	D	Q	H



* LSB and MSB

LSB :- Least Significant Bit

MSB :- Most Significant Bit

1010011

MSB ↙ ↘ LSB

- bit :- 0 or 1
- Byte : 8 bits
- Word = 2 bytes = 16 bits
- 1 Kilo bits = 2^{10} bits = 1024 bits
- 1 Mega bits = 2^{20} bits = 1024 * 1024 bits
- 1 Giga bits = 2^{30} bits = 1024 * 1024 * 1024 bits
- 1 Tera bits = 2^{40} bits = 1024 * 1024 * 1024 * 1024 bits

Example :- Find the exact number of bits of 64 Mega bytes.

$$64 * 2^{20} * 8 = 64 * 1024 * 1024 * 8 \text{ bites}$$

* The weight :-

weight = $(\text{Base})^{(\text{status number})}$

$(\overset{2}{1}\,\overset{1}{8}\,\overset{0}{3}\,.\,\overset{-1}{5}\,\overset{-2}{6})_{10}$ ← status number

Factors, Base

$1*10^2 + 8*10^1 + 3\times10^0 + 5\times10^{-1} + 6\times10^{-2}$

$= 183,56$

Decimal	Octal	HexaDecimal	Binary
0	0	0	0
1	1	1	1
2	2	2	10
3	3	3	11
4	4	4	100
5	5	5	101
6	6	6	110
7	7	7	111
8	10	8	1000
9	11	9	1001
10	12	A	1010
11	13	B	1011
12	14	C	1100
13	15	D	1101
14	16	E	1110
15	17	F	1111
16	20	10	10000
17	21	11	10001
⋮	⋮	⋮	⋮

* Conversion :-

1) From any system to Decimal :- (Sum of weights)

2) From Decimal to Any system :- a) integer → Division

b) fraction → Multibly

1) From binary to decimal :-

(Sum of weights)

$$\text{weight} = (\text{base})^{\text{status number}}$$

Example :-

$$(\overset{3}{1}\overset{2}{0}\overset{1}{1}\overset{0}{0})_2 \xrightarrow{?} (\qquad)_{10}$$

$$0*2^0 + 1*2^1 + 0*2^2 + 1*2^3 = 0 + 2 + 0 + 8 = 10$$

$$(1010)_2 = (10)_{10}$$

Example :-

$$(\overset{3}{1}\overset{2}{0}\overset{1}{1}\overset{0}{1}.\overset{-1}{0}\overset{-2}{1})_2 \longrightarrow (\qquad)_d$$

$$1*2^0 + 1*2^1 + 0*2^2 + 1*2^3 + 0*2^{-1} + 1*2^{-2}$$

$$1 + 2 + 0 + 8 + 0 + \frac{1}{4} = 11.25$$

$$\therefore (1011.01)_2 = (11.25)_d$$

Example :-

$$(\overset{4}{1}\overset{3}{1}\overset{2}{0}\overset{1}{1}\overset{0}{1}.\overset{-1}{1}\overset{-2}{0})_2 \longrightarrow (\quad)_{10}$$

$$1*2^0 + 1*2^1 + 0*2^2 + 1*2^3 + 1\times 2^4 + 1*2^{-1} + 0*2^{-2}$$

$$1 \quad 2 \quad 0 \quad 8 \quad 16 \quad \tfrac{1}{2}$$

$$(27.5)_{10}$$

$$(11011.10)_2 = (27.5)_{10}$$

2) from octal to decimal

Example :-

$$(\overset{1}{7}\overset{0}{6})_8 \longrightarrow (\quad)_{10}$$

$$6*8^0 + 7*8^1 =$$

$$6 + 56 = 62$$

$$(76)_8 = (62)_{10}$$

Example :-

$$(\overset{1}{4}\overset{0}{2}.\overset{-1}{4})_8 \longrightarrow (\quad)_{10}$$

$$2*8^0 + 4*8^1 + 4*8^{-1}$$

$$2 + 32 + \frac{4}{8} = 34.5$$

$$(42.4)_8 = (34.5)_{10}$$

Example:

$$\overset{1}{2}\overset{0}{1}.\overset{-1}{2}$$

$$(21.2)_8 \longrightarrow (\qquad)_{10}$$

$$1*8^0 + 2*8^1 + 2*8^{-1}$$

$$1 + 16 + \frac{2}{8} = 17.25$$

$$(21.2)_8 = (17.25)_{10}$$

3) Hexadecimal to decimal

Examples:

$$(\overset{0}{A}.\overset{-1}{4})_H \longrightarrow (\qquad)_d$$

$$\overset{10}{A}*16^0 + 4*16^{-1}$$

$$10 + \frac{4}{16} = (10.25)$$

$$(A.4)_{16} = (10.25)_{10}$$

Example:-

$$(\overset{1}{1}\overset{0}{B}.\overset{-1}{C})_{16} \longrightarrow (\qquad)_{10}$$

$$\overset{11}{B}*16^0 + 1*16^1 + \overset{12}{C}*16^{-1}$$

$$11 + 16 + \frac{12}{16} = 27.75$$

$$(1B.C)_{16} = (27.75)_{10}$$

4) Decimal to binary

integer $\longrightarrow$ Division

fraction $\longrightarrow$ Multiply

Example:-

$(41)_d \longrightarrow (\qquad\qquad)_2$

2	41
1	20
0	10
0	5
1	2
0	1
1	0

$(101001)_2$

Example:-

$(65)_d \longrightarrow (\qquad\qquad)_2$

2	65
1	32
0	16
0	8
0	4
0	2
0	1
1	0

$(1000001)_2$

Example :-

$(27.4)_d \longrightarrow (\qquad)_2$

Multiply

2	27
1	13
1	6
0	3
1	1
↑ 1	0

$(11011.\overline{0110})$

$0.4 * 2 = 0.8$

$0.8 * 2 = 1.6$

$0.6 * 2 = 1.2$

$0.2 * 2 = 0.4$

$0.4 * 2 = 0.8$

Example :-

$(18.5)_d \longrightarrow (\qquad)_2$

$0.5 \times 2 = 1.0$

$0 \times 2 = 0.0$

2	18
0	9
1	4
0	2
0	1
↑ 1	0

$(10010.1)_2$

5) Decimal to octal

Example:-

$(95)_{10} \longrightarrow (\quad)_8$

8	95
7	11
3	1
1	0

$(137)_8$

Example:-

$(100)_{10} \longrightarrow (\quad)_8$

8	100
4	12
4	1
1	0

$(144)_8$

Example:

$(172.6875)_{10} \longrightarrow (\quad\quad)_8$

8	172
4	21
5	2
2	0

(254.54)

$0.6875 * 8 = 5.5$

$0.5 * 8 = 4.0$

6) Decimal to Hexa decimal

Example:

$(29)_{10} \longrightarrow (\quad\quad)_{16}$

	16	29
D	13	1
	1	0

$(1D)_{16}$

Example:

$(93)_{10} \longrightarrow (\quad\quad)_{16}$

	16	93
D	13	5
	5	0

$(5D)_{16}$

Example :-

$(1020.1)_{10} \longrightarrow (\qquad)_{16}$

	16	1020
C	12	63
F	15	3
	3	0

$0.1 * 16 = 1.6$

$0.6 * 16 = 9.6$

$0.6 * 16 = 9.6$

$(3FC.1\bar{9})$

7) octal to binary

Example :-

$(76)_8 \longrightarrow (\qquad)_2$

$7 \longrightarrow 111$

$6 \longrightarrow 110$

$\therefore (76)_8 \longrightarrow = (111\,110)_2$

Example :- $(15)_8 \longrightarrow (\qquad)_2$ or $(\overset{10}{15})_8 \longrightarrow (\quad)_d$

$1 \longrightarrow 001$

$5 \longrightarrow 101$

$(15)_8 = (001101)_2$

$5 * 8^0 + 1 * 8^1 =$

$5 + 8 = 13$

$(15)_8 = (13)_d$

$(13)_d \longrightarrow (\qquad)_2$

Example:

$$(56.4)_8 \longrightarrow (\qquad\qquad)_2$$

$5 \longrightarrow 101$

$6 \longrightarrow 110$

$4 \longrightarrow 100$

$$(101110.100)_2$$

8) Binary to octal:-

Example:-

$$(10111.1)_2 \longrightarrow (\qquad\qquad)_8$$

$$\downarrow$$

$$(\underbrace{010}_{2}\,\underbrace{111}_{7}.\underbrace{100}_{4})_2 = (27.4)_8$$

9) Hexadecimal to binary

Example:-

$$(AB)_{16} \longrightarrow (\qquad)_2$$

$A \longrightarrow 1010$

$B \longrightarrow 1011$

$$(AB)_{16} = (10101011)_2$$

Example :-

$$(A1.8)_{16} \longrightarrow (\quad)_2$$

$$8 \longrightarrow 1000$$

$$1 \longrightarrow 0001$$

$$A \longrightarrow 1010$$

$$(A1.8)_{16} = (10100001.1)_2$$

10) binary to Hexa decimal

Example :-

$$(10110011.1)_2 \longrightarrow (\quad)_{16}$$

$$\downarrow$$

$$\underbrace{1011}_{B}\ \underbrace{0011}_{3} . \underbrace{1000}_{8}$$

$$(10110011.1)_2 = (B3.8)_{16}$$

11) Octal to hexadecimal :-

$$(76.4)_8 \longrightarrow (\quad)_{16}$$

$$\downarrow$$

$$(111\ 110.100)_2 \longrightarrow (\quad)_{16}$$

$$\downarrow$$

$$(\underbrace{0011}_{3}\ \underbrace{1110}_{E} . \underbrace{1000}_{8})_2 = (3E.8)_{16}$$

12) Hexa decimal to Octal :-

$$(E1.A)_{16} \longrightarrow (\qquad)_8$$

$$\downarrow$$

$$(\underbrace{011}_{3}\,\underbrace{100}_{4}\,\underbrace{001}_{1}\,.\,\underbrace{101}_{5}0)_2 \longrightarrow (\qquad)_8$$

$$(011100001.101)_2 = (341.5)_8$$

* Binary Addition :-

Decimal addition

$$\begin{array}{r} 4 \\ +\ 4 \\ \hline 08 \end{array} \qquad \begin{array}{r} 4 \\ +\ 5 \\ \hline 09 \end{array} \qquad \begin{array}{r} 5 \\ +\ 5 \\ \hline 10 \end{array} \qquad \begin{array}{r} 5 \\ +\ 6 \\ \hline 11 \end{array}$$

Binary

$$\begin{array}{r} 0 \\ +\ 0 \\ \hline 00 \end{array} \qquad \begin{array}{r} 0 \\ +\ 1 \\ \hline 01 \end{array} \qquad \begin{array}{r} 1 \\ +\ 0 \\ \hline 01 \end{array} \qquad \begin{array}{r} 1 \\ +\ 1 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 1 \\ +\ 1 \\ +\ 1 \\ \hline 11 \end{array}$$

octal

$$\begin{array}{r} 3 \\ +\ 3 \\ \hline 6 \end{array} \qquad \begin{array}{r} 3 \\ +\ 4 \\ \hline 7 \end{array} \qquad \begin{array}{r} 4 \\ +\ 4 \\ \hline 10 \end{array} \qquad \begin{array}{r} 4 \\ +\ 5 \\ \hline 11 \end{array} \qquad \begin{array}{r} 5 \\ +\ 5 \\ \hline 12 \end{array}$$

Examples:-

$$\begin{array}{ccccccc}
 & \boxed{1} & & & & & \\
\boxed{1} & \boxed{1} & \boxed{1} & & & & \\
 & 1 & 1 & 1 & 0 & 1 & 1 \\
 & & 1 & 1 & 1 & 0 & 0\ + \\
 & & 1 & 1 & 0 & 0 & 0\ + \\
\hline
1 & 1 & 0 & 1 & 1 & 1 & 1
\end{array}$$

Examples:

$$\begin{array}{ccccccc}
\boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} & & \\
 & 1 & 1 & 0 & 1 & 1 & 0 \\
 & & 1 & 1 & 1 & 1 & 0\ + \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0
\end{array}$$

* How do Computers represent digits ?

- Central processing Unit (CPU)

	CPU	
①	ALU	ALU : Arithmetic Logic Unit (Addition, subtraction, Multiplication, division, Comparison)
②	CU	CU : Control Unit
3	Registers	Rigesters : Memory

* Analysis

input → system → output ??

input ↑ Known, system ↑ Known

* Design

input → system → output

input ↑ Known, system ??, output ↑ Know

in Computer we Can add using adder but there is no subtractor

16 + 16 = 32

10000 + 10000

$$\begin{array}{r} 10000 \\ +\ 10000 \\ \hline 100000 \end{array} \rightarrow 32$$

So subtraction must be changed to addition using complements

* Complements :- (1) Diminished Complement

(2) Radix Complement

(1) Diminished :- Given a number N in base r having n digits the $(r-1)$'s complement of N is $(r^n - 1) - N$

* Binary → 1's complement

* Decimal → 9's complement

* Hexa → 15's comp

* octal → 7's comp.

Example :- what is the 9's complement of $\underbrace{\overset{N}{666}}_{n=3}$ in base $\overset{r}{10}$?

$(r^n - 1) - N$

$(10^3 - 1) - 666$

$999 - 666 = 333$

Example :- What is the 9's complement of 33 in base 10

$(10^2 - 1) - 33$

$(100 - 1) - 33 = 66$

Example:- what is the 1's (first) complement of 1011 in base 2

$$(2^4 - 1) - 1011$$
$$15 - 1011$$
$$1111 - 1011 = 0100$$

$$\begin{array}{r} 1111 \\ -1011 \\ \hline 0100 \end{array}$$

② Radix complement :- Given a number N in base r having n digits, the r's complement of N is $(r^n - N)$ fo $N \neq 0$

Example:- what is the 10's complement of 666 in base 10

$10^3 - 666 = 334$

10's complement = 9's complement + 1

r's complement = $(r-1)$'s + 1

Example:- what is the 2's complement of 1011

2's complement = 1's complement + 1

= 0100 + 1

= 0101

* subtraction with complements :-

(A) using r's complement :-

$M - N = M + r\text{'s complement of } N$

(1) If $M > N$, the end carry is discarded

(2) If $M < N$, no end carry and the result will be negative so to obtain the final answer take the r's complement of the summation

(1) $M > N$

Example:-

$99 - 33 = 99 + 10\text{'s complement of } 33$

$M > N$

10's comp. of $33 = 100 - 33 = 67$

$99 + 67 = \cancel{1}66$

$$\begin{array}{r} {}^{1}99 \\ +67 \\ \hline \cancel{1}66 \end{array}$$

discard the carry

So the answer is 66 only

Example:-

$1011 - 0011$

$M > N$

$1011 + 2\text{'s complement of } 0011$

$1100 + 1$

$1011 + 1101 = 1000$

$$\begin{array}{r} {}^{1\,1\,1}\ \ \\ 1011 \\ +1101 \\ \hline \cancel{1}1000 \end{array}$$

② M < N

Example :- 33 - 99

33 + 10's complement of 99

10's Comp. of 99 = 100 - 99 = 1

33 + 1 = $\underline{\underline{34}}$

negative

∴ Final answer is 10's complement of 34

100 - 34 = $\boxed{(-)\,66}$

Example :-

0011 - 1011

0011 + 2's Comp. of 1011

0011 + 0101 = 1000

$$\begin{array}{r} {\scriptstyle 1\,1\,1\ \ } \\ 0011 \\ +\ 0101 \\ \hline 1000 \end{array}$$

∴ Final answer is the 2's complement of 1000

$$\begin{array}{r} 0111 \\ +\ \ \ \ \ 1 \\ \hline \boxed{(-)\,1000} \end{array}$$

B) using (r-1)'s Complement

1) M > N

Example: using 9's complement find

72532 - 3250

First: Find the 9's complement of 3250

we use n = 5 : the maximum digits of (M, N)

$(r^n - 1) - N$

$(10^5 - 1) - 3250 = 96749$

second: Chang the subtraction to addition

$\therefore 72532 - 3250 = 72532 + 96749$

$$
\begin{array}{r}
72532 \\
+\ 96749 \\
\hline
1\,69281
\end{array}
$$

$\xrightarrow{\quad} 1^{+}$

Third: add the Carry to the number so the final answer is 69281 + 1 = 69282

Example: using 1's complement find

110101 - 100101

1) 1's complement of 100101 is 011010

(2)
$$\begin{array}{r} \overset{1}{1}10101 \\ +\ 011010 \\ \hline 1\ 001111 \end{array}$$

(3) 001111 + 1 (end-around carry)

$\boxed{010000}$ Final answer

(2) M < N

Example: using 9's complement find

$$3250 - 72532$$

(1) 9's complement of 72532 is 27467

(2)
$$\begin{array}{r} 03250 \\ +\ 27467 \\ \hline 30717 \end{array}$$

(3) The final answer is −(9's complement of 30717)

$$= -69282$$

Example: using 1's complement find

$$101011 - 111001$$

(1) 1's complement of 111001 is 000110

(2)
$$\begin{array}{r} 101011 \\ +\ 000110 \\ \hline 110001 \end{array}$$

(3) The final answer − 001110