 Written by Alaa Etaiwi

Chap 3 Differentiation

* $f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$

* If f is diff at $x = c$ Then f is cont at $x = c$

• Diff Rules :-

1- $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

2- $(f(x) \cdot g(x))' = f(x)\, g'(x) + g(x)\, f'(x)$

3- $\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x) f'(x) - g'(x) f(x)}{[g(x)]^2}$ $\quad g(x) \neq 0$

4- $(f \circ g)'(x) = f'(g(x)) \times g'(x)$

$\boxed{4}$ $(f \circ g)'(x) = f'(g(x)) \times g'(x)$

- Derivative & Trigonometric

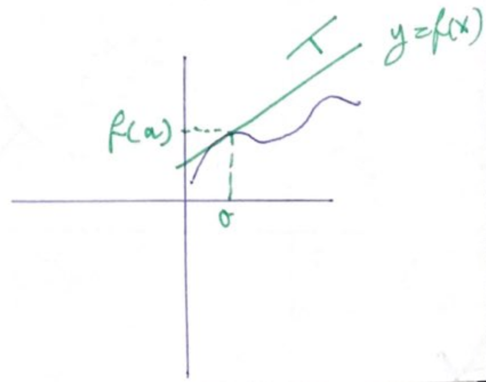
- $(\sin x)' = \cos x$
- $(\cos x)' = -\sin x$
- $(\tan x)' = \sec^2 x$
- $(\cot x)' = -\csc^2 x$
- $(\sec x)' = \sec x \tan x$
- $(\csc x)' = -\csc x \cot x$

Tangent & Normal

- Tangent

$y - y_0 = m(x - x_0)$

$(x_0, y_0) = (a, f(a)) / m = f'(a)$



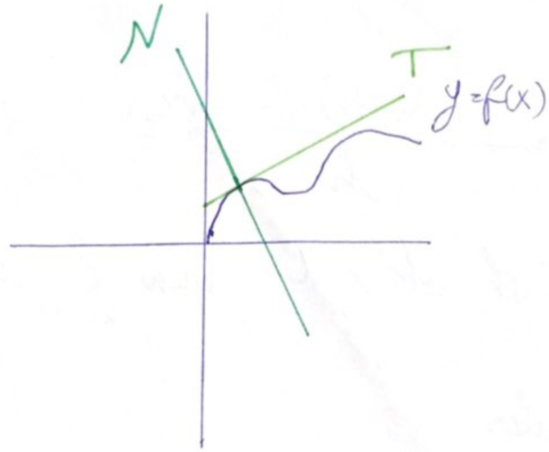
Normal

$y - y_0 = n(x - x_0)$

$(x_0, y_0) = (a, f(a))$

$n = \frac{-1}{m}$

$mn = -1$



Implicit diffrentiation

$\frac{dy}{dx}$

dy → dependent on x   so we diff → respect to x

dx → Independent

linearization

If f is diff function at $x = a$

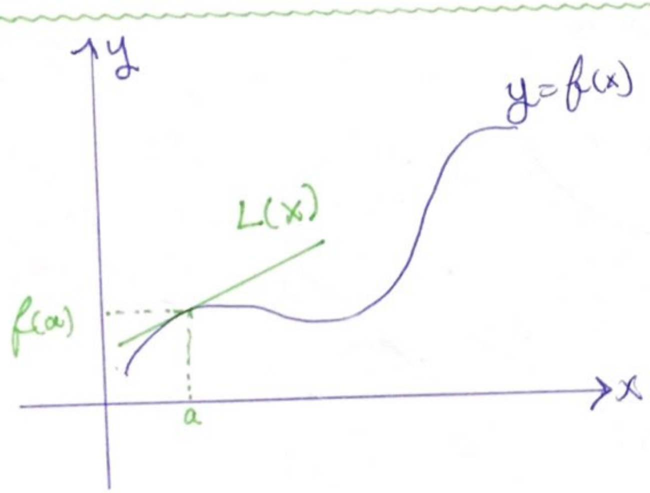
Then The linearization of f at $x = a$ is

linearization

If f is diff function at $x = a$
Then The linearization of f at $x = a$ is defined by :-

$$\boxed{L(x) = f(a) + f'(a)(x-a)}$$

حيث L هي تقريب
لـ y (النقاط القريبة من a)
$L(x)$ approximate $f(a)$ near by $x = a$
$L(x) \approx f(x)$



Prove :-

$y - y_0 = m(x - x_0)$

$(x_0, y_0) = (a, f(a))$

$f'(a) = m$

$y - f(a) = f'(a)(x-a)$

$y = f'(a)(x-a) + f(a)$ #

$L(x)$ is the standard linear approximation of $f(x)$ near $x=a$

* If $y=f(x)$ is diff at $x=a$ Then $\frac{dy}{dx} = f'(x)\Big|_{x=a} = f'(a)$

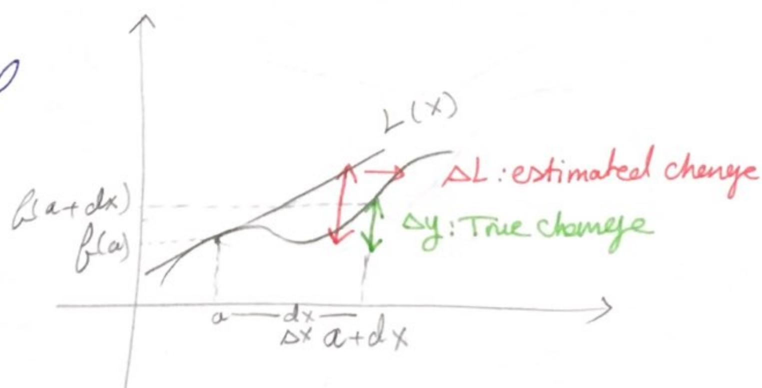
$\rightarrow dy = f'(a)\, dx$

- dependant variable
- Independant Variable $dx = \Delta x$

$\boxed{\underbrace{dy}_{\text{estimated value}} \approx \underbrace{\Delta y}_{\text{True value}} = \Delta f}$

$\boxed{\Delta L \approx \Delta y}$

- result: we use the differential dy to estimate The True change Δy



Error = $|T - E|$