1.3 Part 2

Thursday, November 25, 2021 9:19 AM

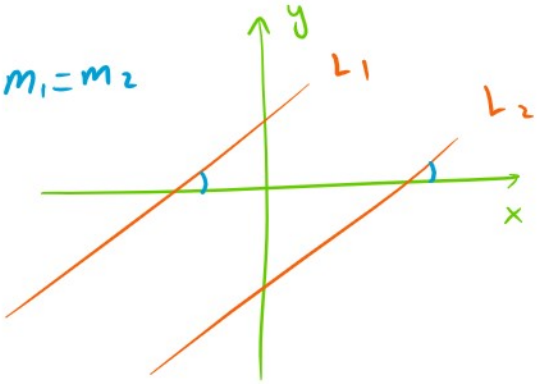
line L

slope m

Parallel $\parallel$

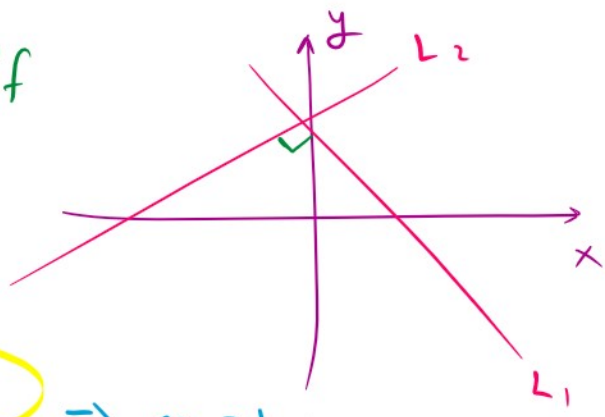
Orthogonal $\perp$
Perpendicular
normal

$L_1 \parallel L_2$ iff $m_1 = m_2$



$L_1 \perp L_2$ iff

$m_1 \cdot m_2 = -1$



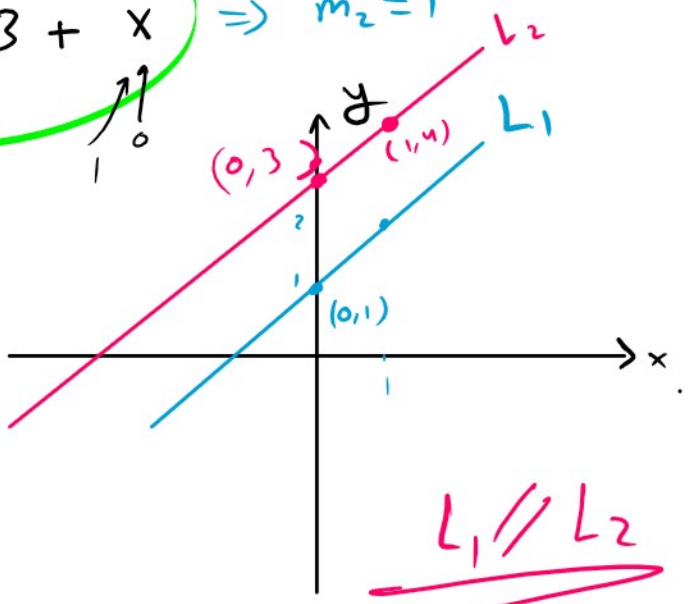
Exp Sketch $L_1: y = x + 1$ $\Rightarrow m_1 = 1$

$L_2: y = 3 + x$ $\Rightarrow m_2 = 1$

$y = ax + b$
a = m

L_1: $(x_1, y_1) = (0, 1)$, $(x_2, y_2) = (1, 2)$

L_2: $(x_1, y_1) = (0, 3)$, $(x_2, y_2) = (1, 4)$



$L_1 \parallel L_2$

$m_1 = m_2 = 1$

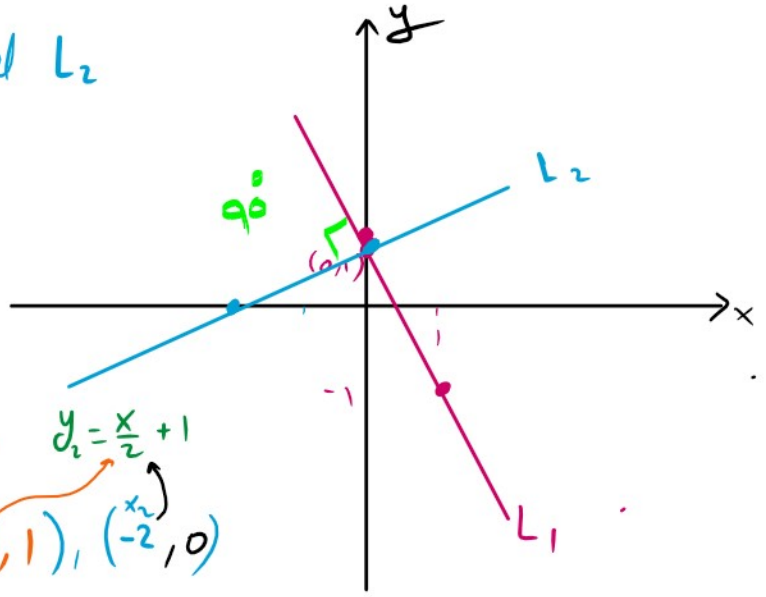
Exp sketch $L_1 : y_1 = 1 - 2x$ $\Rightarrow m_1 = -2$

$L_2 : y_2 = \frac{x}{2} + 1$ $\Rightarrow m_2 = \frac{1}{2}$

$m_1 \neq m_2 \Rightarrow L_1$ not parallel L_2

$m_1 \cdot m_2 = (-2)(\frac{1}{2}) = -1 \Rightarrow$

$L_1 \perp L_2$



$L_1 : y_1 = 1 - 2x$

$(x_1, y_1), (x_2, y_2)$

$(0, 1), (1, -1)$

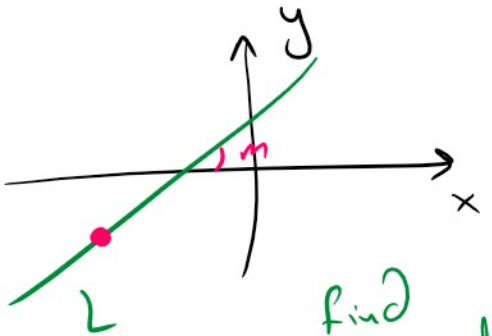
$L_2 : y_2 = \frac{x}{2} + 1$

$(x_1, y_1), (x_2, y_2)$

$(0, 1), (-2, 0)$

$L_1 \perp L_2$

Q: How can we write the equation of any line



find it's equation?

A: We need point through the line and slope

Point (x_1, y_1)

Point (x_1, y_1)

slope m

Equation of line is

$$y - y_1 = m(x - x_1)$$

Exp 2 Find equation of line passes through the point $(1, -2)$ and

① slope is 2

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = 2(x - 1)$$

$$y + 2 = 2x - 2$$

$$-2 \qquad -2$$

$$y = 2x - 4 \rightarrow m = 2$$

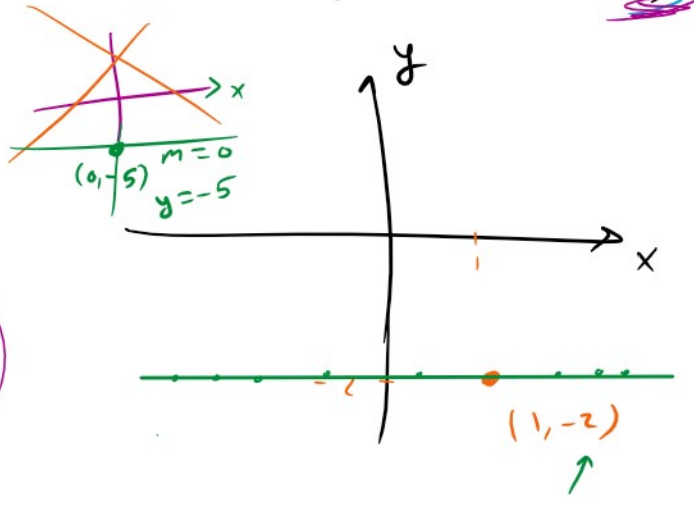
② slope 0

$$y = -2$$

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = 0(x - 1)$$

Point $(1, -2)$



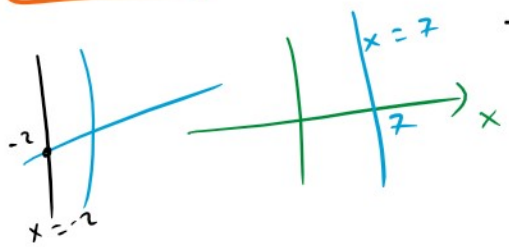
$y - -2 = 0(x - 1)$

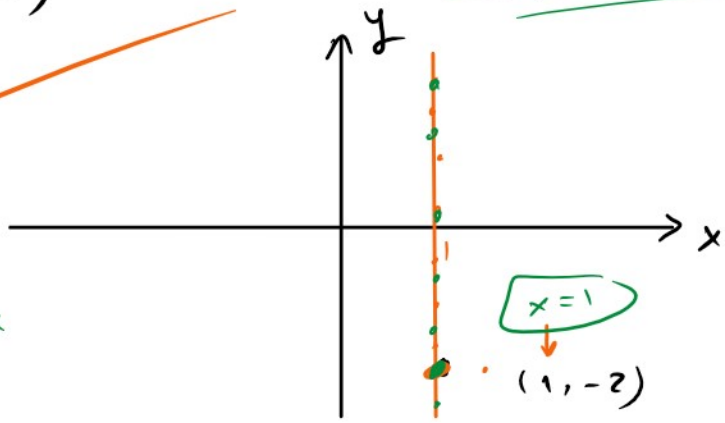
$y + 2 = 0$

$y = -2$

(3) line passes $(1, -2)$ with slope undefined

$x = 1$





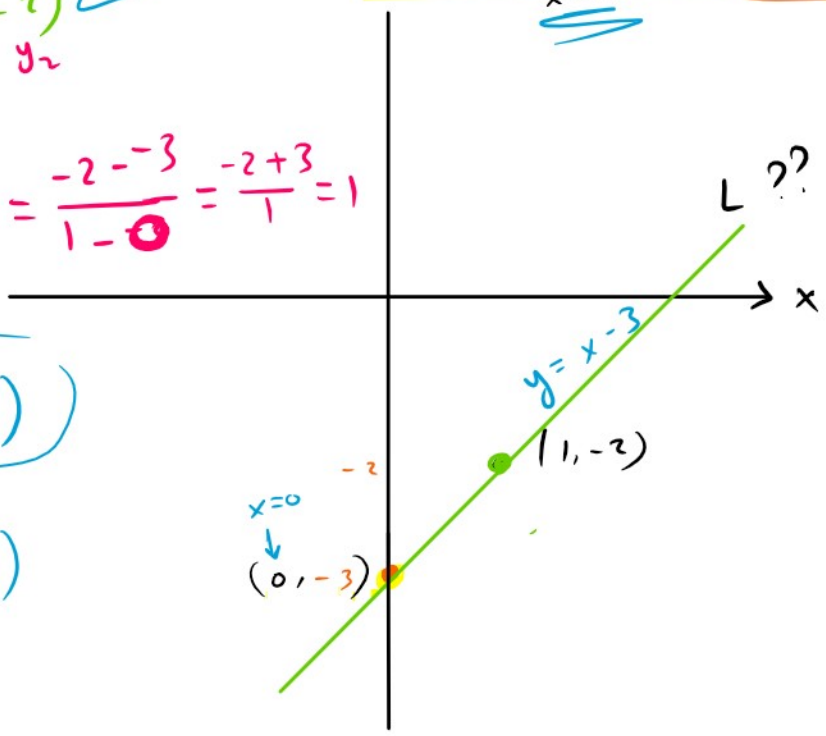
(4) line passes $(1, -2)$ with y-intercept -3

y-intercept $(0, -3)$, $(1, -2)$

x_1 y_1 x_2 y_2

$x = 0$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - -3}{1 - 0} = \frac{-2 + 3}{1} = 1$



$y - y_1 = m(x - x_1)$

$y - -3 = 1(x - 0)$

$y + 3 = x$

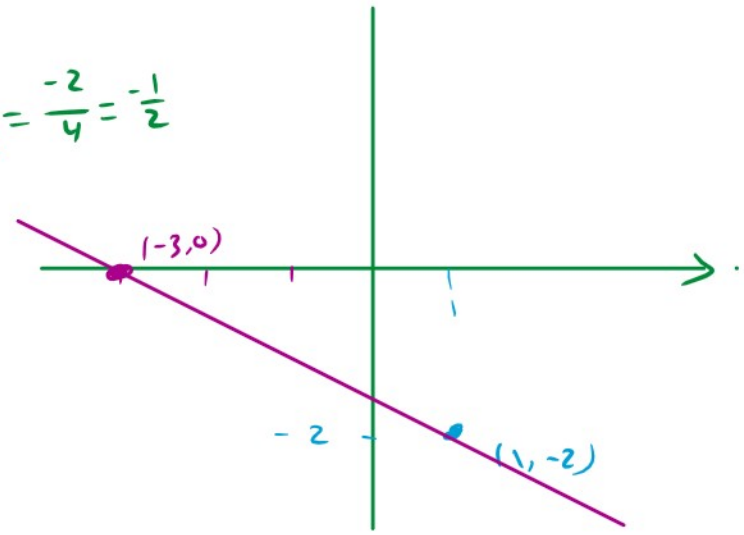
-3 -3

$y = x - 3$

(5) line passes $(1, -2)$ and x-intercept -3 ($y = 0$)

$(-3, 0)$, $(1, -2)$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-6}{1-(-3)} = \frac{-2}{1+3} = \frac{-2}{4} = -\frac{1}{2}$



$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{1}{2}(x - -3)$

$y = -\frac{1}{2}(x + 3)$

$y = -\frac{1}{2}x - \frac{3}{2}$

Exp which lines are // or ⊥

(1) L_1: $5x - 2y = 8$

L_2: $10x - 4y = 8$

$\Rightarrow m_1 = 5x$

$\Rightarrow m_2 = 10$

L_1: $5x - 2y = 8$

$-5x \quad -5x$

$\frac{-2y}{-2} = \frac{8}{-2} - \frac{5x}{-2}$

$y = -4 + \frac{5}{2}x \Rightarrow \boxed{m_1 = \frac{5}{2}}$

L_2: $10x - 4y = 8$

$-10x \quad -10x$

$\frac{-4y}{-4} = \frac{8}{-4} - \frac{10x}{-4}$

$y = -2 + \frac{5}{2}x \Rightarrow \boxed{m_2 = \frac{5}{2}}$

$L_1 // L_2$

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(2) L_1: $2 - 3y = 5 - 6x$

L_2: $4y + 2x = 3$

L_1: $2 - 3y = 5 - 6x$

$-2 \quad -2$

$\frac{-3y}{-3} = \frac{3}{-3} - \frac{6x}{-3}$

$y = -1 + 2x$

$m_1 = 2$

L_2: $4y + 2x = 3$

$-2x \quad -2x$

$\frac{4y}{4} = \frac{3}{4} - \frac{2x}{4}$

$y = \frac{3}{4} - \frac{1}{2}x$

$m_2 = -\frac{1}{2}$

$m_1 \cdot m_2 = (2)(-\frac{1}{2}) = -1$

$\Downarrow$

$L_1 \perp L_2$

$m_1 = 2$ | $m_2 = -\frac{1}{2}$ | $L_1 \perp L_2$

(3) L_1 : $x + y = -1$ $\Rightarrow$ $y = -1 - x$ $\Rightarrow m_1 = -1$

L_2 : $y - 3x = 4$ $\Rightarrow$ $y = 4 + 3x$ $\Rightarrow m_2 = +3$

$m_1 \neq m_2 \Rightarrow L_1$ not parallel to L_2

$m_1 \cdot m_2 \neq -1 \Rightarrow L_1$ not perpendicular to L_2