Chapter 7.1, Problem 44E

Problem

Let *X* and *Y* be sets, let *A* and *B* be any subsets of *X*, and let *C* and *D* be any subsets of *Y*. Determine which of the properties are true for all functions *F* from *X* to *Y* and which are false for at least one function *F* from *X* to *Y*. Justify your answers.

Exercise

For all subsets *A* and *B* of *X*, $F(A - B) = F(A) - F(B)$.

Step-by-step solution

Step 1 of 2

Consider X and Y be sets and consider A and B be any subsets of X.

Also consider the property of function:

For all subset A and B of X,

$$F(A-B)=F(A)-F(B)$$

Step 2 of 2

Objective is to determine this property of function is true or not.

Claim: $F(A-B)\neq F(A)-F(B)$

For this, consider the sets $A=\{1,2,3\}$ and $B=\{1\}$. Then consider, $A-B=\{2,3\}$

Now define a function, $F:X\to Y$ such that, $F(x)=k,$ for all x belongs to X.

Here, k is any fixed constant.

Then,

$$F(A-B)=k$$

And,

$$F(A)-F(B)=\{\ \}$$

This shows that, $\boxed{F(A-B)\neq F(A)-F(B)}$.