

* Summary of tests:-

• $\{a_n\}$ (sequence)

→ If $\lim_{n \rightarrow \infty} a_n = C$ (any finite number) then $\{a_n\}$ converges

to C

→ If $\lim_{n \rightarrow \infty} c_n \rightarrow \infty$ the $\{a_n\}$ diverges.

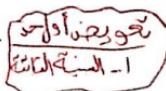
• $\sum a_n$ (series).

① p-series: $\sum \frac{1}{n^p}$

→ if $p > 1$ converges, if $p \leq 1$ diverges.

② Geometric series: $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \dots$

→ if $|r| < 1$, converges to $\frac{a}{1-r}$
if $|r| \geq 1$, diverges.



③ The nth term test:

→ if $\lim_{n \rightarrow \infty} a_n \neq 0$ then $\sum a_n$ diverges.

if $\lim_{n \rightarrow \infty} a_n = 0$ (test fails).

④ The nth partial sum:

→ If $\lim S_n = L$ then $\sum a_n$ converges.

telescoping $\xrightarrow{\text{up to}} \sum_{\text{series}}$ $S_n = a_1 + a_2 + \dots + a_n$
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if $\lim_{n \rightarrow \infty} S_n = \begin{cases} +\infty \\ \text{DNE} \end{cases}$, $\sum a_n$ diverges.

④ Series with nonnegative terms:

- Comparison test (D.C.T.)
- Limit comparison test (L.C.T.)
- Integral test (tre, cont., decreasing)
- Ratio test.
- n-th root test.

⑤ Alternating series test: $\sum (-1)^{n+1} u_n$

→ u_n tre, decreasing, $\lim u_n = 0 \rightarrow \sum (-1)^{n+1} u_n$ converges by alternating series test.

→ If $\lim u_n \neq 0 \rightarrow$ diverges by n-th term test.